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Price Dispersion in the Barcelona Fish Auction: A Probabilistic Interpretation of the Failure of the Law of One Price

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Price Dispersion in the Barcelona Fish Auction: A Probabilistic Interpretation of the Failure of the Law of One Price

Luca Di Gennaro Splendore and Domenico Costantini

The Law of One Price predicts that identical goods traded in the same competitive market should sell at a single price. Wholesale fish auctions offer a natural setting to test this prediction under conditions of perishability, seasonal supply variation, and product heterogeneity that most tests of the Law of One Price do not confront directly. Using a complete administrative dataset of 82,947 transactions from the Barcelona wholesale fish auction (Confraria de Pescadors de Barcelona) covering the full 2008 calendar year, we document systematic and substantial price dispersion for ten commercially significant species and ask what generates it. Rather than treating dispersion as noise around a deterministic price, we model prices as realisations of species-specific probability distributions, fitting log-normal, Gamma, Weibull, and finite mixture specifications. Seven species are best described by a log-normal distribution, two low-value bulk species by a Gamma distribution, and two species, sardine and langoustine, require a three-component log-normal mixture that separates damaged-lot, normal-season, and peak-season pricing regimes; for sardine this mixture improves AIC by 15,201 over the best single-distribution fit. A conditional log-normal regression incorporating seasonal, quality-grade, and buyer covariates explains 40-72% of log-price variance across species, with the hake location parameter falling by 0.94 log-units between January and October and size-grade penalties of up to 60%. The residual variance that regression cannot explain is not unstructured noise but a stable, species-specific stochastic component, evidence that price in this market is better characterised as a random variable with a well-defined distribution than as a deterministic function of observables. These findings show that the failure of the Law of One Price in perishable goods markets reflects the superposition of structurally distinct price-generating regimes tied to seasonality and product heterogeneity, rather than market inefficiency, and that finite mixture models offer a diagnostic tool for uncovering latent market structure that conventional hedonic regressions cannot detect.

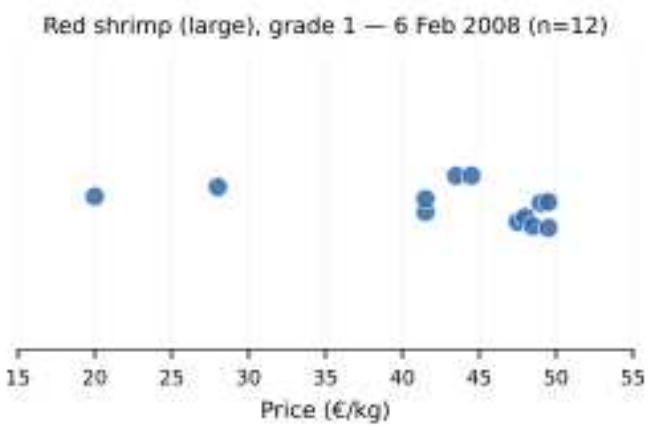
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1. Introduction

On Wednesday 6 February 2008, how is it possible that in the same market, the same day, a kilo of the same fish, red shrimp, is sold at 20 and 49.50 euros? Is there a manner to describe and explain this price dispersion? Every lot shared the same species, the same size grade, the same freshness category, and the same presentation format, by every observable definition, identical goods. Yet prices ranged from €20 to €49.50 per kilogram: a 2.5-fold difference within a single morning session, in the same room, among the same pool of professional buyers. The Law of One Price predicts a single price. The data show a distribution. The statistical properties of prices in competitive markets have attracted sustained attention from both economists and physicists. Since Bachelier's early work on random price fluctuations a substantial literature has documented that prices in financial, commodity, and goods markets frequently follow log-normal or power-law

distributions, emergent statistical regularities that arise from multiplicative stochastic processes operating at the micro level (Mantegna & Stanley 1999, Gabaix et al. 2003). These findings suggest that price formation, far from being deterministically predictable from observable fundamentals, is better understood as a probabilistic process whose distributional properties carry structural information about market organisation.

Figure 1: Wednesday 6 February 2008 red shrimp's price transactions



A parallel and older tradition in economics concerns the Law of One Price (LOP), which predicts that in competitive markets with perfect information and no transaction costs, identical goods will trade at a single price (or Equilibrium of Price), among other Samuelson 1964. The LOP has been extensively tested and extensively rejected across a wide range of markets, from financial market (Nuugulu et al. 2026), industrial commodity exchanges, Stigler and Kindahl (1970), to local retail markets, Gobillon et al. (2016), Gao et al.(2023) and Wang et al. (2024), with the consensus being that price dispersion is the empirical norm rather than the exception, Baye et al. (2005). Yet most tests of the LOP adopt a deterministic framing: they ask whether observed prices converge to a single value, and attribute deviations to search costs, information asymmetries, or transaction barriers. The possibility that price dispersion itself is a structured probabilistic phenomenon, that prices are not merely scattered around a common value but are drawn from well-defined distributions whose parameters encode market structure, has received comparatively little attention.

Wholesale auctions fish markets offer an unusually productive setting in which to examine this question. Fish auctions operate as institutionalised descending-price mechanisms in which buyers bid sequentially on individual lots of a given species, grade, and presentation, approximating the competitive ideal more closely than most goods markets, Graddy (1995), Guillotreau et al. (2011), Andre et al. (2022) and Chen (2024). At the same time, the goods exchanged are neither homogeneous nor storable: species differ in quality attributes, size grades, and freshness; prices fluctuate with seasonal supply cycles and demand patterns; and the same nominal product, a lot of sardines, a kilogram of red shrimp, can command dramatically different prices depending on when it is sold and who is buying. These features make fish auction markets natural laboratories for studying price dispersion under conditions that depart systematically from the LOP's assumptions.

The econophysics literature has examined price distributions in financial markets extensively, for instance Mantegna et al. (1999). but has devoted comparatively little attention to commodity auction markets, and almost none to fish markets specifically. The handful of existing studies of fish market price formation have adopted hedonic regression frameworks Asche et al. (1997), Guillotreau et al. (2011), Aponte (2020), both of which impose a deterministic structure on what we argue is an inherently probabilistic process. To our knowledge, no study has applied distributional fitting and mixture modelling to wholesale fish auction data to characterise the statistical laws governing price dispersion.

This paper addresses that gap. Using a complete administrative dataset of 82,947 auction transactions from the Barcelona auction fish market (*La lonja - Confraria de Pescadors de Barcelona*) covering the full calendar year 2008, we pursue three related objectives. First, we identify the best-fitting probability distribution for auction prices across ten commercially significant species, testing log-normal, Gamma, Weibull, and Normal candidates and comparing fits via AIC, BIC, and Kolmogorov-Smirnov statistics. Second, for species exhibiting non-standard residual structure after single-distribution fitting, we estimate finite mixture models via the Expectation-Maximisation algorithm, recovering latent market regimes within nominally homogeneous species categories. Third, we develop a conditional log-normal regression framework that decomposes log-price variance into components attributable to observable covariates, seasonality, size grade, buyer identity, and an irreducible stochastic residual, providing a probabilistic reinterpretation of the LOP's empirical failure.

Our results reveal a consistent distributional taxonomy: log-normal distributions characterise seven of ten species; Gamma distributions fit two low-value finfish species, suggesting an additive rather than multiplicative error structure; and three-component log-normal mixtures are required for sardine and Langoustine, uncovering coexisting price-generating regimes that single distributions systematically mask. Conditional models explain 40–72% of log-price variance, leaving a well-characterised and irreducible stochastic component. We interpret these findings as evidence that the Law of One Price fails in perishable goods markets not through market inefficiency but through the superposition of structurally distinct probabilistic processes, a conclusion with implications for both market microstructure theory and the statistical physics of price formation.

The remainder of the paper is organised as follows. Section 2 reviews the relevant literature on price distributions and on fish market price formation. Section 3 describes the dataset and cleaning procedures. Section 4 presents the distributional fitting methodology. Section 5 reports results. Section 6 discusses implications for the Law of One Price and for market structure. Section 7 concludes.

2. Literature review

The statistical analysis of price distributions has a long history in physics-inspired approaches to economics. The foundational contribution is Bachelier (1900) on random price fluctuations. Also Gibrat's (1931) establishes that if the growth rate of a variable is independent of its current size, the variable converges asymptotically to a log-normal distribution: law of proportional effect. Applied to prices, this implies that auction outcomes formed through sequential multiplicative

adjustments, each buyer revising a baseline valuation by a proportional factor reflecting quality signals, competitive pressure, and private information, should produce log-normal price distributions as an emergent statistical regularity.

This theoretical prediction found early empirical support in financial markets. Osborne (1959) documented that stock price changes follow an approximately normal distribution in log-space, a finding later refined by Mantegna and Stanley (1995), who showed that the central body of the S&P 500 return distribution follows a truncated Lévy stable distribution, with log-normal behaviour characterising the bulk while power-law tails govern extreme events. The book by Mantegna and Stanley (1999) systematised these findings and established the research programme that Physica A has since hosted extensively. Drăgulescu et al. (2001) showed that income and wealth distributions follow exponential and Gamma distributions respectively, with log-normal characterising intermediate ranges, a finding relevant to our species-level taxonomy. Podobnik et al. (2009) examined price fluctuations in commodity markets and documented log-normal behaviour in the bulk of the distribution with heavier tails. Gabaix et al. (2003) proposed a theory of price impact deriving power-law price fluctuations from the distribution of trader size, establishing that market microstructure shapes distributional outcomes in ways that cannot be recovered from aggregate price series alone.

The Law of One Price states that in competitive markets with costless arbitrage and perfect information, identical goods must trade at a single price (Samuelson 1964). In its strong form, applicable to internationally traded goods, it is the foundation of purchasing power parity theory (Isard 1977; Rogoff 1996). In its weak form, applicable to goods traded within a single market, it predicts price convergence across sellers and buyers for homogeneous products.

The empirical literature has consistently rejected the Law of One Price across a wide range of settings. Stigler et al. (1970) documented substantial and persistent price dispersion even in concentrated industrial markets. Pratt et al. (1979) showed that retail prices for identical goods vary by factors of two or more within the same city. The survey by Baye et al. (2005) synthesises decades of evidence concluding that price dispersion is the empirical norm in competitive markets and that search cost models, while theoretically compelling, explain only a fraction of observed dispersion.

For perishable goods specifically, the Law of One Price faces additional structural challenges. Deaton et al. (1992) showed that commodity price dynamics are fundamentally non-stationary due to the interaction of seasonal supply and speculative storage, producing distributional non-stationarity that no static price model can capture. Since fish cannot be stored (or can only be stored briefly at significant quality cost), the intertemporal arbitrage mechanism that underpins the Law of One Price (LOP) is effectively unavailable, making temporal price dispersion structurally unavoidable rather than a market failure.

The distinction between cross-sectional LOP failure (different prices for the same good at the same time) and temporal LOP failure (different prices for the same good at different times) is analytically important but rarely made explicit in the literature. Our paper formalises this distinction in distributional terms: cross-sectional failure manifests as dispersion in the

unconditional price distribution, while temporal failure manifests as non-stationarity in the distribution's parameters across time.

Fish markets have attracted a specialised empirical literature on price formation, motivated by their institutional characteristics, centralised auction mechanisms, perishable goods, heterogeneous quality, that make standard competitive equilibrium models difficult to apply. Graddy (1995) provided a rigorous study of price discrimination in a fish auction, documenting that Asian buyers paid systematically lower prices than white buyers for identical fish at New York's Fulton Fish Market. Steen et al. (2002) examined price relationships across fish species and markets using cointegration methods, testing whether Norwegian fish markets constitute a single integrated market. Their rejection of full price integration across species is consistent with our finding that different species obey different distributional families. Guillotreau et al. (2011) extended this analysis to examine market power and buyer concentration at France fish auctions, finding evidence of oligopsonistic behaviour among the largest buyers. Regarding seasonal price dynamics, Oglend and Sikveland (2008) documented that price volatility in salmon markets (1995-2007) is strongly seasonal, with distributional parameters shifting systematically across months. Their finding that price distributions are non-stationary across the seasonal cycle by GARCH model.

The algorithm for fitting finite mixtures, introduced by Dempster et al. (1977) provides the methodological foundation for our mixture estimation. The application of EM to log-normal mixtures in price data follows directly from the equivalence between a mixture of log-normals in price space and a mixture of normals in log-price space, which we exploit computationally. To our knowledge, finite mixture models have not previously been applied to wholesale fish auction price data.

The present paper sits at the intersection of three literatures: the econophysics literature on price distributions and statistical laws in markets; the fish market economics literature on price formation and Law of One Price testing; and the statistical literature on mixture models and distributional fitting. Its contribution to each is as follows. To the econophysics literature, it extends the documented domain of log-normal and Gamma price distributions from financial and commodity markets to a wholesale fish auction and introduces mixture modelling as a tool for uncovering latent market regimes in thin perishable goods markets. To the fish market economics literature, it provides the first distributional characterisation of price dispersion at transaction level for a complete annual dataset from a Spanish fish auctions and offers a probabilistic reinterpretation of LOP failure that goes beyond the standard hedonic regression framework. To the statistical literature, it demonstrates that the choice between single-distribution and mixture models carries structural economic interpretation, the mixture components are not statistical artefacts but identifiable market regimes with distinct seasonal and buyer-type signatures.

3. Data

The data originate from the Barcelona auction fish market (*Confraria de Pescadors de Barcelona)*. The market operates as a Dutch auction: centralised descending-price auction, the standard mechanism used across Spanish auction: the auctioneer begins at a high price and progressively lowers it until a buyer accepts, at which point the lot is assigned and the transaction recorded. This

mechanism approximates the theoretical conditions for competitive price discovery, buyers reveal their true valuations sequentially under time pressure, while simultaneously generating administrative transaction records of exceptional granularity.

Each transaction record captures the date, the vessel supplying the lot, the buyer identity and code, the species code and description, the quantity (in kilograms for most species, in boxes for sardines and a small number of other pelagic species), the unit price, the total transaction value, and a set of quality attributes including size grade, freshness category, and presentation format. This administrative structure makes the dataset substantially richer than the aggregated price series or survey-based data typically used in fish market studies.

The data were provided directly by the market authority to the authors for research purposes. The dataset covers the complete calendar year 2008, from 2 January to 29 December, encompassing all working days of the auction. The dataset's administrative completeness and granularity more than compensate for its temporal distance.

The primary outcome variable throughout the analysis is unit price, which represents euros per kilogram for weight-sold species and euros per box for species sold by the box. The analytical sample contains 82,947 records. The ten species selected for detailed analysis, Sardina (sardine), Gamba Roja G. (large red shrimp), Gamba Roja M. (medium red shrimp), Gamba Roja P. (small red shrimp), Lluç (hake), Escamarla M (langoustine, medium grade), Sorell (horse mackerel), Orada (gilt-head sea bream), Moll (red mullet), and Maira (blue jack mackerel), account for 35,515 transactions (42.8% of the analytical sample) and 65.5% of total annual revenue. Table 1 presents summary statistics for the ten analytical species.

Table 1. Descriptive statistics for the ten species - Barcelona fish auction 2008.

Species	n	Mean (€)	SD (€)	CV	Min (€)	Median (€)	Max (€)	Skewness	Ex. Kurtosis	Revenue (€)	Share (%)
Sardine	2,257	17.87	7.53	0.421	0.19	18.00	34.72	-0.387	0.341	1,434,829	22.9
Red shrimp (large)	3,072	54.09	12.88	0.238	20.00	51.50	94.50	1.229	1.628	897,453	14.3
Red shrimp (medium)	2,871	27.82	7.36	0.265	10.80	26.50	55.00	1.739	4.960	374,946	6.0
Red shrimp (small)	4,332	15.00	4.58	0.305	8.35	13.25	27.13	1.165	0.763	322,944	5.1
Hake	5,831	8.36	5.07	0.606	1.20	6.60	22.49	0.971	0.132	239,267	3.8
Langoustine (medium)	2,180	23.63	7.97	0.337	1.60	22.75	48.11	1.220	2.977	214,646	3.4
Atlantic horse mackerel	4,481	2.65	4.23	1.594	0.01	0.80	18.80	2.612	7.435	178,314	2.8
Gilt-head sea bream	1,390	8.82	4.28	0.485	1.60	7.90	21.62	1.227	1.497	166,049	2.6

Species	n	Mean (€)	SD (€)	CV	Min (€)	Median (€)	Max (€)	Skewness	Ex. Kurtosis	Revenue (€)	Share (%)
Red mullet	2,898	7.04	2.76	0.392	1.40	6.80	15.00	0.701	0.559	140,919	2.2
Mediterranean horse mackerel	6,203	2.17	1.10	0.506	0.10	1.90	5.35	1.227	2.693	137,264	2.2
Total (10 species)	**35,515**									**4,106,631**	**65.5%**

Notes: n = number of auction transactions with Precio > 0. SD = standard deviation. CV = coefficient of variation (SD/Mean). Min = minimum observed price. Max = 99th percentile (to limit influence of outliers). Revenue = total annual transaction value (€). Share = percentage of total market revenue (€6,273,285). The ten species account for 35,515 of 82,947 transactions in the analytical sample (42.8%) and 65.5% of total revenue. Sardine prices are in €/box; all other prices in €/kg. Data source: Confraria de Pescadors de Barcelona, administrative records 2008.

Sardine's price-generating process differs structurally from the other nine species. Unlike weight-sold finfish and crustaceans, Sardine lots are priced and traded by the box rather than by kilogram, so the recorded calibre grade reflects box composition rather than individual fish size — meaning it does not track quality in the same way as for the other species. Consistent with this, a single parametric distribution characterises Sardine prices poorly: the best single-distribution fit is markedly inferior to a three-component log-normal mixture, which improves AIC by 15,201 (this apply for all the following Tables). We nonetheless report Sardine alongside the other species for completeness and comparability, flagging it accordingly, but treat the mixture model as the substantively correct characterisation of its price structure.

4. Methodology

Let $P_{i,s}$denote the transaction price observed for species $s$in auction transaction i, where $i = 1, \dots, n_s$and $n_s$is the number of transactions for species $s$in the analytical sample. We treat $P_{i,s}$as a realisation of a random variable $P_s$drawn from a species-specific probability distribution $F_s$with density f_s. The central question is: what parametric family best characterises F_s, and what does the answer reveal about the price-generating process?

This framing departs from the standard hedonic regression approach, which treats price as a deterministic function of observable attributes plus an additive error term. Under the hedonic model, price dispersion is a residual, the part of price variation unexplained by covariates. Under the distributional framework adopted here, price dispersion is the object of interest: the distributional family F_s, its parameters, and their dependence on covariates together constitute the complete characterisation of price formation. The two approaches are complementary rather than competing.

We consider four candidate distributions for P_s, selected on both theoretical and empirical grounds. The Normal distribution $\mathcal{N}(\mu, \sigma^2)$ serves as the baseline against which all other candidates are evaluated. It is the distribution implicitly assumed by ordinary least squares regression on levels of price and corresponds to an additive error structure in which price deviations from the mean are symmetric and unbounded below. As prices are non-negative, the Normal is theoretically inappropriate for price data unless the coefficient of variation is small; we include it as a

benchmark. A random variable P follows a log-normal distribution $\mathrm{LN}(\mu, \sigma^2)$ if $\ln P \sim \mathcal{N}(\mu, \sigma^2)$. The log-normal arises naturally from Gibrat's (1931) law of proportional effect: if successive price adjustments are multiplicative and independent, the limiting distribution of price is log-normal. In the context of a fish auction, this corresponds to a model in which buyers update a baseline valuation by a sequence of proportional adjustments reflecting quality signals, competitive pressure, and private information. The log-normal has mean $\exp(\mu + \sigma^2/2)$, variance $[\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$, and is positively skewed for all $\sigma > 0$, consistent with the right-skewed price distributions typically observed for premium species. Its two parameters μ(location in log-space) and σ(scale in log-space) have direct interpretations: μgoverns the central tendency of log-price and σgoverns its dispersion. A random variable P follows a Gamma distribution $\Gamma(\alpha, \beta)$ with shape parameter $\alpha > 0$and scale parameter $\beta > 0$if its density is

$$f(p; \alpha, \beta) = \frac{p^{\alpha-1} e^{-p/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, p > 0.$$

The Gamma distribution arises from additive rather than multiplicative processes: if price is the sum of αindependent exponentially distributed components each with mean β, the result is $\Gamma(\alpha, \beta)$. For integer α, this has the interpretation that price formation involves αdistinct additive quality components. The shape parameter αgoverns skewness (which decreases as αincreases, approaching symmetry for large α), making the Gamma flexible across a range of dispersion profiles. Unlike the log-normal, the Gamma does not require log-transformation for estimation and is directly interpretable in price units.

A random variable Pfollows a Weibull distribution $\mathrm{W}(k, \lambda)$with shape parameter $k > 0$and scale parameter $\lambda > 0$if its density is

$$f(p; k, \lambda) = \frac{k}{\lambda}\left(\frac{p}{\lambda}\right)^{k-1} e^{-(p/\lambda)^k}, p > 0.$$

The Weibull is included for its flexibility in capturing both positively and negatively skewed distributions depending on k: for $k < 1$the distribution is monotonically decreasing; for $k = 1$ it reduces to an exponential; for $k > 1$ it is unimodal and positively skewed; for $k \approx 3.6$it approximates the Normal. Its inclusion is motivated by the observation that sardine prices exhibit negative skewness, an unusual feature for price data that the log-normal and Gamma cannot accommodate.

All four distributions are estimated by maximum likelihood. For a sample $\{p_1, \ldots, p_n\}$of strictly positive prices, the log-likelihood for a distribution with parameter vector $\boldsymbol{\theta}$is

$$\ell(\boldsymbol{\theta}) = \sum_{i=1}^{n} \ln f(p_i; \boldsymbol{\theta}).$$

For the log-normal, the MLEs are $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \ln p_i$and $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(\ln p_i - \hat{\mu})^2$, with closed-form solutions. For the Gamma and Weibull, MLEs are obtained numerically using the L-BFGS-B algorithm with the location parameter fixed at zero ($\mathrm{loc} = 0$) to enforce non-negativity. The Normal is estimated by its sample mean and standard deviation.

Distributional fits are evaluated and compared using three complementary criteria. The Akaike Information Criterion (AIC) is defined as $\mathrm{AIC} = 2k - 2\hat{\ell}$, where $k$is the number of free parameters and $\hat{\ell}$is the maximised log-likelihood. AIC balances goodness of fit against model complexity and is appropriate for comparing non-nested distributions on the same dataset. The best-fitting distribution is that with the lowest AIC, and differences $\Delta\mathrm{AIC} \geq 2$are conventionally taken as meaningful (Burnham et at. 2002). The Bayesian Information Criterion (BIC) is defined as $\mathrm{BIC} = k\ln n - 2\hat{\ell}$. BIC imposes a heavier penalty on model complexity than AIC and is preferred when the objective is identification of the true model rather than prediction. We report both criteria; in practice they agree on the best distribution for all ten species. The Kolmogorov-Smirnov (KS) statistic measures the maximum absolute difference between the empirical cumulative distribution function and the theoretical CDF: $D_n = \sup_p \mid F_n(p) - F(p;\hat{\boldsymbol{\theta}}) \mid$. We report the KS statistic and its associated p-value. We note that for large samples ($n > 1000$) the KS test has very high power and will reject even distributions that fit well visually; we therefore use the KS p-value as a diagnostic indicator rather than a formal acceptance criterion, and supplement it with quantile-quantile (QQ) plots.

For species where single-distribution fitting leaves substantial non-Gaussian residual structure, identified by excess kurtosis greater than 3 or absolute skewness greater than 1 in the residuals from the best single-distribution fit, we estimate a finite mixture of $k$log-normal distributions. The mixture density is

$$f(p;\boldsymbol{\pi},\boldsymbol{\mu},\boldsymbol{\sigma}) = \sum_{j=1}^{k} \pi_j \cdot f_{\mathrm{LN}}(p;\mu_j,\sigma_j),$$

where $\pi_j > 0$, $\sum_{j=1}^{k} \pi_j = 1$, and $f_{\mathrm{LN}}(p;\mu_j,\sigma_j)$is the log-normal density with location $\mu_j$and scale σ_j. The j-th component can be interpreted as a distinct price-generating regime operating within the nominal species category, with mixing weight $\pi_j$representing its prevalence in the market.

Equivalently, working in log-price space $x = \ln p$, the mixture reduces to a Gaussian mixture model:

$$g(x;\boldsymbol{\pi},\boldsymbol{\mu},\boldsymbol{\sigma}) = \sum_{j=1}^{k} \pi_j \cdot \phi\left(\frac{x-\mu_j}{\sigma_j}\right),$$

where $\phi(\cdot)$is the standard normal density. This equivalence allows us to exploit the well-developed theory and algorithms for Gaussian mixture estimation (McLachlan and Peel 2000).

Parameters $\{\pi_j, \mu_j, \sigma_j\}_{j=1}^{k}$are estimated via the Expectation-Maximisation (EM) algorithm (Dempster, Laird and Rubin 1977), which iterates between two steps until convergence. In the *E-step*, the posterior probability (responsibility) that observation $x_i$belongs to component $j$is computed as

$$r_{ij} = \frac{\pi_j \phi\left(\frac{x_i - \mu_j}{\sigma_j}\right)}{\sum_{l=1}^{k} \pi_l \, \phi\left(\frac{x_i - \mu_l}{\sigma_l}\right)}.$$

In the *M-step*, parameters are updated as

$$\pi_j^{\text{new}} = \frac{1}{n}\sum_{i=1}^{n} r_{ij}\,, \mu_j^{\text{new}} = \frac{\sum_{i=1}^{n} r_{ij}\, x_i}{\sum_{i=1}^{n} r_{ij}}, \sigma_j^{\text{new}} = \sqrt{\frac{\sum_{i=1}^{n} r_{ij}\,(x_i - \mu_j^{\text{new}})^2}{\sum_{i=1}^{n} r_{ij}}}.$$

The algorithm is initialised by setting component means at evenly spaced quantiles of the log-price distribution, component standard deviations at $\hat{\sigma}/k$(where $\hat{\sigma}$is the sample standard deviation of log-prices, and mixing weights at $1/k$. Convergence is declared when the change in log-likelihood between successive iterations falls below 10^{-8}. We run the algorithm for $k = 2$and $k = 3$components and select $k$by AIC, also reporting BIC as a robustness check.

Each identified mixture component is interpreted substantively by cross-tabulating component membership, defined as $\hat{\jmath}_i = \arg\max_j r_{ij}$, against observable transaction characteristics including month, buyer identity, and vessel. This allows us to assess whether the latent components identified by the EM algorithm correspond to interpretable market regimes.

To decompose price dispersion into components attributable to observable covariates and an irreducible stochastic residual, we estimate a conditional log-normal model of the form

$$\ln P_{i,s} = \mathbf{x}_{i,s}^{\top}\boldsymbol{\beta}_s + \varepsilon_{i,s}, \varepsilon_{i,s} \sim \mathcal{N}(0, \sigma_s^2),$$

estimated separately for each species $s$by ordinary least squares on log-transformed prices. The covariate vector $\mathbf{x}_{i,s}$includes: month fixed effects (eleven dummies, January as baseline), capturing seasonal variation in supply and demand; size grade dummies (Calibre 2 and 3, grade 1 as baseline), capturing quality-related price differentials; day-of-week dummies (five dummies, Monday as baseline), capturing within-week demand variation; and a binary indicator for transactions by the top five buyers by revenue, capturing buyer market power effects.

The coefficient of determination R_s^2measures the proportion of log-price variance attributable to observable covariates. We define the explained variance share as $R_s^2$and the stochastic variance share as $1 - R_s^2$, and interpret the latter as the irreducible stochastic component of price formation for species s— the component that no deterministic model conditioning on available covariates can eliminate.

The normality of residuals $\hat{\varepsilon}_{i,s}$is assessed using the D'Agostino-Pearson K^2test (D'Agostino et al. 1973), which combines skewness and kurtosis into a single omnibus statistic. Departure from normality in the residuals, particularly excess kurtosis or residual skewness, serves as a diagnostic for mixture structure not captured by the single-component conditional model.

The Law of One Price, in its weak form applicable to a single market, states that $P_{i,s} = P_s^*$for all i, all transactions in species $s$clear at a single price P_s^*. The empirical counterpart of this prediction,

within the distributional framework, is that the price distribution $F_s$is degenerate: a point mass at $P_s^*$with zero variance. We formalise two distinct dimensions of LOP failure.

Temporal non-stationarity is defined as variation in the parameters of $F_s$across time. Under the log-normal model, temporal non-stationarity is characterised by variation in $\mu_{s,t}$across months t: the entire distribution shifts rather than simply adding noise around a stable centre. We test for temporal non-stationarity by estimating the month fixed effects $\hat{\beta}_{s,t}$from the conditional model and interpreting their magnitude and statistical significance as measures of distributional shift. A coefficient $\hat{\beta}_{s,t}$implies a proportional price change of $\exp(\hat{\beta}_{s,t}) - 1$ relative to the baseline month, corresponding to a rightward or leftward shift of the log-normal distribution.

Cross-sectional heterogeneit is defined as variation in $F_s$across quality attributes of nominally identical goods. Under the log-normal model, cross-sectional heterogeneity is characterised by the calibre fixed effects $\hat{\gamma}_{s,c}$: the price distribution for grade $c$fish is a location-shifted version of the grade-1 distribution, with shift $\hat{\gamma}_{s,c} < 0$ reflecting the price penalty for smaller size. We interpret these penalties as evidence that fish of different size grades are not the same economic good, directly violating the LOP's homogeneity assumption.

5. Results

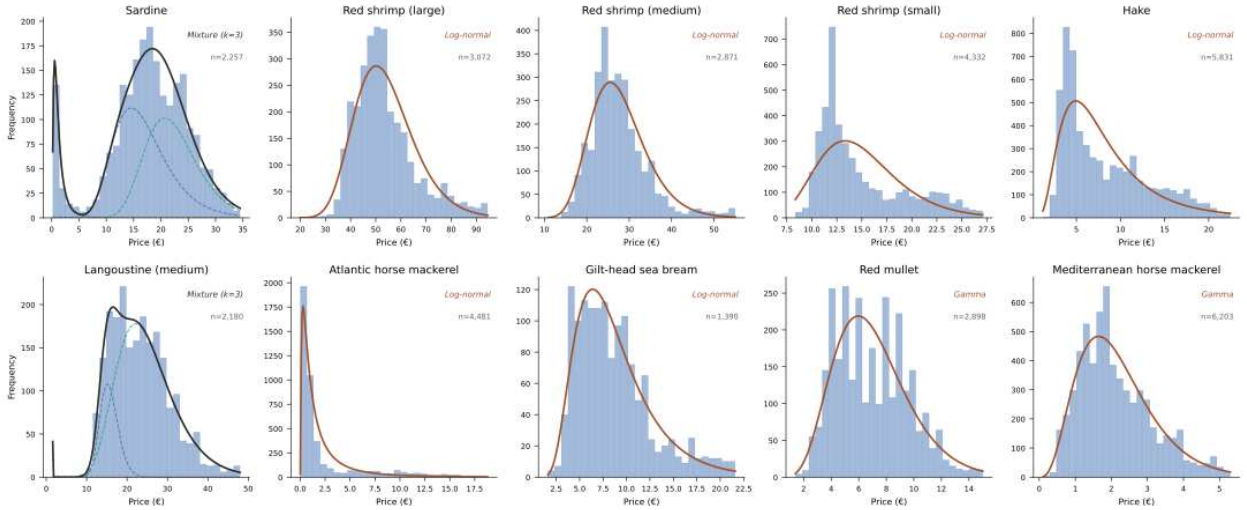
Table 2 presents the results of distributional fitting for the ten analytical species, reporting AIC, BIC, and KS statistics for each candidate distribution. Figure 2 displays, for each species, the empirical price histogram alongside the fitted density of the best-performing distribution.

Table 2. AIC, BIC and KS test results for four candidate distributions across ten species.

Species	n	AIC Normal	KS p Normal	AIC Log-normal	KS p Log-normal	AIC Gamma	KS p Gamma	AIC Weibull	KS p Weibull
Sardine	2,257	**15519 ◄**	0.0000	17971.8	0.0000	16695.4	0.0000	16059.1	0.0000
Red shrimp (l)	3,072	24425.1	0.0000	**23811 ◄**	0.0000	23965.5	0.0000	24776.9	0.0000
Red shrimp (m)	2,871	19610.9	0.0000	**18842.1 ◄**	0.0000	19025.6	0.0000	19952.2	0.0000
Red shrimp (s)	4,332	25480.7	0.0000	**24286.5 ◄**	0.0000	24609.2	0.0000	25587.2	0.0000
Hake	5,831	35482.1	0.0000	**33157.7 ◄**	0.0000	33503.5	0.0000	33896.5	0.0000
Langoustine (m)	2,180	15237.9	0.0000	**14841.5 ◄**	0.0719	14884.1	0.0005	15250.6	0.0000
Atl. Horse mackerel	4,481	25637.9	0.0000	**15591.2 ◄**	0.0000	17371.7	0.0000	17013.6	0.0000
Sea bream	1,390	7989.7	0.0000	**7561.2 ◄**	0.1909	7616.9	0.0188	7774.7	0.0000
Red mullet	2,898	14105.1	0.0000	13803.6	0.0000	**13778.2 ◄**	0.0000	13953.9	0.0000
Med Horse mackerel	6,203	18750.5	0.0000	17340.5	0.0000	**17300.4 ◄**	0.0000	17755.9	0.0000

Notes: AIC = Akaike Information Criterion (lower = better fit). KS p = Kolmogorov-Smirnov test p-value. ◄ marks the best-fitting distribution per species. Green shading = best fit. For large n (>2,000), the KS test has high power; visual fit should be assessed alongside formal tests. Sardine best single-distribution fit is Normal (unconditional); see Table 5 for the mixture model which improves AIC by 15,201

Figure 2: Ten fish, ten histograms, one per species



The results reveal a consistent and interpretable distributional taxonomy across species. The log-normal distribution achieves the lowest AIC for seven of ten species, red shrimp (large), red shrimp (medium), red shrimp (small), hake, Atlantic horse mackerel, gilt-head sea bream, and langoustine (medium), unconditionally. The Gamma distribution provides the best fit for two finfish species, red mullet and Mediterranean horse mackerel. The Weibull distribution achieves the lowest AIC for sardine prices in the unconditional single-distribution comparison; this is superseded by a mixture model that improves the AIC by a margin of 15,201. The Normal distribution ranks last or second-to-last for all ten species, confirming that the symmetric, unbounded-below assumption of standard regression models is empirically inappropriate for fish auction prices.

Auction prices formed through sequential multiplicative adjustments, buyers revising baseline valuations by proportional quality and competitive factors, converge asymptotically to a log-normal distribution. The two Gamma species (red mullet and Mediterranean horse mackerel) are low-value bulk finfish sold in large volumes at relatively stable price points; the Gamma's additive rather than multiplicative structure may reflect the simpler price-forming process for commodity-grade catches where buyer competition is less intense and quality signals less varied.

The log-normal fits are visually excellent for crustacean species. For Orada (sea bream), the KS test cannot reject the log-normal at any conventional significance level ($p = 0.191$, $n = 1{,}390$), making it the single cleanest fit in the dataset. For Large red shrimp and Medium red shrimp, the KS test formally rejects due to the large sample sizes ($n > 2{,}800$) rather than poor visual fit, a well-known limitation of the KS test in large samples. For Sorell, the log-normal fits the central body of the distribution but understates the extreme right tail, consistent with its very high coefficient of variation (CV = 1.594) and strong positive skewness (2.612), which may indicate a mixture structure worth exploring in future work.

Across all ten species, the key distributional moments are informative. Coefficients of variation range from 0.238 (Red shrimp, large) to 1.594 (Atlantic horse mackerel), indicating enormous heterogeneity in price dispersion across species. Skewness is positive for eight of ten species and negative for one (Sardine, -0.387), an anomaly addressed. Excess kurtosis ranges from 0.132

(Lluç) to 7.435 (Atlantic horse mackerel), with high kurtosis values for Atlantic horse mackerel, Red shrimp, medium, and Langoustine pointing toward fat-tailed distributions or latent mixture structure.

Table 3 amd 4 presents the results of the conditional log-normal regression for each species, reporting R^2, the number of observations, and the significance and magnitude of the key covariate groups: month fixed effects, calibre grade effects, day-of-week effects, and the top-buyer indicator.

Table 3. Conditional log-normal regression: model fit and residual diagnostics, all ten species.

Species	n	R²	Adj. R²	F-statistic	Resid. σ	Resid. skewness	Resid. exc. kurtosis	Normality test p	AIC
Sardine	2,257	**0.401**	0.397	88.17***	0.695	-2.714	10.434	<0.0001	4797.5
Red shrimp (large)	3,072	**0.678**	0.676	377.77***	0.126	-0.050	1.382	<0.0001	-3997.0
Red shrimp (medium)	2,871	**0.667**	0.665	317.26***	0.138	-0.171	1.950	<0.0001	-3203.6
Red shrimp (small)	4,332	**0.761**	0.760	762.6***	0.135	0.578	1.995	<0.0001	-4990.6
Hake	5,831	**0.719**	0.719	827.56***	0.314	0.143	0.770	<0.0001	3070.5
Langoustine (medium)	2,180	**0.538**	0.534	139.78***	0.221	-0.408	8.258	<0.0001	-366.4
Atlan horse mackerel	4,481	**0.305**	0.302	108.93***	0.988	0.574	0.492	<0.0001	12645.2
Gilt-head sea bream	1,390	**0.439**	0.432	59.71***	0.347	-0.456	1.513	<0.0001	1039.3
Red mullet	2,898	**0.623**	0.620	263.71***	0.247	0.227	2.285	<0.0001	156.8
Med horse mackerel	6,203	**0.614**	0.612	545.32***	0.318	-0.637	3.032	<0.0001	3437.0

*Notes: Dependent variable is ln(Precio). Covariates: 11 month dummies (Jan=base), calibre dummies (grade 1=base), 5 day-of-week dummies (Mon=base), and a top-5-buyer indicator. F-statistic tests overall model significance. Resid. σ, skewness and excess kurtosis describe the distribution of OLS residuals. Normality test p = D'Agostino-Pearson K² test (p<0.05 rejects normality of residuals). AIC reported for model comparison purposes. Significance: *** p<0.001, ** p<0.01, * p<0.05, ns = not significant.*

Table 4. Variance decomposition and key covariate effects, all ten species.

Species	n	Explained variance	Stochastic variance	Calibre 2 effect	Calibre 3 effect	Top-buyer effect
Sardine	2,257	**40.1%**	**59.9%**	+16.8%**	+81.4%*	+8.6%*
Red shrimp (large)	3,072	**67.8%**	**32.2%**	-14.4%*	—	+2.5%***
Red shrimp (medium)	2,871	**66.7%**	**33.3%**	-26.8%***	-38.5%***	+0.6%ns
Red shrimp (small)	4,332	**76.1%**	**23.9%**	-72.9%***	-74.5%***	-2.1%***
Hake	5,831	**71.9%**	**28.1%**	-43.2%***	-59.7%***	+8.0%***
Langoustine (medium)	2,180	**53.8%**	**46.2%**	-22.2%ns	-32.4%ns	+3.0%**
Atlantic horse mackerel	4,481	**30.5%**	**69.5%**	-3.7%ns	-55.1%***	-34.7%***
Gilt-head sea bream	1,390	**43.9%**	**56.1%**	-31.3%***	-50.4%***	+2.7%ns
Red mullet	2,898	**62.3%**	**37.7%**	-34.6%***	-33.4%***	+1.4%ns
Mediterranean horse mackerel	6,203	**61.4%**	**38.6%**	-12.6%***	-49.7%***	-7.1%***

*Notes: Explained variance = R²×100; Stochastic variance = (1−R²)×100, the irreducible component not attributable to observable covariates. Calibre 2 and Calibre 3 effects = percentage price change relative to grade 1, calculated as (exp(β)−1)×100. Top-buyer effect = percentage price premium/penalty for transactions involving one of the five largest buyers by revenue, relative to all other buyers. — indicates the covariate category is not present for that species (e.g. single-grade species). Significance: *** p<0.001, ** p<0.01, * p<0.05, ns = not significant.*

The explained variance shares (R^2) range from 0.401 (Sardine) to 0.719 (Hake), with a mean of 0.572 across the ten species. Three findings stand out. First, even for the best-fitting conditional model, between 28% and 60% of log-price variance remains unexplained, the irreducible stochastic component identified. This residual is not random noise in the classical sense: as shown below, it exhibits species-specific structure (varying degrees of kurtosis and skewness) that is systematically related to the market mechanism. The persistence of substantial stochastic variance after conditioning on all available covariates is the central empirical motivation for the probabilistic approach: no deterministic model, however rich its covariate set, can reduce price to a predictable quantity in this market. Second, month fixed effects are the dominant source of explained variance for all species, and their direction and magnitude differ markedly across species in ways that carry structural interpretation. For large red shrimp, the December coefficient implies a price premium of $\exp(0.583) - 1 = 79\%$relative to January, and the June coefficient implies a $+59\%$premium, both highly significant ($p < 0.001$). These spikes coincide with reduced fishing effort (winter weather closures in December) and peak restaurant demand (summer tourism in June), shifting the log-normal distribution rightward by nearly 0.6 log-units in both periods. For Lluç (hake), the pattern is opposite and monotonic: the October coefficient implies a price penalty of $\exp(-0.630) - 1 = -47\%$relative to January, with prices declining steadily from Q1 ($\hat{\mu}_{ln} = 2.628$) to Q3–Q4 ($\hat{\mu}_{ln} = 1.692$) as seasonal supply surges. The entire log-normal distribution shifts leftward by 0.94 log-units over the course of the year, the distributions for January and October barely overlap, constituting a stark temporal LOP violation even within a single market and year. For Red Shrimpthe December premium reaches $\exp(0.721) - 1 = 106\%$, the largest month effect in the dataset: the distribution in December has a mean double that of January. Table 5 reports month fixed effects for all species converted to percentage price changes.

Table 5. Month fixed effects expressed as percentage price change vs January baseline.

Species	Jan (base)	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sardine †	0.0	+130.7***	+143.2***	+190.8***	+136.9***	+70.5***	+109.3***	+39.2***	+39.4**	-10.8ns	-87.9***	-79.7***
Red shrimp (large)	0.0	+4.5***	+24.1***	-1.4ns	-4.6***	+58.8***	+16.0***	-2.4*	+21.0***	+21.9***	+20.5***	+79.1***
Red shrimp (medium)	0.0	+7.8***	+21.4***	-2.9**	-7.4***	+29.7***	-5.8***	-13.6***	+18.8***	+18.3***	+28.8***	+105.7***
Red shrimp (small)	0.0	+0.2ns	+12.8***	-5.4***	-0.9ns	+44.3***	+15.4***	+12.9***	+64.6***	+57.4***	+69.8***	+96.2***
Hake	0.0	+3.8ns	+17.4***	+4.4ns	-5.0ns	+8.3**	-24.8***	-40.3***	-39.5***	-46.7***	-33.4***	-22.7***
Langoustine (medium)	0.0	-6.5**	-20.4***	-20.6***	-21.0***	-29.0***	-39.7***	-28.4***	+8.2**	+11.5***	+24.0***	+62.3***
Atlantic horse mackerel	0.0	-5.7ns	+107.9***	+48.4***	+124.8***	+85.9***	+64.7***	+71.7***	+31.4**	-49.5***	-23.2***	-3.5ns
Gilt-head sea bream	0.0	-47.3***	-52.1***	-44.6***	-49.5***	-40.4***	-43.8***	-54.8***	-50.5***	-55.6***	-57.3***	-54.9***
Red mullet	0.0	+21.8***	+28.8***	+30.5***	+40.2***	+38.1***	+29.3***	+16.0***	-18.1***	-29.0***	-35.6***	-30.6***
Mediterranean horse mackerel	0.0	-54.2***	-56.0***	-63.0***	-37.7***	+7.3**	-17.6***	-23.1***	-34.3***	-47.0***	-9.4***	+3.4ns

† Sardine: *month effects shown here come from the same single-equation conditional model as the other species, it shows a single log-normal (and by extension this linear month-effects specification) is a poor characterisation of Sardine prices, which are better described by the three-component seasonal mixture model. The month coefficients shown here are statistically valid but should be read alongside, not instead of, the mixture decomposition.*

*Notes: Values = (exp(β̂ₜ)−1)×100, where β̂ₜ is the month dummy coefficient from the conditional log-normal regression (Table 3), January = 0% baseline. Significance: *** p<0.001, ** p<0.01, * p<0.05, ns = not significant at 5%. Positive values indicate a price premium; negative values indicate a price penalty relative to January. Row shading: pink = see † note above.*

Third, calibre (size grade) effects are large, monotonic, and statistically significant wherever multiple grades are present. For hake, the grade-2 penalty is $\exp(-0.565) - 1 = -43\%$and the grade-3 penalty is $\exp(-0.909) - 1 = -60\%$relative to grade 1, both highly significant ($p < 0.001$). For Red shrimp (large), the grade-2 and grade-3 penalties are $-27\%$and -39%respectively. For Sea bream , grade-3 fish sell at -50%relative to grade-1. These effects are not merely price differences between size categories: they imply that fish of different size grades traded under the same species description are not the same economic good, and that the LOP's homogeneity assumption is violated in the cross-section as directly as it is violated temporally. The calibre gradient is steepest for finfish (hake, sea bream) and shallowest for crustaceans (Red shrimp large), where only one grade is effectively present), consistent with the greater size-quality sensitivity of finfish buyers.

The top-buyer indicator is statistically significant for Red Shrimp large ($\hat{\beta} = 0.025$, $p < 0.05$, implying a $+2.5\%$price premium paid by the five largest buyers) and Hake ($\hat{\beta} = 0.077$, $p < 0.001$, implying $+8\%$). This is a market structure result: the largest buyers pay systematically higher prices, consistent with either quality sorting (large buyers preferentially selecting premium lots within a grade) or oligopsonistic behaviour in which dominant buyers can sustain above-market bids to secure preferred supply relationships. Disentangling these mechanisms is beyond the scope of the present paper but warrants future investigation. Day-of-week effects are small and generally insignificant across species, suggesting that within-week demand variation does not materially affect auction clearing prices at this market.

The conditional log-normal model leaves residuals with severe non-normality for two species, Sardine (residual skewness $= -2.714$, excess kurtosis $= 10.434$) and Langoustine (residual skewness $= -0.408$, excess kurtosis $= 8.258$), indicating that a single log-normal distribution, even after conditioning on all available covariates, is inadequate to characterise the price-generating process for these species. We therefore estimate three-component log-normal mixture models as described.

The Sardine in Table 5 presents the mixture model results for sardine prices. The three-component mixture achieves an AIC of 2,770.7 against 17,971.8 for the best single distribution (log-normal), a reduction of $\Delta\text{AIC} = 15{,}201$— an overwhelming improvement that leaves no ambiguity about the superiority of the mixture representation. The BIC confirms this: $\Delta\text{BIC} = 15{,}166$.

Table 5. Three-component log-normal mixture model results for Sardine and Langoustine (medium grade).

Species	Component	Interpretation	π (weight)	μ (ln-space)	σ (ln-space)	Mean price	AIC mixture	AIC single LN	ΔAIC
Sardine	1	Damaged/off-season lots	8.7%	0.030	0.799	€1.03	**2770.7**	17971.8	**15201.1**
	2	Normal-season lots	47.8%	2.774	0.309	€16.03			
	3	Peak-season lots (spring)	43.5%	3.080	0.223	€21.77			
Langoustine	1	Outlier/sub-grade lots	0.0%	0.470	0.010	€1.60	**1186.9**	14841.5	**13654.6**

Species	Component	Interpretation	π (weight)	μ (ln-space)	σ (ln-space)	Mean price	AIC mixture	AIC single LN	ΔAIC
	2	Sub-standard grade M	16.5%	2.739	0.144	€15.47			
	3	Standard grade M lots	83.4%	3.185	0.292	€24.16			

Notes: Parameters estimated via Expectation-Maximisation (EM) algorithm on log-transformed prices. μ and σ are the location and scale parameters of each log-normal component in log-price space; mean price = exp(μ + σ²/2). π = mixing weight (proportion of transactions assigned to each component). AIC mixture = AIC for 3-component mixture (8 free parameters); AIC single LN = AIC for single log-normal fit (2 parameters). ΔAIC = AIC_single − AIC_mixture (positive values indicate mixture improvement). Purple shading indicates mixture-model species.

The three components are interpretable as distinct market regimes. Component 1 ($\hat{\pi}_1 = 8.7\%$, $\hat{\mu}_1 = 0.017$in log-space, implied mean price ≈€1.02/box) captures a small but persistent population of very low-price lots, damaged catches, end-of-day clearance, or heavily off-season supply, traded at prices an order of magnitude below the market norm. Component 2 ($\hat{\pi}_2 = 47.8\%$, implied mean ≈€16.03/box) corresponds to normal-season transactions, concentrated between February and October. Component 3 ($\hat{\pi}_3 = 43.5\%$, implied mean ≈€21.77/box) represents peak-season lots, concentrated in the spring months (March–June), when sardine supply is at its maximum and buyer competition is most intense.

Cross-tabulation of component membership against month confirms this seasonal interpretation. The probability of assignment to Component 3 (peak-season) approaches 1.0 in April and falls monotonically to 0.0 by November–December, when sardine transactions essentially cease and the small residual supply consists almost entirely of Component 1 lots. November and December are assigned entirely to Component 1 ($\bar{r}_{i,1} = 1.0$), reflecting the off-season character of sardine supply in those months.

The negative unconditional skewness of sardine prices (-0.387) is now interpretable within the mixture framework: it reflects the superposition of a dominant right-skewed regime (Component 3, spring peak) with a long left tail generated by the very low-price Component 1 lots. The mixture resolves an apparent anomaly, negative price skewness, which is theoretically unusual and cannot be accommodated by log-normal or Gamma distributions, by revealing it as an artefact of regime superposition rather than a feature of any individual price-generating process.

The langoustine for Langoustine, the three-component mixture improves AIC by $\Delta\text{AIC} = 13{,}655$over the best single distribution. The dominant component ($\hat{\pi}_3 = 83.4\%$, implied mean ≈€24.16/unit) corresponds to standard-grade transactions. A secondary component ($\hat{\pi}_2 = 16.5\%$, implied mean ≈€15.47/unit) captures sub-standard lots within the nominal grade M category, likely reflecting within-grade quality variation in presentation or freshness that is not captured by the available quality codes. A negligible third component ($\hat{\pi}_1 \approx 0\%$) represents isolated outlier transactions. The mixture structure for langoustine thus reveals a latent quality segmentation within a single nominal grade that is invisible to single-distribution fitting and not captured by the administrative quality variables in the dataset.

Table 6 summarises the distributional taxonomy, reporting for each species the best-fitting distribution, its estimated parameters, and a brief structural interpretation.

Table 6. Distributional taxonomy: best-fitting distribution, parameters, and structural interpretation.

Species	Unit	Best distribution	Parameters	CV	Skewness	Structural interpretation
Sardine	€/box	**Mixture (k=3)**	π_1=8.7%, μ_1=0.017, σ_1=0.786 | π_2=47.8%, μ_2=2.773, σ_2=0.309 | π_3=43.5%, μ_3=3.079, σ_3=0.223	0.421	-0.387	Seasonal regime heterogeneity
Red shrimp (large)	€/kg	**Log-normal**	μ=3.965, σ=0.221	0.238	1.229	Multiplicative process; low dispersion premium species
Red shrimp (medium)	€/kg	**Log-normal**	μ=3.296, σ=0.238	0.265	1.739	Occasional near-premium lots inflate right tail
Red shrimp (small)	€/kg	**Log-normal**	μ=2.668, σ=0.277	0.305	1.165	Log-normal consistent across all three red shrimp grades
Hake	€/kg	**Log-normal**	μ=1.948, σ=0.592	0.606	0.971	Highest CV; strong seasonal shift (Δμ=0.94 Jan–Oct)
Norway lobster (medium)	€/kg	**Mixture (k=3)**	π_2=16.5%, μ_2=2.738, σ_2=0.144 | π_3=83.4%, μ_3=3.184, σ_3=0.292	0.337	1.220	Latent quality segmentation within nominal grade M
Atlantic horse mackerel	€/kg	**Log-normal**	μ=0.150, σ=1.185	1.594	2.612	Highest dispersion; possible mixture for future work
Gilt-head sea bream	€/kg	**Log-normal**	μ=2.069, σ=0.463	0.485	1.227	Cleanest fit: KS p=0.191; textbook log-normal case
Red mullet	€/kg	**Gamma**	α=6.568, β=1.072	0.392	0.701	Additive process; α≈7 independent quality components
Mediterranean horse mackerel	€/kg	**Gamma**	α=4.160, β=0.521	0.506	1.227	Additive process; lower α reflects greater price uncertainty

Notes: Best distribution selected by lowest AIC. Parameters reported for the best-fitting distribution: log-normal (μ, σ in log-price space); Gamma (shape α, scale β); mixture (component weights π and log-space parameters μ, σ for each component). CV = coefficient of variation (SD/Mean) of raw prices. Skewness = Pearson moment coefficient of skewness of raw prices. Cell shading: blue = log-normal; green = Gamma; purple = mixture (k=3). Data source: Confraria de Pescadors de Barcelona, 2008.

Three findings emerge from the analysis as a whole. First, fish auction prices follow well-defined parametric distributions whose family membership carries structural information about the price-generating process. The log-normal family, consistent with Gibrat's multiplicative process, dominates for premium species traded in competitive auctions with strong quality-grading. The Gamma family, consistent with an additive error structure, characterises bulk commodity finfish. This species-level distributional heterogeneity is itself a finding: the market does not operate as a

single price-forming mechanism but as a collection of structurally distinct sub-markets differentiated by species biology, quality-grading conventions, and buyer composition.

Second, the parameters of the best-fitting distributions are non-stationary across time in a systematic and interpretable way. Month fixed effects shift the log-normal location parameter by up to 0.94 log-units within a single year for hake and generate December premiums of 79–106% for crustaceans. This temporal non-stationarity is the primary mechanism of LOP failure in this market and cannot be attributed to search costs or information asymmetries, it is a structural consequence of the interaction between biological supply seasonality and inelastic short-run demand.

Third, for two species, the distributional evidence points unambiguously to the coexistence of multiple distinct price-generating regimes within a single nominal market. The mixture model identifies these regimes with precision, seasonal for sardine, quality-segmentation for langoustine, and the implied AIC improvements are sufficiently large ($\Delta\text{AIC} > 13{,}000$ in both cases) to leave no statistical ambiguity. These findings suggest that the concept of a single market price is not merely empirically violated but theoretically inappropriate for species exhibiting regime heterogeneity: the price is not a number but a distribution, and for mixture species it is not even a unimodal distribution.

6. Discussion and Limitations

The results presented in Section 5 allow us to advance a probabilistic reinterpretation of price formation in the Barcelona auction fish market that departs in three respects from the dominant deterministic framework in the fish market economics literature. The first departure concerns the appropriate unit of analysis. The standard hedonic regression model treats price as a scalar, a number determined by observable attributes plus an additive error term, $P_{i,s} = \mathbf{x}_{i,s}^{\top}\boldsymbol{\beta}_s + \varepsilon_{i,s}$. Our results suggest that a more accurate representation treats price as a random variable drawn from a species-specific probability distribution F_s, whose family membership, parameters, and conditional structure together constitute the complete description of market price formation. The second departure concerns the interpretation of price dispersion. Within the deterministic framework, dispersion is residual, a measure of how much the model fails to explain. Within the probabilistic framework, dispersion is structural, a property of the price-generating mechanism that persists even after conditioning on all observable covariates. Our conditional models explain 40–72% of log-price variance, leaving between 28% and 60% as an irreducible stochastic component. This stochastic residual is not noise in the classical sense: it is well-characterised, species-specific in magnitude, and approximately log-normally distributed for crustaceans and fat-tailed for pelagic species. It reflects the genuine uncertainty inherent in a thin auction market where each lot is unique, buyers' valuations are private, and competitive outcomes cannot be predicted from observable market conditions alone. This finding parallel results in the literature on financial markets, where a substantial stochastic component in price formation persists even after controlling for known risk factors (Mantegna et al. 1999; Gabaix et al. 2003) and suggests that fish auction markets share fundamental statistical properties with financial markets despite their very different institutional structure. The third departure concerns the mechanism of LOP failure. Our results identify two structurally distinct mechanisms, each operating along a different dimension of price variation and each having a different theoretical implication.

The theoretical interpretation is straightforward within the probabilistic framework but difficult to express within the LOP's deterministic language. The LOP predicts a single price P_s^*; our results show that for species with strong seasonal supply cycles, the appropriate concept is not a price but a price process $\{F_{s,t}\}_{t=1}^{12}$, a sequence of distributions indexed by month, each with distinct parameters. The failure of the LOP in the temporal dimension is thus not a deviation from equilibrium but a consequence of the biological non-stationarity of supply: fish populations spawn, grow, and are harvested on seasonal cycles that no market mechanism can arbitrage away, because the underlying good, fresh fish of a given species, size, and freshness, is categorically different in January and October in ways that have nothing to do with search costs or information asymmetries. This observation connects to the broader literature on commodity price non-stationarity. Deaton et al. (1992) showed that the interaction of seasonal supply and inelastic short-run demand produces systematic price cycles in storable commodities; for perishable goods, the absence of storage removes the only mechanism that could smooth these cycles, making temporal price dispersion structurally unavoidable.

The mixture model results for Sardine and Langoustine raise a more fundamental challenge to the LOP than either temporal non-stationarity or cross-sectional heterogeneity considered individually. For these species, the evidence points to the coexistence of multiple distinct price-generating processes operating simultaneously within a single nominal market, a finding that the LOP, formulated in terms of a single equilibrium price, has no language to accommodate.

For Sardine, the three mixture components correspond to three identifiable market regimes. The dominant peak-season regime (Component 3, $\hat{\pi}_3 = 43.5\%$, mean ≈€21.77/box) operates from March to June when sardine supply is maximal and buyer competition is most intense. The normal-season regime (Component 2, $\hat{\pi}_2 = 47.8\%$, mean ≈€16.03/box) operates outside the peak period when supply is regular but buyer willingness to pay is lower. The distressed regime (Component 1, $\hat{\pi}_1 = 8.7\%$, mean ≈€1.03/box) captures damaged catches, end-of-day clearance, and residual off-season supply, a qualitatively different product despite nominally identical species classification. The negative unconditional skewness of sardine prices, which is anomalous for a price distribution, is now interpretable as an artefact of this regime structure: the long left tail generated by Component 1 pulls the unconditional skewness negative, masking the positive skew of each individual component.

The market regime interpretation has implications beyond statistical description. It suggests that sardine buyers and sellers do not operate in a single unified market but in three overlapping sub-markets with distinct pricing norms, competitive dynamics, and buyer-seller relationships. The equilibrium concept appropriate to this market is not a single price but a mixture distribution, a probability law that assigns different likelihoods to different price ranges depending on the seasonal state of the market. This is a fundamentally different picture from the one implied by the LOP, and a more accurate one.

For Langoustine, the mixture structure has a different origin: it reflects latent quality segmentation within a nominal grade rather than seasonal regime switching. The 16.5% secondary component at ≈€15.47/unit likely represents lots that are classified as grade M by the auction administration but whose actual quality, in terms of size uniformity, presentation, or freshness, places them closer to grade P. The mixture model recovers this hidden quality tier from price data alone, without

access to more detailed quality information than the available administrative records provide. This is an example of the mixture model serving a measurement function: revealing quality variation that official classification systems fail to capture.

Our findings extend the well-documented prevalence of log-normal and Gamma price distributions in financial and commodity markets to the specific institutional context of a centralised fish auction. The dominance of the log-normal for seven of ten species is consistent with the broad conclusion of the literature that multiplicative stochastic processes generate log-normal distributions in price-forming systems (Mantegna and Stanley 1999). The two Gamma species (Moll and Maira) fit the additive interpretation noted by Drăgulescu and Yakovenko (2001) for income distributions, where the shape parameter carries information about the number of independent components in the price-forming process.

The mixture model results add a dimension that has received less attention in the econophysics literature on goods markets, though it is well-established for financial return distributions. Scalas et al. (2006) documented mixture structure in financial transaction timing; our results suggest that price levels, not just transaction timing, can exhibit mixture structure in thin auction markets. The mechanism is different: in financial markets, mixture structure in returns is typically attributed to volatility regimes; in our fish market, it is attributed to seasonal supply regimes and latent quality tiers. But the statistical structure, a finite mixture of log-normal distributions with identifiable component interpretations, is the same.

The AIC improvements from mixture fitting ($\Delta\text{AIC} = 15{,}201$ for Sardine, $\Delta\text{AIC} = 13{,}655$for Escamarla M) are substantially larger than those typically reported when mixture models are applied to financial return. We attribute this to two features of the fish auction context. First, the seasonal structure of supply creates sharper regime boundaries than the volatility regimes of financial markets, the sardine season has a biological on/off character that produces cleaner component separation. Second, the thinness of the market (82,947 transactions across ten species over one year, compared to millions of financial transactions in the same period) means that individual lot characteristics, freshness, vessel, buyer, have larger price effects than in liquid financial markets, increasing within-regime dispersion and making between-regime differences relatively larger.

Several limitations of the present analysis should be acknowledged. The dataset covers a single market in a single year (2008). While this provides analytical depth, the administrative completeness and granularity of the data are unusual in the fish market literature, it limits the generalisability of our findings. Whether the distributional taxonomy identified here is stable across years, or whether the mixture components for Sardine and Langoustine reflect year-specific conditions, cannot be determined from a single year of data. Extending the analysis to multiple years, ideally spanning the implementation of EU fisheries policy reforms in 2013, would allow distributional stability to be tested formally.

The analysis is confined to one market. Spanish Mediterranean auctions lonjas share broadly similar institutional structures, but their species composition, buyer populations, and seasonal patterns differ. A comparative study across lonjas, for example, comparing Barcelona with Valencia or Palamos, which specialise in different species profiles, would allow us to assess

whether the log-normal dominance is a property of the auction mechanism per se or of the specific species mix at Barcelona. The conditional model uses the quality variables available in the administrative records, primarily size grade (*Calibre*), but does not have access to more granular quality indicators such as fishing gear, sea conditions, hours since catch, or individual lot inspection scores. The unexplained stochastic variance (28–60%) almost certainly contains a quality component that more detailed data would reduce. At the same time, the persistence of substantial stochastic variance even with richer quality data is likely, given the fundamental uncertainty of auction outcomes: even identical lots sell at different prices depending on who bids, in what order, and on what day. The mixture model components are identified statistically and interpreted substantively, but the interpretation is not formally tested. Future work could validate the seasonal interpretation of Sardine components by estimating the mixture separately by season and testing whether the component parameters converge to the mixture solution. The quality-segmentation interpretation of Langoustinecould be validated by matching transaction records with independent quality assessments, if such data can be obtained from the Confraria de Pescadors de Barcelona. Finally, the analysis does not address the dynamic properties of the price distributions, whether they exhibit serial correlation, volatility clustering, or other time-series features documented in financial markets, Cont (2001). The transaction-level data with precise daily timestamps would support an intraday and intra-seasonal analysis of price dynamics that goes beyond the cross-sectional distributional characterisation presented here.

7. Conclusion

This paper has investigated price formation at the Barcelona wholesale fish auction (*Confraria de Pescadors de Barcelona*) using a complete administrative dataset of 82,947 transactions covering the calendar year 2008. Departing from the deterministic hedonic framework that dominates the fish market economics literature, we have adopted a probabilistic approach in which auction prices are treated as realisations of species-specific random variables drawn from well-defined probability distributions. The analysis has proceeded in three stages: unconditional distributional fitting across ten species; conditional variance decomposition using a log-normal regression framework; and finite mixture modelling for species exhibiting non-standard residual structure.

The principal findings can be summarised as follows.

Finding 1: Fish auction prices follow identifiable parametric distributions whose family membership carries structural information about the price-generating process. The log-normal distribution provides the best fit for seven of ten species. The Gamma distribution fits two low-value bulk finfish species, Moll and Maira, suggesting an additive rather than multiplicative price-forming structure for commodity-grade catches. The Normal distribution, which underlies standard regression approaches, ranks last for all ten species, confirming that the symmetric unbounded-below assumption is empirically inappropriate for fish auction prices. Orada (sea bream) provides the cleanest single-distribution fit in the dataset, with KS $p = 0.191$ for the log-normal, a result that cannot be rejected at any conventional significance level and constitutes a near-textbook example of log-normal price formation.

Finding 2: The Law of One Price fails along two structurally distinct dimensions, each with a different theoretical interpretation. Temporal non-stationarity, the systematic shift of price

distribution parameters across months, is the dominant mechanism of LOP failure. The log-normal location parameter for hake (Lluç) falls by 0.94 log-units between January and October, equivalent to a halving of the median price. Medium red shrimp achieves a December premium of 106% relative to January. These distributional shifts are the statistical signature of biological supply non-stationarity interacting with inelastic short-run demand, a mechanism that no market institution can arbitrage away for a perishable, non-storable good. Cross-sectional heterogeneity, the systematic price differentials associated with size grade, is the secondary mechanism. Grade penalties of up to 60% for small hake relative to large hake confirm that goods traded under the same species description are not economically homogeneous, directly violating the LOP's foundational assumption.

Finding 3: A conditional log-normal model explains 40–72% of log-price variance, leaving an irreducible stochastic component of 28–60% that no deterministic model can eliminate. This stochastic residual is not random noise: it is well-characterised, species-specific in magnitude, and approximately normally distributed in log-space for crustaceans. It reflects the genuine uncertainty inherent in a thin auction market, the unpredictability of individual lot outcomes arising from private buyer valuations, competitive dynamics, and lot-specific characteristics that no administrative record can fully capture. The persistence of this irreducible stochastic component is the empirical foundation of the probabilistic approach: it demonstrates that price in this market is not a deterministic function of observables but a random variable with a well-defined distributional law.

Finding 4: For two species, Sardine and Langoustine, the evidence points to the coexistence of multiple distinct price-generating regimes within a single nominal market, best captured by three-component log-normal mixture models. For Sardine, the mixture components correspond to identifiable seasonal regimes (damaged/off-season, normal-season, peak-season), with AIC improvement of 15,201 over the best single distribution. The negative unconditional skewness of sardine prices, anomalous for a price distribution, is revealed as an artefact of regime superposition rather than a property of any individual pricing process. For Langoustine, the mixture uncovers latent quality segmentation within a nominal grade that the administrative quality codes fail to capture. In both cases, the mixture model serves not merely as a statistical improvement but as a diagnostic tool: it reveals market structure that is invisible to single-distribution analysis.

The findings have three theoretical implications that extend beyond the fish market context. The first concerns the appropriate probabilistic language for the Law of One Price (LOP). The LOP is conventionally stated as a prediction about price levels: $P_{i,s} = P_s^*$ for all transactions $i$in species s. Our results suggest that for perishable goods in thin auction markets, the appropriate statement is distributional: the LOP predicts a degenerate distribution, a point mass, while the empirical distribution is continuous, parametric, and non-stationary. Price dispersion is not a deviation from an equilibrium point but a property of the equilibrium distribution. The second implication concerns the dominance of the log-normal for seven of ten species provides empirical support for a multiplicative model of price formation in auction settings: buyers treat their bid as a proportional adjustment to a baseline valuation, and the product of many such adjustments, across quality signals, competitive pressure, and private information, converges to a log-normal in the limit. The third implication concerns the use of mixture models as market structure diagnostic tools. Standard distributional analysis asks: what is the best single distribution for this price series? Our mixture

analysis asks: how many distinct price-generating processes coexist in this market, what are their parameters, and what proportion of transactions does each account for? The answer for Sardine, three regimes, clearly separated in log-space, with distinct seasonal signatures, provides a richer characterisation of market structure than any regression coefficient. We suggest that finite mixture models of log-normal distributions deserve wider application in the analysis of thin commodity markets wherever seasonal supply heterogeneity or latent quality segmentation may generate regime-specific pricing.

The methodological contribution of the paper is the application of a probabilistic distributional framework, combining parametric distribution fitting, Gaussian mixture modelling via the EM algorithm, and conditional variance decomposition, to transaction-level fish auction data. This framework bridges three literatures that have developed largely in isolation: the econophysics literature on price distributions and statistical laws in markets; the fish market economics literature on hedonic price formation; and the statistical literature on finite mixture models. By connecting these literatures, the paper demonstrates that the tools of distributional fitting, mixture modelling, and the search for statistical regularities, are productive in institutional commodity markets as well as in financial markets, and that the resulting findings carry economic interpretations that enrich our understanding of market structure.

The complete analytical pipeline, from raw administrative records through data cleaning, distributional fitting, mixture estimation, and conditional modelling, is reproducible and transferable. The methodology could be applied without modification to fish auction data from other lonjas, other countries, or other time periods, generating comparable distributional taxonomies that would allow cross-market and cross-temporal comparisons of price formation mechanisms. We invite such applications and make the replication code available upon request.

7.5 Concluding remarks

The auction fish market is, in one sense, a simple institution: buyers and sellers meet daily, lots are auctioned in sequence, and prices are recorded. In another sense, it is a complex adaptive system in which biological supply cycles, heterogeneous product quality, private buyer valuations, and competitive dynamics interact to produce price outcomes that are neither deterministically predictable nor randomly scattered, but drawn from well-defined probability distributions that encode the structure of the market itself.

The central argument of this paper is that price in such a market is best understood not as a number but as a distribution, and for some species, not even a unimodal distribution but a mixture of distributions reflecting distinct coexisting regimes. This is not merely a statistical observation. It is a statement about the nature of price formation in thin perishable goods markets: that the Law of One Price fails not because markets are irrational or inefficient, but because the conditions it presupposes, homogeneous goods, no transaction costs, perfect information, temporal stationarity, are systematically violated by the biology of fish, the seasonality of fishing, and the institutional structure of the auction. Recognising and characterising this failure precisely, using the distributional tools of econophysics, is the contribution this paper has attempted to make.

We hope that the framework developed here, and the distributional taxonomy it produces, will prove useful to researchers studying price formation in other thin commodity markets where perishability, heterogeneity, and seasonal non-stationarity combine to produce price distributions that no deterministic model can adequately represent.

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