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Milling force-induced elastic deformation modeling and physical constraint transfer prediction of thin-walled grid conical shell

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Abstract

The thin-walled grid conical shell is affected by the coupling effect of cutting load and local weak stiffness in the process of grid milling, which easily leads to elastic tool deflection along the shell normal, resulting in the actual material removal thickness less than the theoretical cutting depth, and the wall thickness error is formed after unloading springback. This phenomenon will directly cause uneven wall thickness distribution in the component. The key difficulty lies in the coupling between the local stiffness mutation caused by the distribution of the ribs, the fluctuation of the cutting load at the tool position, and the limited real milling samples. In view of the above problems, this paper proposes a cutting force-induced tool deflection prediction method for grid conical shell milling process. Firstly, the deflection response of conical shell under normal load is analyzed based on Donnell shell theory, and the periodic “rib-skin” structure is equivalent to a stiffened conical shell with direction-dependent stiffness characteristics. The analytical relationship between cutting load, local structural stiffness and normal deflection is established. Secondly, taking the finite element simulation sample as the source domain and the real milling experiment sample as the target domain, the physical constraint transfer learning model is constructed to realize the synergy between the normal cutting force prediction and the elastic deformation analytical calculation under the condition of small samples. Finally, the proposed method is verified by finite element simulation and milling test. The results show that the method can effectively improve the accuracy and stability of the elastic deformation prediction for thin-walled grid conical shell, and can provide the manufacturing process modeling basis for the milling error prediction, process parameter optimization and deformation compensation of thin-walled stiffened curved surface components.

Keywords: Grid conical shell; Elastic deformation; Equivalent stiffness; Transfer learning; Physical constraint

1 Introduction

The grid conical shell has the advantages of both the efficient bearing characteristics of the conical shell and the lightweight advantages of the grid stiffened structure. It is a key bearing component in the fields of aerospace and deep-sea exploration, such as the interstage section of the launch vehicle, the bearing shell of the return capsule and the pressure-resistant structure of the submersible[1][2]. In the whole manufacturing process of this type of components, due to the large size of the shell, the thin wall

thickness and the complex rib distribution, the local support stiffness near the tool point changes with the machining position and the material removal state. Under the action of cutting load, it is prone to produce normal elastic yield, so that the actual material removal is less than the theoretical cutting depth, and finally the wall thickness and surface accuracy decrease. Therefore, revealing the coupling relationship between “cutting load-local stiffness-elastic deformation” is of great significance for milling accuracy control, process parameter optimization and subsequent error compensation of thin-walled grid conical shell. The existing prediction methods of machining deformation are mainly divided into two categories: one is the prediction method based on the mechanical theory to establish the mechanism model; the other is a data-driven method that combines machining process data, measurement data, and historical process data. Scholars in various countries have used the above two methods to study the prediction of machining deformation.

The prediction method based on the mechanism relationship can be divided into analytical method and finite element method. The analytical method usually establishes the theoretical expression of machining deformation based on plate and shell theory and energy method, and reveals the influence of load, structural stiffness and process parameters on deformation from the mechanism level. Li et al. [2] established a semi-analytical deformation prediction model considering both the processing-induced residual stress and the initial residual stress of the blank, which is used to analyze the stress redistribution and deformation response during the processing of thin-walled parts. Based on the energy principle, Yang et al. [4] studied the material removal sequence optimization problem of integral aerospace structural parts, and pointed out that the initial elastic strain energy release is an important essence of residual stress induced machining deformation. Wang et al. [5] constructed an analytical deformation prediction model based on energy method for the milling process of multi-frame components, and incorporated the body residual stress, machining-induced residual stress and structural characteristics into a unified analysis framework. Huang et al. [6] further established an analytical model for the machining deformation of aerospace integral components from the perspective of energy conversion. Ju et al. [7] used the Ritz method to predict the machining deformation induced by turning residual stress for the turning process of thin-walled circular parts, which provided a theoretical basis for the deformation analysis of rotary thin-walled structures. Based on the theory of elastic mechanics, Zheng et al. [8] established a prediction model of residual stress evolution and machining deformation of slab under non-uniform milling conditions, and analyzed the influence of local material removal thickness change on the superposition effect of deformation. Yang et al. [9] established an analytical model for machining deformation based on thin plate bending theory for asymmetric thin-walled structural parts, and considered the influence of neutral layer position change and machining residual stress redistribution on deformation. Ma et al. [10] established an analytical prediction model for thin-walled workpieces considering residual stress accumulation and redistribution effects for the layer-by-layer milling process, and clarified the

deformation evolution law under the condition of layer-by-layer material removal. The above research provides an important basis for the mechanism modeling of machining deformation of thin-walled parts. However, most models are mainly oriented to thin plates, frame beams or general thin-walled structures, and the adaptability to curved surface components such as grid conical shells with periodic distribution of stiffeners and local stiffness mutation characteristics is still insufficient.

The finite element method can describe the deformation evolution process under complex machining conditions in detail by discrete structure and considering the coupling effect of clamping, material removal and cutting load. Wang et al. [11] combined the voxel model with the finite element method to realize the rapid prediction of cutting force-induced deformation in five-axis milling. Xi et al [12] considered the influence of material removal on the overall stiffness matrix, and established a cutting force induced deformation prediction model. Li et al. [13] constructed a thin-walled part-tool coupling deformation prediction model based on the static substructure method, which improved the efficiency of finite element calculation. Ge et al.[14] proposed an efficient prediction method for dynamic deformation of side milling of thin-walled parts to characterize the time-varying characteristics of structural stiffness during machining. Tang et al. [15] used ABAQUS to establish the finite element model of TC4 thin-walled parts milling, and analyzed the influence of process parameters and tool geometric parameters on milling force and deformation. Lassila et al. [16] evaluated the influence of material removal and stiffness change on the machining error of thin-walled parts under different cutting strategies by finite element model. Lin et al. [17] established a finite element model of the removal body based on the tool-workpiece meshing relationship, and used the birth-death element method to simulate material removal and stiffness update. Nguyen et al.[18] established and verified a three-dimensional finite element model for thin-walled aluminum alloy milling, and analyzed the relationship between cutting force and deflection response. The finite element method has advantages in the description of complex boundary conditions and process loads. However, it still has problems such as high modeling cost, strong parameter dependence and limited computational efficiency when predicting large number of tool positions, multiple machining cycles and multiple working conditions. It is difficult to directly support the rapid error prediction of the machining site.

For the thin-walled grid conical shell, the machining error is not only affected by the overall structural stiffness, but also significantly controlled by the spatial distribution of the ribs, the residual thickness of the skin, the relative position of the tool point and the transverse and longitudinal ribs, and the fluctuation of the cutting load. Although the traditional analytical model has strong interpretability, it is difficult to accurately describe the deformation difference between the central area of the grid, the adjacent area of the ribs and the boundary area if the stiffness contribution of the local ribs and the orthogonal direction coupling effect are ignored. Although the finite element model can obtain high local simulation accuracy, it is difficult to predict large-scale tool sites efficiently in the real milling process.

Therefore, it is necessary to establish a manufacturing process prediction method that can not only reflect the local stiffness mechanism, but also adapt to the real cutting load changes.

The traditional mechanism model is difficult to balance the prediction accuracy, computational efficiency and adaptability of complex working conditions. With the continuous accumulation of on-machine measurement and historical process data, data-driven methods have been gradually introduced into the research of machining deformation prediction. This type of method provides a new idea for the rapid prediction of machining deformation of complex thin-walled parts by mining the nonlinear mapping relationship between process parameters, structural characteristics and deformation response. Sun et al. [19][20] combined the surrogate model with the Bayesian framework, and proposed a hybrid-driven prediction method for machining errors of thin-walled parts to reduce the dependence of the pure mechanism model on the modeling accuracy of complex machining processes. Furthermore, a hybrid surrogate model considering uncertainty is constructed for rapid estimation of machining deformation of thin-walled parts. Ni et al. [21] proposed a mechanism information neural network, which introduced the thin shell theory and Fourier series into the network constraints, and realized the prediction of machining deformation of annular thin-walled parts. Bai et al. [22] integrated static cutting parameters with dynamic cutting force signals to establish a hybrid deep learning model to predict the milling dimensional accuracy of thin-walled structural parts. Huang et al.[23] used GA-BP neural network to predict the multi-pass machining accuracy of thin-walled parts based on the characteristics of cutting force, stiffness and multi-pass error accumulation. Sun et al. [24] proposed a sparse Bayesian learning method with engineering knowledge constraints for in-situ prediction of machining errors of thin-walled blades. Yu et al. [25] proposed a transfer learning machining error prediction method for the problem of small sample and working condition transfer in the milling process of thin-walled parts. Geng et al.[26] constructed a PSO-LSTM deformation prediction model based on digital twin technology to realize online prediction of thin-walled parts processing deformation.

In the grid conical shell milling scenario, the real processing experiment has high cost, long processing cycle and insufficient sample coverage. The pure data-driven model is prone to the problem of insufficient cross-round and cross-regional generalization ability. Furthermore, the black box model directly predicts the deformation from the tool point characteristics, and it is difficult to ensure that the prediction results meet the basic physical relationships such as cutting load balance, local stiffness constraint and shell deflection response. To improve the reliability of the model under the condition of limited experimental samples, it is necessary to fuse the finite element simulation samples, the shell analytical model and the real machining process data to form a hybrid prediction framework with physical consistency.

The data-physical hybrid driving method provides a new idea for predicting the machining process of complex thin-walled components. Zhao et al. [27] proposed a machining deformation prediction

method based on the latent variable model of physical information. The deformation force is used as the characterization signal of the residual stress state, and the physical knowledge is introduced to realize the deformation prediction of the part. Zhu et al.[28] constructed a digital twin-driven machining framework for thin-walled parts manufacturing, and realized machining condition monitoring and optimization through the interaction between physical space and virtual space. Zhang et al. [29] proposed the autonomous milling process defined by digital twin, which combines high-fidelity modeling, knowledge-driven low-latency simulation and online optimization control for the milling deformation control of thin-walled parts. Wu et al.[30] proposed a deformation prediction and compensation optimization method for thin-walled parts based on on-machine measurement data and surrogate stiffness model. Guo et al. [31] further proposed a data-mechanism-causal fusion model for deformation control of thin-walled rotary parts. Li et al. [32] constructed a simulation and compensation framework for machining deformation of thin-walled parts driven by edge digital twin. The above research emphasizes the importance of physical knowledge and process data fusion, but the modeling of stiffness mutation and cutting force induced elastic deformation of rib pattern in thin-walled grid conical shell milling still needs to be further studied.

In view of the above problems, this paper proposes a cutting force-induced elastic deformation modeling and physical constraint transfer prediction method for the milling process of thin-walled grid conical shell. The main contributions of this paper include: (1) Based on Donnell shell theory, the analytical model of elastic deformation of thin-walled grid conical shell under normal cutting load is established; (2) Considering the directional enhancement effect of transverse and longitudinal stiffeners on local stiffness, a local equivalent stiffness model affected by stiffeners is constructed; (3) Taking the finite element simulation as the source domain and the real milling experiment as the target domain, a physical constraint transfer learning framework for normal cutting force prediction is established, and the predicted load is input into the analytical model to obtain the local elastic deformation. (4) The prediction accuracy and stability of the proposed method are verified by grid conical shell milling experiment, on-line monitoring of cutting force and on-machine measurement of wall thickness / profile. The main framework of this paper is shown in Fig. 1.

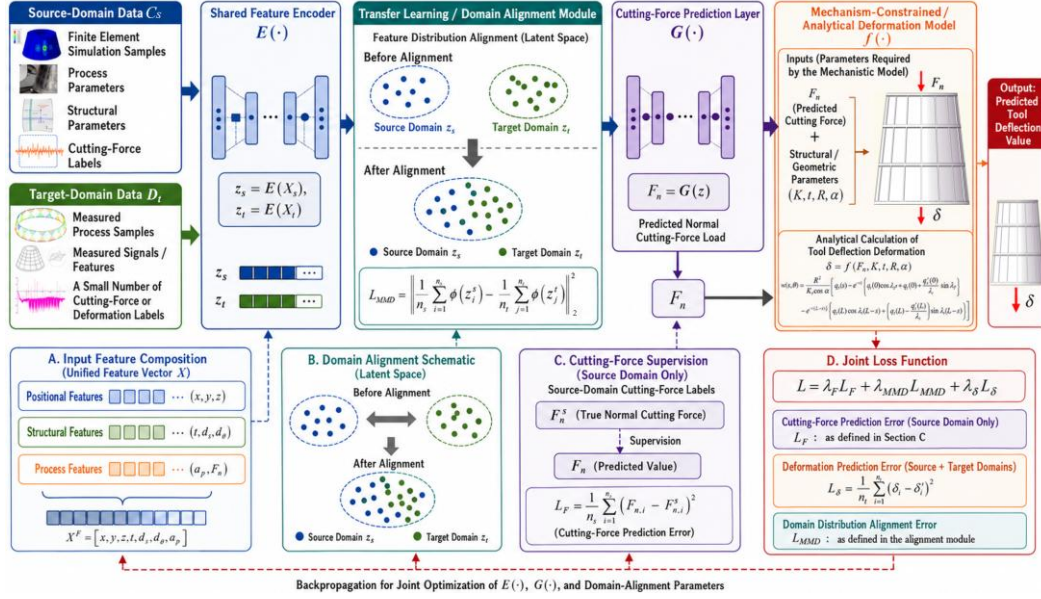


Fig. 1 The main process of the method in this paper

2 Analytical modeling of elastic deformation of grid conical shell induced by cutting force

In the process of grid milling of thin-walled grid conical shell, due to the weak local stiffness, the elastic displacement along the normal direction of the shell is easy to occur under the action of cutting force, resulting in the actual removal of material thickness less than the theoretical value. After the cutting is completed, the workpiece rebounds, causing the actual surface of the workpiece to deform in the thickness direction relative to the theoretical surface. Since the elastic deflection deformation is much smaller than the workpiece deformation, the cutting force is the dominant factor of the deformation, and the elastic deformation can be approximately equivalent to the normal deflection deformation of the thin-walled grid conical shell. Therefore, based on the theoretical analysis of the conical shell deflection deformation, the analytical model of the deflection deformation of the conical shell with ribs is established by further considering the enhancement effect of the grid ribs on the local stiffness.

2.1 Deflection deformation analytical model of conical shell

The geometric diagram of the thin-walled conical shell is shown in Fig. 2. The coordinate system (s, θ, z) is defined on the middle surface of the conical shell, s, θ, z represents the generatrix direction, the circumferential direction and the normal direction of the middle surface of the conical shell, respectively. The corresponding displacement is u, v, w , the small end radius and the large end radius of the truncated conical shell is R_1, R_2 , and the generatrix length, the thickness of the conical shell and the half cone angle of the conical shell is L, h, α .

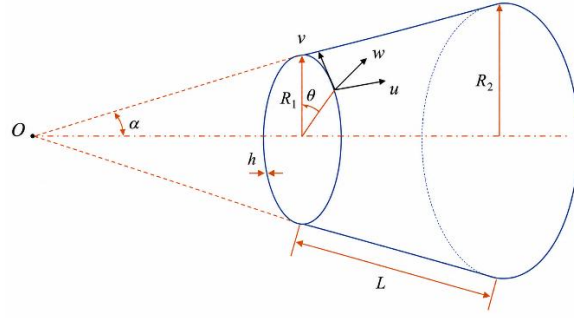


Fig. 2 Geometric sketch of thin-walled conical shell

According to Donnell's shell theory, the geometric equation of conical shell can be expressed as:

$$\begin{cases} \varepsilon_s = \frac{\partial u}{\partial s} \\ \varepsilon_\theta = \frac{1}{s \cdot \sin \alpha} \left(\frac{\partial v}{\partial \theta} + u \sin \alpha - w \cos \alpha \right) \\ \varepsilon_{s\theta} = \frac{\partial v}{\partial s} - \frac{v}{s} + \frac{1}{s \cdot \sin \alpha} \cdot \frac{\partial u}{\partial \theta} \\ k_s = -\frac{\partial^2 \omega}{\partial s^2} \\ k_\theta = -\frac{1}{s} \cdot \frac{\partial \omega}{\partial s} - \frac{1}{s^2 \cdot \sin^2 \alpha} \cdot \frac{\partial^2 \omega}{\partial \theta^2} \\ k_{s\theta} = -\frac{1}{\sin \alpha} \cdot \frac{\partial}{\partial s} \left(\frac{1}{s} \cdot \frac{\partial w}{\partial \theta} \right) \end{cases} \quad (1)$$

where ε_s and ε_θ respectively represent the strain in the generatrix direction and the circumferential direction; $\varepsilon_{s\theta}$ is the shear strain; k_s and k_θ respectively represent the curvature change caused by stretching in the generatrix direction and the circumferential direction; $k_{s\theta}$ is the curvature change caused by torsion.

The physical equation of the shell element can be expressed as:

$$\begin{cases} N_s = A \left[\frac{\partial u}{\partial s} + \frac{v}{s} \left(\frac{1}{\sin \alpha} \cdot \frac{\partial v}{\partial \theta} + u + \frac{w}{\tan \alpha} \right) \right] \\ N_\theta = A \left[\frac{1}{s} \left(\frac{1}{\sin \alpha} \cdot \frac{\partial v}{\partial \theta} + u + \frac{w}{\tan \alpha} \right) + v \cdot \frac{\partial u}{\partial s} \right] \\ N_{s\theta} = N_{\theta s} = \frac{(1-\nu)A}{2} \left[\frac{\partial v}{\partial s} + \frac{1}{s} \left(\frac{1}{\sin \alpha} \cdot \frac{\partial u}{\partial \theta} - v \right) \right] \\ M_s = D \left[\frac{\partial w}{\partial s} + \frac{v}{s} \left(\frac{1}{s \cdot \sin^2 \alpha} \cdot \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial w}{\partial s} \right) \right] \\ M_\theta = D \left[\frac{1}{s} \left(\frac{1}{s \cdot \sin^2 \alpha} \cdot \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial w}{\partial s} \right) + v \cdot \frac{\partial^2 w}{\partial s^2} \right] \\ M_{s\theta} = M_{\theta s} = \frac{(1-\nu)D}{s \cdot \sin \alpha} \cdot \left(\frac{\partial^2 w}{\partial s \partial \theta} - \frac{1}{s} \cdot \frac{\partial w}{\partial \theta} \right) \end{cases} \quad (2)$$

Where $N_s, N_\theta, N_{s\theta}, N_{\theta s}$ are the internal force and tangential shear force of the shell element along the

generatrix and circumferential direction ; $M_s, M_\theta, M_{s\theta}, M_{\theta s}$ are the internal bending moment and torque of the shell element along the generatrix direction and the circumferential direction ; $A = \frac{Eh}{1-\mu^2}$ is tensile stiffness ; $D = \frac{Eh^3}{12(1-\mu^2)}$ is bending stiffness ; the elastic modulus is E ; the Poisson's ratio is μ .

The equilibrium differential equation of conical shell can be expressed as:

$$\begin{cases} \frac{\partial N_s}{\partial s} + \frac{N_s - N_\theta}{s} + \frac{1}{s \cdot \sin \alpha} \cdot \frac{\partial N_{s\theta}}{\partial \theta} = q_1 \\ \frac{\partial N_{s\theta}}{\partial s} + \frac{2N_{s\theta}}{s} + \frac{1}{s \cdot \sin \alpha} \cdot \frac{\partial N_\theta}{\partial \theta} = q_2 \\ \frac{1}{s} \cdot \frac{\partial^2}{\partial s^2} (s \cdot M_s) - \frac{1}{s} \cdot \frac{\partial M_\theta}{\partial s} + \frac{1}{s^2 \cdot \sin^2 \alpha} \cdot \frac{\partial^2 M_\theta}{\partial \theta^2} \\ + \frac{2}{s^2 \sin \alpha} \cdot \frac{\partial^2 (s \cdot M_{s\theta})}{\partial s \partial \theta} + \frac{\cos \alpha}{s \cdot \sin \alpha} \cdot N_\theta = q_3 \end{cases} \quad (3)$$

Where q_1, q_2, q_3 are the components of the external load of the conical shell in the generatrix direction, circumferential direction and normal direction.

Since the deflection deformation of the conical shell is mainly concentrated in the normal direction of the middle surface during the machining process, the deformation of the middle surface is calculated according to the normal load. $q_1 = q_2 = 0$ in Eq. (3), The Eq. (2) is substituted into Eq. (3), and the analytical formula of the deflection deformation of the conical shell that can be characterized by the deformation of the middle surface is sorted out:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ q_3 \end{bmatrix} = 0 \quad (4)$$

Where $L_{ij} (i, j = 1, 2, 3)$ is a differential operator, and its specific expression is:

$$\begin{cases}
L_{11} = \frac{12D}{h^2} \left(\frac{\partial^2}{\partial s^2} + \frac{1}{s} \frac{\partial}{\partial s} - \frac{1}{s^2} \right) + \frac{6(1-\mu)D}{h^2 s^2 \cdot \sin^2 \alpha} \cdot \frac{\partial^2}{\partial \theta^2} \\
L_{12} = \frac{6D(1+\mu)}{h^2 s \cdot \sin \alpha} \cdot \frac{\partial^2}{\partial s \partial \theta} - \frac{6D(2-\mu)}{h^2 s^2 \cdot \sin \alpha} \cdot \frac{\partial}{\partial \theta} \\
L_{13} = -\frac{12\mu D \cos \alpha}{h^2 s \cdot \sin \alpha} \cdot \frac{\partial}{\partial s} + \frac{12D \cos \alpha}{h^2 s^2 \cdot \sin \alpha} \\
L_{21} = \frac{6D(1+\mu)}{h^2 s \cdot \sin \alpha} \cdot \frac{\partial^2}{\partial s \partial \theta} + \frac{6D(2-\mu)}{h^2 s^2 \cdot \sin \alpha} \cdot \frac{\partial}{\partial \theta} \\
L_{22} = \frac{6D(1-\mu)}{h^2} \left(\frac{\partial^2}{\partial s^2} + \frac{1}{s} \cdot \frac{\partial}{\partial s} - \frac{1}{s^2} \right) + \frac{12D}{h^2 s^2 \cdot \sin^2 \alpha} \cdot \frac{\partial^2}{\partial \theta^2} \\
L_{23} = L_{32} = -\frac{12D \cos \alpha}{h^2 s^2 \cdot \sin^2 \alpha} \cdot \frac{\partial}{\partial \theta} \\
L_{31} = -\frac{12D\mu \cos \alpha}{h^2 s \cdot \sin \alpha} \cdot \frac{\partial}{\partial s} - \frac{12D \cos \alpha}{h^2 s^2 \cdot \sin \alpha} \\
L_{33} = \frac{12D \cos^2 \alpha}{h^2 s^2 \cdot \sin^2 \alpha} + D \cdot \frac{\partial^4}{\partial s^4} + \frac{(3-\mu)D}{s^3 \cdot \sin^2 \alpha} \cdot \frac{\partial^3}{\partial s \partial \theta^2} + \frac{2(3-\mu)D}{s^4 \cdot \sin^2 \alpha} \cdot \frac{\partial^2}{\partial \theta^2} - \frac{D}{s^2} \cdot \frac{\partial^2}{\partial s^2} + \frac{D}{s^3} \cdot \frac{\partial}{\partial s}
\end{cases} \quad (5)$$

2.2 Equivalent stiffness modeling of grid conical shell considering rib stiffness effect

The grid conical shell is a periodic stiffened structure composed of stiffeners and skins. The spatial distribution of stiffeners, the section size and the coupling between stiffeners and skins will significantly change the local stiffness distribution of the shell, which in turn affects the normal deflection deformation law. Therefore, based on the analytical model of conical shell deflection, this paper introduces the influence of the stiffness of the ribs, and carries out the equivalent stiffness field analysis of the grid conical shell. The periodic “rib-skin” composite structure is equivalent to the orthotropic conical shell with direction-dependent stiffness characteristics, which lays a foundation for the subsequent establishment of the analytical model of the deflection deformation of the grid conical shell considering the stiffness effect of the ribs. Assuming that the rib width is t_r , the rib height is h_r , the rib spacing in the generatrix direction is b , the circumferential rib spacing is a , the skin thickness is h , and the rib is rigidly connected to the skin, as shown in Fig. 3. Taking the middle surface of the skin as the reference surface, the distance d between the center of the rib and the middle surface of the skin is:

$$d = \frac{h}{2} + \frac{h_r}{2} \quad (6)$$

The single rib with respect to the middle surface of the skin's equivalent inertia moment I_{eq} is:

$$I_{eq} = \frac{t_r h_r^3}{12} + t_r h_r d^2 \quad (7)$$

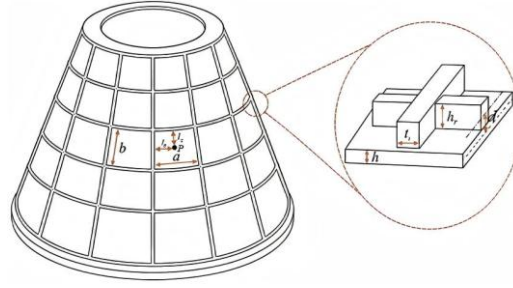


Fig. 3 Schematic diagram of grid conical shell structure

For orthotropic conical shells, the bending moment-curvature relationship can be expressed as:

$$\begin{bmatrix} M_s \\ M_\theta \\ M_{s\theta} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} k_s \\ k_\theta \\ 2k_{s\theta} \end{bmatrix} \quad (8)$$

Where D_{11} and D_{22} are the bending stiffness of the generatrix and the circumferential direction respectively; D_{12} is the Poisson coupling stiffness between the busbar and the circumferential direction, reflecting the additional bending effect caused by the bending of the single-direction ribs in the orthogonal direction ; D_{66} is the torsional stiffness, reflecting the resistance of the conical shell to the torsional bending.

Using the concept of unit cell homogenization, the influence of the bus bar and the circumferential stiffener on the skin stiffness is evenly distributed along the pitch perpendicular to its extension direction, and the bending stiffness in both directions can be expressed as:

$$\begin{cases} D_{11} = D + \frac{EI_{eq}}{b} \\ D_{22} = D + \frac{EI_{eq}}{a} \end{cases} \quad (9)$$

Where D is the bending stiffness of the grid skin. Assume that the two types of ribs have the same section parameters, they have the same rib height and rib width. Considering the combined effect of the bus direction and the circumferential direction, the equivalent coupling and torsional stiffness can be approximated as:

$$\begin{cases} D_{12} = \mu \sqrt{D_{11} D_{22}} \\ D_{66} = \frac{1-\mu}{2} \sqrt{D_{11} D_{22}} \end{cases} \quad (10)$$

With a point P on the skin as the center, a local element with a span of and in the generatrix direction and the circumferential direction is established in the grid are l_s ($l_s \square b$) and l_θ ($l_\theta \square a$). Then the curvature of this region under unit deflection w_0 can be approximately expressed as:

$$k_s \approx \frac{w_0}{l_s^2}, \quad k_\theta \approx \frac{w_0}{l_\theta^2}, \quad k_{s\theta} \approx \frac{w_0}{l_s l_\theta} \quad (11)$$

The local bending strain energy U can be expressed as:

$$U = \frac{1}{2} \iint_A (D_{11}k_s^2 + 2D_{12}k_s k_\theta + D_{22}k_\theta^2 + 4D_{66}k_{s\theta}^2) dA \quad (12)$$

The local area $A = l_s l_\theta$ in the formula. The equivalent stiffness K satisfies:

$$U = \frac{1}{2} K w_0^2 \quad (13)$$

The analytical expression of the equivalent stiffness K of the grid conical shell can be obtained by combining Eq. (11),(12)and (13):

$$K = l_s l_\theta \left(\frac{D_{11}}{l_s^4} + \frac{2D_{12} + 4D_{66}}{l_s^2 l_\theta^2} + \frac{D_{22}}{l_\theta^4} \right) \quad (14)$$

2.3 Analytical modeling of deflection deformation of grid conical shell

In order to further establish the analytical expression suitable for the deflection deformation of the grid conical shell, the equivalent stiffness parameter is substituted into the deflection deformation control equation of the conical shell, and the quantitative relationship between the cutting load, the local equivalent stiffness and the normal deflection is constructed. Based on the analytical model of the deflection of the smooth conical shell, combined with the equivalent stiffness correction of the grid ribs, the analytical modeling of the deflection deformation of the thin-walled grid conical shell is further carried out. The conical shell is discretized into several micro-elements along the generatrix direction, and each micro-element is approximated as a cylindrical shell element with a slowly changing radius. Taking the local infinitesimal near the generatrix coordinate, the equivalent radius of the second local infinitesimal can be taken as:

$$R_i = R_1 + s_i \sin \alpha \quad (15)$$

In the cylindrical shell element, under the condition of only considering the axisymmetric normal load $q_i(s)$, the normal displacement of the conical shell causes the circumferential mid-plane to elongate, and the initial circumferential curvature $k_{\theta,i}$ of the local cylindrical shell element can be expressed as:

$$k_{\theta,i} = \frac{\cos \alpha}{R_i} \quad (16)$$

The corresponding circumferential membrane force $N_{\theta,i}$ can be expressed as:

$$N_{\theta,i} = K_i \varepsilon_{\theta,i} = K_i k_{\theta,i} w_i = K_i \frac{\cos \alpha}{R_i} w_i \quad (17)$$

Where K_i is the equivalent bending stiffness.

According to the normal equilibrium condition of the cylindrical shell element, the deflection control equation of the i element can be obtained:

$$D_{11} \frac{d^4 w_i}{ds^4} + K_i \frac{\cos^2 \alpha}{R_i^2} w_i = q_i(s) \quad (18)$$

According to the principle of deformation superposition, the local micro-element deflection w_i is decomposed into the sum of the main response deflection in the membrane and the response deflection of the end boundary layer:

$$w_i = w_{a,i} + w_{b,i} \quad (19)$$

Where $w_{a,i}$ is the main response deflection in the membrane, $w_{b,i}$ is the response deflection of the end boundary layer. when far away from the boundary, the effect of the curved boundary layer is weak and $w_{b,i}$ can be ignored. Eq. (18) is transformed into the equilibrium relationship in the membrane:

$$K_i \frac{\cos^2 \alpha}{R_i^2} w_{a,i}(s) = q_i(s) \quad (20)$$

Thus, the main response in the membrane is obtained:

$$w_{a,i}(s) = \frac{R_i^2}{K_i \cos^2 \alpha} q_i(s) \quad (21)$$

Substituting Eq. (19) and Eq.(21) into Eq. (18), the boundary layer response satisfies the homogeneous equation:

$$D_{11} \frac{d^4 w_{b,i}}{ds^4} + K_i \frac{\cos^2 \alpha}{R_i^2} w_{b,i} = 0 \quad (22)$$

Define the characteristic parameters:

$$\lambda_i = \left(\frac{K_i \cos^2 \alpha}{4D_{11} R_i^2} \right)^{1/4} \quad (23)$$

Substitute Eq. (23) into Eq. (22), then it can be expressed as:

$$w_{b,i}(s) = e^{-\lambda_i s} (C_{1i} \cos \lambda_i s + C_{2i} \sin \lambda_i s) + e^{-\lambda_i (L_i - s)} (C_{3i} \cos \lambda_i (L_i - s) + C_{4i} \sin \lambda_i (L_i - s)) \quad (24)$$

Where L_i represents the length of the micro element.

In the milling process of thin-walled grids, it is assumed that both ends are fixed, and the boundary conditions can be expressed as:

$$\begin{cases} w(0) = 0, w'(0) = 0 \\ w(L) = 0, w'(L) = 0 \end{cases} \quad (25)$$

Substituting Eq. (25) into Eq. (24), the constants can be obtained:

$$\begin{cases} C_{1i} = -w_{a,i}(0) \\ C_{2i} = -w_{a,i}(0) - \frac{1}{\lambda_i} w'_{a,i}(0) \\ C_{3i} = -w_{a,i}(L) \\ C_{4i} = -w_{a,i}(L) - \frac{1}{\lambda_i} w'_{a,i}(L) \end{cases} \quad (26)$$

By combining Eq. (19), Eq. (24) and Eq. (26), an explicit approximation formula for the deflection of thin-walled latticed conical shells can be obtained:

$$w_i(s, \theta) = \frac{R_i^2}{K_i \cos^2 \alpha} \left[q_i(s) - e^{-\lambda_i s} \left(q_i(0) \cos \lambda_i s + \left(q_i(0) + \frac{q'_i(0)}{\lambda_i} \right) \sin \lambda_i s \right) - e^{-\lambda_i (L_i - s)} \left(q_i(L_i) \cos \lambda_i (L_i - s) + \left(q_i(L_i) - \frac{q'_i(L_i)}{\lambda_i} \right) \sin \lambda_i (L_i - s) \right) \right] \quad (27)$$

3 Physical constraint transfer model for milling force prediction and elastic deformation calculation

3.1 The overall framework of physical constraint transfer prediction model

In the process of grid milling, the normal elastic deformation of the thin-walled grid conical shell is determined by the cutting load, the local residual wall thickness, the relative position of the tool point, the rib and the cutting depth. Among them, the normal cutting force is not only the direct external load that induces the tool deformation, but also the key intermediate variable that connects the machining state and the mechanical response of the shell. If the tool deflection is predicted directly from the tool position characteristics, the model may obtain the fitting effect in the local sample range, but it is difficult to ensure the physical consistency between the load, the stiffness constraint and the deflection response. Therefore, in this paper, the transfer learning network is used to predict the normal cutting force under complex machining conditions, and the analytical model is used to calculate the tool deformation of the grid cone shell driven by the cutting load, thereby constructing a chain framework of “machining state feature-cutting force prediction-elastic deformation analysis” for the manufacturing process.

In this paper, a physical-constrained transfer learning deformation prediction model (PCTD) is established. The finite element simulation data and real milling experimental data are used as the source of double-domain samples. The stable prediction of small sample target domain is realized by sharing feature coding, domain distribution alignment, cutting load prediction and mechanism constraint calculation, as shown in Fig. 4. In the model, the shared feature encoder is used to extract the potential features of the processing state shared by the source domain and the target domain. The domain alignment module is used to reduce the distribution deviation between the simulation sample and the experimental sample. The cutting load predicts the normal cutting force at the output tool position of the layer; the mechanism constraint branch inputs the predicted load into the mentioned equivalent stiffness analytical model to calculate the cutting force-induced elastic deformation.

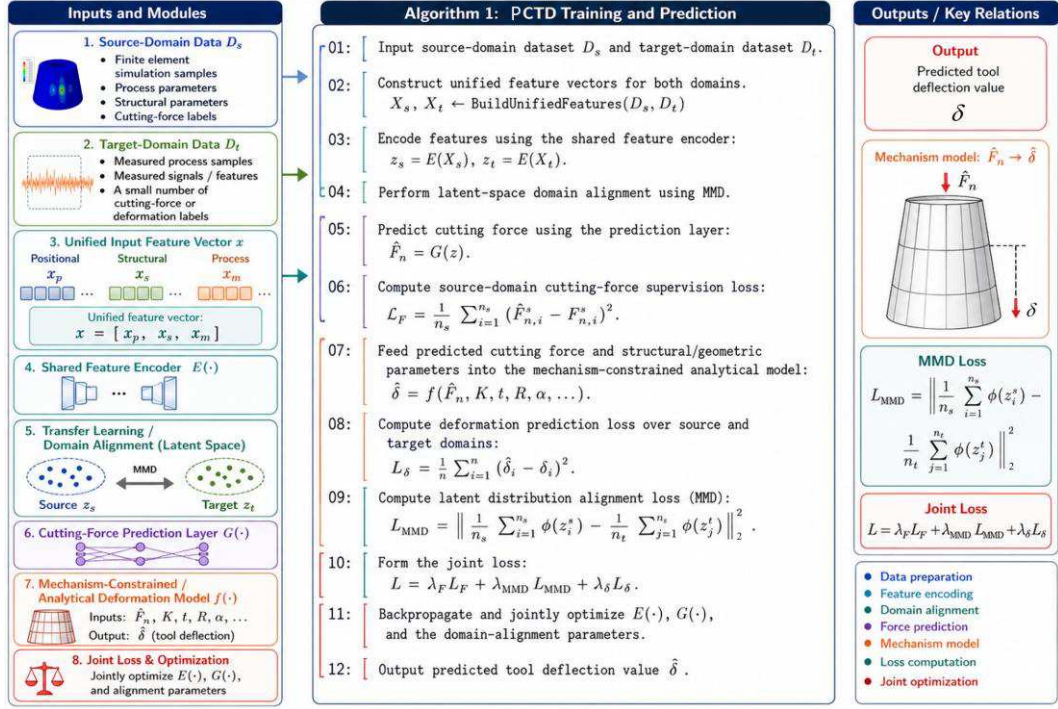


Fig. 4 PCTD model components

In order to accurately characterize the geometric state, stiffness state and load state of the local area of the thin-walled grid conical shell during the cutting process, the input variables of the model are summarized into three categories: position feature, structural feature and process feature, and the output variable is the elastic deflection value. The position feature is used to represent the spatial distribution of the cutter location point on the conical shell surface. The three-dimensional coordinates (x, y, z) of the tool location point are selected as the position feature and used as the reference for multi-source information fusion. The structural feature is used to represent the stiffness state of the local area where the tool location is located. The local wall thickness t and the minimum distance (d_s, d_θ) from the cutter location to the transverse and longitudinal ribs are defined as the structural feature. The process characteristics are used to represent the material removal state and stress level of the current process, and the cutting depth a_p and normal cutting force F_n are selected as the process characteristics.

Compared with the traditional data-driven deformation prediction model, the essential difference of the framework is that the functional division is clearer: the transfer learning network is responsible for the mapping of cutting force load under complex machining conditions, and the analytical model is responsible for the calculation of the deflection of the grid conical shell under load. Thus, a complete link of “input state characteristics-transfer learning network-prediction of cutting load-deflection analytical model-output of elastic deformation” is formed. The link not only utilizes the adaptability of transfer learning to complex working conditions and small sample target domain, but also retains the

physical interpretation ability of thin shell theory and equivalent stiffness model to structural deformation response, which can avoid the free extrapolation of deformation results completely by black box network.

3.2 Dual-domain sample construction and phased training strategy

The construction of dual-domain samples for cutting force prediction is the premise for the transfer learning load prediction model to play a role. Aiming at the problem that the normal cutting force is difficult to obtain stably under the target working condition, the source domain samples and the target domain samples are used for the basic load law learning and the real working condition deviation correction respectively. The source domain samples are derived from historical processing force measurement data and finite element simulation sample data, which have the characteristics of large sample size, wide working condition coverage and controllable variable combination. The target domain sample is derived from the real milling test of thin-walled grid conical shell. Although the number of samples is limited, it can truly reflect the factors such as clamping state, tool state, material difference, measurement noise and path local disturbance.

In terms of source domain sample construction, input and label are organized around the “position-structure-process-load” mapping relationship. The input features mainly include the coordinates (x, y, z) of the tool location point, the local wall thickness t , the minimum distance (d_s, d_θ) from the cutter location point to the transverse and longitudinal ribs, and the output label is the normal cutting force F_n under the corresponding working conditions.

In the aspect of target domain sample construction, the correspondence between the real force measurement data and the tool location point state is mainly solved. Firstly, the spatial coordinates of the tool point and the machining sequence are obtained according to the machining code. Secondly, the cutting force signal is collected by dynamometer or on-line monitoring system, and the normal cutting force characteristics are extracted according to the tool point passing time, sampling time window and cutting contact interval. Finally, combined with the measured local wall thickness, structural characteristics and process parameters, the target domain load prediction samples are formed. The wall thickness deviation after processing is mainly used for the verification and correction of the back-end analytical results, and is not used as the direct output target of the front-end transfer learning network.

Assuming that the input feature of the i tool position for cutting force prediction is X_i^F , and the corresponding normal cutting force label is Y_i^F , it can be expressed as:

$$X_i^F = [x, y, z, t, d_s, d_\theta, a_p], Y_i^F = F_n \quad (28)$$

After the prediction of the front-end model is obtained F_n , the back-end analytical model is input together with K, t, R, α and other physical parameters to form the final elastic deformation δ output as

follows:

$$\delta = f(F_n, K, t, R, \alpha) \quad (29)$$

In order to ensure that the two-domain samples can enter the same transfer learning network, it is necessary to carry out unified preprocessing of the source domain and the target domain samples. The process mainly includes the steps of uniform numbering of tool location points, spatial coordinate registration, cutting force timing matching, abnormal load elimination, feature normalization and training /verification/test set division. Among them, timing matching is used to ensure that the cutting force sampling fragment has the same physical meaning as the corresponding tool position; feature normalization is used to eliminate the influence of different dimensional variables on network training. Outlier elimination is used to weaken the influence of instantaneous interference and measurement noise on load prediction. The dual-domain sample construction and training strategy for cutting force prediction is shown in Fig. 5.

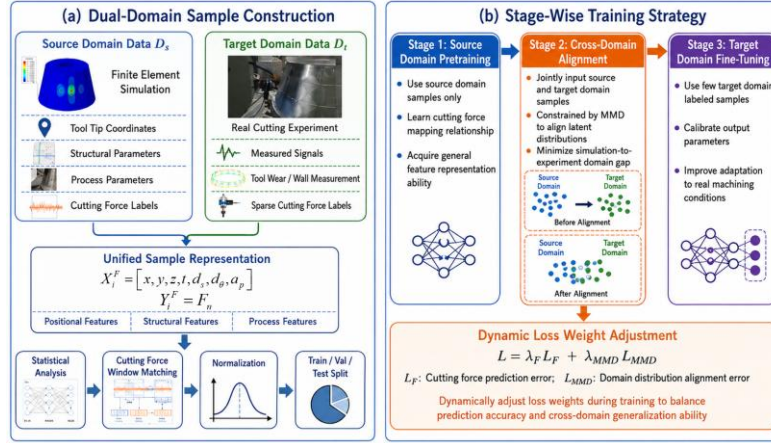


Fig. 5 Dual-domain sample construction and phased training

The model training adopts a phased strategy of source domain pre-training, cross-domain alignment and target domain fine-tuning. In the first stage, the 8-dimensional input features of the fully connected layer are projected into the high-dimensional space by using the structure of multi-layer perceptron and residual nonlinear mapping. The MLP layer composed of LayerNorm, ReLU activation function and Dropout is used to normalize, nonlinearly enhance and slightly randomly inactivate the intermediate features, so as to extract features and improve the adaptability of the model to small sample data. In the second stage, two ResNet residual blocks are used to further extract high-level nonlinear features. The domain alignment module introduces the Maximum Mean Discrepancy (MMD) as the inter-domain distribution measure. The calculation expression of the maximum mean difference is:

$$L_{MMD} = \left\| \frac{1}{n_s} \sum_{i=1}^{n_s} \phi(Z_i^s) - \frac{1}{n_t} \sum_{j=1}^{n_t} \phi(Z_j^t) \right\|_2^2 \quad (30)$$

Where Z_i^s and Z_j^t respectively represent the potential features of the source domain and the target domain samples extracted by the shared encoder, $\phi(\cdot)$ is the kernel space mapping function, n_s and n_t are the

number of samples in the source domain and the target domain, respectively. By minimizing L_{MMD} , the distribution offset between the cutting force prediction features of the source domain and the target domain can be reduced, and the prediction reliability under real working conditions can be improved. The cross-domain alignment principle is shown in Fig. 6. In the third stage, the network is fine-tuned by using a small number of measured cutting force samples in the target domain to correct the load deviation in the real grid conical shell processing.

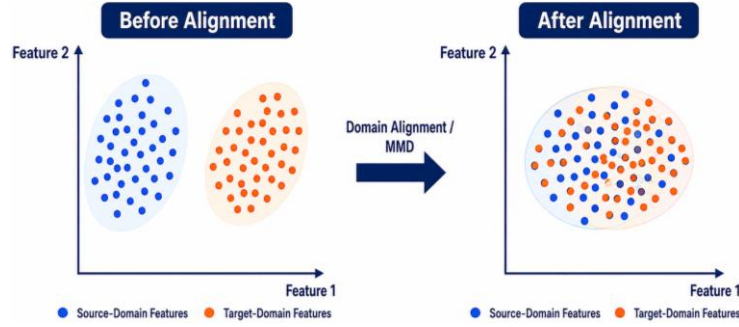


Fig. 6 Cutting force prediction feature space cross-domain alignment diagram

3.3 Joint loss function construction

In order to ensure the overall reliability of the framework of “cutting force transfer prediction-deformation mechanism analysis”, it is still difficult to meet the demand of accurate deformation prediction of thin-walled grid conical shell under complex working conditions by only improving the prediction accuracy of a single module. Firstly, the front-end transfer learning network needs to accurately characterize the mapping relationship between the cutter location features and the normal cutting force to obtain a stable and reliable prediction load. Secondly, there are differences in working conditions, boundary constraints and measurement noise between the source domain and the target domain, so it is necessary to reduce the feature distribution offset by cross-domain alignment. At the same time, the elastic deformation output by the back-end analytical model needs to be consistent with the measured wall thickness error or profile error. Therefore, it is necessary to construct a joint loss function, which takes the load prediction accuracy, cross-domain transfer ability and analytical deformation consistency as the joint optimization objectives, and collaboratively constrains the transfer learning load prediction module and the mechanism analytical deformation output.

The joint loss function consists of three parts : (1) The final output of the transfer learning network is F_n , therefore, the load prediction loss is the main loss item of the front-end network training; (2) In the process of migrating from the source domain to the target domain, the domain alignment loss L_{MMD} is used to reduce the difference in the distribution of potential features between the two domains. (3) The output of the mechanism analysis model is δ that the deformation consistency loss L_δ is used to correct

the deviation between the analytical value and the measured value of the final elastic deflection. Therefore, the expression of the joint loss function is:

$$L = \lambda_F L_F + \lambda_{MMD} L_{MMD} + \lambda_\delta L_\delta \quad (31)$$

Where λ_F , λ_{MMD} and λ_δ are L_F , L_{MMD} and L_δ corresponding loss coefficient, L_F and L_δ the expression is:

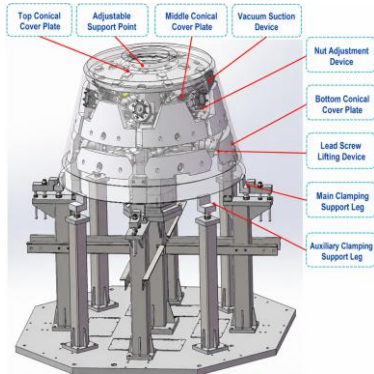
$$\begin{cases} L_F = \frac{1}{n_s} \sum_{i=1}^{n_s} (F_{n,i} - F'_{n,i})^2 \\ L_\delta = \frac{1}{n_t} \sum_{i=1}^{n_t} (\delta_i - \delta'_i)^2 \end{cases} \quad (32)$$

Where $F'_{n,i}$ is the simulation of cutting force, δ'_i is the measured elastic deflection value. In the pre-training stage of the source domain, L_F should be considered as the main optimization goal, so that the network first establishes a stable cutting force prediction ability; in the cross-domain alignment stage, the weight L_{MMD} should be increased to enhance the ability of the source domain load knowledge to migrate to the target domain; in the fine-tuning stage of the target domain, the target domain L_F should be the main domain, and combined with a small amount of deformation measurement data introduced into L_δ , the analytical deformation results driven by the predicted load should be corrected as a whole. In the above way, the model training is always carried out around the prediction of cutting force, and the final deformation result is calculated by the mechanism analysis model.

4. Experimental verification and process prediction performance analysis

4.1 Experimental design and data acquisition

To verify the effectiveness of the proposed prediction method, a thin-walled grid conical shell processing test bench was built on a five-axis gantry machine tool, as shown in Fig. 7. The cone shell blank was formed by integral spinning process. The material was 6061 aluminum alloy, the initial thickness was 10 mm, the large end diameter was 1100 mm, the small end diameter was 800 mm, and the height was 500 mm. The tool used in milling is a cemented carbide flat-end milling cutter with a diameter of 10 mm, a spindle speed of 5000 r/min, and a feed rate of 5000 mm/min. The main experimental process was as follows: Firstly, the wall thickness of the grid conical shell was roughened, and the overall wall thickness of the blank was 6mm. Secondly, the grid milling of the conical shell was carried out. The main grid distribution on the surface of the conical shell is in the form of three rows of grids, 16 grids in each row, a total of 48 grids. The residual wall thickness of the grid rib of the final test piece was 2.5mm, the width of the rib is 10mm, and the height of the rib was 3.5mm. The experiment was carried out in seven rounds of milling, from the main body thickness of 6mm to the thickness of the grid rib of 2.5mm, and the cutting depth of each round was 0.5mm.



(a) Structure diagram of the test bench



(b) Milling experiment of thin-walled conical shell

Fig. 7 Thin-walled grid conical shell processing test bench

The data acquisition of the seven-round grid conical shell milling process is carried out respectively. The data set is composed of the three-dimensional coordinates of the tool location point, the local wall thickness, the minimum distance from the tool location point to the middle surface of the transverse and longitudinal ribs, the cutting depth and the normal cutting force. The tool location point coordinates can be extracted from the NC machining code, and the corresponding feature information can be matched based on this. Local wall thickness can be obtained by electromagnetic ultrasonic on-machine scanning wall thickness measurement method. The cutting force is obtained by a rotary tool holder dynamometer ; the minimum distance from the tool point to the middle surface of the transverse and longitudinal ribs is calculated in combination with the line laser on-machine scanning profile measurement method. The specific data acquisition process is shown in Fig. 8.

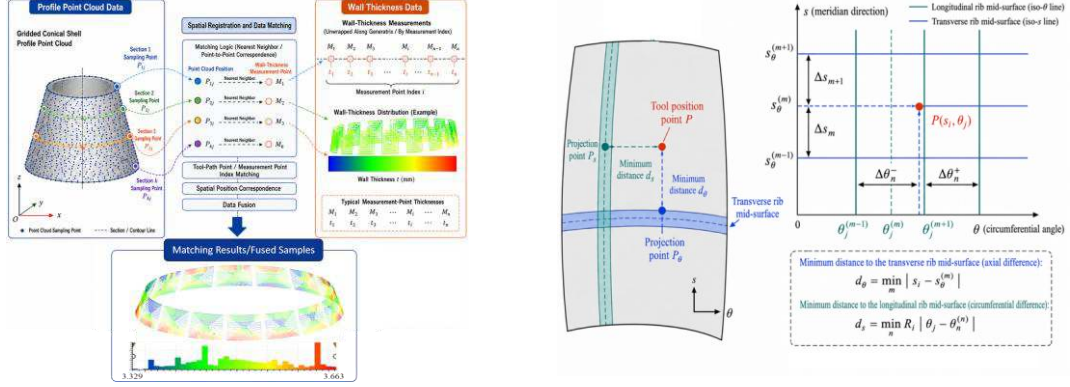


(a) Electromagnetic ultrasonic wall thickness on-machine scanning (b) Milling force on-line monitoring (c) Line laser profile on-machine scanning

Fig. 8 Data acquisition of thin-walled grid conical shell

In order to realize the unified characterization of the local geometric state and wall thickness information of the tool location point, the three-dimensional coordinates of each tool location point are extracted according to the NC program based on the conical shell workpiece coordinate system and the tool location point data. For the wall thickness measurement data, combined with the sampling workpiece coordinates corresponding to the wall thickness data, the position expression of the wall thickness measurement point in the conical shell parameter space is established. The wall thickness matching process is shown in Fig. 9 (a). Then, the coordinates of the tool location points are mapped to the axial-circumferential parameter domain of the conical shell, and the nearest neighbor search is used to establish the correspondence between the tool location points and the measured wall thickness, so as to obtain the local wall thickness at each tool location point. For the area with sparse distribution of measurement points, the IDW weighted interpolation criterion is further constructed by combining the axial distance and circumferential distance of adjacent measuring points to improve the reliability of wall thickness mapping. For the solution of the minimum distance from the tool location point to the middle surface of the transverse reinforcement and the longitudinal reinforcement, the spatial distribution characteristics of the transverse reinforcement and the longitudinal reinforcement are identified by using the profile scanning point cloud. The middle surface of the transverse reinforcement is approximately expressed as a circumferential curve distributed along the circumferential direction, and the middle surface of the longitudinal reinforcement is expressed as a spatial curve extending along the generatrix direction. The distance solution process is shown in Fig. 9 (b). Each tool location point is projected into the conical shell parameter space. The minimum distance is determined by the difference between the axial coordinates of the tool location point and the axial position of the middle surface of each transverse rib. The calculation of the middle surface distance of the transverse reinforcement is based on the difference between the circumferential angle of the cutter location point and the circumferential position of the middle surface of each longitudinal reinforcement, and the actual spatial distance is converted by

combining the local radius. Therefore, the position features and structural features of the conical shell data set are unified into the data set samples.



(a) Profile data and wall thickness data matching (b) The minimum distance from the tool location point to the middle surface of the transverse and longitudinal ribs.

Fig. 9 Location and structure feature data set construction

For the cutting force signal, according to the sampling frequency and the tool point processing time, the continuous cutting force sequence is expressed as a one-dimensional time series signal, and it is segmented by the sliding window method. The cutting force extraction process is shown in Fig. 10(a). For the adjacent windows, a certain overlap area is set to ensure the smoothness and timing continuity of the cutting force feature extraction between the continuous tool sites. Each sliding window corresponds to a local cutting force sample. Characteristic parameters such as mean value, peak value, root mean square value and normal cutting force component can be extracted to form a load characterization vector matching the tool position. The cutting force characteristics of the window are fused with the position characteristics and structural characteristics, and the elastic deflection can be solved according to the difference between the wall thickness and the cutting depth of the corresponding tool position of the two adjacent wheels. Finally, a total of 5600 samples were obtained and labeled from 7 sets of experiments. The data under different cutting rounds were randomly reorganized to construct the target domain data sets C1-C4, and each data set contained 1400 samples.

Based on the finite element simulation, the source domain data set for milling force prediction is constructed to provide basic training samples for the transfer learning model. Firstly, the finite element model was established based on the geometric size, wall thickness distribution, layout characteristics of transverse and longitudinal ribs of the grid conical shell, and the bottom support, fixture constraints and local contact relationship are equivalently set according to the actual clamping method, so as to ensure that the simulation model can reflect the stress state and local stiffness change of the workpiece in the milling process, as shown in Fig. 10(b). Taking the tool location point as the basic calculation unit, according to the characteristic parameters such as the coordinates of the tool location point, the local wall thickness, the cutting depth and the minimum distance between the tool location point and the middle surface of the transverse and longitudinal ribs extracted from the machining program, the milling simulation is carried out under different machining areas, different local stiffness and different process parameters. In the simulation process, by defining the tool-workpiece contact relationship, material removal area and cutting load direction, the equivalent normal cutting force, radial cutting force or resultant force load response at each tool point are obtained, so as to establish the source domain mapping relationship between “machining position-local structure state-process parameters-cutting force”. Referring to the data set construction form of the target domain, the source domain sample set C5-C16

is constructed from multiple rounds of simulation data, and each sample set also contains 1400 samples. Through the construction of finite element simulation data, a source domain data set with clear physical meaning and wide working condition coverage can be formed, which provides data support for pre-training of transfer learning model, cross-domain feature alignment and small sample correction of target domain.

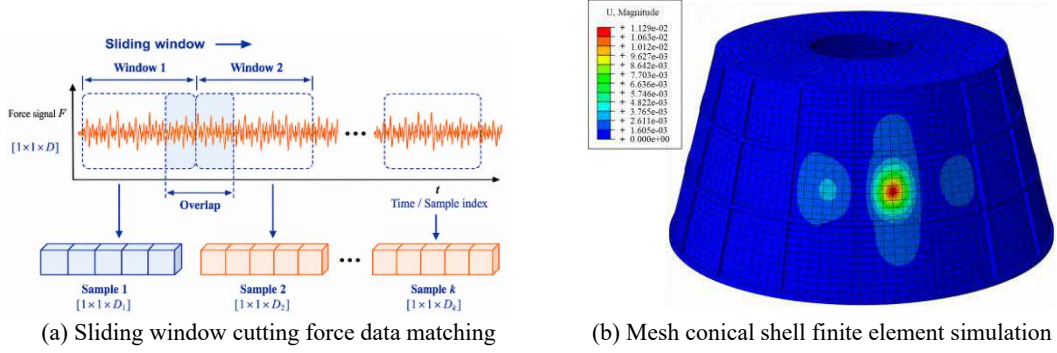


Fig. 10 Source domain and target domain data set construction

4.2 Model training and elastic deformation prediction accuracy verification!

Before the model training, it is necessary to set the hyperparameters and optimization parameters of the model. The training and testing environment is the Pytorch 1.13.0 framework, the optimizer uses Adam, and the hyperparameters are set as shown in Table.1. The real cutting experimental samples corresponding to the data set C1-C4 are used as the target domain data set ; the finite element simulation samples corresponding to the data sets C5-C16 are used as the source domain data sets for pre-training and transfer alignment. The data set composed of the target domain and the source domain is randomly divided into a training set and a verification set at a ratio of 6:4, and four training modes are constructed. Three target domain datasets are selected as the validation set, all source domain datasets are used as the training set, and the remaining target domain datasets are used as the test set. The data partition is shown in Table.2.

Table.1 Hyperparameters of model training

Hyperparametric	Batch size	Round	Learning rate	Activation function	Discard rate
Value	0.05	60	0.001	ReLU	0.05

Table.2 Training mode division

Pattern	Target domain training set	Source domain training set	Test set
M1	C2, C3, C4	C5-C16	C1
M2	C1, C2, C3	C5-C16	C4
M3	C1, C3, C4	C5-C16	C2
M4	C1, C2, C4	C5-C16	C3

In order to systematically evaluate the effectiveness and advantages of the proposed PCTD model in the prediction of the elastic deformation of thin-walled grid conical shell, this paper further constructs three comparative models of CNN, PINN and TL, and carries out comparative analysis under the same test set conditions. The CNN model consists of an input layer, a two-layer convolution layer, a two-layer pooling layer and a regression output layer. The target domain samples consistent with the PCTD model are used as input, and the data mapping relationship between machining features and elastic deformation is directly established. The PINN model introduces the mechanism constraint layer on the basis of the CNN network structure. Firstly, the cutting force is predicted by the CNN branch, and then the predicted cutting force is input into the physical constraint layer to calculate the elastic deflection, but its training process only depends on the target domain data. The TL model retains the transfer learning structure on the basis of the PCTD framework, uses the source domain and target domain data for joint training,

removes the branch of mechanism constraints, and directly outputs the prediction results of elastic deformation. Through the above model settings, the prediction performance of each model under the condition of small sample target domain can be compared and analyzed from different perspectives such as pure data-driven, physical constraint embedding, transfer learning and physical-transfer fusion. If the deviation range between the predicted elastic deformation value and the actual measured value is $\pm 0.04\text{mm}$, it can be determined that the prediction result is accurate. Finally, the accuracy of the prediction result is evaluated by the ratio of the number of accurately predicted cutter location points to the number of cutter location points in the test set. After completing the training of the four models, the prediction accuracy results on the test set are shown in Table.3.

Table 3 The prediction accuracy results of the four methods in the test set

Pattern	M1(%)	M2(%)	M3(%)	M4(%)
PCTD	93.05	95.27	94.98	94.12
CNN	90.21	91.37	91.56	89.85
PINN	91.47	92.26	92.31	92.55
TL	90.69	91.55	91.98	92.35

Taking pattern M2 as an example, the accuracy and loss of PCTD and CNN in the test set are shown in Fig. 11. The accuracy of PCTD model is improved rapidly in the early stage of training, and tends to be stable in fewer training rounds. Although the loss of the CNN model continues to decline, the accuracy improvement is relatively slow, and it is still lower than the PCTD model after stabilizing in the later period.

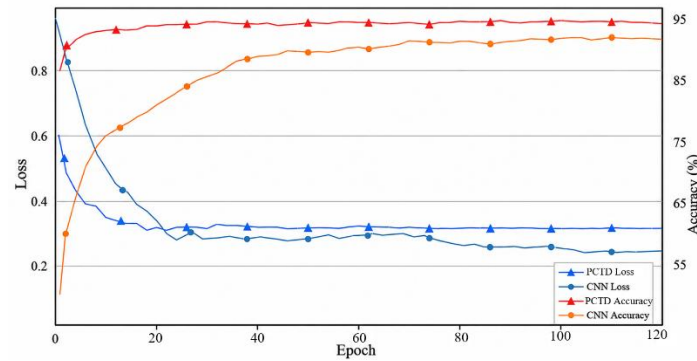


Fig. 11 PCTD and CNN accuracy set loss curve

For each trained model, further verification was carried out from two aspects: (1) A part of the current round of conical shell mesh milling data was used as the training set, and the remaining data was used as the test set to evaluate the prediction performance of the model under known working conditions ; (2) The conical shell mesh data of the semi-finishing stage is used as the training set, and the data in the finishing stage was used as the test set to test the generalization prediction ability of the model for the elastic deformation under the condition of unknown processing rounds. The scatter distribution of the test results of each model is shown in Fig. 12, and the test set prediction curve of the first 400 samples is shown in Fig. 13.

In general, the four models can characterize the change trend of elastic deformation to a certain extent, but their prediction accuracy and local response ability are significantly different. Although the CNN model can basically track the overall change rule of deformation, there are obvious prediction deviations in the low deformation area, the local high deformation area and the rapid fluctuation stage of deformation, which indicates that it is difficult to fully reflect the nonlinear deformation characteristics under complex machining conditions only by data-driven method. After introducing physical constraints into the PINN model, the concentration of scatter distribution and the smoothness of prediction curve are improved to some extent, but there are still problems of response lag and insufficient stability in the local peak and valley positions. In contrast, the TL model further improves the feature transfer ability by means of joint training of the source domain and the target domain, and its scatter distribution is more

convergent. The tracking effect of the prediction curve on the overall deformation change is also better than that of the model trained only by the target domain data. The prediction results of the PCTD model show the best consistency, and its scatter distribution is closest to the 45 ° ideal reference line. At the same time, it can accurately describe the overall change trend and the dynamic evolution process of local peak and valley values in the test curve. The results show that after the fusion of transfer learning and physical constraints, the model can effectively improve the feature representation ability and prediction robustness in different deformation intervals, so as to obtain higher comprehensive prediction accuracy.

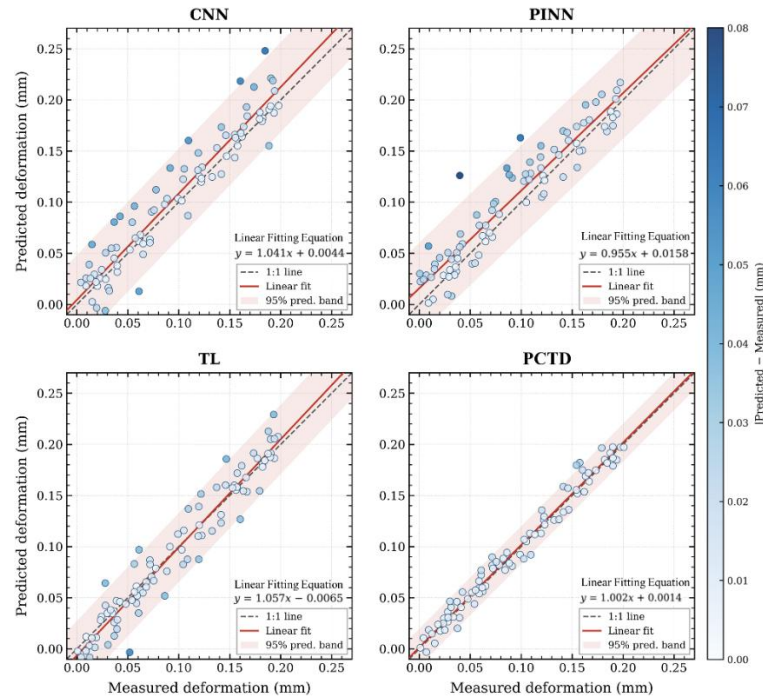
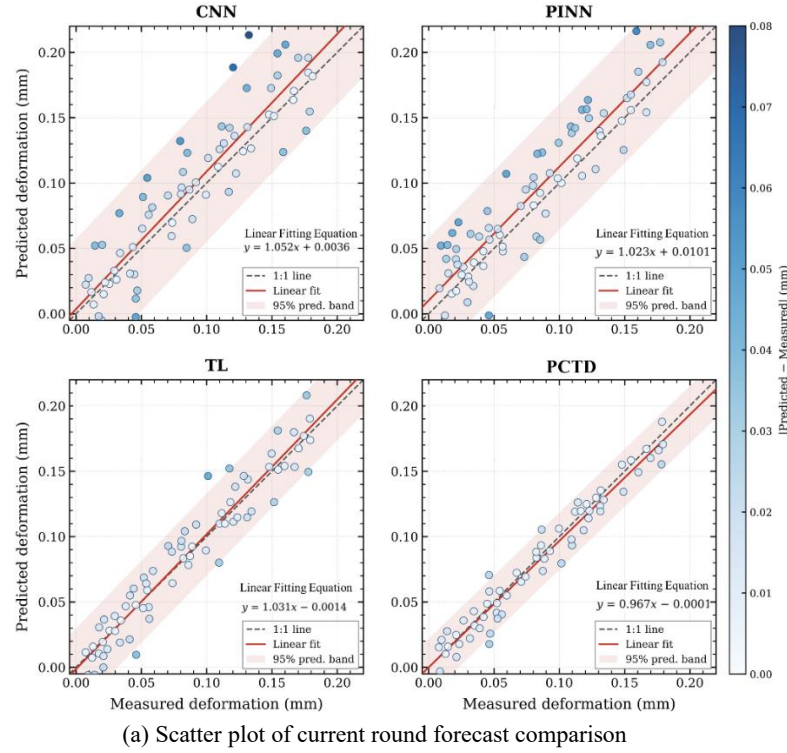
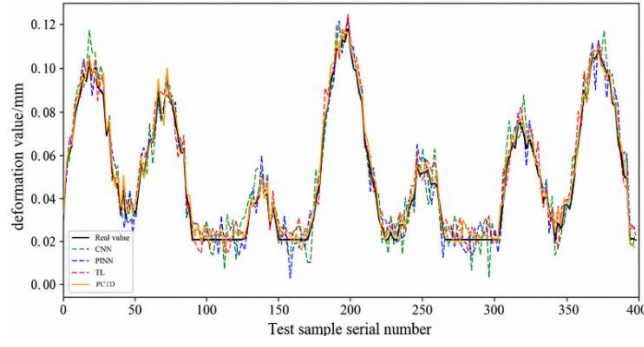
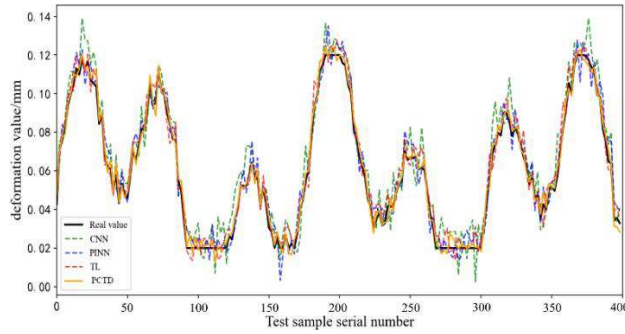


Fig. 12 Comparison of scatter plot of real value prediction value



(a) Comparison of current round prediction curves



(b) Comparison of prediction curves of unknown rounds

Fig. 13 Comparison of test set prediction curves (the first 400 samples)

In the process of network model training and performance evaluation, this paper uses root mean squared error (RMSE), mean absolute error (MAE) and coefficient of determination as the evaluation indexes of regression model prediction performance. Among them, RMSE and MAE are used to characterize the degree of deviation between the predicted value and the true value of the model, and to measure the fitting ability of the model to the change rule of the target variable. It can be seen from the results shown in Fig. 14 that among the four comparison models, the RMSE and MAE of the PCTD model are the smallest, and the R^2 value is the largest, indicating that it has the best prediction accuracy and comprehensive generalization ability in the prediction of elastic deformation. The results of CNN and PINN models are further compared. Under the condition of only using the target domain data for training, the RMSE and MAE of the model are effectively reduced and significantly improved after the introduction of physical constraints, indicating that the embedding of physical information can enhance the model's ability to characterize the elastic deformation law, thereby improving the stability and accuracy of the prediction results.

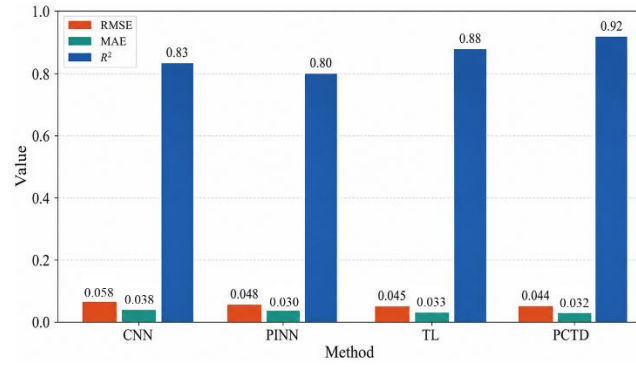


Fig. 14 Comparison of four methods to predict performance indicators

5 Conclusion

In this paper, a manufacturing process prediction method combining equivalent stiffness analytical model and physical constraint transfer learning is proposed for the prediction of cutting force-induced elastic deformation in the milling process of thin-walled grid conical shell. This method takes the normal cutting force as the key intermediate variable to establish the relationship between the machining state characteristics, local structural stiffness and elastic deformation, and realizes the transfer of the finite element source domain knowledge to the real milling target domain. The main conclusions are as follows:

(1) An analytical model for the normal elastic deformation of thin-walled grid conical shell considering the stiffness effect of the ribs is established. By equivalent the periodic “rib-skin” structure to a stiffened conical shell with direction-dependent stiffness characteristics, the equivalent stiffness model is introduced to characterize the influence of local stiffness change on the normal deflection response during grid milling.

(2) A physical constraint transfer learning model for normal cutting force prediction is constructed. The model uses finite element simulation data to learn the load law of wide working conditions, and corrects the target domain through real milling experimental samples. Under the condition of small samples, the coupling of cutting force prediction and analytical calculation of tool deflection is realized, and the prediction stability in complex machining areas is improved.

(3) The experimental verification of thin-walled grid conical shell milling shows that the prediction accuracy of PCTD model in different test modes reaches 93.05%-95.27 %, which is better than the comparison models such as CNN, PINN and TL. At the same time, the comparison results of RMSE, MAE and determination coefficient further show that the introduction of intermediate variables of cutting load and analytical physical constraints can effectively improve the prediction ability of the model for low stiffness regions and unknown processing rounds.

(4) The proposed method reveals the coupling relationship of “cutting load-local stiffness-tool deformation” in the milling of grid conical shell from the perspective of manufacturing process, which can provide a basis for the prediction of machining error and precision control of thin-walled stiffened curved components.

CRedit authorship contribution statement

Xingliang Chai: Writing–original draft, Methodology, Investigation. **Haibo Liu:** Funding acquisition. **Fangyu Chen:** Investigation. **Xuehang Lin:** Formal analysis. **Rui Fang:** Experiment. **Chunhui Wu:** Formal analysis. **Yongqing Wang:** Funding acquisition.

Data availability The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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