Online Resource 1: Supplementary Information for "Mechanics-Informed Semantic-Geometry Framework for Staged Lightweight Design of a Box-Type Thin-Walled Structure"

S1. Case Loads, Boundary Conditions, and Mapping

Tables S1 and S2 list the prescribed resultants, heat inputs, supports, and thermal conditions. Each force or heat-power entry is the total assigned to its labeled region; mapping preserves the resultant or total power after remeshing. Q_SLIDER_BLOCK is one heat source spanning the union of the four corner regions GREEN0–GREEN3; its 500 W is distributed once over their combined matched shell area and is not applied separately to each region. Thus, the seven heat-source entries in Table S1 prescribe a total of 3500 W. The ambient-film conditions in Table S2 are applied to both shell faces, F1 and F2.

Table S1 Mechanical and thermal load definitions

ID	Domain / scope	Region label / red–green–blue (RGB)	Load definition	Distribution	Description
F_CUT_TOP	Mechanical / Face	PURPLE / (155, 0, 255)	Resultant 1500 N; (0, −1, 0)	Uniform	Cutting force on top-beam front face
F_CUT_BOTT_F	Mechanical / Face	YELLOW_0 / (255, 255, 0)	Resultant 750 N; (0, −1, 0)	Uniform	Cutting force on bottom-beam front face
F_CUT_BOTT_B	Mechanical / Face	YELLOW_1 / (255, 255, 1)	Resultant 750 N; (0, −1, 0)	Uniform	Cutting force on bottom-beam back face
F_DRIVE_CUT	Mechanical / Face	RED / (255, 0, 0)	Resultant 5000 N; (1, 0, 0)	Uniform	Drive/cutting force on bottom-beam center face
F_GRAV_PAYLOAD_F	Mechanical / Face	YELLOW_0 / (255, 255, 0)	Resultant 5000 N; (0, 0, −1)	Uniform	Payload gravity on bottom-beam front-side edge face
F_GRAV_PAYLOAD_B	Mechanical / Face	YELLOW_1 / (255, 255, 1)	Resultant 5000 N; (0, 0, −1)	Uniform	Payload gravity on bottom-beam back-side edge face
F_GRAV_PAYLOAD_TOP	Mechanical / Face	PURPLE / (155, 0, 255)	Resultant 2000 N; (0, 0, −1)	Uniform	Payload gravity on top-beam front face
Q_GUIDE_TOP	Thermal / Face	PURPLE / (155, 0, 255)	Total heat power 500 W	Uniform	Top-beam guide-rail friction heat
Q_GUIDE_BOTTOM_F	Thermal / Face	YELLOW_0 / (255, 255, 0)	Total heat power 500 W	Uniform	Bottom-beam front guide-rail friction heat
Q_GUIDE_BOTTOM_B	Thermal / Face	YELLOW_1 / (255, 255, 1)	Total heat power 500 W	Uniform	Bottom-beam back guide-rail friction heat
Q_SLIDER_BLOCK	Thermal / Face	GREEN0–GREEN3 union	Total heat power 500 W across the union	Uniform over the combined matched area	Slider-block friction heat at four corner regions
Q_MOTOR_LEFT	Thermal / Face	BLUE0 / (0, 155, 255)	Total heat power 500 W	Uniform	Left motor-mover loss heat
Q_MOTOR_RIGHT	Thermal / Face	BLUE1 / (1, 155, 255)	Total heat power 500 W	Uniform	Right motor-mover loss heat
Q_MAGNET_TRACK	Thermal / Face	RED / (255, 0, 0)	Total heat power 500 W	Uniform	Motor magnet-track loss heat

Table S2 Mechanical and thermal boundary conditions

ID	Domain / scope	Region label / red–green–blue (RGB)	Condition	Distribution	Description
BC_GUIDE_BLOCKS_LB	Mechanical / Face	GREEN0 / (0, 255, 0)	Locked components; $u_x = u_y = 0$	Lock	Lower-left guide-block support
BC_GUIDE_BLOCKS_LT	Mechanical / Face	GREEN1 / (0, 255, 1)	Locked components; $u_x = u_y = 0$	Lock	Upper-left guide-block support
BC_GUIDE_BLOCKS_RT	Mechanical / Face	GREEN2 / (0, 255, 2)	Locked components; $u_x = u_y = 0$	Lock	Upper-right guide-block support
BC_GUIDE_BLOCKS_RB	Mechanical / Face	GREEN3 / (0, 255, 3)	Locked components; $u_x = u_y = 0$	Lock	Lower-right guide-block support
BC_MOTOR_LEFT	Mechanical / Face	BLUE0 / (0, 155, 255)	Locked component; $u_z = 0$	Lock	Left linear-motor mover support
BC_MOTOR_RIGHT	Mechanical / Face	BLUE1 / (1, 155, 255)	Locked component; $u_z = 0$	Lock	Right linear-motor mover support
H_AMBIENT_COEFF	Thermal / Face	WHITE / (203, 210, 239)	Convection; $h = 5$ W m^{-2} K^{-1}	Uniform	Uniform effective ambient-film coefficient
H_AMBIENT_TEMP	Thermal / Face	WHITE / (203, 210, 239)	Ambient temperature; $T_\infty = 295.15$ K	Uniform	Ambient reference temperature
T_SOLVER_INIT	Thermal / Model and nodes	INIT_TEMP / (0, 0, 0)	Initialization; $T_0 = 295.15$ K	Uniform	Initial field for the thermal solution
T_STRESS_FREE	Thermoelastic / Model	—	Reference temperature; $T_{\text{ref}} = 295.15$ K	Uniform	Stress-free temperature for thermal expansion

Note: listed translational degrees of freedom are locked; shell rotations are not prescribed directly. The common 295.15 K value serves as both initialization and stress-free reference.

S2. Stage 2 Sampling Ranges and Round 1 Diagnostic

Table S3 maps the ten scalar variables to the implementation labels retained in Figs. 8 and S1. In Table S3, b∈{2,4}; wb, hb, and db denote the current mapped width, height, and depth of Box b, respectively, and mb denotes the candidate-specific geometry margin used in the corresponding side-chamfer bound. The index j runs over the eight chamfer variables, and pj,max is the candidate-specific upper bound defined in the corresponding row. For each candidate, the primary dimensions d4 and h2 are mapped first; the dependent box dimensions, geometry margins, and pj,max values are then recomputed before the chamfer variables are checked.

Table S3 Case-specific Stage 2 semantic variables and sampling intervals

Semantic group	Scalar variable	Round 1 interval and bound (mm)	Round 2 interval (mm)	Implementation ID in Figs. 8 and S1
Box 4 depth	d_4	[235,290]	[238,275]	box4_depth(y)
Rear/top chamfer	TB_α	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.38d_4, 0.20w_4)$	[15,85]	b4_rear_top_cutY
Rear/top chamfer	TB_β	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.45h_4, 80)$	[10,65]	b4_rear_top_cutZ
Rear/side chamfer	TLTR_α	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.12w_4, 220)$	[40,170]	b4_rear_side_cutX
Rear/side chamfer	TLTR_β	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.98m_4, 0.35d_4)$	[2,25]	b4_rear_side_cutY
Box 2 height	h_2	[200,280]	[250,280]	box2_height(z)
Lower/side chamfer	BLBR_α	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.12w_2, 220)$	[20,170]	b2_lower_side_cutX
Lower/side chamfer	BLBR_β	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.98m_2, 0.45h_2)$	[35,125]	b2_lower_side_cutZ
Lower/front chamfer	BF_α	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.38d_2, 190)$	[20,170]	b2_lower_front_cutY
Lower/front chamfer	BF_β	$[0.02p_{i,\max}, p_{i,\max}]$; $p_{i,\max} = \min(0.45h_2, 130)$	[20,120]	b2_lower_front_cutZ

The following definitions describe the Round 1 range-refinement and energy-localization screen. All lower-is-better metrics were independently min–max normalized over the 200-candidate population; a zero-range metric was assigned zero.

The range-refinement score is defined in Equation (S1):

$$S_{\mathrm{ME},i} = \frac{U_{\mathrm{m,max},i}{}^{*} + C_{\mathrm{m},i}{}^{*} + D_{\mathrm{T},i}{}^{*}}{3} + 0.15V_i{}^{*}. \quad \text{(S1)}$$

The coefficient 0.15 makes normalized volume a secondary regularizer: its maximum contribution is 0.15, compared with a unit range for the mean primary-response term. It separates candidates with similar global responses without allowing volume to dominate displacement, compliance, or thermal-distortion performance. The coefficient was selected through preliminary ranking tests and retained throughout the reported Stage 2 screen. Severe local energy localization is screened by Equation (S2):

$$\kappa_{\mathrm{E},i} = \frac{\max_{e\in\mathcal{E}_i{}^{\mathrm{FE}}} W_{e,i}}{|\mathcal{E}_i{}^{\mathrm{FE}}|^{-1}\sum_{e\in\mathcal{E}_i{}^{\mathrm{FE}}} W_{e,i}}. \quad \text{(S2)}$$

Here $W_{e,i}$ is the strain energy of element e in candidate i, and $\mathcal{E}_i{}^{\mathrm{FE}}$ is that candidate's FE-element set. The Round 1 cutoff $\kappa_{\mathrm{E}} = 52$ retained 104 of the 200 candidates. Because κ_{E} uses raw element strain energy, it reflects local energy concentration together with element area and thickness. It is applied consistently to candidates reconstructed with the common target mesh. The volume coefficient 0.15 was retained in both round-specific scores. The cutoff 52 was used only for the Round 1 diagnostic and was not imposed as a hard Round 2 feasibility threshold; in Round 2, κE entered the independently normalized ordering score in Equation (9) of the main text.

Fig. S1 presents the complete Round 1 diagnostic used to identify influential variables and focus the Stage 2 Round 2 intervals.

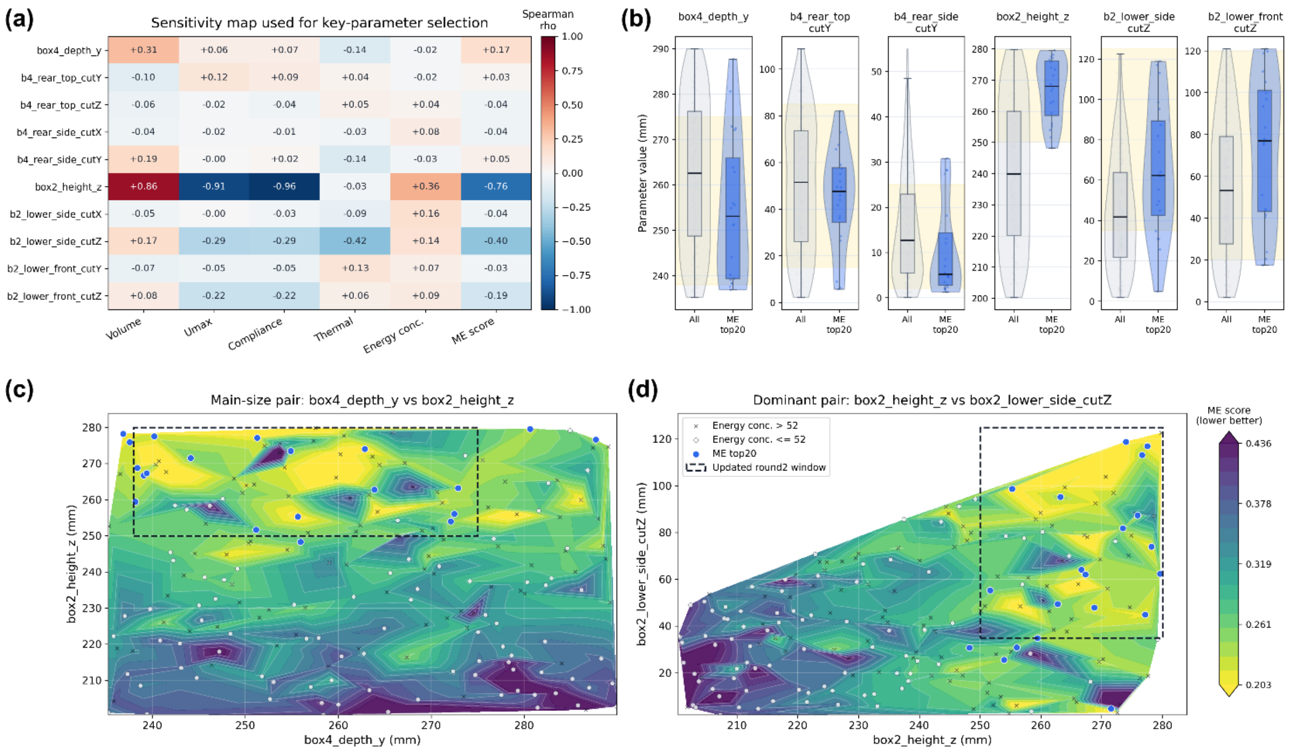

Fig. S1 Stage 2 Round 1 material-efficiency diagnostic. (a) Parameter–metric Spearman correlation; (b) sampled response distributions; and (c,d) material-efficiency responses and focused Round 2 windows. Crosses and open markers indicate Round 1 values of the strain-energy-localization index above and at or below 52, respectively

For traceability, the selected Stage 2 Round 2 handoff design (Candidate 0066), whose panel and chamfer dimensions are rounded to 0.1 mm for reporting, is represented by $d_4 = 248.1$ mm, $h_2 = 271.1$ mm, $\mathrm{TB}_\alpha = 28.2$ mm, $\mathrm{TB}_\beta = 34.2$ mm, $\mathrm{TLTR}_\alpha = 151.2$ mm, $\mathrm{TLTR}_\beta = 7.3$ mm, $\mathrm{BLBR}_\alpha = 63.5$ mm, $\mathrm{BLBR}_\beta = 114.9$ mm, $\mathrm{BF}_\alpha = 86.8$ mm, and $\mathrm{BF}_\beta = 82.3$ mm.

S3. Stage 3 Moving Morphable Void (MMV) Parameterization and Trial Controller

The main text summarizes the Stage 3 workflow and acceptance equations. The following equations define the surface-attached Polar-R representation, clipping and symmetry mapping, and dynamic step multiplier.

For opening i, N_θ equally spaced polar directions define the boundary samples:

$$\theta_l = \frac{2\pi l}{N_\theta}, \qquad l = 0, \dots, N_\theta - 1,$$
$$r_{i,l} = r_{\min} + r_{\mathrm{s}}\mathrm{softplus}(\eta_{i,l}), \tag{S3}$$
$$\mathrm{softplus}(\eta) = \ln(1 + \mathrm{e}^{\eta}), \qquad r_{\mathrm{s}} = 1 \ \mathrm{mm}, \qquad r_{\min} = 1 \ \mathrm{mm}.$$

$$\mathbf{p}_{i,l} = \mathbf{c}_i + \mathbf{R}(\varphi_i) r_{i,l} \begin{bmatrix} \cos\theta_l \\ \sin\theta_l \end{bmatrix}. \tag{S4}$$

In Equations (S3)–(S4), $\mathbf{c}_i$ and φ_i are the opening-center vector and local orientation, $\mathbf{R}(\varphi_i)$ is the host-plane rotation matrix, and $\eta_{i,l}$ is a dimensionless softplus preimage. The scale $r_{\mathrm{s}} = 1\ mm$ makes Equation (S3) dimensionally consistent, and the 1 mm lower radius prevents a singular parameterization. Physical radius updates are projected to the admissible domain before the latent variable is recovered by the inverse-softplus map. The case study used $N_\theta = 12$ and initialized each eligible seed with a 30 mm mean radius. The audit log stores seed centers, master ordering, operator targets, and signed directions for the ordered radius–shape–center cycle. Radius updates change the overall scale; the shape operator uses $e_l(\psi) = \cos[2(\theta_l - \psi)]$ in the softplus preimages; and the center operator translates $\mathbf{c}_i$ while retaining the radial field.

Symmetry is represented explicitly rather than by optimizing both sides independently. A seed on $x \le 0$ is the master and its paired opening is generated by reflection about $x = 0$; the mirrored instance reuses the master variables. In Equation (S5), $\mathcal{P}_i$ is the raw host-plane footprint reconstructed from Equation (S4), $\tilde{\mathcal{P}}_i$ is the clipped footprint passed to geometry transcription, $h_{(i)}$ maps opening i to its semantic host surface, $\mathcal{D}_{h(i)}{}^{\text{adm}}$ is that host's admissible design patch, and $\mathcal{M}_x$ is reflection about the symmetry plane:

$$\mathcal{P}_{\text{slave}} = \mathcal{M}_x(\mathcal{P}_{\text{master}}), \qquad \tilde{\mathcal{P}}_i = \mathcal{P}_i \cap \mathcal{D}_{h(i)}{}^{\text{adm}}. \tag{S5}$$

Equation (S6) updates a dynamic multiplier for radius, shape, or center steps. At multiplier s_k, the radius expansion and contraction factors are $1 + 0.06 s_k$ and $1 - 0.03 s_k$, the shape step is $0.14 s_k$ in the dimensionless softplus preimages, and the center step is $0.18 s_k$ times the current mean radius. Let a_k denote the decision defined by Equation (11) in the main text and $o_k \in \{\text{R}, \text{S}, \text{C}\}$ denote radius, shape, or center. For every FE-evaluated trial, $\Delta J_k = J_{\text{acc}}{}^{(k-1)} - J_{\text{trial}}{}^{(k)}$, so a positive value denotes improvement. For an invalid or failed trial, a_k is set to zero and ΔJ_k is not evaluated. The four branches are mutually exclusive:

$$s_{k+1} = \begin{cases} \min(\gamma_+ s_k, s_{\max}) & a_k = 1, o_k = \text{R}, \Delta J_k \ge \delta_{\text{near}} \\ \max(\gamma_- s_k, s_{\min}) & a_k = 0 \\ \max(\gamma_- s_k, s_{\min}) & a_k = 1, o_k = \text{R}, \Delta J_k < \delta_{\text{near}} \\ s_k & a_k = 1, o_k \in \{\text{S}, \text{C}\} \end{cases} \tag{S6}$$

Settings were $s_0 = 1.8$, $s_{\min} = 0.12$, $s_{\max} = 4.5$, $\gamma_+ = 1.18$, $\gamma_- = 0.55$, and $\delta_{\text{near}} = 1.2 \times 10^{-4}$. The run used a 30-trial sequence and an early-stop rule of three consecutive rejections at the minimum step. It completed all 30 trials with 25 acceptances and five rejections; the multiplier after trial 30 was $0.207 > s_{\min}$. The reported geometry is the last accepted design in this sequence. Algorithm S2 gives the trial-generation, acceptance, and rollback pseudocode; the ordered target and signed-direction sequence is stored in the Stage 3 audit log. For the Equation (11) acceptance test, $\tau_J = 0.0015$ is the dimensionless tolerance on the normalized objective score; it is not a compliance tolerance and does not relax the independent compliance ceiling or the validity requirement.

S4. Detailed Trial Generators

Algorithms S1 and S2 expand the stage-specific trial generators invoked by Algorithm 1 in the main text. They document proposal, reconstruction, acceptance, rollback, and stopping logic. Stage 1 does not quantize rib orientations to a prescribed angle set; the 18° criterion in Algorithm S1 is only a local near-collinearity trigger for optional topology collapse. Rib-thickness proposals use the Phase C relaxed integer-thickness projection defined in Section 3.1.1 of the main text; the selected state retains the resulting generally non-integer thickness values.

Algorithm S1 Stage 1 proposal, acceptance, and topology-collapse sweep

Step	Operation
Input: $\mathcal{S}_{\text{acc}}$, phase p, phase-entry references, Equation (8) acceptance weights (also used as surrogate driver scales), bounds, L_p, and the applicable global-compliance guard. The input also includes the relaxed integer-thickness projection settings; no standard-angle or direction-family term is used.	
1	Reconstruct and solve $\mathcal{S}_{\text{acc}}$; evaluate the global compliance, active box responses, and incumbent Equation (8) acceptance merit.
2	Evaluate $J^{(p)} = \frac{1}{\lvert\mathcal{B}_{\text{act}}\rvert}\sum_{b \in \mathcal{B}_{\text{act}}} J_b^{(p)}$ from Equation (8) for acceptance bookkeeping; do not differentiate it to generate the continuous proposal.
3	Form the continuous coordinate and thickness blocks $\mathbf{x}$ and $\mathbf{t}$.

Step	Operation
4	For $z \in \{\mathbf{x}, \mathbf{t}\}$, compute the derivatives used by the continuous proposal surrogate. Use $\frac{dC_\mathrm{m}}{dz} = 2\mathbf{u}_\mathrm{m}{}^\mathrm{T}\frac{\partial \mathbf{f}_\mathrm{m}}{\partial z} - \mathbf{u}_\mathrm{m}{}^\mathrm{T}\frac{\partial \mathbf{K}}{\partial z}\mathbf{u}_\mathrm{m}$ for the global-compliance block, the fixed-load analytic thickness derivative, and centered reconstructed differences for node coordinates. Thickness derivatives are formed for the continuous pre-projection variables; the relaxed integer-thickness projection is applied when the reconstructed trial is formed in Step 8.
5	For $z \in \{\mathbf{x}, \mathbf{t}\}$, normalize each active surrogate-response gradient as $\mathbf{g}_{r,z} = \frac{\nabla_z \hat{X}_r}{\mathrm{med}(\lvert\nabla_z \hat{X}_r\rvert) + \varepsilon_{\mathrm{g},z}}$ with $\varepsilon_{\mathrm{g},z} = 10^{-12}\ \ \mathrm{mm}^{-1}$; use the phase-normalized global compliance as the compliance response and omit node count, the collapse-trigger fraction, and rib-thickness regularity from the continuous direction because they change only through discrete topology changes or relaxed integer-thickness projection updates.
6	Compute the normalized surrogate proposal direction $\mathbf{d}_z = -\frac{\sum_{r\in\mathcal{I}_\mathrm{cont}{}^{(p)}} w_\mathrm{r}^{(p)}\mathbf{g}_{r,z}}{\left\lVert\sum_{r\in\mathcal{I}_\mathrm{cont}{}^{(p)}} w_\mathrm{r}^{(p)}\mathbf{g}_{r,z}\right\rVert_\infty + \varepsilon_\mathrm{d}}$, $\varepsilon_\mathrm{d} = 10^{-12}$ for $z \in \{\mathbf{x}, \mathbf{t}\}$. This direction is not the negative gradient of Equation (8).
7	Set $m_\mathrm{x} = \min(0.04L_p, 18\ \ mm)$, $m_{\mathrm{t},j} = 0.06t_j$, and $\xi = 1$.
8	Generate the coordinate trial $\mathbf{x}_\mathrm{trial} = \Pi_\mathrm{x}(\mathbf{x}_\mathrm{acc} + \xi m_\mathrm{x}\mathbf{d}_\mathrm{x})$; the coordinate projection enforces coordinate bounds, host-plane admissibility, and protected regions without snapping rib directions to a standard-angle set. Generate the corresponding continuous thickness trial with $\xi m_{\mathrm{t},j} d_{\mathrm{t},j}$. Clip it to the admissible thickness bounds and apply the relaxed integer-thickness projection defined in Section 3.1.1 of the main text before reconstruction and FE evaluation.
9	Check bounds, labels, protected regions, connectivity, geometry, and remeshing; reconstruct and solve the trial using the post-projection thickness values and evaluate its Equation (8) merit and phase guard.
10	Accept only if the trial is valid, satisfies the phase guard, and has a non-increasing reconstructed Equation (8) merit; otherwise set $\xi \leftarrow \xi/2$ and retry. Reject when $\xi < 10^{-4}$.
11	In Phases B and C, enumerate and sort the near-pair and degree-two triggers defined by $0.10L_p$ and 18°. The 18° value measures only local deviation from collinearity and does not constrain absolute rib orientation.
12	Test collapse candidates in order. Accept each only after reconstruction, a non-increasing Equation (8) merit, and the phase guard; regenerate the list after each accepted collapse.
13	Write the trial to the audit log. Transfer only if the receiving guard holds; otherwise retain the accepted state.
Output: Accepted Stage 1 state and ordered trial log.	

Algorithm S2 Stage 3 MMV trial generation and acceptance

Step	Operation
Input: Opening-free Stage 2 state; ordered hosts and cavities; 30-trial schedule Π; Equations (10)–(11) and (S6); geometry rules.	
1	Sort eligible hosts. Place 30 mm seeds at the patch centroid or maximum-clearance interior point; align φ with the first local axis.
2	Retain $x \leq 0$ masters; regenerate mirrored slaves; canonicalize contacts with the tolerance in θ_{FE}; check cavity rules.
3	Evaluate iteration 0; store V_{ref} and $D_{\mathrm{T,ref}}$; set $\mathcal{S}_{\mathrm{acc}}{}^{(0)}$, $J_{\mathrm{acc}}{}^{(0)}$, $s_0 = 1.8$, and $k = 1$.
4	Read the ordered trial instruction $\Pi_k = (\mathcal{I}_k, o_k, \delta_k)$ with payload $\delta_k = \begin{cases} \sigma_k & o_k = \mathrm{R} \\ \{\psi_{i,k}: i \in \mathcal{I}_k\} & o_k = \mathrm{S} \\ \{\mathbf{d}_{i,k}: i \in \mathcal{I}_k\} & o_k = \mathrm{C} \end{cases}$. The audit log stores the complete 30-trial sequence.
5	Set $\mathcal{S}_{\mathrm{trial}}{}^{(k)}$ as a copy of $\mathcal{S}_{\mathrm{acc}}{}^{(k-1)}$.
6	Radius update on $\mathcal{I}_k$: for $\sigma_k = +1$ multiply by $1 + 0.06 s_k$; for $\sigma_k = -1$ multiply by $1 - 0.03 s_k$; project radii and invert the softplus map.
7	Shape update: set $\eta_{i,l}{}' = \eta_{i,l} + 0.14 s_k \cos\left[2\left(\theta_l - \psi_{i,k}\right)\right]$ for each $i \in \mathcal{I}_k$.
8	Center update: compute $\bar{r}_i = \frac{1}{N_\theta} \sum_{l=0}^{N_\theta - 1} r_{i,l}$ and set $\mathbf{c}_i{}' = \mathbf{c}_i + 0.18 s_k \bar{r}_i \mathbf{d}_{i,k}$, with $\left\|\mathbf{d}_{i,k}\right\|_2 = 1$.
9	Apply Equations (S3)–(S4), regenerate mirrors, clip footprints by Equation (S5), and canonicalize contacts.
10	Check hosts, protection, symmetry, diameter, cavity rules, connectivity, and transcription.
11	After geometry and remeshing pass, set $g_k = 1$ and solve. On solver success, evaluate $J_{\mathrm{trial}}{}^{(k)}$ and apply Equation (11); on any failure, set $a_k = 0$ and reject.
12	On acceptance, replace the incumbent and score; otherwise retain the accepted state.
13	Update s_{k+1} by Equation (S6) and write the available trial data and decision to the audit log.
14	Stop after 30 trials or three consecutive rejections at $s_{\min}$; otherwise set $k \leftarrow k + 1$ and return to Step 4.
Output: Last accepted Stage 3 state and ordered trial log.	