

Terrain-Aware Convex Feasible Region Planning for Robust Wheeled-Legged Locomotion

Zhen Fu

Southern University of Science and Technology

Wei Zhang

zhangw3@sustech.edu.cn

Southern University of Science and Technology

Research Article

Keywords: Wheeled-legged robots, Perceptive locomotion, Terrain-aware planning, Convex trajectory optimization, Nonlinear model predictive control, Whole-body control

Posted Date: September 23rd, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10577407/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Terrain-Aware Convex Feasible Region Planning for Robust Wheeled-Legged Locomotion

Zhen Fu¹ and Wei Zhang^{1,2*}

^{1*}School of Automation and Intelligent Manufacturing (AIM), Southern
University of Science and Technology, Shenzhen, 518055, China.

²LimX Dynamics, Shenzhen, 518055, China.

*Corresponding author(s). E-mail(s): zhangw3@sustech.edu.cn;
Contributing authors: 12131084@mail.sustech.edu.cn;

Abstract

Wheeled-legged robots combine efficient rolling with articulated terrain adaptation, but standard perceptive nonlinear model predictive control (NMPC) uses short horizons and heuristic base references that can become kinematically inconsistent on geometrically discontinuous terrain. This study presents a terrain-aware base-reference planner based on convex feasible regions (CFRs). Foothold polygons extracted from an elevation map are transformed into phase-wise horizontal and vertical base regions using conservative reachability approximations, Minkowski sums, and set intersections. A convex B-spline quadratic program constrains its control points with these regions and supplies a long-horizon geometric reference to an existing kinodynamic NMPC and whole-body controller. The planner runs onboard in typically less than 1 ms and below 2 ms in the reported worst case. On a W1 wheeled quadruped, the proposed pipeline completed 200/200 staircase trials (100%; 95% Wilson CI [98.1, 100%]), including 100 ascents and 100 descents, whereas the baseline completed 157/220 trials (71.36%; CI [65.1, 76.9%]). It also completed 100/100 mid-stair stopping trials, compared with 48/101 for the baseline. These results indicate that phase-wise geometric guidance can improve reliability on the tested staircase without enlarging the NMPC horizon, while cross-platform performance and operation near infeasible CFRs remain to be validated.

Keywords: Wheeled-legged robots, Perceptive locomotion, Terrain-aware planning, Convex trajectory optimization, Nonlinear model predictive control, Whole-body control

Categories: (3), (5)

1 Introduction

Wheeled-legged robots combine the energy efficiency and speed of wheeled vehicles with the obstacle-traversal capability of legged platforms, making them well suited for navigating complex real-world environments such as industrial facilities, urban infrastructure, and disaster sites [1–8]. Their ability to alternate between rolling and stepping modes supports applications such as multi-story delivery, industrial inspection, and disaster response, where stairs, curbs, and uneven surfaces are prevalent. However, operating in constrained and structurally disjointed environments, such as steep staircases, remains challenging. In these scenarios, the robot must maintain kinematic consistency with discrete terrain contacts across multiple future steps while managing dynamic, nonholonomic rolling motions.



Fig. 1: The W1 wheeled quadrupedal robot (LimX Dynamics) used as the hardware platform. The system features 16 actuated DoFs (12 leg joints and 4 active wheels).

Recent perceptive locomotion frameworks based on Nonlinear Model Predictive Control (NMPC) [3, 6, 9, 10] have demonstrated strong agility on continuous uneven terrain. However, on discrete topographies like stairs, three specific gaps emerge. First, many deployed perceptive NMPC pipelines rely on heuristic base references that do not encode kinematic feasibility relative to the discrete footholds, leading to leg over-extensions when the terrain geometry is tight; methods that instead optimize footholds and base motion jointly, such as TAMOLS [10], make different scope and computation

trade-offs and are discussed in Section 2. Second, the short prediction horizons required for real-time execution (0.5–1.0 s) cannot explicitly reason about kinematic feasibility across multiple upcoming contact phases. Third, for wheeled-legged robots, rolling contact adds complexity beyond the point-foot case: contact locations evolve continuously along the rolling direction, and consistency between the base motion and the wheel-ground contacts must ultimately be enforced by the downstream tracking controllers, so the reference-generation layer must remain compatible with these rolling-contact controllers in real time. Existing trajectory optimization approaches [4, 5, 11, 12] either require prohibitive computation or use simplified models that ignore these kinematic limits. A detailed discussion is provided in Section 2.

To address these gaps, we propose a *phase-aware convex base-feasibility representation* that converts multi-phase terrain geometry into real-time convex constraints for base-reference generation. The core idea is to map discrete foothold regions into continuous convex feasible regions (CFRs) for the robot base using Minkowski sum operations and conservative reachability bounds. For each locomotion phase, the intersection of per-leg sets yields a horizontal convex polygon and a vertical interval that conservatively approximate base positions compatible with the stance-leg reachability model. This representation transforms long-horizon geometric reasoning—which is difficult to include in a short-horizon NMPC—into linear constraints that can be solved in real time.

Two supporting designs make this representation practical. First, we formulate base trajectory generation as a convex B-spline quadratic program (QP), using phase-assigned control-point constraints to provide conservative geometric guidance with solve times under 2 ms. Second, we integrate this planner into a decoupled pipeline with an existing low-rank contact-relaxation NMPC [3] and a no-slip whole-body controller (WBC), where the planner supplies the long-horizon reference and the NMPC handles dynamic tracking over a short horizon.

Rather than extending the NMPC horizon or smoothing heuristic references, the proposed method constructs a phase-wise geometric corridor for the robot base directly from the contact schedule and foothold regions. This corridor supplies phase-assigned control-point constraints for long-horizon reference generation on discrete terrain with rolling contacts.

The main contributions are:

- **Phase-aware convex base-region construction:** We map discrete foothold regions into per-phase base regions via Minkowski sums and convex set intersections. The resulting linear constraints connect discrete terrain geometry to continuous base-reference planning for wheeled-legged robots with rolling contacts.
- **Real-time B-spline trajectory optimization:** We generate long-horizon base references through a convex QP whose control points are bounded by the phase-wise CFRs, executing in under 2 ms onboard. Combined with an existing low-rank NMPC and a no-slip WBC, this forms a decoupled pipeline in which the planner provides geometric foresight that the short-horizon NMPC lacks.
- **Hardware validation:** Experiments on a quadrupedal wheeled-legged robot show 200/200 successful continuous stair-traversal trials and 100/100 successful mid-stair

stopping trials, compared with 157/220 and 48/101 for the standard perceptive NMPC baseline. Additional terrains are demonstrated qualitatively.

Regarding the scope of these contributions, the kinodynamic NMPC with low-rank contact relaxation and the whole-body controller are adopted from prior work [3, 13]. The novelty of this paper lies in the CFR-based reference generation layer—the phase-wise feasible-region construction and the B-spline optimization built on it—and in its real-time integration with these existing components.

2 Related Work

2.1 Wheeled-Legged Locomotion

Wheeled-legged robots have attracted growing attention due to their ability to combine wheeled efficiency with legged terrain adaptability. Bjelonic et al. [1] proposed a whole-body motion control and planning framework that demonstrated dynamic rolling locomotion on a wheeled quadruped. Subsequent work extended this to online trajectory optimization for hybrid rolling-stepping gaits [6] and whole-body MPC with automatic gait discovery [3]. On the trajectory optimization side, Medeiros et al. [4] formulated a single-rigid-body planner for wheeled-legged robots on rough terrain, while De Viragh et al. [12] introduced linearized zero-moment-point constraints for stable wheel-legged trajectories. Bellegarda and Byl [5] explored trajectory optimization that accounts for wheel slip. More recent studies have considered real-time whole-body optimization for skid-steered wheeled-legged vehicles [14] and online wheel-leg hybrid planning for obstacle traversal [15]. These approaches address unified motion generation or contact-sequence planning, whereas the present work focuses on lightweight phase-wise base-region construction for an externally supplied contact schedule. Recent work has also explored wheeled-legged loco-manipulation [2] and learning-based navigation and locomotion for wheeled-legged systems [16]; neither directly studies the conservative base-reference construction considered here.

2.2 Model Predictive Control for Legged Locomotion

MPC has become a dominant approach for dynamic legged locomotion. Di Carlo et al. [17] demonstrated that convex MPC with a simplified single-rigid-body model can achieve agile quadruped locomotion at real-time rates. To preserve full kinematics, Sleiman et al. [13] proposed a unified MPC framework based on centroidal dynamics with full-body kinematics, enabling loco-manipulation tasks. Grandia et al. [9] extended NMPC to perceptive locomotion by incorporating elevation-map information directly into the optimization. Jenelten et al. [10] further developed terrain-aware motion optimization (TAMOLS) that jointly optimizes footholds and body motion. These NMPC-based approaches generally use short prediction horizons (typically 0.5–1.0 s) to maintain real-time performance. On structured discrete terrain such as stairs, that horizon and the use of heuristic base references can limit anticipation of multi-phase kinematic constraints. TAMOLS addresses terrain awareness within a nonlinear program, whereas our planner retains an externally supplied contact schedule and adds

a separate convex long-horizon geometric layer; the two methods therefore make different scope and computation trade-offs. Corbères et al. [18] likewise couple whole-body MPC with online selection of contact surfaces over multiple future steps, whereas the present planner keeps an externally supplied contact schedule and constructs convex base regions from given foothold polygons.

Separately, monolithic trajectory optimization methods [11, 19] can synthesize long-horizon multi-contact motions with full kinematics, but their nonlinear programs are generally more computationally demanding than the QP considered here. Recent gait planning based on Monte Carlo tree search [20] offers an alternative for non-gaited systems but does not directly address the base-region problem considered here.

2.3 Geometric Feasibility and Safe Region Planning

Computing safe, feasible regions for robot motion has a rich history. Bretl and Lall [21] formalized static equilibrium testing for multi-contact legged robots by characterizing the set of feasible center-of-mass positions. Orsolino et al. [22] extended this notion to actuation-aware feasible regions that account for both kinematic reachability and force limits. Caron and Kheddar [23] used convex feasibility constraints for multi-contact walking pattern generation. In the planning domain, Deits and Tedrake [24] proposed IRIS for computing large convex obstacle-free regions via semidefinite programming, while Tonneau et al. [25] leveraged reachability analysis for efficient acyclic contact planning.

These methods primarily target point-foot legged robots in quasi-static or slow dynamic regimes and often characterize feasible center-of-mass positions under equilibrium or actuation limits. The use of lightweight convex base-region construction for high-speed wheeled-legged locomotion on discrete terrain has received comparatively limited attention. Our work addresses this gap by constructing Minkowski sum-based CFRs for base-reference planning and embedding them into a real-time convex B-spline optimization that interfaces with a kinodynamic NMPC for hybrid rolling-stepping locomotion.

2.4 Learning-Based Locomotion

Recent learning-based approaches have demonstrated strong terrain-traversal capabilities. Lee et al. [26] trained locomotion policies over challenging terrain using simulation, while Miki et al. [27] combined proprioceptive and exteroceptive sensing with reinforcement learning for perceptive locomotion. Jiang et al. [2] applied learning to whole-body loco-manipulation on wheeled-legged platforms. Such methods can implicitly learn complex terrain negotiation, whereas optimization-based approaches expose geometric constraints and modeling assumptions directly. We view the two directions as complementary; this work follows the latter route and provides interpretable, conservative reachability bounds rather than an end-to-end learned policy.

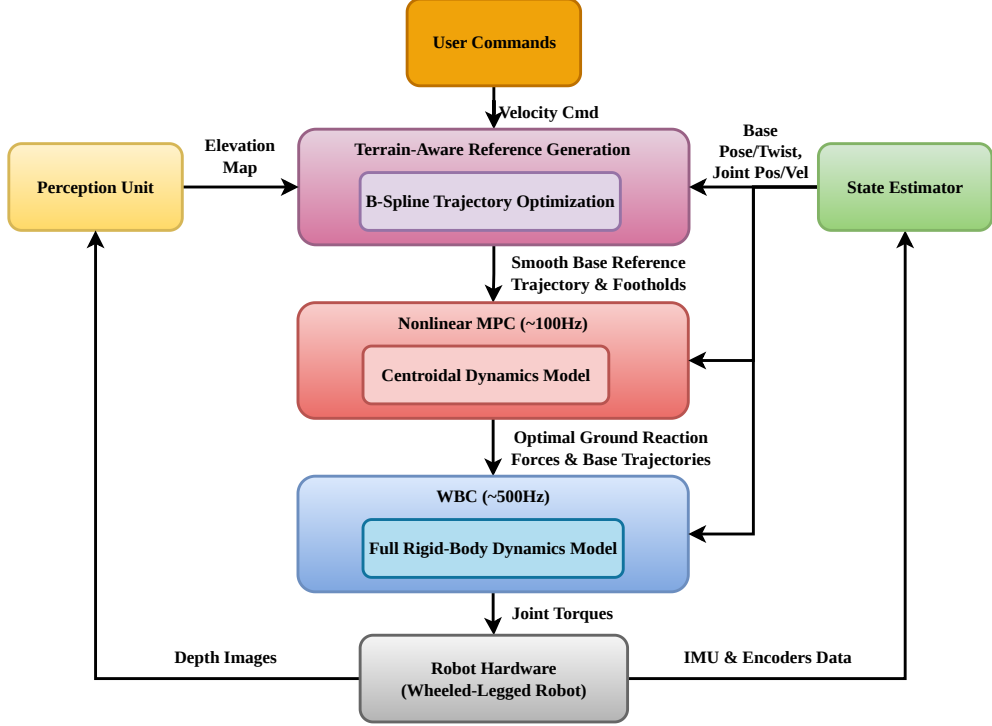


Fig. 2: Overview of the hierarchical control architecture. The perception unit constructs the elevation map and foothold regions, the terrain-aware planner generates a 6-DoF base reference using the gait schedule, the NMPC (~ 100 Hz) computes ground reaction forces and motion plans, and the WBC (~ 500 Hz) maps them to joint torques while enforcing no-slip rolling constraints. State-estimator feedback closes the loop for all planning and control layers.

3 System Architecture

This section describes the hardware platform and the hierarchical control pipeline. The terrain-aware planner and tracking controller are formulated in Sections 4 and 5, respectively.

3.1 Hardware Platform

The target platform is the W1 quadrupedal wheeled-legged robot developed by LimX Dynamics, shown in Figure 1. The system is modeled as a free-floating base with four identical articulated limbs. Each limb has three actuated joints: hip abduction/adduction, hip flexion/extension, and knee flexion/extension. Instead of point feet, each limb terminates in an actively driven, torque-controlled wheel.

The robot has a total of 16 actuated DoFs: 12 leg joints and 4 wheel joints. To avoid quaternion singularities, the configuration space is defined by $\mathbf{q} = [\mathbf{q}_b^T, \mathbf{q}_l^T, \mathbf{q}_w^T]^T \in \mathbb{R}^{23}$,

where $\mathbf{q}_b \in \mathbb{R}^7$ is the floating base pose (3D position and unit quaternion), $\mathbf{q}_j \in \mathbb{R}^{12}$ represents the leg joint angles, and $\mathbf{q}_w \in \mathbb{R}^4$ denotes the wheel rotation angles. The corresponding generalized velocity vector is $\mathbf{v} \in \mathbb{R}^{22}$. The redundant actuation provides a large kinematic workspace but requires a decoupled control strategy to manage the hybrid continuous-discrete states.

3.2 Hierarchical Control Architecture

To balance long-horizon geometric planning with high-frequency full-body control, we use a decoupled hierarchical pipeline (Figure 2). Each layer operates at a distinct frequency and communicates through explicit interfaces: the perception module outputs elevation maps and convex foothold polygons; the base planner receives these along with the contact schedule and outputs a 6-DoF base reference trajectory; the NMPC receives this reference and computes ground-reaction forces and end-effector trajectories; and the WBC maps these outputs to joint torques. The closed-loop pipeline relies on continuous feedback from an onboard state estimator and consists of four modules:

- **Terrain Perception and Foothold Extraction:** Processes onboard depth data to build a local 2.5D elevation map and extracts collision-free convex foothold regions for each leg (Section 4.1).
- **Terrain-Aware Base Planner:** Constructs CFRs from the footholds and generates a geometrically constrained base reference trajectory $\mathbf{p}_b^{ref}$ via B-spline optimization (Sections 4.2–4.4).
- **Kinodynamic NMPC (~100 Hz):** Computes ground-reaction forces and base/end-effector trajectories that track $\mathbf{p}_b^{ref}$ using a centroidal-dynamics model with low-rank contact relaxation (Section 5.1).
- **Full-Body WBC (~500 Hz):** Maps the NMPC outputs to joint torques $\boldsymbol{\tau}$ using the full rigid-body model and enforces instantaneous no-slip constraints at the wheel-ground contacts (Section 5.2).

4 Terrain-Aware Base Reference Generation

This section presents the terrain-aware reference generation layer. Rather than relying on the NMPC to implicitly resolve spatial geometry on disjointed terrains (e.g., stairs), this planner bridges discrete foothold constraints with the continuous motion of the robot’s base through convex feasible region construction and B-spline optimization.

4.1 Elevation Mapping and Foothold Extraction

The perceptive pipeline processes depth data to maintain a local 2.5D elevation map [28, 29] around the robot. Given a nominal base velocity command, we use the Raibert heuristic [30] to estimate the nominal foothold for the i -th leg, located below the corresponding hip joint along the gravity direction at the expected mid-stance time. As shown in Figure 3, the perception module identifies the closest valid planar surface relative to this nominal position and computes the corresponding projection point [9]. From this projection, the algorithm generates a bounded convex region that

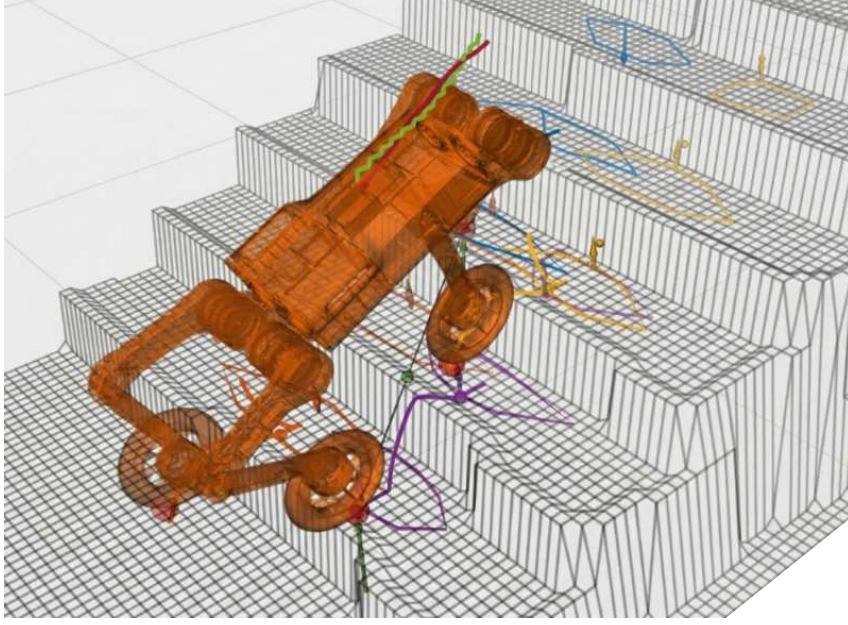


Fig. 3: Visualization during stair traversal. The background grid is the 2.5D elevation map. Colored polygons on the steps are the extracted foothold regions ($\mathcal{S}_{i,j}$). The curves above the robot show the optimized base and end-effector trajectories. Green arrows at the wheel-terrain interfaces indicate the ground reaction forces ($\mathbf{f}_c$) computed by the NMPC.

respects the geometric limits of the terrain (e.g., step edges). This collision-free stepping area is a planar convex polygon $\mathcal{S}_{i,j} \subset \mathbb{R}^3$ on the identified surface, extracted separately for the contact of leg i scheduled in locomotion phase j : because the same leg contacts different surfaces across successive phases (e.g., successive treads of a staircase), one region is constructed for every upcoming contact event in the gait schedule. While $\mathcal{S}_{i,j}$ bounds the allowable contact locations, it only constrains the end-effector and cannot be directly used for base trajectory planning.

4.2 Kinematic Reachability Mapping

To formulate the optimization problem with respect to the base trajectory, the discrete foot-placement constraints must be mapped into the base spatial domain. We address the horizontal and vertical directions separately to maintain computational efficiency. This decoupling yields per-axis bounds tightened by inward margins, rather than a certificate for the coupled full-body dynamics.

4.2.1 Horizontal Reachability via Minkowski Sum

Let $\bar{\mathcal{S}}_{i,j}$ denote the 2D horizontal projection of the foothold polygon $\mathcal{S}_{i,j}$. If the relative pose between the base and the foot were entirely fixed, the allowable horizontal region for the base would be identical in shape to $\bar{\mathcal{S}}_{i,j}$. However, exploiting the kinematic workspace of the articulated leg significantly expands this allowable region.

Let $\bar{\mathcal{W}}_{kin}$ be a conservative 2D convex set of admissible foot displacements relative to the hip, and let $\bar{\mathbf{p}}_{hip,i}$ be the horizontal vector from the base origin to hip i . If the foot position lies in $\bar{\mathcal{S}}_{i,j}$ and the hip-to-foot displacement lies in $\bar{\mathcal{W}}_{kin}$, the corresponding horizontal base-position set is

$$\mathcal{B}_{i,j} = (\bar{\mathcal{S}}_{i,j} \oplus (-\bar{\mathcal{W}}_{kin})) - \bar{\mathbf{p}}_{hip,i}. \quad (1)$$

The reflection $-\bar{\mathcal{W}}_{kin}$ is required because the optimization variable is the base position rather than the foot displacement; it is identical to $\bar{\mathcal{W}}_{kin}$ only when the approximation is centrally symmetric. Operationally, the construction projects the 3D foothold polygon, expands it by the reflected conservative workspace, shifts it by the base-to-hip offset, and intersects the resulting per-leg regions over all stance legs. The result approximates the reachable base set under the sampled workspace model; it is tightened by the inward margins but does not account for every coupled postural effect.

4.2.2 Vertical Reachability Bounds

In addition to horizontal constraints, the base height z_b must remain within the modeled reach of all stance legs. For stance leg i in phase j , let $z_{f,i,j}$ be the wheel-center elevation at the scheduled contact with $\mathcal{S}_{i,j}$ (the contact-surface elevation offset by the wheel radius), let $[l_{min}, l_{max}]$ denote conservative vertical hip-to-wheel-center distances along gravity, and let $\Delta z_{hip,i}$ be the signed vertical offset from hip i to the base-frame origin. The admissible base-height interval derived from leg i in phase j is

$$z_{f,i,j} + l_{min} + \Delta z_{hip,i} \leq z_b \leq z_{f,i,j} + l_{max} + \Delta z_{hip,i} \quad (2)$$

where l_{min} and l_{max} are chosen conservatively to avoid configurations near full extension or retraction. This interval captures base heights consistent with the selected vertical reach limits for leg i .

4.3 Phase-wise Convex Feasible Region Construction

During locomotion phase j , a subset of legs, denoted by $\mathcal{C}_j$, maintains terrain contact. To require simultaneous membership in the modeled stance-leg sets, the horizontal CFR for phase j , denoted by $\mathcal{Q}_j^{xy}$, is computed as

$$\mathcal{Q}_j^{xy} = \bigcap_{i \in \mathcal{C}_j} \mathcal{B}_{i,j} \quad (3)$$

Because intersections preserve convexity, $\mathcal{Q}_j^{xy}$ is a convex polygon when it is nonempty.

Simultaneously, the vertical feasible interval for phase j is obtained by intersecting the per-leg height bounds from Eq. (2):

$$\mathcal{Q}_j^z = \left[\max_{i \in \mathcal{C}_j} z_{i,j}^{min}, \min_{i \in \mathcal{C}_j} z_{i,j}^{max} \right] \quad (4)$$

where $z_{i,j}^{min} = z_{f,i,j} + l_{min} + \Delta z_{hip,i}$ and $z_{i,j}^{max} = z_{f,i,j} + l_{max} + \Delta z_{hip,i}$. On discrete terrain such as stairs, where different stance legs contact surfaces at different elevations, this intersection tightens the admissible height range and keeps the base reference away from heights that would exceed the modeled vertical reach of any stance leg. The complete phase-wise CFR is then defined as the Cartesian product $\mathcal{Q}_j = \mathcal{Q}_j^{xy} \times \mathcal{Q}_j^z$, which remains a convex set.

Remark (Feasibility check and fail-soft output): Before solving the B-spline QP, the planner checks each phase-wise CFR $\mathcal{Q}_j$. If the area of $\mathcal{Q}_j^{xy}$ is below 0.001 m^2 or $\mathcal{Q}_j^z$ is empty, the implementation raises an infeasibility flag and passes the heuristic base reference used by the baseline pipeline. The same fallback applies when the B-spline QP of Section 4.4 is reported infeasible or the solver fails to converge: the planner raises the flag and outputs the heuristic reference. This behavior maintains an output to downstream controllers but does not restore CFR feasibility and is therefore a fail-soft implementation detail rather than a safety mechanism. It was not triggered in the reported 15 cm-high, 30 cm-deep staircase trials; operation near an empty CFR is outside the validated scope.

4.4 B-Spline Trajectory Optimization

Having constructed the phase-wise CFRs, we parameterize the base motion as a k th-order 3D B-spline. A B-spline lies in the convex hull of the control points active over each knot interval. This property converts control-point region membership into a finite set of linear constraints and yields an efficiently solvable convex QP. The curve is written as

$$\mathbf{P}(u) = \sum_{c=0}^{n-1} \mathbf{P}_c B_{c,k}(u) \quad (5)$$

where $\mathbf{P}_c = [x_c, y_c, z_c]^T$, $c = 0, \dots, n-1$, are the coordinates of the n control points, and $B_{c,k}(u)$ are the B-spline basis functions. The optimization variables are stacked as $\mathbf{x} = [x_0, \dots, x_{n-1}, y_0, \dots, y_{n-1}, z_0, \dots, z_{n-1}]^T \in \mathbb{R}^{3n}$, and phase-wise region membership is imposed on the control points.

For a control point assigned to phase j , horizontal membership in $\mathcal{Q}_j^{xy}$ is formulated with half-space inequalities. Let the polygon vertices be ordered counterclockwise and indexed cyclically. For the edge from (x_v^j, y_v^j) to (x_{v+1}^j, y_{v+1}^j) , the interior half-space is

$$(y_v^j - y_{v+1}^j)x_v^j + (x_{v+1}^j - x_v^j)y_v^j \leq (y_v^j - y_{v+1}^j)x_c + (x_{v+1}^j - x_v^j)y_c \quad (6)$$

Applying this to all edges of $\mathcal{Q}_j^{xy}$ yields a set of linear inequality constraints on the x_c, y_c components. The vertical constraint from Eq. (4) is enforced as simple box

constraints on the z_c components:

$$\max_{i \in \mathcal{C}_j} z_{i,j}^{min} \leq z_c \leq \min_{i \in \mathcal{C}_j} z_{i,j}^{max} \quad \forall c \in \text{phase } j \quad (7)$$

For a knot interval whose active basis functions all multiply control points constrained to the same $\mathcal{Q}_j$, the convex-hull property implies that the corresponding curve interval remains in $\mathcal{Q}_j$. At a phase boundary, basis-function support may span control points assigned to adjacent CFRs; unless those transition control points also satisfy the intersection of the adjacent CFRs, control-point membership alone is not a trajectory-wide feasibility proof. The present implementation uses phase-assigned control-point constraints, so the CFR should be interpreted as a conservative geometric guide, with formal certification limited to intervals sharing a common CFR.

Base-reference generation is formulated as a convex QP that balances trajectory smoothness with tracking of a nominal velocity reference. The objective consists of two terms:

1) **Smoothness Cost** (J_{smooth}): We penalize the integral of the squared second derivative over the parameter domain $[0, u_{max}]$ to promote smooth base motion:

$$J_{smooth} = \int_0^{u_{max}} \left\| \frac{d^2 \mathbf{P}(u)}{du^2} \right\|^2 du = \mathbf{x}^T \mathbf{H}_1 \mathbf{x} \quad (8)$$

Since the derivative of a B-spline is a linear combination of its control points, the Hessian $\mathbf{H}_1$ is constant, positive semi-definite, and can be precomputed from the knot vector.

2) **Tracking Cost** (J_{track}): To ensure the robot progresses consistently along the macroscopic commanded direction, we penalize the spatial deviation between the B-spline curve and a nominal reference trajectory $\mathbf{p}_{ref}(u)$ at M uniformly sampled evaluation points:

$$J_{track} = \sum_{m=1}^M \|\mathbf{P}(u_m) - \mathbf{p}_{ref}(u_m)\|^2 = \mathbf{x}^T \mathbf{H}_2 \mathbf{x} + \mathbf{g}^T \mathbf{x} + \text{const} \quad (9)$$

where $\mathbf{H}_2$ and the gradient vector $\mathbf{g}$ are constructed by evaluating the basis functions at each discrete parameter u_m .

Combining these objectives, the final QP is formulated as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \omega_1 J_{smooth} + \omega_2 J_{track} \\ \text{s.t.} \quad & (x_c, y_c) \in \mathcal{Q}_j^{xy}, \quad z_c \in \mathcal{Q}_j^z \quad \forall c \in \text{phase } j \\ & \mathbf{P}(0) = \mathbf{p}_{current}, \quad \dot{\mathbf{P}}(0) = \mathbf{v}_{current} \end{aligned} \quad (10)$$

where ω_1 and ω_2 are positive weights. The initial conditions anchor the spline to the measured base state, with $\dot{\mathbf{P}}(0)$ matched to the current base velocity through the planner's parameter-to-time scaling. Since $\mathcal{Q}_j^{xy}$ is a convex polygon and $\mathcal{Q}_j^z$ is a convex interval, the region constraints are linear and the objective Hessian is positive

semidefinite. The resulting problem is a convex QP; strict convexity is not required by the formulation. Solved using OSQP [31], it executes in under 2 ms onboard.

To construct the full 6D base reference $\mathbf{p}_b^{ref}$ for the NMPC, the 3D B-spline provides the constrained positional trajectory (x, y, z) . The reference yaw is aligned with the trajectory tangent, while pitch and roll are adapted to the local terrain gradient (e.g., aligning base pitch with the staircase slope).

Remark: The horizontal and vertical constraints are treated independently; the Cartesian product of the two projected bounds is not a certified inner approximation of the coupled three-dimensional workspace, and the inward margins are applied to absorb, empirically, the coupling neglected by this decomposition. The workspace was sampled at the nominal level base orientation, so commanded pitch and roll rotate the hip positions and further perturb the mapping; these orientation-induced deviations are likewise left to the margins within the tested staircase inclination. The reachability parameters $\bar{W}_{kin}$, l_{min} , and l_{max} are reduced from sampled workspace limits. These bounds reduce exposure to joint limits but do not by themselves prove full-body kinematic or dynamic feasibility.

Implementation on W1: On the W1 platform, $\bar{W}_{kin}$ was obtained by forward-kinematics sampling over the joint range, yielding a horizontal projection of approximately $0.55 \text{ m} \times 0.35 \text{ m}$ (longitudinal $\times$ lateral). A 15% inward margin produced an effective approximation of approximately $0.47 \text{ m} \times 0.30 \text{ m}$. The sampled vertical range was $[0.25 \text{ m}, 0.55 \text{ m}]$; a 10% inward margin yielded $l_{min} = 0.275 \text{ m}$ and $l_{max} = 0.495 \text{ m}$. The reduced bounds were checked against the sampled W1 configurations used to configure the reported experiments.

5 Decoupled Perceptive Kinodynamic Control

This section describes how the base reference $\mathbf{p}_b^{ref}$ from the terrain-aware planner is tracked by a two-layer controller: an NMPC that computes forces and trajectories, and a WBC that produces joint torques.

5.1 Kinodynamic NMPC with Low-Rank Contact Relaxation

We adopt the kinodynamic NMPC formulation of Bjelonic et al. [3] and Sleiman et al. [13]. This OCS2-based formulation balances centroidal-dynamics fidelity and real-time performance, handles wheeled contacts via low-rank contact relaxation, and runs at 100 Hz with a 1.0 s horizon. We use $\mu = 0.6$ for the modeled rubber-wheel-concrete contact. The NMPC operates at 100 Hz using a kinematic-centroidal dynamics model [13, 32] that preserves leg kinematics while excluding wheel joints to reduce the optimization dimension. The centroidal dynamics are:

$$\dot{\mathbf{h}}_{com} = \begin{bmatrix} m\mathbf{g} + \sum_{i=1}^{N_c} \mathbf{f}_{c,i} \\ \sum_{i=1}^{N_c} (\mathbf{p}_{c,i} - \mathbf{p}_{com}) \times \mathbf{f}_{c,i} \end{bmatrix} \quad (11)$$

where $\mathbf{h}_{com} \in \mathbb{R}^6$ is the centroidal momentum, $\mathbf{f}_{c,i}$ and $\mathbf{p}_{c,i}$ are the contact force and position of the i -th leg, and m , $\mathbf{g}$ are the robot mass and gravity vector.

Algorithm 1 Terrain-Aware Base Reference Generation

Require: Elevation map $\mathcal{M}$, gait schedule $\{\mathcal{C}_j\}$, commanded velocity $\mathbf{v}_{cmd}$

Ensure: Base reference trajectory $\mathbf{p}_b^{ref}$ and status flag

```
1: for each scheduled phase  $j$  do
2:   for each leg  $i \in \mathcal{C}_j$  do
3:     Extract convex foothold polygon  $\mathcal{S}_{i,j}$  from  $\mathcal{M}$  for the scheduled contact
4:     Compute  $\mathcal{B}_{i,j}$  from Eq. (1)
5:     Compute vertical bounds:  $[z_{i,j}^{min}, z_{i,j}^{max}]$  from Eq. (2)
6:   end for
7:    $\mathcal{Q}_j^{xy} \leftarrow \bigcap_{i \in \mathcal{C}_j} \mathcal{B}_{i,j}$  {Horizontal CFR}
8:    $\mathcal{Q}_j^z \leftarrow [\max_i z_{i,j}^{min}, \min_i z_{i,j}^{max}]$  {Vertical CFR}
9:   if  $\text{area}(\mathcal{Q}_j^{xy}) < \epsilon$  or  $\mathcal{Q}_j^z = \emptyset$  then
10:
11:     return heuristic reference and infeasibility flag
12:   end if
13: end for
14: Formulate B-spline QP with control points  $\mathbf{x}$  constrained in  $\{\mathcal{Q}_j\}$ 
15: Solve:  $\min_{\mathbf{x}} \omega_1 J_{smooth} + \omega_2 J_{track}$  s.t. CFR constraints
16: if QP infeasible or solver failure then
17:
18:   return heuristic reference and infeasibility flag
19: end if
20: Construct 6-DoF reference  $\mathbf{p}_b^{ref}$  from B-spline + terrain orientation
```

The NMPC objective directly tracks the planner’s base reference $\mathbf{p}_b^{ref}$:

$$\min_{\mathbf{x}(\cdot), \mathbf{u}(\cdot)} \int_0^{T_h} \left(\|\mathbf{p}_b(\mathbf{x}(t)) - \mathbf{p}_b^{ref}(t)\|_{\mathbf{Q}}^2 + \|\mathbf{u}(t)\|_{\mathbf{R}}^2 \right) dt \quad (12)$$

subject to the centroidal dynamics, joint limits, and friction cones. Since the planner handles long-horizon geometric reasoning, the NMPC operates over a short horizon ($T_h = 1.0$ s).

To further reduce computational cost, we adopt the low-rank contact relaxation from [3]. As illustrated in Figure 4, standard point-foot formulations enforce a 3-DoF zero-velocity constraint at the contact. For wheeled contacts, we drop the longitudinal (rolling direction) component, retaining only:

$$\begin{bmatrix} \mathbf{n}^T \\ \mathbf{t}_{lat}^T \end{bmatrix} \mathbf{v}_{P_{c,i}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

where $\mathbf{v}_{P_{c,i}}$ is the contact point velocity, $\mathbf{n}$ the terrain normal, and $\mathbf{t}_{lat}$ the lateral tangent. This allows the NMPC to treat the end-effector as a gliding contact along the rolling direction without optimizing wheel joint states, deferring the enforcement of instantaneous rolling physics to the WBC.

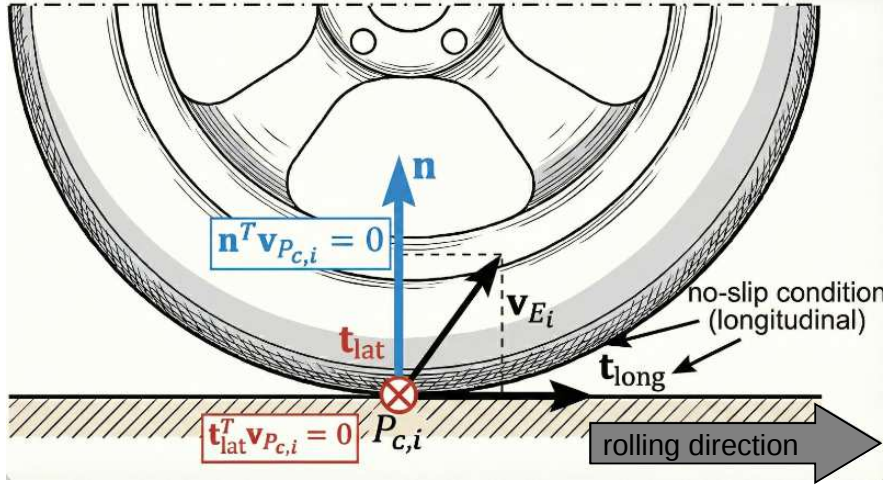


Fig. 4: Low-rank contact relaxation in the NMPC. The normal ($\mathbf{n}$) and lateral ($\mathbf{t}_{lat}$) no-slip constraints are enforced; the longitudinal ($\mathbf{t}_{long}$) constraint is relaxed, treating the wheel as a gliding contact.

The NMPC outputs base trajectories, wheel center trajectories, and ground reaction forces for the WBC (Figure 3).

5.2 Full-Body WBC with No-Slip Constraints

The WBC runs at 500 Hz and maps the NMPC outputs to joint torques using the full rigid-body model ($\mathbf{v} \in \mathbb{R}^{22}$, including wheel joints), following a TSID formulation [33, 34]. Below we detail the two components specific to the wheeled-legged decoupling.

5.2.1 Contact Motion Constraints

The no-slip condition requires zero velocity at the instantaneous wheel-ground contact. We enforce this at the acceleration level using the spatial slip Jacobian $\mathbf{J}_c$:

$$\mathbf{J}_c \dot{\mathbf{v}} + \dot{\mathbf{J}}_c \mathbf{v} = \mathbf{0} \quad (14)$$

Remark: Both our formulation and that of [1] enforce rolling at the acceleration level, but differ in the contact point definition. In [1], the constraint uses a wheel-fixed contact point with Jacobian $\mathbf{J}_{C_i}(\theta_i)$ that depends on the wheel angle, requiring an explicit centripetal term $\ddot{\mathbf{r}}_{IC_i}$ on the right-hand side. Our formulation uses the spatial slip Jacobian $\mathbf{J}_c$ evaluated at the geometric contact interface (instantaneous wheel-ground intersection), where $\dot{\mathbf{J}}_c \mathbf{v}$ implicitly absorbs the centripetal terms, simplifying the constraint.

5.2.2 Wheel Center Tracking

A key design choice is that the WBC tracks the NMPC-planned trajectories at the *wheel center*, not at the wheel-ground contact point. Since the NMPC excludes wheel joints and treats contacts as gliding, its outputs $\mathbf{r}_{des}$ and $\dot{\mathbf{r}}_{des}$ represent the intended wheel center motion. The tracking objective minimizes the operational-space acceleration error:

$$\left\| \mathbf{J}_e \dot{\mathbf{v}} + \dot{\mathbf{J}}_e \mathbf{v} - \ddot{\mathbf{r}}_{des}^{cmd} \right\|_{\mathbf{Q}}^2 \quad (15)$$

where $\mathbf{J}_e$ is the Jacobian at the wheel center and the desired acceleration follows a PD law:

$$\ddot{\mathbf{r}}_{des}^{cmd} = \mathbf{k}_p(\mathbf{r}_{des} - \mathbf{r}) + \mathbf{k}_d(\dot{\mathbf{r}}_{des} - \dot{\mathbf{r}}) \quad (16)$$

By simultaneously tracking the wheel center ($\mathbf{J}_e$ task) and enforcing no-slip at the ground ($\mathbf{J}_c$ constraint), the QP resolves the kinematic redundancy, producing the wheel-joint motion and torques required to maintain rolling.

6 Experimental Results

We evaluate the proposed framework through hardware experiments on the LimX Dynamics W1 robot. The experiments compare the framework with a standard perceptive NMPC baseline on constrained discrete terrain and provide qualitative demonstrations on continuous and irregular terrain.

6.1 Implementation Details

The framework is deployed onboard using an x86 computing unit (Intel Core i7, 16 GB RAM), with all modules running on CPU without GPU acceleration. The pipeline uses separate CPU threads: elevation mapping runs asynchronously, and terrain-aware base-reference generation runs at 100 Hz immediately before each NMPC update. The reduced-order kinodynamic NMPC is implemented using OCS2 [35], and the B-spline QP is solved using OSQP [31]. Key system parameters are summarized in Table 1.

All staircase experiments use a trotting gait in which diagonal leg pairs alternate contact. The contact sequence $\{\mathcal{C}_j\}$ is specified by the platform gait scheduler rather than optimized by the proposed planner, and the same externally provided schedule is used for both stair ascent and descent. This sequence is shared by the CFR planner and the NMPC.

The planner horizon spans all upcoming contact phases provided by the platform gait scheduler, which exceeds the 1.0 s NMPC prediction horizon; control points are assigned to phases according to the scheduled phase timing. The B-spline QP has $3n = 45$ decision variables for $n = 15$ control points and $O(nE)$ horizontal half-space constraints, where E is the average number of polygon edges per assigned phase. With typical $E \leq 8$, there are approximately 120 horizontal inequalities, 30 vertical box inequalities, and 6 initial-state equalities, for approximately 156 scalar constraints. The QP is typically solved in under 1 ms, with the reported worst case below 2 ms. The NMPC runs at 100 Hz with a typical solve time of ~ 8 ms per iteration, and the WBC runs at 500 Hz with a typical solve time of ~ 1 ms. All modules met their respective real-time deadlines during the reported experiments.

The tuning procedure involved a limited set of platform-dependent adjustments. Reachability margins were calibrated once from forward-kinematics workspace samples. The B-spline weights were first tuned on flat ground to balance smoothness and commanded-velocity tracking and were then retained for stair traversal. NMPC cost weights were inherited from the existing low-rank contact-relaxation framework, with adjustments to the W1 base-tracking terms.

Table 1: System Parameters

Module	Parameter	Value
B-spline Planner	Spline order k	5
	Number of control points n	15
	Smoothness weight ω_1	1.0
	Tracking weight ω_2	10.0
Control Frequency	B-spline planner	< 2 ms / solve (typ. < 1 ms)
	NMPC (OCS2)	100 Hz
	WBC	500 Hz
NMPC	Prediction horizon T_h (s)	1.0
	Friction coefficient μ	0.6
Gait	Contact pattern	Trot (diagonal leg pairs)
	Schedule source	External platform gait scheduler

6.2 Comparison on Highly Constrained Terrain

To evaluate the contribution of the terrain-aware geometric planner, we compare against a baseline representing the standard perceptive NMPC paradigm [3, 9] adapted to the wheeled-legged platform. The baseline uses the same kinodynamic NMPC and WBC but replaces the CFR planner with a heuristic base reference derived from the elevation map and commanded velocity. Specifically, the baseline follows the reference-generation procedure of Grandia et al. [9] (Section III-E): user-commanded 2D linear velocity and angular rate are extrapolated under constant input; the smoothed torso-reference layer is interpolated at each hip location; and a least-squares fit through the four hip positions produces a 6-DoF base reference. A Raibert-style feedback term adjusts the first upcoming nominal foothold. This heuristic is smooth on continuous terrain but does not encode base reachability relative to discrete footholds. Both configurations are evaluated on a staircase with 15 cm step height and 30 cm tread depth over repeated hardware trials.

The CFR planner is the only module changed between the two configurations, so the comparison isolates its effect within this control pipeline. Success rate is the primary metric because a failed trial ends in a fall or inability to complete the maneuver; no continuous secondary metric is reported consistently over the full trial set.

For traversal, the baseline completed 95/120 ascent and 62/100 descent trials; the proposed method completed 100/100 trials in each direction. During ascent stopping, the baseline completed 48/101 trials and the proposed method completed

100/100. The baseline totals aggregate all recorded staircase trials executed with the heuristic-reference configuration during pipeline development and evaluation, whereas the proposed-method trials were conducted subsequently; the full campaign spanned approximately one month, and the unequal group sizes reflect this record-based allocation rather than a designed equal split. Traversal success denotes completion of the full staircase without falling. In stopping trials, the commanded forward velocity is stepped to zero during ascent; success requires the robot to settle into a stable stance on the staircase, maintain it for at least 5 s, and then resume and complete the ascent once the velocity command is restored. Because trials used the same platform and staircase and the two configurations were tested sequentially rather than interleaved, the inferential tests rely on an independence assumption.

Table 2: Success Rate Comparison on Stair Traversal Hardware Trials

Method	Stair Traversal	Mid-Stair Stopping
Standard Perceptive NMPC	71.36% (157/220) 95% CI [65.1, 76.9]%	47.52% (48/101) 95% CI [38.1, 57.2]%
Proposed (CFR + NMPC)	100% (200/200) 95% CI [98.1, 100]%	100% (100/100) 95% CI [96.3, 100]%

Note: The baseline uses heuristic base references from the elevation map; the proposed method adds the CFR-based B-spline planner while retaining the same NMPC and WBC. Brackets denote 95% Wilson score intervals. Treating trials as independent Bernoulli outcomes, two-sided Fisher exact tests give $p = 1.54 \times 10^{-20}$ for traversal and $p = 1.81 \times 10^{-20}$ for stopping; direction-stratified traversal tests give $p = 9.1 \times 10^{-8}$ (ascent, 95/120 vs. 100/100) and $p = 9.3 \times 10^{-14}$ (descent, 62/100 vs. 100/100).

6.2.1 Dynamic Continuous Stair Traversal

The robot is commanded to ascend and descend the staircase at a forward velocity of 1.0 m/s. As summarized in Table 2, the proposed framework completed 200/200 trials, while the baseline completed 157/220. The 95% Wilson lower bound for the proposed success probability is 98.1%. The phase-wise CFR constraints guided the planned base reference during both ascent and descent (Figure 5a). Baseline failure modes are discussed in Section 6.3.

6.2.2 Mid-Stair Stopping Maneuver

A more demanding test is the mid-stair stopping maneuver: while ascending, the commanded forward velocity is abruptly stepped to zero on the staircase. This requires dissipation of forward momentum while maintaining a viable stance across disjoint elevations. The proposed framework completed 100/100 trials, whereas the baseline completed 48/101; the 95% Wilson lower bound for the proposed success probability is 96.3%. The observed difference is consistent with the long-horizon geometric reference improving the short-horizon controller’s ability to decelerate without reaching a kinematic limit (Figure 5b), although the experiment does not isolate individual failure mechanisms quantitatively.



(a) Dynamic Continuous Stair Traversal



(b) Mid-Stair Stopping

Fig. 5: Staircase experiments with the proposed framework. (a) The robot traverses a staircase (15 cm step height) with continuous forward momentum during ascent and descent; the sequence highlights front-leg transitions over successive step edges. (b) The robot executes an abrupt halt during ascent and maintains a static stance on disjoint steps after deceleration. Both maneuvers use the CFR-constrained base reference and rolling-contact constraints in the WBC.

The stopping comparison therefore supports the use of multi-phase geometric guidance for abrupt transitional behavior on the tested staircase.

6.3 Baseline Failure Analysis

To provide insight beyond aggregate success rates, Figure 6 compares representative NMPC-planned wheel trajectories during stair descent. Under the heuristic reference (top), the planned trajectory intersects the modeled step edge (red), indicating inconsistency between the planned wheel path and the terrain geometry. In the corresponding proposed-method sequence (bottom), the planned trajectory clears the edge before descending to the lower tread (green). During stopping failures, we also observed one or more legs approaching full extension. These observations are consistent with insufficient geometric anticipation in the baseline, but they are qualitative failure examples rather than a frequency-weighted causal analysis. Step heights beyond the tested 15 cm specification can narrow CFR intersections and may reduce reliability; this regime was not evaluated.

6.4 Qualitative Demonstrations on Other Terrains

Beyond discrete staircases, we operated the framework on continuous and irregular terrains. Representative snapshots are shown in Figure 7. These trials are qualitative demonstrations only; quantitative benchmarking is confined to the staircase comparison in Sections 6.2 and 6.3.

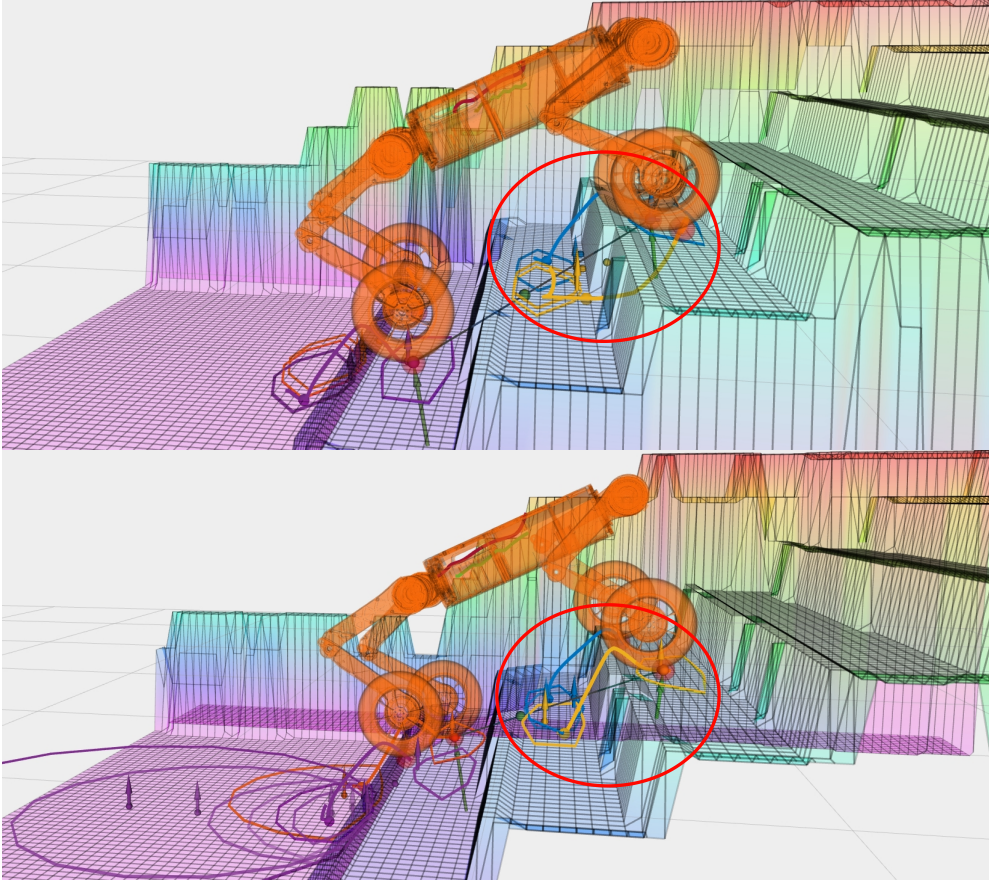


Fig. 6: Comparison of NMPC-planned wheel trajectories during stair descent. Top: under the heuristic baseline reference, the planned wheel trajectory intersects the step edge (red circle), indicating a geometric inconsistency between the planned motion and the discrete terrain. Bottom: with the CFR-based planner, the NMPC generates a trajectory that clears the step edge and descends to the lower surface in a geometrically feasible manner (green circle). This comparison illustrates how the proposed planner provides the geometric foresight that the baseline lacks.

6.4.1 Curbs and Slopes

The robot traverses a concrete ramp approximately 3 m long with a slope of up to approximately 25° . The framework adapts base pitch to the terrain gradient, while the NMPC enforces its modeled friction-cone constraints. The low-rank contact relaxation permits continuous wheel motion along the slope without a forced stepping transition.

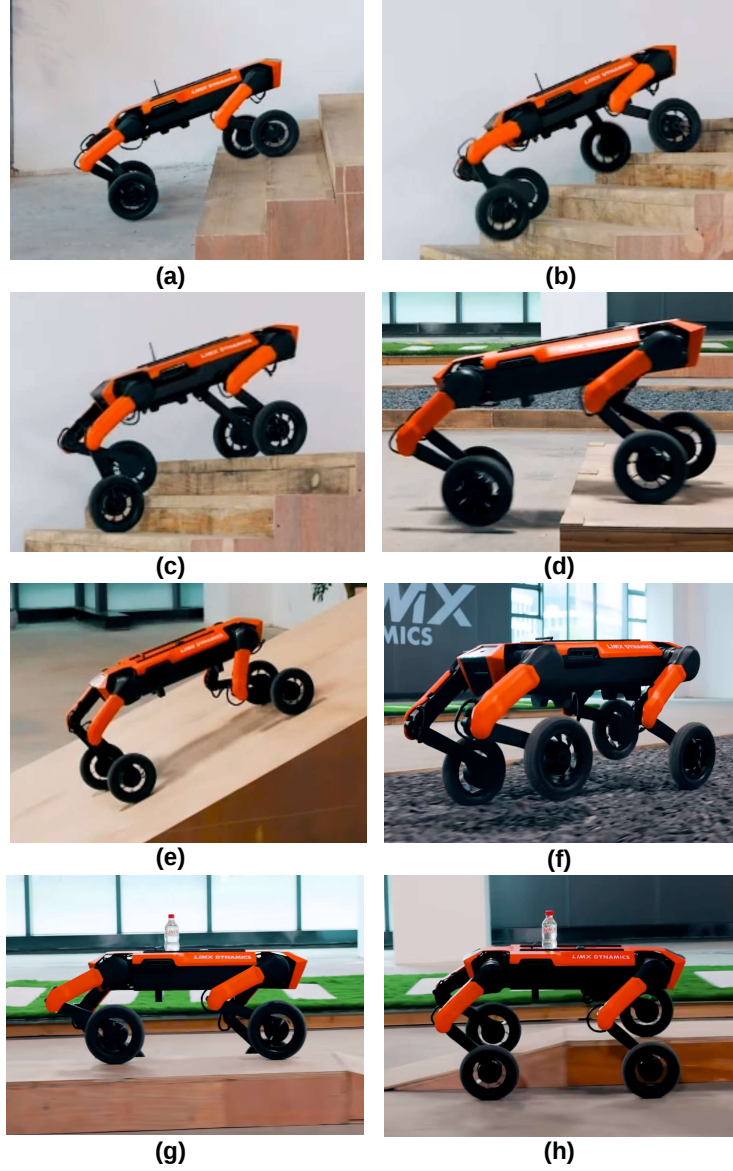


Fig. 7: Qualitative hardware demonstrations: (a)-(b) continuous stair ascent and descent; (c)-(d) discrete step traversal; (e)-(f) rolling over gravel; and (g)-(h) traversal of speed bumps. The bottle placed on the base in (g)-(h) is a visual indicator in the illustrated run, not a quantitative stability metric.

6.4.2 Gravel and Speed Bumps

The relaxed formulation treats end-effectors as gliding contacts along the longitudinal axis, allowing continuous wheel rotation over local terrain variation. On loose gravel with approximately 5 cm aggregate diameter and speed bumps approximately 5 cm high, the robot maintained continuous rolling without a commanded stepping transition in the illustrated runs.

6.4.3 Narrow Elevated Tracks

In a qualitative narrow-track run, the robot traversed an elevated bridge approximately 30 cm wide, close to its lateral stance width. The planner constructed narrow CFRs from the track boundaries, and the robot remained on the track without a commanded stepping transition. No lateral-error metric is reported for this demonstration.

6.5 Qualitative Parameter Sensitivity

The following observations come from implementation tuning rather than a controlled factorial study. Increasing the relative smoothness weight produces smoother trajectories but can increase deviation from the commanded motion; increasing the relative tracking weight can introduce sharper transitions. The selected values ($\omega_1 = 1.0$, $\omega_2 = 10.0$) were retained across the reported terrains.

Increasing n from 15 to 25 produced limited visible trajectory improvement while approximately doubling solve time in the tuning runs. Reducing n to 10 produced visible approximation artifacts. We therefore used $n = 15$ as a practical trajectory-quality and computation trade-off.

The reachability margin also affects mobility. Reducing the margin from 15% to 5% increased joint-limit warnings during tuning runs involving stair descent, whereas increasing it to 25% yielded narrower motions and lower traversal speed. These observations motivate the selected 15% value but do not constitute a quantitative ablation.

7 Discussion

The controlled comparison changes only the base-reference generator and shows a large difference in completion rate on the tested staircase. The effect is present in both ascent (95/120 versus 100/100) and descent (62/100 versus 100/100), and is larger in the mid-stair stopping task (48/101 versus 100/100). Together with the representative trajectory comparison, these results support the interpretation that a multi-phase geometric reference reduces conflicts between the commanded base motion and discrete contact geometry. They do not establish universal reliability: all trials used one robot, one staircase geometry, one externally supplied trot contact pattern, and one principal speed.

The formal scope is correspondingly limited. Each CFR is a conservative decoupled approximation derived from sampled horizontal and vertical reachability. B-spline convex-hull containment applies within intervals whose active control points share a common CFR; it does not automatically certify cross-phase intervals. The NMPC

and WBC must still satisfy dynamic, friction, state-estimation, and model-consistency requirements. In addition, the fail-soft heuristic output used when a CFR is empty was not triggered in the reported trials and should not be interpreted as a validated recovery controller.

Other limitations include the assumptions of known static terrain and accurate 2.5D elevation mapping; sensitivity to reflective surfaces, poor depth measurements, and lateral terrain discontinuities; the absence of online contact-schedule optimization; and the lack of a quantitative evaluation on the additional ramp, gravel, speed-bump, and narrow-track scenarios. Trial-level temporal grouping was not modeled in the Fisher tests, so their reported p -values rely on the stated independence assumption; in particular, the baseline trials preceded the proposed-method trials rather than being interleaved, so session-level effects are not controlled. Future evaluation should therefore include multiple staircase geometries, speeds, experimental sessions, and quantitative tracking and joint-margin metrics.

Transfer to another wheeled-legged platform is plausible because the CFR layer requires a horizontal reachability approximation, vertical reach limits, and base-to-hip offsets. Nevertheless, such transfer would require a new workspace calibration, robot model, and controller tuning and has not been experimentally demonstrated. Full system code cannot currently be released because it depends on proprietary W1 software. Algorithm 1 and Table 1 document the planner structure and reported parameters; a standalone CFR-planner release and trial-level data archive would materially improve reproducibility.

8 Conclusions

This study presented a terrain-aware CFR planner that converts discrete foothold geometry and conservative reachability approximations into a real-time convex B-spline base-reference problem. The planner augments an existing kinodynamic NMPC and WBC without enlarging the NMPC horizon and runs onboard in typically less than 1 ms, with the reported worst case below 2 ms.

On the tested W1 platform and staircase, the proposed pipeline completed 200/200 traversal trials and 100/100 mid-stair stopping trials, compared with 157/220 and 48/101 for the baseline. These results indicate that phase-wise geometric guidance improves reliability in this experimental setting. Future work should address cross-phase certification, contact-schedule optimization, operation near empty CFRs, quantitative evaluation across additional terrain geometries, and transfer to other wheeled-legged machines.

Acknowledgements. During preparation of this manuscript, the authors used OpenAI Codex (GPT-5, accessed 16 July 2026) for language editing and consistency checking. The authors reviewed and edited the output and take full responsibility for the content of this publication.

References

- [1] Bjelonic, M., Bellicoso, C.D., Viragh, Y., Sako, D., Tresoldi, F.D., Jenelten, F.,

- Hutter, M.: Keep rollin’—whole-body motion control and planning for wheeled quadrupedal robots. *IEEE Robotics and Automation Letters* **4**(2), 2116–2123 (2019) <https://doi.org/10.1109/LRA.2019.2899750>
- [2] Jiang, K., Fu, Z., Guo, J., Zhang, W., Chen, H.: Learning whole-body locomanipulation for omni-directional task space pose tracking with a wheeled-quadrupedal-manipulator. *IEEE Robotics and Automation Letters* **10**(2), 1481–1488 (2025) <https://doi.org/10.1109/LRA.2024.3519856>
- [3] Bjelonic, M., Grandia, R., Harley, O., Galliard, C., Zimmermann, S., Hutter, M.: Whole-body MPC and online gait sequence generation for wheeled-legged robots. In: 2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 8388–8395 (2021). <https://doi.org/10.1109/IROS51168.2021.9636371>. IEEE
- [4] Medeiros, V.S., Jelavic, E., Bjelonic, M., Siegwart, R., Meggiolaro, M.A., Hutter, M.: Trajectory optimization for wheeled-legged quadrupedal robots driving in challenging terrain. *IEEE Robotics and Automation Letters* **5**(3), 4172–4179 (2020) <https://doi.org/10.1109/LRA.2020.2990720>
- [5] Bellegarda, G., Byl, K.: Trajectory optimization for a wheel-legged system for dynamic maneuvers that allow for wheel slip. In: 2019 IEEE 58th Conference on Decision and Control (CDC), pp. 7776–7781 (2019). <https://doi.org/10.1109/CDC40024.2019.9029417>. IEEE
- [6] Bjelonic, M., Sankar, P.K., Bellicoso, C.D., Vallery, H., Hutter, M.: Rolling in the deep—hybrid locomotion for wheeled-legged robots using online trajectory optimization. *IEEE Robotics and Automation Letters* **5**(2), 3626–3633 (2020) <https://doi.org/10.1109/LRA.2020.2979661>
- [7] Dietrich, A., Bussmann, K., Petit, F., Kotyczka, P., Ott, C., Lohmann, B., Albu-Schäffer, A.: Whole-body impedance control of wheeled mobile manipulators: Stability analysis and experiments on the humanoid robot rollin’justin. *Autonomous Robots* **40**(3), 505–517 (2016) <https://doi.org/10.1007/s10514-015-9438-z>
- [8] Geilinger, M., Winberg, S., Coros, S.: A computational framework for designing skilled legged-wheeled robots. *IEEE Robotics and Automation Letters* **5**(2), 3674–3681 (2020) <https://doi.org/10.1109/LRA.2020.2978444>
- [9] Grandia, R., Jenelten, F., Yang, S., Farshidian, F., Hutter, M.: Perceptive locomotion through nonlinear model-predictive control. *IEEE Transactions on Robotics* **39**(5), 3402–3421 (2023) <https://doi.org/10.1109/TRO.2023.3275384>
- [10] Jenelten, F., Grandia, R., Farshidian, F., Hutter, M.: Tamols: Terrain-aware motion optimization for legged systems. *IEEE Transactions on Robotics* **38**(6), 3395–3413 (2022) <https://doi.org/10.1109/TRO.2022.3186804>

- [11] Winkler, A.W., Bellicoso, C.D., Hutter, M., Buchli, J.: Gait and trajectory optimization for legged systems through phase-based end-effector parameterization. *IEEE Robotics and Automation Letters* **3**(3), 1560–1567 (2018) <https://doi.org/10.1109/LRA.2018.2798285>
- [12] De Viragh, Y., Bjelonic, M., Bellicoso, C.D., Jenelten, F., Hutter, M.: Trajectory optimization for wheeled-legged quadrupedal robots using linearized ZMP constraints. *IEEE Robotics and Automation Letters* **4**(2), 1633–1640 (2019) <https://doi.org/10.1109/LRA.2019.2896721>
- [13] Sleiman, J.-P., Farshidian, F., Minniti, M.V., Hutter, M.: A unified MPC framework for whole-body dynamic locomotion and manipulation. *IEEE Robotics and Automation Letters* **6**(3), 4688–4695 (2021) <https://doi.org/10.1109/LRA.2021.3068908>
- [14] Xu, M., Yang, Z., Xu, C., Wang, K., Zhang, Y., Qin, Y.: Whole-body trajectory optimization for skid steer wheeled-legged vehicles. *Chinese Journal of Mechanical Engineering* **39**, 100128 (2026) <https://doi.org/10.1016/j.cjme.2025.100128>
- [15] Yu, H., Li, X., Guan, S., Tao, Z., Feng, H., Zhang, S., Fu, Y.: Planning and control for wheel-leg hybrid locomotion in wheeled biped robots for obstacle traversal. *IEEE Transactions on Automation Science and Engineering* **22**, 18382–18394 (2025) <https://doi.org/10.1109/TASE.2025.3587848>
- [16] Lee, J., Bjelonic, M., Reske, A., Wellhausen, L., Miki, T., Hutter, M.: Learning robust autonomous navigation and locomotion for wheeled-legged robots. *Science Robotics* **9**(89), 9641 (2024) <https://doi.org/10.1126/scirobotics.adi9641>
- [17] Di Carlo, J., Wensing, P.M., Katz, B., Bledt, G., Kim, S.: Dynamic locomotion in the MIT cheetah 3 through convex model-predictive control. In: 2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), pp. 1–9 (2018). <https://doi.org/10.1109/IROS.2018.8594448>. IEEE
- [18] Corbères, T., Mastalli, C., Merkt, W., Shim, J., Havoutis, I., Fallon, M., Mansard, N., Flayols, T., Vijayakumar, S., Tonneau, S.: Perceptive locomotion through whole-body MPC and optimal region selection. *IEEE Access* **13**, 69062–69080 (2025) <https://doi.org/10.1109/ACCESS.2025.3558093>
- [19] Mastalli, C., Budhiraja, R., Merkt, W., Saurel, G., Hammoud, B., Naveau, M., Carpentier, J., Righetti, L., Vijayakumar, S., Mansard, N.: Crocoddyl: An Efficient and Versatile Framework for Multi-Contact Optimal Control. In: *IEEE International Conference on Robotics and Automation (ICRA)*, pp. 2536–2542 (2020). <https://doi.org/10.1109/ICRA40945.2020.9196673>
- [20] Amatucci, L., Kim, J.-H., Hwangbo, J., Park, H.-W.: Monte carlo tree search gait planner for non-gaited legged system control. In: *2022 IEEE International Conference on Robotics and Automation (ICRA)*, pp. 4701–4707 (2022). <https://doi.org/10.1109/ICRA46638.2022.9832444>

- [21] Bretl, T., Lall, S.: Testing static equilibrium for legged robots. *IEEE Transactions on Robotics* **24**(4), 794–807 (2008) <https://doi.org/10.1109/TRO.2008.2001360>
- [22] Orsolino, R., Focchi, M., Caron, S., Raiola, G., Barasuol, V., Semini, C.: Feasible region: an actuation-aware extension of the support region. *IEEE Transactions on Robotics* **36**(4), 1239–1255 (2020) <https://doi.org/10.1109/TRO.2020.2983318>
- [23] Caron, S., Kheddar, A.: Multi-contact walking pattern generation based on model preview control of 3D COM accelerations. In: *IEEE-RAS International Conference on Humanoid Robots (Humanoids)*, pp. 550–557 (2016). <https://doi.org/10.1109/HUMANOIDS.2016.7803329>
- [24] Deits, R., Tedrake, R.: Computing large convex regions of obstacle-free space through semidefinite programming. In: *Algorithmic Foundations of Robotics XI*. Springer Tracts in Advanced Robotics, vol. 107, pp. 109–124. Springer, Cham (2015). https://doi.org/10.1007/978-3-319-16595-0_7
- [25] Tonneau, S., Del Prete, A., Pettré, J., Park, C., Manocha, D., Mansard, N.: An efficient acyclic contact planner for multipied robots. *IEEE Transactions on Robotics* **34**(3), 586–601 (2018) <https://doi.org/10.1109/TRO.2018.2819658>
- [26] Lee, J., Hwangbo, J., Wellhausen, L., Koltun, V., Hutter, M.: Learning quadrupedal locomotion over challenging terrain. *Science Robotics* **5**(47), 5986 (2020) <https://doi.org/10.1126/scirobotics.abc5986>
- [27] Miki, T., Lee, J., Hwangbo, J., Wellhausen, L., Koltun, V., Hutter, M.: Learning robust perceptive locomotion for quadrupedal robots in the wild. *Science Robotics* **7**(62), 2822 (2022) <https://doi.org/10.1126/scirobotics.abk2822>
- [28] Fankhauser, P., Hutter, M.: A universal grid map library: Implementation and use case for rough terrain navigation. *Robot Operating System (ROS) The Complete Reference (Volume 1)*, 99–120 (2016)
- [29] Miki, T., Wellhausen, L., Grandia, R., Jenelten, F., Homberger, T., Hutter, M.: Elevation mapping for locomotion and navigation using GPU. In: *2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 2273–2280 (2022). <https://doi.org/10.1109/IROS47612.2022.9981507> . IEEE
- [30] Raibert, M.H.: *Legged Robots that Balance*. The MIT Press, Cambridge, MA (1985)
- [31] Stellato, B., Banjac, G., Goulart, P., Bemporad, A., Boyd, S.: Osqp: An operator splitting solver for quadratic programs. *Mathematical Programming Computation* **12**(4), 637–672 (2020) <https://doi.org/10.1007/s12532-020-00179-2>

- [32] Dai, H., Valenzuela, A., Tedrake, R.: Whole-body motion planning with centroidal dynamics and full kinematics. In: 2014 IEEE-RAS International Conference on Humanoid Robots, pp. 295–302 (2014). IEEE
- [33] Sentis, L., Khatib, O.: A whole-body control framework for humanoids operating in human environments. In: 2006 IEEE International Conference on Robotics and Automation (ICRA), pp. 2641–2648 (2006). <https://doi.org/10.1109/ROBOT.2006.1642100> . IEEE
- [34] Del Prete, A., Mansard, N., Ramos, O.E., Stasse, O., Nori, F.: Implementing torque control with high-ratio gear boxes and without joint-torque sensors. International Journal of Humanoid Robotics **13**(01), 1550044 (2016) <https://doi.org/10.1142/S0219843615500449>
- [35] Farshidian, F., et al.: OCS2: An open source library for optimal control of switched systems. <https://github.com/leggedrobotics/ocs2>. Accessed: 2 August 2026

Declarations

Funding. This research received in-kind support from LimX Dynamics through the provision of the W1 robot, proprietary software, experimental facilities, and engineering support. The authors declare that no other funds, grants, or support were received during the preparation of this manuscript.

Competing interests. LimX Dynamics, the developer of the W1 robot used in this study, supported this study by providing the robot, proprietary software, experimental facilities, and engineering support. Wei Zhang is affiliated with LimX Dynamics. The authors declare no other competing interests.

Ethics approval. Not applicable. This study did not involve human participants or animals.

Data availability. Data supporting the reported aggregate trial outcomes are available from the corresponding author on reasonable request. Full platform logs are not publicly available because they contain proprietary W1 platform information.

Code availability. The implementation code is not publicly available because it depends on proprietary W1 platform software.

Author contributions. Conceptualization: Z.F. and W.Z.; methodology, software, validation, investigation, and writing—original draft preparation: Z.F.; writing—review and editing: Z.F. and W.Z.; supervision: W.Z. All authors read and approved the final manuscript.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [supplementvideo.mp4](#)