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## Research Article

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# MoSI: A Sensor-Fused Margin of Stability at Interfaces for Modular Robots

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## Abstract

Lattice-based modular robots form adaptive structures from repeated units for self-reconfiguration and programmable matter. As these structures grow, loads are transferred through changing interfaces, making stability assessment difficult. Geometric support-polygon and center-of-mass checks are efficient but may overlook localized interface loading, whereas full mechanical-equilibrium solvers require more detailed models and greater computational effort. We introduce a sensor-fused Margin of Stability at Interfaces (MoSI) that combines an edge-weighted, scale-normalized geometric margin with force-sensitive resistor differentials and inertial measurement unit tilt. The geometric margin is computed for the complete configuration, while local sensing differentiates the monitored interfaces. Three variants provide progressively different fusion behaviors, ranging from smooth continuous correction to fuzzy-weighted and sign-adjusted responses. Hardware cantilever experiments over 10 repeated trials show that the fused margins approach or cross the zero-margin threshold at the observed critical four-block condition, while the geometry-only margin remains positive. A hardware-correlated Webots simulation model reproduces the decreasing sensor-fused margin trend and identifies the same lowest-margin condition, with root mean square error = **0.018** and mean absolute error = **0.013** over four paired growth stages. Scalability-oriented simulations of a V-bridge, J-pillar, branch overhang, and progressively loaded tower-supported plate further

demonstrate consistent responses to localized overload, lever-arm growth, asymmetric support loss, and rotational deformation without configuration-specific retuning. The interface scores are compatible with hierarchical box-based evaluation and event-driven distributed minimum aggregation. Overall, the proposed sensor-fused margin of stability at interfaces provides an interpretable stability score that reduces the overconfidence of geometry-only assessment without requiring a full mechanical-equilibrium model.

**Keywords:** Modular robots, Programmable matter, Stability assessment, Sensor fusion, Fuzzy logic, Distributed algorithms

**Categories:** (2), (5), and (6)

## 1 Introduction

Modular self-reconfigurable robotic systems (MSRRs) are a promising route toward programmable matter, where distributed robotic units rearrange to adapt to tasks and environments [1, 2]. By leveraging distributed control, sensing, and actuation, these systems can achieve adaptability and scalability, and platforms such as *Blinky Blocks* demonstrate this potential in real modular hardware [1]. However, the practical usefulness of such systems depends not only on reaching a desired shape, but also on maintaining physical stability as the structure grows, extends, and transfers load through changing interfaces.

A central challenge for autonomous shape transformation is therefore ensuring stability during and after reconfiguration. Many self-reconfiguration algorithms focus on coordination, connectivity maintenance, and collision avoidance [3, 4], while mechanical risks are often treated through simplified assumptions about static contacts, rigid connections, and load distribution. In practice, tipping, excessive interface loading, and structural breakage can become limiting factors for reliable deployment, especially when modular structures form cantilevers, bridges, pillars, or other load-bearing configurations.

Prior work has incorporated mechanical reasoning into distributed modular-robot algorithms. Support-polygon and center-of-mass (COM) methods provide efficient geometric stability checks by verifying whether the projected COM remains inside the support region [5, 6]. Other approaches use distributed force-equilibrium models to predict connector overload or loss of balance with stronger mechanical detail [7]. These methods address important parts of the stability problem, but they leave a gap between fast geometric balance decisions and more detailed model-based breakage prediction. In particular, purely geometric checks may remain overconfident when local interface loading or small rotational deformation appears before the projected COM leaves the support polygon.

To address this gap, we introduce a sensor-fused Margin of Stability at Interfaces (MOSI). The key idea is to preserve the interpretability and efficiency of geometry-based stability assessment while correcting it with local physical evidence from the interfaces where load is transmitted. Instead of treating local sensing as a separate

diagnostic layer, MOSI integrates force-sensitive resistor (FSR) differential measurements and inertial measurement unit (IMU) tilt estimates directly into a continuous configuration-space margin. This provides a lightweight interface-level score that can indicate proximity to instability before a purely geometric balance test becomes critical.

The main contribution of this paper to stability assessment in lattice-based modular robots is threefold.

- *Geometry-aware MOSI margin.* We propose a lightweight MOSI score that augments the support-polygon/COM stability principle with edge-specific weighting and effective-radius normalization, yielding a continuous margin sensitive to shape and scale. This geometric baseline is evaluated in hardware and Webots to show how the margin evolves across increasingly unstable configurations.
- *Sensor-fused interface assessment.* We integrate heterogeneous sensing, namely force-sensitive resistor (FSR) differential measurements and inertial measurement unit (IMU) tilt estimates, so that the margin reflects both geometric support and measured physical loading at interfaces. Hardware cantilever trials show that these local sensing cues reduce the overconfidence of a geometry-only baseline near failure.
- *Three explicit fusion variants.* We define and compare three formulations: MOSI-Direct  $h_{\text{MoSI,dir}}(x)$ , which directly fuses geometry with sensor penalties; MOSI-Flip  $h_{\text{MoSI,flip}}(x)$ , which conservatively flips the geometric contribution under unstable fuzzy sensor patterns; and MOSI-Weighted  $h_{\text{MoSI,wt}}(x)$ , which adapts sensor influence using fuzzy membership degrees. The variants are compared experimentally over repeated trials to analyze their relative conservativeness near the observed breakage condition.

The interpretation of  $h_{\text{MoSI}}(x)$  is inspired by safety-margin and barrier-function reasoning, where a scalar function separates admissible states from states that should

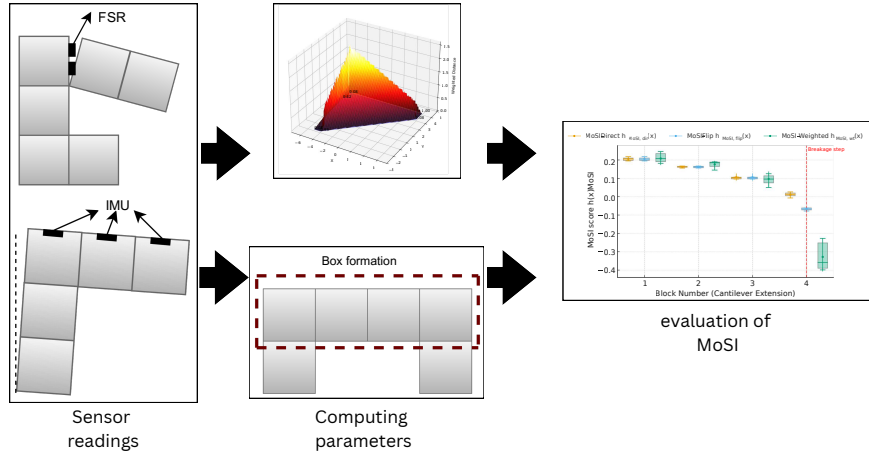


**Fig. 1** An overview of the proposed stability-aware modular robotic system. The figure outlines the transition from local physical sensing, including force-sensitive resistor (FSR) readings and inertial measurement unit (IMU) tilt, to the evaluation of the sensor-fused MOSI score  $h_{\text{MoSI}}(x)$ .

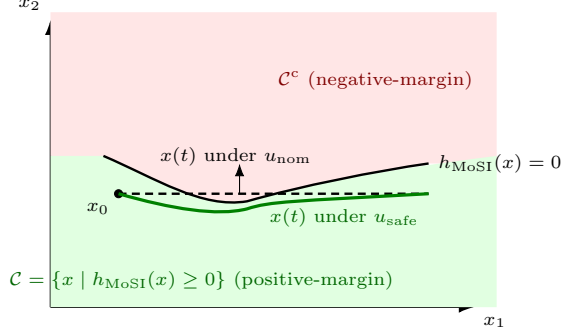


**Fig. 2** The schematic illustrates how a future lightweight command filter could adjust the nominal command  $u_{\text{nom}}$  to a safer command  $u_{\text{safe}}$ .

be avoided. In our case,  $h_{\text{MoSI}}(x)$  is not used to claim formal forward invariance, but to provide an interpretable stability margin for modular configurations:  $h_{\text{MoSI}}(x) > 0$  indicates a positive remaining buffer,  $h_{\text{MoSI}}(x) \approx 0$  indicates proximity to the selected critical threshold, and  $h_{\text{MoSI}}(x) < 0$  indicates that the sensor-fused evidence has crossed that threshold. Negative values are interpreted as threshold violations rather than as calibrated measures of post-instability severity. Accordingly, once the score becomes negative, its sign is operationally relevant, while differences in negative magnitude are not used to rank configurations that have already crossed the threshold. Fig. 2 illustrates this configuration-space interpretation.

The remainder of this paper is organized as follows. Section 2 reviews related work on distributed stability and breakage detection in MSRRs. Section 3 formulates MOSI, presents the three fusion variants, and justifies interface-level evaluation within the hierarchical box abstraction. Section 4 describes the distributed recursive computation algorithm, analyzes its complexity, and presents the hardware implementation. Section 5 extends the framework into Webots simulation and evaluates additional configurations. Finally, Section 6 concludes the paper and outlines directions for future work.

## 2 Related Work

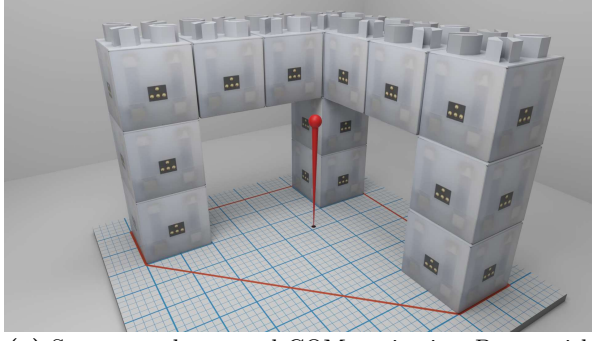
Modular self-reconfigurable robots have been studied as variable-morphology systems capable of adapting their shape, connectivity, and function to changing tasks and environments [8, 9]. Surveys of modular and reconfigurable robotic systems highlight recurring challenges in module design, docking, actuation, communication, distributed control, and scalability [10, 11]. These challenges become especially important for lattice-based platforms and programmable matter, where large numbers of modules may form cantilevers, bridges, pillars, and other load-bearing structures. In such configurations, successful reconfiguration depends not only on connectivity and collision avoidance, but also on whether the evolving structure can safely transmit load through its changing interfaces.

A common way to reason about static balance is geometric. In legged robots, mobile manipulators, and modular structures, stability is often assessed by constructing a support polygon and verifying whether the center-of-mass (COM) projection lies inside it. Recent fully distributed implementations for modular robots exchange local data and merge support polygons efficiently, enabling fast balance decisions under static loading and ideal contact assumptions [5, 6]. These approaches are attractive because they are computationally lightweight, interpretable, and naturally compatible with distributed modular-robot computation. However, the resulting decision is usually geometric and often binary: a configuration is classified as balanced or unbalanced according to the position of the projected COM relative to the support region. Even when a continuous distance-to-boundary margin is computed, geometry alone does not directly measure local interface loading, connector stress, contact imbalance, or small rotational deformation. As a result, a configuration can retain a positive geometric margin while already exhibiting physical signs of impending failure at a critical interface.

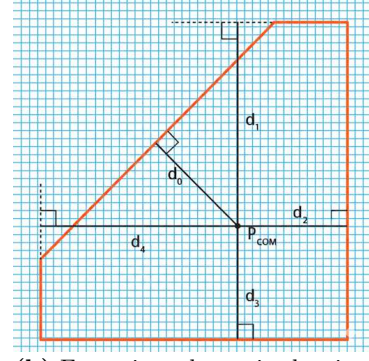
Complementary model-based approaches address this limitation by reasoning explicitly about transmitted forces and moments. Distributed equilibrium methods can propagate local mechanical quantities through a modular structure and predict connector overload or loss of balance before executing a reconfiguration step [7]. Such formulations provide a stronger mechanical basis than geometry-only checks because they evaluate inter-module connections where bending moments, axial forces, and load transfer are physically expressed. Their limitation is that they require more detailed modeling of contact, connector behavior, mass distribution, and load paths. They may also introduce higher computational and communication costs, especially when equilibrium updates must be repeated during growth or online reconfiguration. This creates a practical gap between fast geometric checks and detailed mechanical breakage predictors.

Sensor-based approaches provide another route for reducing the gap between geometric balance checks and detailed equilibrium models. In self-reconfigurable systems, fuzzy or rule-based safety layers have been used to combine heterogeneous signals and gate actions under uncertainty [12]. In our earlier sensor-fusion breakage-detection work, force-sensitive resistor (FSR) and inertial measurement unit (IMU) signals were used to detect instability-related changes in a modular-robot cantilever experiment [13]. That work demonstrated the usefulness of local physical sensing for breakage detection, but it did not formulate a continuous geometry-aware stability margin, compare multiple fusion variants, or embed the sensor evidence into a hierarchical interface-level evaluation. The present work extends this direction by using FSR and IMU cues not merely as diagnostic signals, but as terms in a stability margin that corrects a geometric support score without requiring a full mechanical equilibrium model.

A separate line of work concerns the representation of growing modular structures. Module-addition-aware hierarchies of overlapping, axis-aligned boxes have been used to support dynamic shape recognition as a modular assembly grows or changes [14]. This abstraction compresses a large lattice configuration into a smaller set of connected substructures and supports efficient incremental updates. However, the box hierarchy



(a) Support polygon and COM projection  $P_{\text{COM}}$  with vertical drop.



(b) Footprint schematic showing distances  $d_i$  and edge-length weights  $\gamma_i$ .

**Fig. 3** Geometry used in the MOSI geometric term  $D_w$ : (a) support polygon and COM projection; (b) distance and weighting diagram used to compute  $D_w = \min_i(\gamma_i d_i)/R_{\text{eff}}$ .

is primarily a shape-recognition and organization tool. It does not by itself evaluate whether the interfaces between box clusters are mechanically close to instability. This leaves an opportunity to use the hierarchy not only for representing shape, but also for localizing where stability assessment should be performed.

Other work has addressed stability and robustness through configuration design rather than online margin evaluation. For example, FreeSN improves assembly strength using strut-node modules that form load-bearing triangular substructures [15], while flying modular systems search for configurations satisfying aggregate force and moment requirements before action [16]. These approaches reinforce the importance of mechanical feasibility in modular systems, but they operate at the level of module architecture or global configuration selection rather than lightweight interface-level monitoring.

Taken together, existing work provides geometric balance tests, distributed force-equilibrium breakage predictors, sensor-based safety cues, configuration-design strategies, and hierarchical shape abstractions. However, these elements remain largely separate. Geometry-based methods are efficient but can be overconfident near local interface failure; equilibrium-based methods are mechanically richer but more model-dependent; configuration-design methods improve structural feasibility but do not provide an online local margin; and prior sensor-fusion approaches do not provide a continuous geometry-aware score within a box-cluster hierarchy. This gap motivates the MOSI formulation introduced in Section 3, which combines a normalized support margin with local FSR and IMU evidence at the interfaces where load transfer concentrates.

### 3 MoSI: A Margin of Stability at Interfaces for Modular Robots

#### 3.1 Definition of MoSI $h_{\text{MoSI}}(x)$

The MoSI score  $h_{\text{MoSI}}(x)$  quantifies the proximity of a modular robot configuration to mechanical instability by combining geometric support information with local sensor-derived cues. It builds on the support-polygon/COM stability principle used in distributed balance detection [5], but replaces a binary inside/outside decision with a continuous, sensor-corrected interface margin.

At the conceptual level, the available margin can be written as

$$h_{\text{MoSI}}(x) = D_w - f(\Delta F) - g(\theta). \quad (1)$$

**Remark:** Eq. (1) is an intuitive decomposition: the geometric support margin  $D_w$  is reduced by normalized instability cues derived from force imbalance and tilt. The implemented MoSI variants are given in Eqs. (6), (7), and (10), where all terms are dimensionless and bounded to prevent unbounded compensation between factors.

The geometric term  $D_w$  represents the weighted and normalized signed distance from the projected center of mass  $P_{\text{COM}}$  to the closest edge of the support polygon, as illustrated in Fig. 3. For each polygon edge  $i$ , let  $d_i$  denote the signed perpendicular distance from  $P_{\text{COM}}$  to the corresponding edge line, with  $d_i > 0$  when  $P_{\text{COM}}$  lies inside the support half-plane and  $d_i < 0$  when it lies outside. Each edge is assigned a length-based weight  $\gamma_i$ , which provides a simple geometric proxy for the stabilizing contribution of that support edge. The weighted margin is then defined as

$$D_w = \frac{\min_i (\gamma_i d_i)}{R_{\text{eff}}}, \quad (2)$$

where

- $d_i$  is the signed perpendicular distance from  $P_{\text{COM}}$  to edge  $i$ ,
- $\gamma_i = \ell_i / \sum_j \ell_j$ , with  $\ell_i$  the length of edge  $i$ ,
- $R_{\text{eff}} = \sqrt{A/\pi}$  is the effective radius of the support polygon with area  $A$ .

The normalization by  $R_{\text{eff}}$  makes  $D_w$  comparable across structures with different footprint sizes. Positive values indicate that the COM projection remains inside the weighted support region, values near zero indicate proximity to a support boundary, and negative values indicate that the projection has crossed outside the corresponding support half-plane. As illustrated in Fig. 4, the resulting margin varies smoothly across the footprint, with higher values near central regions and lower values near weak or nearby support edges.

The FSR differential  $\Delta F_{\text{ADC}}$  captures lateral force imbalance at the evaluated interface, particularly in cantilever-like configurations where asymmetric contact pressure can indicate the onset of rotational loading. Unlike a single FSR reading, which is affected by absolute load, contact offset, and baseline drift, the differential suppresses common-mode loading and emphasizes contact-pressure imbalance between the two



The tilt angle  $\theta$  is computed as the angle between the IMU local  $z$ -axis and world  $+Z$ . If  $H$  is the vertical distance from the IMU to the ground and  $L_0$  is the module edge length, we define the normalized height  $\tilde{h} = H/L_0$ . The tilt cue is then

$$\tilde{h} \sin \theta,$$

which approximates the horizontal displacement induced by tilt, normalized by the module size. Larger tilt angles or higher sensor placement increase this value, indicating a stronger rotational contribution to local instability.

Together,  $D_w$ ,  $\sigma(\Delta F)$ , and  $\tilde{h} \sin \theta$  provide complementary information:  $D_w$  captures geometric support, the FSR cue captures asymmetric interface loading, and the IMU cue captures rotational deformation. A high sensor cue can therefore reduce the stability margin even when the COM projection remains geometrically admissible. This is the key mechanism by which MOSI reduces the overconfidence of geometry-only stability assessment.

The following subsections define three concrete fusion variants based on this principle. Each variant preserves the same geometric term and sensor cues, but differs in how aggressively local physical evidence modifies the final margin.

### 3.2 Sensor Fusion Variants

To capture different levels of conservativeness in the presence of local physical evidence, we implement and compare three sensor-fusion strategies within the MOSI framework. Each variant uses the configuration-level geometric margin  $D_w$ , computed once from the support polygon and projected center of mass of the complete structure and shared by all monitored interfaces. Interface-level differentiation is introduced through the local FSR imbalance cue  $\sigma(\Delta F)$  and IMU tilt cue  $\tilde{h} \sin \theta$ . The three variants use the same geometric and sensing inputs but combine them differently to examine how strongly local physical evidence should modify the geometry-only margin while the support polygon still indicates a positive buffer.

1) *MOSI-Direct*: The direct variant applies the sensor cues as smooth penalties on the geometric margin. It is defined as

$$h_{\text{MOSI,dir}}(x) = D_w (1 - \sigma(\Delta F)) - \tilde{h} \sin \theta. \quad (6)$$

Here,  $\sigma(\Delta F) \in [0, 1]$  is the sigmoid-mapped differential FSR cue, and  $\tilde{h} \sin \theta$  is the dimensionless tilt-induced displacement term. The FSR term reduces the usable geometric margin multiplicatively, while the IMU term is subtracted as an additive rotational penalty. This variant is the least conservative of the three because it degrades the margin continuously without changing the sign of the geometric contribution unless the combined penalties are large enough.

2) *MOSI-Flip: Fuzzy-Based Sign Adjustment*. The second variant introduces a conservative sign adjustment based on a two-input fuzzy classifier. The first input is the normalized FSR imbalance score

$$s_F = 100 \sigma(\Delta F),$$

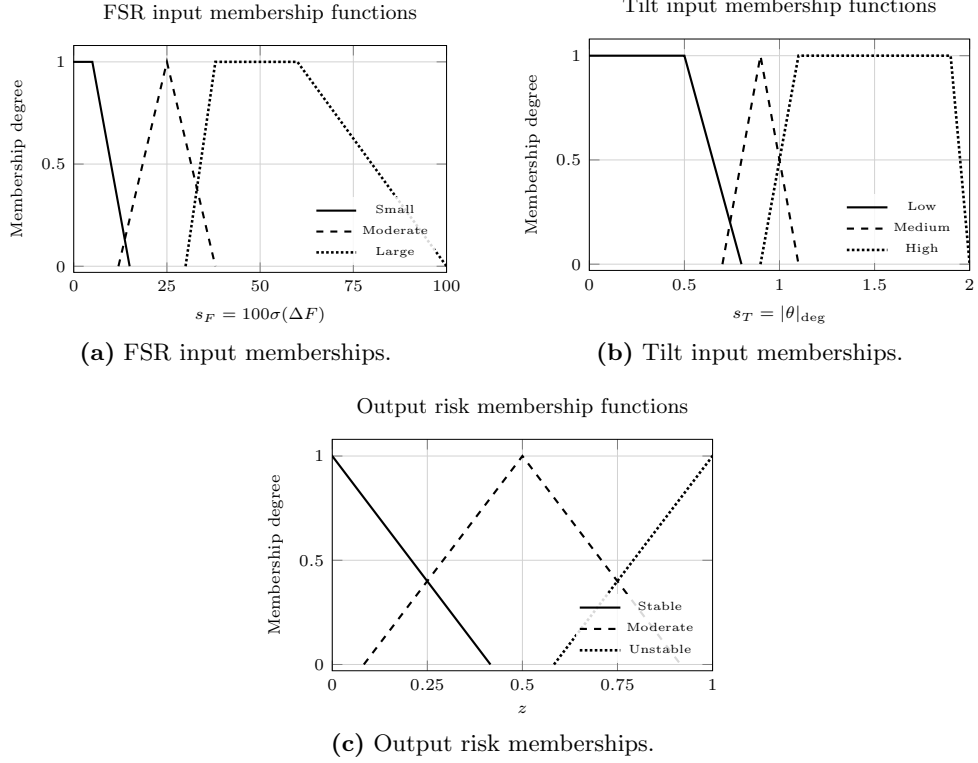


**Fig. 5** Fuzzy membership functions used by the MoSI-Flip and MoSI-Weighted variants: (a) FSR risk input  $s_F = 100\sigma(\Delta F)$ , (b) tilt input  $s_T = |\theta|_{\text{deg}}$ , and (c) output fuzzy risk score  $z$ . The same membership functions are used in the hardware and Webots evaluations.

expressed as a percentage in  $[0, 100]$ . The second input is the absolute tilt angle

$$s_T = |\theta|_{\text{deg}},$$

expressed in degrees for the fuzzy classifier. The final MoSI equation still uses the dimensionless tilt penalty  $\tilde{h} \sin \theta$ , but the fuzzy classifier uses  $s_T$  because the membership breakpoints were selected over the small-angle tilt range observed in the hardware and simulation trials.

Each input is mapped to three overlapping trapezoidal or shoulder membership functions, shown in Fig. 5(a) and Fig. 5(b). The breakpoints were selected as fixed design priors that partition each sensor domain into low, transition, and high-risk regimes. Slightly negative lower bounds in the Small/Low sets account for noise around zero, while overlapping regions avoid hard switching between neighboring classes. These membership functions were kept fixed across the hardware and Webots experiments.

The FSR memberships are defined by Small  $[-37.5, -4.167, 5, 15]$ , Moderate  $[12, 25, 25, 38]$ , and Large  $[30, 38, 60, 100]$ . The tilt memberships are Low

**Table 1** Fuzzy association matrix used by MoSI-Flip.

| FSR / Tilt | Low      | Medium   | High     |
|------------|----------|----------|----------|
| Small      | Stable   | Stable   | Unstable |
| Moderate   | Stable   | Moderate | Unstable |
| Large      | Moderate | Unstable | Unstable |

$[-5.625, -0.625, 0.5, 0.8]$ , Medium  $[0.7, 0.9, 0.9, 1.1]$ , and High  $[0.9, 1.1, 1.9, 2]$ . The output memberships are Stable  $[-0.416, 0, 0, 0.416]$ , Moderate  $[0.083, 0.5, 0.5, 0.916]$ , and Unstable  $[0.583, 1, 1, 1.416]$ .

The fuzzy rule base is given in Table 1. The output linguistic classes are Stable, Moderate, and Unstable, with output membership functions shown in Fig. 5(c). Centroid-of-area defuzzification produces a scalar risk score  $z \in [0, 1]$ .

The sign-adjusted margin is defined as

$$h_{\text{MoSI,flip}}(x) = \begin{cases} -|D_w|(1 - \sigma(\Delta F)) - \tilde{h} \sin \theta, & z \geq \tau, \\ D_w(1 - \sigma(\Delta F)) - \tilde{h} \sin \theta, & z < \tau, \end{cases} \quad (7)$$

with  $\tau = 0.6$ . The absolute value in the first branch ensures that an unstable fuzzy classification always produces a non-positive geometric contribution, regardless of whether the original geometric margin  $D_w$  is positive or negative. In particular, if the projected COM has already crossed the support boundary and  $D_w < 0$ , the sign adjustment cannot incorrectly convert the negative geometric margin into a positive contribution.

The threshold activates the conservative adjustment only under strongly unstable fuzzy patterns, such as a large FSR imbalance or the combination of moderate FSR imbalance and high tilt. The purpose of this variant is therefore to provide a conservative override mechanism: when the sensor pattern indicates a critical interface condition, the geometric contribution is prevented from preserving or recovering a positive margin by itself.

3) *MoSI-Weighted: Fuzzy-Membership Weighted Fusion*. The third variant uses the same fuzzy membership functions, but avoids a binary sign flip. Instead, it computes continuous sensor weights from the membership degrees. For the FSR input, the weight is

$$w_{\text{fsr}} = \alpha_s \mu_{\text{small}}^F + \alpha_m \mu_{\text{moderate}}^F + \alpha_l \mu_{\text{large}}^F, \quad (8)$$

and for the tilt input, the weight is

$$w_{\text{imu}} = \beta_l \mu_{\text{low}}^T + \beta_m \mu_{\text{medium}}^T + \beta_h \mu_{\text{high}}^T, \quad (9)$$

where  $\mu^F$  and  $\mu^T$  denote the FSR and tilt membership degrees, respectively. The coefficients were chosen as monotone fixed design weights,

$$\{\alpha_s, \alpha_m, \alpha_l\} = \{0.1, 0.5, 0.8\}, \quad \{\beta_l, \beta_m, \beta_h\} = \{0.1, 0.5, 0.8\},$$

so that higher-risk membership classes apply stronger penalties. These weights were kept fixed after selection and were not retuned between experiments.

The resulting weighted variant is

$$h_{\text{MoSI,wt}}(x) = D_w - w_{\text{fsr}} \sigma(\Delta F) - w_{\text{imu}} \tilde{h} \sin \theta. \quad (10)$$

Unlike MOSI-Flip, this formulation does not impose a discrete override. Instead, sensor influence increases smoothly with the degree of membership in higher-risk classes. This makes MOSI-Weighted the most continuously adaptive variant: a single sensor can express partial instability without triggering a hard transition, while simultaneous high FSR and tilt evidence can drive the margin strongly downward. The nominal parameters defined above were kept fixed across the reported hardware and Webots evaluations. Their sensitivity with respect to the repeated hardware cantilever trials is examined in Section 4.4.

### 3.3 Hierarchical MOSI Evaluation Justification

Our evaluation strategy begins by clustering the modular structure into axis-aligned 3D box volumes, following the hierarchical box decomposition used for shape recognition in lattice-based modular robots [14]. As in the referenced method, adjacent boxes may share common blocks to preserve continuity between overlapping regions. These shared blocks are not treated as duplicated physical modules and contribute only once to the mass and center-of-mass calculations. This abstraction compacts large assemblies and supports fast, incremental updates during growth, making it a practical substrate for online stability checks. Rather than analyzing every individual block, we evaluate the MOSI score ( $h_{\text{MoSI}}(x)$ ) only at the interfaces between adjacent box regions. This significantly reduces computation while aligning with structural principles of mechanical stress concentration: stress and instability tend to emerge at boundaries rather than within uniformly supported interiors.

1) *Interface and section-cut argument.* The reason for evaluating interfaces rather than all interior modules can be expressed by treating each box cluster as a substructure. When a cluster  $B_k$  is isolated from the rest of the modular configuration, interactions between modules strictly inside  $B_k$  contribute to internal load redistribution, while the mechanical compatibility of  $B_k$  with the rest of the structure is governed by the forces and moments transmitted through its boundary. This is analogous to the section-cut method in structural mechanics, where internal shear forces and bending moments are exposed by cutting the structure at the location where the transmitted demand is to be evaluated [17].

This argument does not imply that interior modules carry no load. Rather, it means that the stability of one compact substructure relative to another is expressed through the interface resultants transmitted across their shared boundary. Therefore, evaluating  $h_{\text{MoSI}}(x)$  at inter-cluster interfaces provides a physically meaningful reduction of the full module-level problem: interior modules remain important for mass, center-of-mass, and shape aggregation, while interface modules capture where the load path is transferred between substructures.

2) *Structural mechanics intuition.* This interface-centered view is also consistent with classical stress-concentration reasoning. Localized increases in stress occur near geometric discontinuities such as holes, notches, shoulders, or other abrupt changes in load path [18]. Although modular-robot interfaces are not identical to classical notch geometries, they share the key mechanical feature that motivates the analogy: both represent abrupt transitions in load-path geometry where transmitted demand can concentrate. In cantilever-like modular configurations, adding modules farther from the support increases the lever arm and amplifies the moment transmitted at the fixed interface. Thus, the mechanically critical locations are not arbitrary interior points, but the boundaries where one substructure supports another.

3) *Relation to modular-robot breakage criteria.* This interface-centered reasoning is directly supported by recent distributed breakage detection for Blinky Blocks. In [19], a modular structure is partitioned into a fixed part  $G$  and a potentially separating part  $H$ , and breakage is evaluated at the interface face connecting the two parts. The considered failure modes include vertical sliding, rotational sliding, and rotational debonding at vertical and lateral connectors. This is the same structural principle used here: the mechanically relevant question is not whether every interior module should be checked independently, but whether a supported substructure can remain attached through the boundary connection that transmits its load.

The same work shows that several breakage conditions can be evaluated from aggregate quantities associated with the subtree supported by a given interface, such as the number of modules in the separated group and sums of their relative positions with respect to the interface. For example, using a reference point on the interface face, quantities of the form

$$S_x^f = \sum_{i \in H} x_i - N_H x_O, \quad S_y^f = \sum_{i \in H} y_i - N_H y_O$$

capture how the mass of the supported group is distributed relative to the connection. These aggregate quantities are then compared with breakage thresholds for sliding or rotational debonding. Thus, loads farther from the interface affect the breakage decision through their lever-arm contribution, while the decision itself remains localized at the connection transmitting that demand.

Our hierarchical MOSI evaluation follows the same interface-centered principle, but replaces hard breakage tests with a continuous sensor-fused margin. Interior modules contribute to shape, mass, and center-of-mass aggregation, while  $h_{\text{MOSI}}(x)$  is evaluated at inter-box boundaries where load transfer between compact substructures is concentrated. In this sense, MOSI does not attempt to reproduce the full breakage model of [19]; instead, it uses the same structural insight, namely that stability-relevant information can be aggregated over a supported substructure and evaluated at the interface through which that substructure is attached.

**Remark.** The interface-level evaluation considered in this work applies to fully occupied box regions composed of uniform, defect-free modules and connectors under quasi-static, gravity-dominated loading. Within this scope, the interior of each compact box is treated as a mechanically coherent region, while stability is evaluated at the

boundaries through which load is transferred between adjacent box regions. Manufacturing defects, nonuniform connector properties, internal voids, and localized external forces are not represented by the present model.

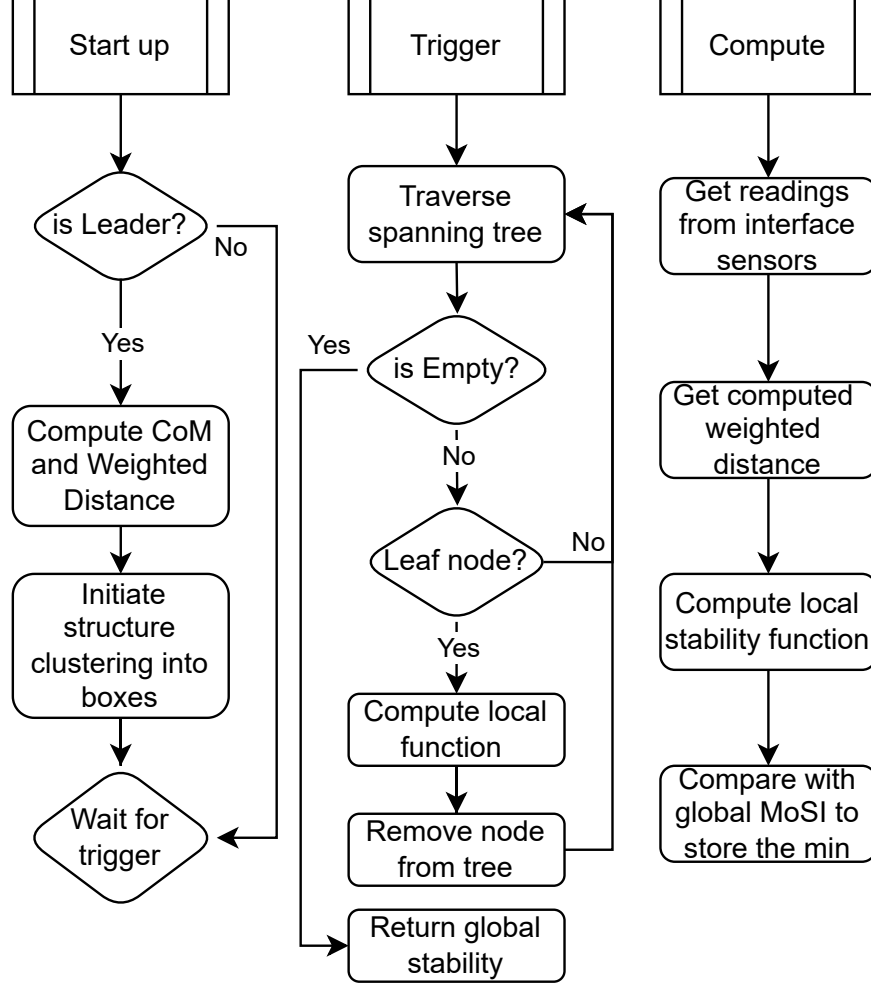


**Fig. 6** Flowchart of the distributed computation and recursive aggregation process for MoSI. Local evaluations occur at the cluster level using COM, FSR, and IMU data, while results are aggregated up the spanning tree to yield a global MoSI measure.

## 4 Distributed Computation of MoSI: Aggregation, Complexity, and Hardware

### 4.1 Distributed Computation and Recursive Aggregation of MoSI

To scale stability evaluation across large modular structures, we model the assembly as a spanning tree of box clusters. Each box cluster maintains local geometric and sensing information for its incident interfaces, and computes a local interface margin for each relevant boundary. The tree is obtained from the adjacency graph induced by the box hierarchy of [14]: each box cluster is a vertex, shared box boundaries define edges, and a breadth-first or depth-first traversal rooted at a selected support/base cluster yields the aggregation tree. When modules are added, the affected box clusters and their adjacency links are updated incrementally, and the local portion of the aggregation tree is repaired accordingly. The cluster-level score is then defined conservatively as the minimum over its evaluated interfaces:

$$h_{\text{MoSI}}^{B_k}(x) = \min_{e \in \mathcal{I}(B_k)} H_{\text{MoSI},e}(x),$$

where  $\mathcal{I}(B_k)$  denotes the set of interfaces incident to cluster  $B_k$ . The global score is obtained by recursively propagating these local minima along the spanning tree:

$$h_{\text{MoSI}}^{\text{global}}(x) = \min_k h_{\text{MoSI}}^{B_k}(x).$$

The aggregation proceeds as follows:

1. Each leaf cluster computes the minimum  $H_{\text{MoSI},e}(x)$  over its local interfaces.
2. Each parent cluster waits for the scores reported by its children, compares them with its own local interface score, and stores the minimum.
3. This process continues recursively toward the root, which obtains the global or region-level MoSI estimate.

This design supports event-driven local computation and scalable aggregation. Since the reduction operator is a minimum, the global score preserves the most critical interface rather than averaging risk over the structure. The recursive flow also matches the hierarchical box representation: local interface margins are computed at box boundaries, while the tree only transmits compact scalar summaries upward.

The computation process is illustrated in Fig. 6, showing the sequence from per-interface evaluation to recursive aggregation along the spanning tree.

### 4.2 Complexity: Cold-Start and Event-Driven Updates

Let  $X, Y, Z$  be the bounding-box extents used by the box clustering in [14],  $D$  the diameter of the ground-contact graph used by the support-polygon routine [5],  $N = |\mathcal{B}|$  the number of box clusters,  $E_I$  the number of evaluated inter-box or support interfaces, and  $h$  the height of the aggregation tree shown in Fig. 6. In a cold-start run, the

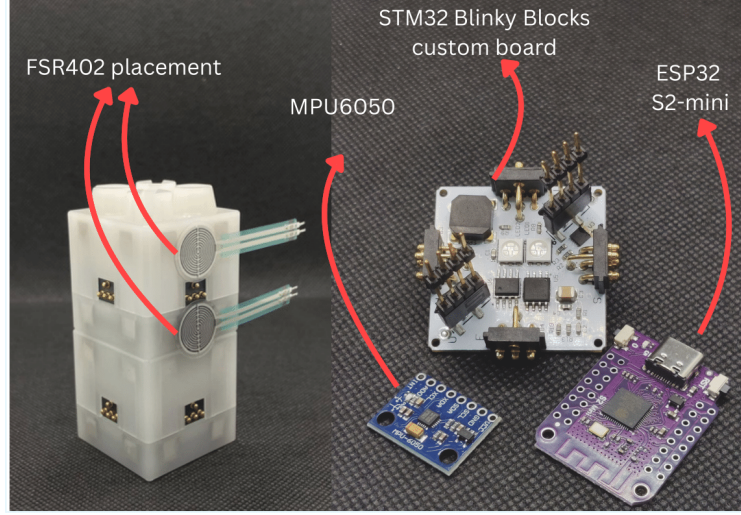


**Fig. 7** Sensor-augmented hardware used in the cantilever experiments. An MPU6050 on the tip block measures tilt; two lateral FSR402s on the base block measure strain differential. Signals are acquired by an ESP32-S2 inside the base block (IMU via I<sup>2</sup>C), while a stationary ESP32 reads the FSR voltage dividers, computes the MoSI score  $h_{\text{MoSI}}(x)$ , and logs data in real time.

system performs one pass of box clustering, one support-polygon computation, one evaluation of the relevant interface margins, and one tree reduction:

$$T_{\text{cold}} = O(X+Y+Z) + O(D) + O(E_I) + O(N).$$

For a lattice with bounded 6-neighborhood connectivity,  $E_I = O(N)$ , so the cold-start complexity reduces to

$$T_{\text{cold}} = O(X+Y+Z + D + N).$$

In the event-driven regime, updates are triggered by module addition/removal, interface activation, footprint change, or a sensor threshold crossing. Only the affected box clusters  $\Delta\mathcal{B}$  and their incident interfaces  $\Delta\mathcal{E}$  need to be recomputed. Because the

**Table 2** Operations, triggers, and asymptotic costs.

| Stage                                       | Trigger                     | Work                                                    | Msgs./lat.                 |
|---------------------------------------------|-----------------------------|---------------------------------------------------------|----------------------------|
| Box clustering [14]                         | cold start; topology change | $O(X+Y+Z)$                                              | local                      |
| Support polygon [5]                         | footprint change            | $O(D)$                                                  | local                      |
| Interface $H_{\text{MoSI},e}(x)$ evaluation | sensor or geometry event    | $O(E_I)$ cold start; $O( \Delta\mathcal{E} )$ per event | local                      |
| Tree min-update                             | local score update          | $O(h)$ path update; $O(N)$ full reduction               | $O(h)$ path; $2(N-1)$ full |
| Global minimum broadcast                    | global minimum changes      | $O(N)$                                                  | $N-1$ msg.; $O(h)$         |

lattice has bounded degree,  $|\Delta\mathcal{E}| = O(|\Delta\mathcal{B}|)$ . The local recomputation cost is therefore

$$T_{\text{local}} = O(|\Delta\mathcal{B}| + |\Delta\mathcal{E}|).$$

After local recomputation, the updated minimum is propagated through the aggregation tree. In the worst case, a full convergecast to the root and broadcast back to all clusters uses  $2(N - 1)$  messages, since the tree has  $N - 1$  edges, and completes in  $O(h)$  communication rounds. For a purely local event whose update only affects the path from the modified cluster to the root, the propagation cost is  $O(h)$  messages and  $O(h)$  rounds, with a full broadcast required only if the global minimum changes and must be disseminated.

Thus, the worst-case event-driven cost is

$$T_{\text{event}} = O(|\Delta\mathcal{B}| + |\Delta\mathcal{E}|) + O(h),$$

with optional  $O(N)$  communication only when the updated global minimum is broadcast to all clusters. In common growth steps,  $|\Delta\mathcal{B}| = O(1)$ , so the local recomputation is constant-time and the update is dominated by propagation along the aggregation tree. Table 2 summarizes the main triggers and costs.

### 4.3 Hardware Implementation

We implemented a controlled hardware benchmark that isolates the mechanically critical case studied in this work: a growing cantilever supported at a fixed base interface. This setting provides a repeatable way to increase lever arm, interface loading, and tilt while keeping the sensing layout and geometry well defined.

Experiments were performed by adding one block at a time to form cantilever configurations of increasing length. Each configuration was repeated 10 times to assess repeatability and to compute the three fused MoSI variants described in Section 3. This constrained but controlled setting acts as a hardware benchmark for the central failure mode of interest: the geometric margin decreases gradually as the cantilever extends, while local interface loading can rise sharply near the critical configuration. The hardware results are complemented by the larger Webots validation study presented in Section 5.

Fig. 8 presents the distribution of  $h_{\text{MoSI}}(x)$  values for the three MoSI variants during progressive cantilever extension. All three methods capture the expected decline in stability as the cantilever grows, reflecting the increasing moment arm and outward shift of the center of mass. The differences between variants show how the same geometric and sensor inputs can be fused with different levels of conservativeness.

The MoSI-*Direct* formulation  $h_{\text{MoSI,dir}}(x)$  shows a smooth and predictable decay, remaining close to zero at the final configuration without crossing strongly into the negative region. This behavior makes it a useful baseline for monitoring gradual stability loss because it reacts continuously to increasing FSR imbalance and tilt without imposing a discrete override.

The MoSI-*Flip* formulation  $h_{\text{MoSI,flip}}(x)$  follows a similar decay profile at shorter cantilever lengths, but becomes more conservative when the fuzzy classifier detects

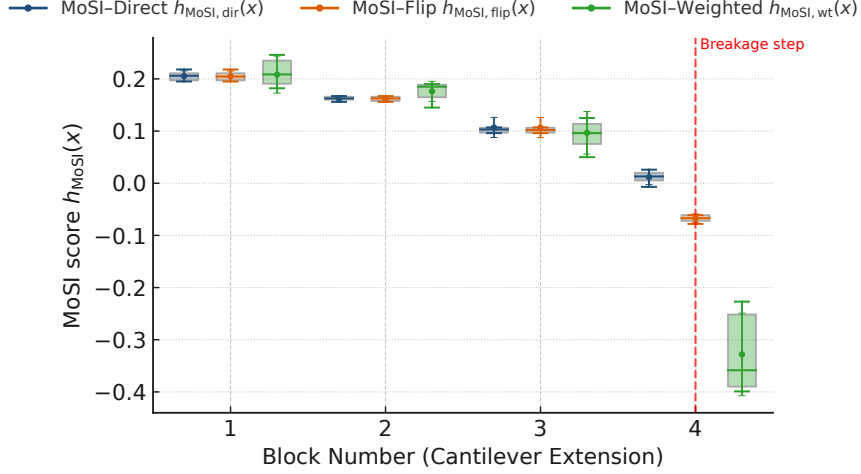


**Fig. 8** Box plots of the MOSI score  $h_{\text{MoSI}}(x)$  across 10 runs for each cantilever step. Boxes show median and IQR; whiskers show the 5th–95th percentile range; dots with error bars denote mean  $\pm$  s.d. The red dashed line marks the observed breakage step at the four-block condition.

unstable sensor patterns. At the four-block condition, the sign adjustment pushes the score below zero, indicating that the sensor evidence is strong enough to override the still-positive geometric margin. This behavior is useful when early rejection of critical interface states is preferred.

The *MoSI-Weighted* formulation  $h_{\text{MoSI, wt}}(x)$  produces the steepest decline because it amplifies the influence of FSR and IMU cues according to fuzzy membership degrees. This makes it the most sensitive of the three variants near the observed breakage condition. The trade-off is that the weighted formulation can also exhibit greater variability, reflecting stronger dependence on sensor fluctuations and transient loading effects.

Overall, the three formulations share the same downward trend with cantilever growth, but differ in smoothness and conservativeness. *MoSI-Direct* provides gradual degradation, *MoSI-Flip* introduces a conservative transition under unstable fuzzy patterns, and *MoSI-Weighted* produces the strongest sensor-driven warning. These differences allow the MOSI framework to be adapted to different modular-robotic objectives, such as steady monitoring, conservative rejection, or heightened sensitivity to early instability cues.

To make the advantage of the fused formulation explicit, we compare the MOSI outputs against a geometry-only baseline computed from the same hardware trials. Building on the support-polygon/COM stability principle used in distributed balance detection [5], we augment the binary inside/outside decision into a continuous, edge-weighted geometric margin  $D_w$ . The FSR and IMU terms are reported separately as instability indicators, not as standalone stability margins: higher values indicate stronger evidence of interface loading or tilt. The role of MOSI is to reduce the over-confidence of this geometric baseline under physical loading by incorporating measured interface-level evidence.

**Table 3** Hardware component-level comparison over 10 cantilever trials.  $D_w$  is the geometry-only baseline margin. FSR and IMU are instability indicators used to correct this baseline in the fused MOSI variants. Values are means over 10 runs.

| Quantity                           | 1 block | 2 blocks | 3 blocks | 4 blocks |
|------------------------------------|---------|----------|----------|----------|
| Baseline $D_w$                     | 0.258   | 0.203    | 0.149    | 0.080    |
| FSR cue $\sigma(\Delta F)$         | 0.182   | 0.157    | 0.189    | 0.524    |
| IMU cue $\tilde{h} \sin \theta$    | 0.006   | 0.014    | 0.014    | 0.039    |
| MOSI-Direct $h_{\text{MoSI,dir}}$  | 0.205   | 0.162    | 0.107    | 0.012    |
| MOSI-Flip $h_{\text{MoSI,flip}}$   | 0.205   | 0.161    | 0.107    | -0.067   |
| MOSI-Weighted $h_{\text{MoSI,wt}}$ | 0.208   | 0.176    | 0.097    | -0.328   |

Table 3 makes the limitation of the geometry-only baseline explicit. As the cantilever is extended from one to four blocks, the baseline margin  $D_w$  decreases monotonically from 0.258 to 0.080, reflecting the expected reduction in geometric support. However, even at the observed breakage step,  $D_w$  remains positive. A purely geometric interpretation could therefore still classify the configuration as having a small remaining buffer to instability. In contrast, the FSR cue changes much more sharply at the same step, increasing from 0.189 at three blocks to 0.524 at four blocks. The IMU cue also increases, from 0.014 to 0.039. Together, these sensor trends indicate that the support interface is experiencing substantially stronger asymmetric loading and rotational deformation, even though the geometric margin has not yet crossed zero.

The fused MOSI variants convert this hidden physical evidence into a reduced stability margin. MOSI-Direct lowers the four-block score to 0.012, placing the configuration close to the instability boundary while preserving a smooth degradation trend. MOSI-Flip becomes more conservative by crossing below zero at the same step (-0.067), indicating that the fuzzy sensor pattern is strong enough to override the positive geometric margin. MOSI-Weighted produces the strongest warning (-0.328) because the high-risk membership degrees amplify the sensor penalties; however, this stronger response should be interpreted together with its higher variability, which reflects the greater sensitivity of this variant to sensor fluctuations and transient loading. This comparison supports the central claim of the paper: local interface sensing reduces the overconfidence of geometry-only stability assessment, while the three fusion variants provide different levels of conservativeness for detecting imminent failure.

#### 4.4 Parameter Selection and Sensitivity Analysis

This subsection evaluates whether the conclusions obtained from the repeated hardware cantilever trials depend on a narrow choice of the nominal MOSI parameters defined in Section 3. The sigmoid parameters  $k$  and  $x_0$  control how the normalized FSR differential is converted into a bounded loading cue. The threshold  $\tau$  determines when the fuzzy classifier activates the conservative sign adjustment in MOSI-Flip. The coefficients  $\alpha_s, \alpha_m, \alpha_l$  and  $\beta_l, \beta_m, \beta_h$  determine the relative penalties associated with the low-, medium-, and high-risk fuzzy memberships in MOSI-Weighted. Once

selected, the nominal parameter values were held fixed across all reported hardware trials and Webots configurations.

The nominal FSR mapping uses  $k = 10$  and  $x_0 = 0.2$  in Eq. (5). The MOSI-Flip threshold is  $\tau = 0.6$ , and the weighted-fusion coefficients are

$$\{\alpha_s, \alpha_m, \alpha_t\} = \{0.1, 0.5, 0.8\}, \quad \{\beta_t, \beta_m, \beta_h\} = \{0.1, 0.5, 0.8\}.$$

The monotone ordering of these coefficients ensures that membership in a higher-risk class cannot produce a smaller penalty than membership in a lower-risk class.

To determine whether the experimental conclusions depend on a narrow parameter choice, we performed a one-at-a-time sensitivity analysis. Each parameter was varied while all remaining parameters were held at their nominal values, and the three MOSI variants were recomputed for all 10 hardware trials at each of the four cantilever lengths. The sigmoid steepness was evaluated at

$$k \in \{6.7, 8, 10, 12, 13.3\},$$

while the sigmoid midpoint was evaluated at

$$x_0 \in \{0.16, 0.18, 0.20, 0.22, 0.24\}.$$

Each individual FSR and IMU membership coefficient was varied by  $\pm 0.1$ , while preserving the monotone ordering of the low-, medium-, and high-risk weights. The FSR and tilt membership breakpoint domains were scaled by factors in

$$\{0.8, 0.9, 1.0, 1.1, 1.2\}.$$

Scaling was applied to the complete input domain rather than to individual trapezoid vertices, thereby preserving the ordering and overlap of the low-, transition-, and high-risk membership regions. Because  $\tau$  activates a discrete sign adjustment, it was examined separately using an extended sweep over

$$\tau \in [0, 1]$$

with increments of 0.01.

For every perturbation, three criteria were evaluated:

1. the block-mean score remained monotonically decreasing from the one-block to the four-block condition;
2. the four-block condition remained the lowest-margin configuration; and
3. the critical condition remained close to or below the zero-margin threshold.

These criteria assess whether the experimental conclusion is preserved rather than requiring exact invariance of the numerical scores.

For all tested values of  $k$ ,  $x_0$ , the membership weights, and the structured breakpoint scales, all three variants retained a monotonically decreasing block-level trend

and identified the four-block condition as the lowest-margin state. Across these non-threshold perturbations, the mean four-block scores remained within

$$0.004 \leq h_{\text{MoSI},\text{dir}} \leq 0.019, \quad -0.065 \leq h_{\text{MoSI},\text{flip}} \leq -0.031,$$

and

$$-0.389 \leq h_{\text{MoSI},\text{wt}} \leq -0.255.$$

Thus, MoSI-Direct consistently remained close to the zero-margin threshold, while the Flip and Weighted variants remained negative at the critical condition. Although the Weighted score exhibited the largest numerical variation, the ordering of the four cantilever conditions and the identification of the critical condition were unchanged.

The sigmoid midpoint  $x_0$  produced the largest variation in the MoSI-Direct critical score, which changed from 0.004 to 0.019 over the tested range. This behavior is expected because  $x_0$  directly shifts the loading range over which the FSR sigmoid transitions. Variations in  $k$  produced a smaller change, with the four-block Direct score remaining between 0.011 and 0.012. The Weighted formulation showed its largest variation under changes to the high-risk FSR coefficient and the FSR membership breakpoint scale, but remained negative throughout both sweeps.

The extended  $\tau$ -sensitivity analysis revealed a clear separation between the earlier and critical cantilever conditions. Across the 30 trials corresponding to blocks 1–3, the largest observed fuzzy score was

$$z_{\text{max}}^{1:3} = 0.4996,$$

whereas the 10 four-block trials produced

$$0.8545 \leq z^4 \leq 0.8610.$$

Consequently, every threshold satisfying

$$0.50 \leq \tau \leq 0.85$$

caused all four-block trials to activate the sign adjustment while none of the earlier conditions activated it. Thresholds below 0.50 can produce premature activation in some noncritical three-block trials. At  $\tau = 0.86$ , only four of the ten four-block trials activate the adjustment, while at  $\tau = 0.90$ , none of them activate it. The nominal value  $\tau = 0.6$  therefore lies within a broad separation interval, approximately 0.10 above the largest noncritical fuzzy score and 0.25 below the smallest critical score, rather than near an isolated decision boundary.

At the nominal parameter setting, the geometry-only margin decreases from 0.149 at three blocks to 0.080 at four blocks, a reduction of 0.069. Over the same transition,  $h_{\text{MoSI},\text{dir}}$  decreases by approximately 0.095,  $h_{\text{MoSI},\text{flip}}$  decreases by approximately 0.172, and  $h_{\text{MoSI},\text{wt}}$  decreases by approximately 0.433. The fused variants therefore preserve the geometric trend while expressing a stronger response to the sharp increase in measured interface loading and tilt at the critical condition.

Overall, the sensitivity results show that the identification of the four-block condition and the qualitative ordering of the cantilever states are preserved under bounded perturbations of the sigmoid, membership, and weighting parameters. The extended threshold analysis also shows that the nominal sign-adjustment threshold lies within a nontrivial interval separating the observed noncritical and critical trials. These findings establish local robustness for the evaluated hardware dataset, but they do not constitute proof of global parameter invariance or remove the need for recalibration when the sensor layout, module design, connector properties, or operating conditions are changed.

## 5 Scalability-Oriented Evaluation of MoSI in Webots

### 5.1 Scalable Simulation Framework and Model Fidelity

The hardware cantilever experiments provide a controlled validation of the proposed sensor-fusion principle at a single mechanically critical interface. To evaluate whether the same MoSI formulation can be extended to larger and more structurally diverse modular assemblies, we developed a physics-based Webots simulation framework. The purpose of this section is therefore not limited to reproducing the hardware experiment, but to examine the scalability of MoSI across configurations containing different geometries, load-transfer paths, and numbers of monitored interfaces.

All simulations were performed in *Webots* (R2025a) [20]. Module dimensions, mass, connector placement, gravity, and sensor locations were matched to the measured hardware values summarized in Table 4. The same configuration-level geometric computation and interface-associated sensor-processing pipeline used in hardware were retained in simulation. For each monitored interface, the simulated IMU provides the local tilt angle  $\theta$ , while the FSR pair provides the top-bottom imbalance cue. These quantities are combined with the shared geometric margin  $D_w$  to compute the local  $h_{\text{MoSI,dir}}(x)$  score using Eq. (6). The minimum interface score can then be propagated across the structure to obtain the configuration-level result.

Unless stated otherwise, the simulations consider quasi-static configurations on planar ground using rigid-body dynamics and simplified contact models. The Webots environment therefore preserves the principal geometry, mass distribution, connector layout, sensing locations, and computation flow of the physical system, while not claiming a fully calibrated, contact-accurate digital twin. This level of fidelity is sufficient for evaluating whether the same geometry-sensor formulation produces consistent behavior as the assembly grows and the number and arrangement of monitored interfaces change.

The simulated configurations were selected to progressively extend the evaluation beyond the single-interface hardware cantilever. They include cantilever growth, bridge-like support, pillar extension, asymmetric branching, and distributed plate loading, representing instability mechanisms previously examined in modular-robot breakage studies [7]. Together, these configurations allow MoSI to be evaluated under localized overload, increasing lever arms, multiple simultaneous interfaces, asymmetric growth, and progressive rotational deformation.

The scalability study focuses on the common MOSI formulation, MOSI-Direct  $h_{\text{MOSI,dir}}(x)$ , defined in Eq. (6). This variant contains the shared geometric, FSR, and IMU computation used by all three formulations and therefore provides the clearest basis for evaluating transfer across structures without introducing variant-specific fuzzy decision rules. The MOSI-Flip and MOSI-Weighted formulations use the same input pipeline but differ in their fusion logic. Their parameter behavior and robustness are evaluated using the repeated hardware trials in Section 4.4.

**Table 4** Geometric and material parameters used in the Webots model.

| Quantity                  | Parameter         | Value                   |
|---------------------------|-------------------|-------------------------|
| <i>Blinky</i> Block width | $a$               | 41 mm                   |
| Mass                      | $m$               | 61.2 g                  |
| Gravity                   | $g$               | $9.81 \text{ m s}^{-2}$ |
| Magnetic force (side)     | $F_{\text{mag}L}$ | 15.0 N                  |
| Magnetic force (top)      | $F_{\text{mag}V}$ | 12.0 N                  |

To validate the simulator against hardware, we recreated the cantilever growth experiment: blocks are added one at a time, the system is allowed to settle, and the per-interface MOSI-Direct score  $h_{\text{MOSI,dir}}(x)$  is computed using the same pipeline as in hardware. Figure 9 shows the simulated scene with the support polygon, COM projection, and interface-level visualization.

Figure 10 overlays  $h_{\text{MOSI,dir}}(x)$  versus block index for the cantilever. The hardware curve shows the mean  $\pm$  95%  $t$ -CI over 10 trials. Although Webots physics is deterministic, we obtain a simulation distribution by injecting zero-mean sensor noise before computing the FSR cue, the tilt cue, and the final  $h_{\text{MOSI,dir}}(x)$  score. The injected

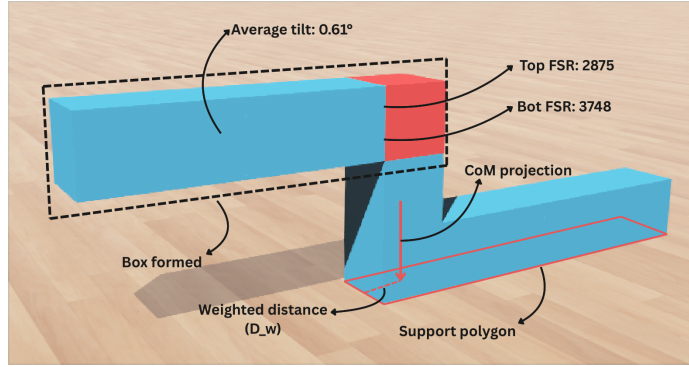


**Fig. 9** Webots cantilever scene mirroring the physical setup. The support polygon and COM projection define the weighted geometric margin  $D_w$  to the nearest edge. The dashed rectangle marks the box used in the hierarchical representation. At the critical interface, simulated FSR imbalance and IMU tilt feed the per-interface MOSI-Direct score  $h_{\text{MOSI,dir}}(x)$ , and the block is colored red to indicate a low margin.

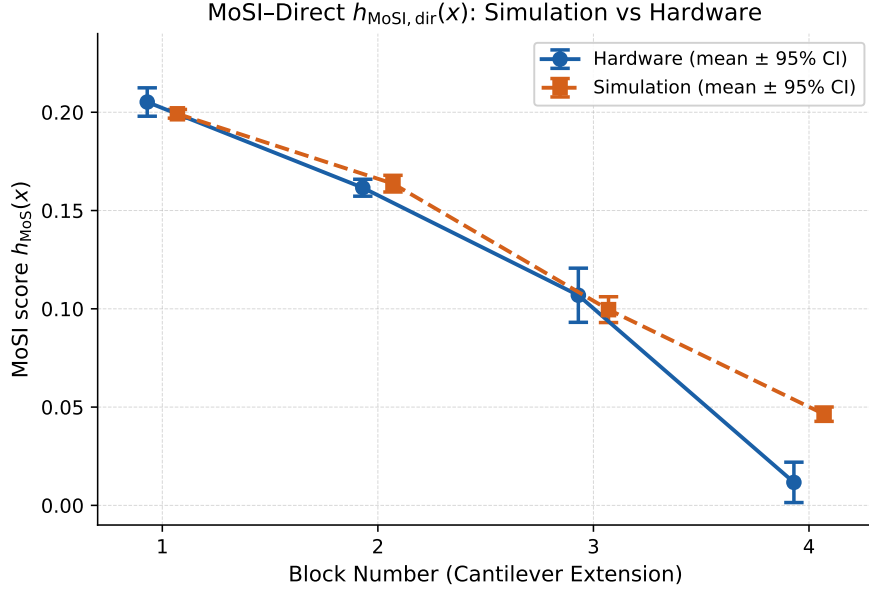


**Fig. 10** Validation of the base MoSI variant, MoSI-Direct  $h_{\text{MoSI,dir}}(x)$ , on the cantilever experiment. Hardware results are reported as mean  $\pm$  95%  $t$ -CI over  $n = 10$  runs ( $df = 9$ ); simulation results are reported as mean  $\pm$  95%  $t$ -CI over  $n = 61$  Monte Carlo sensor-noise runs ( $df = 60$ ). The comparison is descriptive because only four paired block steps are available. The hardware and simulation curves show the same monotonic decay and identify the same most-critical interface, with pointwise errors summarized by RMSE = 0.018 and MAE = 0.013.

noise is 2% on the FSR divider signals and 2% on the IMU tilt input. Because only four paired block steps are available, the comparison is treated as descriptive rather than inferential. The relevant validation criteria are therefore trend agreement, pointwise error, and critical-interface identification: both curves decrease monotonically with cantilever length, both identify the four-block case as the lowest-margin condition, and the pointwise discrepancy remains small (RMSE = 0.018, MAE = 0.013). Small differences between hardware and simulation values of  $D_w$  are expected because the hardware margin is affected by sensor mass, wiring, contact imperfections, block alignment, and the effective support polygon, whereas Webots uses an idealized rigid-body model with nominal geometry and simplified contact assumptions. Therefore, the validation focuses on trend agreement and critical-interface identification rather than exact equality of each intermediate component.

## 5.2 Generalization Across Additional Configurations

After validating the Webots implementation against the physical cantilever experiment, we evaluated whether the same MoSI-Direct formulation transfers to geometrically and mechanically distinct modular-robot configurations. Four complementary scenarios were considered: a V-bridge, a J-pillar, a branch overhang, and a tower-supported plate subjected to progressive corner loading. Rather than exhaustively

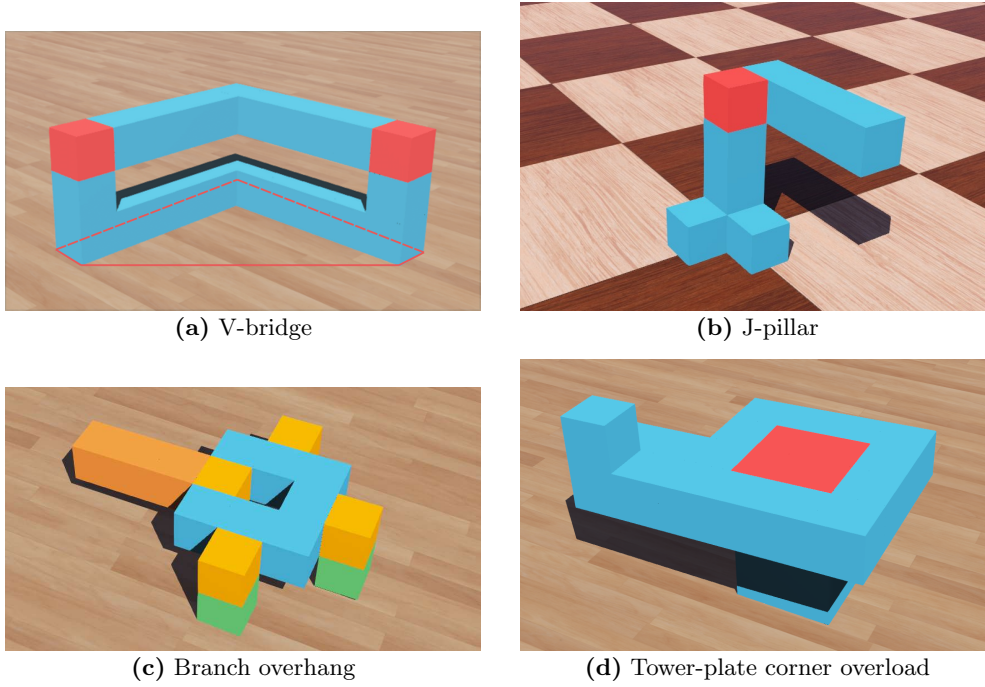


**Fig. 11** Additional Webots configurations used to evaluate the transferability of MoSI-Direct beyond the hardware-correlated cantilever experiment. The V-bridge isolates localized interface loading, the J-pillar produces coupled degradation after module addition, the branch overhang evaluates asymmetric growth across multiple monitored regions, and the tower-plate experiment produces a progressive transition from a positive to a negative fused margin.

cataloguing possible robot shapes, these configurations were selected to expose different sources of instability: localized interface overload, coupled degradation of geometric and sensor cues, asymmetric growth across several monitored regions, and progressively increasing eccentricity and rotation.

The same geometric computation, FSR mapping, IMU contribution, and MoSI parameters were used in all simulations without configuration-specific retuning. Representative steady-state results are summarized in Table 5, while Fig. 11 illustrates the four configurations.

The V-bridge was selected to separate global geometric support from localized interface loading. Its two legs form a broad support region, giving both monitored leg interfaces a relatively large geometric margin of  $D_w = 0.190$ . However, each leg exhibits an FSR differential of approximately 873 ADC counts, corresponding to  $\sigma(\Delta F) = 0.533$ . The resulting scores are reduced to 0.086 and 0.087, despite the negligible measured tilt of  $0.01^\circ$ – $0.06^\circ$ . This configuration therefore demonstrates that favorable global support geometry does not guarantee uniformly safe load transfer and that MoSI can identify locally critical interfaces that would not be revealed by the support polygon alone.

The V-bridge result can also be compared directly with the critical step of the hardware-correlated cantilever simulation. The two configurations produce similar

**Table 5** Summary of the additional Webots configurations using the same MoSI-Direct formulation and parameters. For the branch overhang, the reported FSR cue corresponds to the lower branch support, which exhibited the largest loading change. The terminal tilt of the extended branch was measured after mechanical failure and settling and is therefore not interpreted as the tilt at failure onset. The tower-plate FSR cue remained fixed at  $\sigma(\Delta F) = 0.533$  across all stages.

| Configuration   | Condition / Interface | $D_w$  | $\sigma(\Delta F)$ | Tilt [°]          | $h_{\text{MoSI,dir}}(x)$ |
|-----------------|-----------------------|--------|--------------------|-------------------|--------------------------|
| V-bridge        | LEG_LEFT              | 0.190  | 0.533              | 0.06              | 0.086                    |
| V-bridge        | LEG_RIGHT             | 0.190  | 0.533              | 0.01              | 0.087                    |
| J-pillar        | Initial               | 0.260  | 0.121              | 1.39              | 0.179                    |
| J-pillar        | After addition        | 0.190  | 0.333              | 2.01              | 0.046                    |
| Branch overhang | Base                  | 0.110  | 0.124              | 2.85              | 0.030                    |
| Branch overhang | Base + 2 modules      | -0.042 | 0.837              | 0.39 <sup>†</sup> | -0.047                   |
| Tower-plate     | Stage 1               | 0.716  | 0.380              | 0.71              | 0.309                    |
| Tower-plate     | Stage 2               | 0.506  | 0.426              | 1.79              | 0.172                    |
| Tower-plate     | Stage 3               | 0.413  | 0.533              | 3.19              | 0.075                    |
| Tower-plate     | Stage 4               | 0.325  | 0.635              | 6.33              | -0.091                   |

<sup>†</sup> The extended branch had already crossed the support boundary and mechanically failed before settling. The reported terminal tilt therefore describes the final settled pose and not the transient rotational response at failure onset.

mapped FSR cues, with  $\sigma(\Delta F) = 0.533$  at the V-bridge legs and  $\sigma(\Delta F) = 0.520$  at the critical cantilever interface. Nevertheless, the V-bridge retains a larger geometric margin,  $D_w = 0.190$ , compared with  $D_w = 0.110$  for the cantilever. Consequently, its fused scores remain higher at 0.086–0.087, compared with 0.049 for the cantilever. The resulting offset of approximately 0.037–0.038 is therefore explained by the additional geometric support available in the V-bridge rather than by a weaker loading response. This comparison illustrates that the FSR cue scales the available geometric margin instead of acting as an independent force threshold.

The J-pillar evaluates a different mechanism in which all three inputs deteriorate after an upper module is added. The initial configuration produces a positive score of 0.179, with  $D_w = 0.260$ , a small mapped FSR cue of 0.121, and a tilt of 1.39°. After the addition, the geometric margin decreases to 0.190, the mapped FSR cue increases to 0.333, and the tilt rises to 2.01°, reducing the score to 0.046. Unlike the V-bridge, no single term dominates the result. The J-pillar therefore illustrates the intended fusion behavior, where several moderate degradations jointly indicate a near-critical condition.

The final J-pillar score is also numerically close to the critical cantilever result. Its value of 0.046 lies within 0.003 of the cantilever score of 0.049. The two configurations reach this low-margin range through different combinations of geometry and sensor evidence. The cantilever has a smaller geometric margin and a larger mapped FSR cue, whereas the J-pillar combines a larger remaining geometric margin with greater tilt and simultaneous degradation across all three inputs. Their numerical proximity does not imply identical mechanical states. Instead, it shows that the same unmodified formulation assigns comparably low remaining margins to two geometrically distinct near-critical configurations.

The branch-overhang configuration extends the evaluation to asymmetric growth across several plausible load-transfer regions. Four regions are monitored simultaneously: the left root support, the upper and lower branch supports, and the root of the lateral extension. In the 17-module base configuration, the projected COM remains inside the support polygon, producing  $D_w = 0.110$ , and the global minimum score remains positive at

$$h_{\text{MoSI,dir}}^{\text{global}}(x) = 0.030.$$

The structure therefore retains only a small positive margin before further asymmetric growth.

Adding two modules shifts the projected COM in the direction of the overhang and beyond the support boundary. The geometric margin changes sign from  $D_w = 0.110$  to  $D_w = -0.042$ , while the global fused score decreases to

$$h_{\text{MoSI,dir}}^{\text{global}}(x) = -0.047.$$

The lower branch support also exhibits the strongest redistribution of contact loading, with its FSR differential rising from 0.124 to 0.837. By comparison, the remaining monitored regions exhibit substantially smaller loading changes, allowing the distributed minimum operation to identify the lower support as the most critical region.

The terminal tilt decreases from  $2.85^\circ$  in the base configuration to  $0.39^\circ$  after the module addition. This decrease must not be interpreted as an improvement in stability. The added modules caused the projected COM to cross the support boundary, after which the structure mechanically failed and settled into a new pose. The reported  $0.39^\circ$  value was computed over this settled post-failure window and therefore does not represent the rotational response immediately before or during failure. In this experiment, the unsafe transition is established by the negative geometric margin and the pronounced local FSR increase, while the terminal tilt is retained only to describe the final settled state. Characterizing the transient tilt at failure onset would require a separate time-resolved analysis rather than a terminal-window average.

The branch-overhang experiment consequently demonstrates two complementary capabilities. First, the geometric term identifies when asymmetric growth moves the projected COM outside the support polygon. Second, the distributed interface measurements identify the lower branch support as the region experiencing the strongest load redistribution. The experiment therefore extends the previous single-interface cases by demonstrating simultaneous evaluation of several structurally relevant regions during an asymmetric failure transition.

Finally, the tower-plate experiment provides a controlled progressive loading sequence. Modules are added in successive stages near one corner of a plate supported by a compact central tower. The ground-support footprint remains unchanged, while the projected COM moves toward its boundary and the upper structure develops increasing rotational deformation.

Across the four stages, the geometric margin decreases monotonically from  $D_w = 0.716$  to 0.506, 0.413, and 0.325. At the same time, the measured tilt increases from  $0.71^\circ$  to  $1.79^\circ$ ,  $3.19^\circ$ , and  $6.33^\circ$ . These coupled changes produce the fused-margin progression

$$0.309 \rightarrow 0.172 \rightarrow 0.075 \rightarrow -0.091.$$

Stage 3 represents a near-critical but still positive condition, whereas Stage 4 crosses the fused stability boundary. Importantly, the final geometry-only margin remains positive at  $D_w = 0.325$ . The negative MOSI score is instead produced by the interaction between the reduced geometric margin and the increasing rotational penalty. This result shows why the support polygon alone is insufficient for configurations that remain geometrically supported while undergoing substantial physical deformation.

Collectively, the four configurations demonstrate that the same MOSI-Direct formulation responds consistently to distinct instability mechanisms without parameter retuning. The V-bridge exposes localized interface overload under favorable global support and remains above the critical cantilever score because of its larger geometric margin. The J-pillar reaches a score within 0.003 of the critical cantilever through coupled degradation across all three inputs. The branch overhang identifies asymmetric support-boundary crossing and localized load redistribution, with its terminal tilt explicitly treated as a post-failure measurement. Finally, the tower-plate experiment demonstrates a gradual transition from a stable state to a negative sensor-fused margin as eccentricity and rotation increase. These simulations were therefore selected for their complementary mechanical roles rather than merely for their geometric differences.

### 5.3 Limitations

First, the hardware validation focuses on a controlled cantilever benchmark rather than a full large-scale self-reconfiguration sequence. This choice was intentional: as demonstrated in Section 4, the cantilever setting produces the key divergence between the geometry-only baseline and the sensor-fused MOSI margins. The repeated trials therefore provide a focused hardware test of the proposed interface-level margin. However, broader validation across larger assemblies, multiple simultaneous interfaces, and longer reconfiguration sequences remains necessary to characterize performance at system scale.

Second, both the hardware and Webots evaluations focus mainly on quasi-static or settled configurations. Rapid dynamic motions, impacts, slip, and abrupt friction changes may introduce contact effects that are not fully captured by the current margin. Similarly, sensor drift, saturation, imperfect calibration, or changes in contact placement can affect the FSR and IMU cues. These cases may require adaptive filtering or online recalibration before MOSI is used as a real-time decision variable in more dynamic settings.

Third, the fuzzy membership functions, sigmoid mapping, and fusion weights were kept fixed across the experiments to preserve interpretability and avoid case-by-case retuning. However, these parameters may need systematic calibration when transferred to different module designs, connector stiffnesses, sensor placements, or surface conditions. Future work should therefore investigate data-driven parameter selection and sensitivity analysis across a wider range of geometries and loading conditions.

Finally, MOSI is used here as a stability assessment metric rather than as a formally verified controller. While the margin is inspired by safety-margin and barrier-function reasoning, this work does not prove forward invariance or closed-loop safety under arbitrary reconfiguration actions. A natural extension is to integrate MOSI into an online action-selection or safety-filtering layer and evaluate whether it can reliably

delay, reject, or modify reconfiguration moves that would reduce the interface margin below a chosen threshold.

## 6 Conclusion

We presented MOSI, a sensor-fused stability margin  $h_{\text{MOSI}}(x)$  for lattice-based modular robots that combines a normalized geometric support margin with local FSR and IMU cues evaluated at structural interfaces. The central motivation was that a geometry-only margin can remain positive even when local interface loading and tilt indicate that the structure is approaching instability. By evaluating MOSI at interfaces between axis-aligned box clusters and aggregating the minimum score hierarchically, the proposed formulation provides a lightweight and physically interpretable stability proxy compatible with distributed modular-robot computation.

Hardware cantilever experiments showed that all three MOSI variants decrease as the cantilever grows, while revealing different operational trade-offs. MOSI-Direct provides smooth degradation, MOSI-Flip introduces a conservative transition under unstable fuzzy sensor patterns, and MOSI-Weighted gives the strongest sensor-driven warning at the cost of higher variability. The component-level comparison further showed that the geometry-only margin remained positive at the critical four-block condition, whereas the fused variants moved the score toward, or below, the instability boundary by incorporating interface-level sensing.

In simulation, a hardware-correlated Webots model using the same sensing and computation pipeline reproduced the decreasing MOSI-Direct trend and identified the same lowest-margin cantilever condition observed in hardware. The scalability-oriented evaluation was then extended to a V-bridge, J-pillar, branch overhang, and progressively loaded tower-supported plate without configuration-specific retuning. These configurations demonstrated consistent responses to localized interface loading, coupled degradation of geometry and sensing cues, asymmetric support-boundary crossing, and increasing rotational deformation. Collectively, the results show that MOSI reduces the overconfidence of geometry-only assessment across structurally distinct configurations while retaining a lightweight formulation that does not require a full mechanical-equilibrium model.

Future work will extend validation to larger assemblies, multiple simultaneous interfaces, closed-chain structures, and dynamic reconfiguration tasks. We also aim to complete an on-module distributed implementation, improve systematic calibration of  $f(\Delta F)$  and  $g(\theta)$  across platforms, and integrate MOSI into an online decision layer capable of delaying, rejecting, or modifying reconfiguration actions that would reduce the per-interface margin below a chosen safety threshold.

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## **Statements and Declarations**

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### **Competing Interests**

The authors have no relevant financial or non-financial interests to disclose.

### **Author Contributions**

All authors contributed equally to this work.

### **Data Availability**

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

### **Code Availability**

The codes used for the experiments of this study are available from the corresponding author upon reasonable request.

### **Ethics Approval**

Not applicable.