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Feature driven optimization design for free-form bulge-formed joint

based on Kriging model

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Abstract:

This paper proposes a feature layering-based forward-reverse modeling method aimed at optimizing free-form bulge-formed joints. Beginning with the physical model of the joint, surface features are decomposed into six key global and local shape parameters on the basis of feature layering to establish an initial parametric surface, which is then discretized into point clouds and reconstructed into a Non-Uniform Rational B-Splines (NURBS) model utilizing the Least Square Progressive and Iterative Approximation (LSPIA) algorithm. An orthogonal experimental design is employed to sample the parameter space. Subsequently, a Kriging model is constructed to capture the nonlinear relationships between the shape parameters and joint performance, including mechanical properties and fairness. Through comparison with polynomial response surface models and Support Vector Machine (SVM) models the accuracy of the Kriging model is demonstrated, which is thereby facilitating the optimization of the joint surface design. The optimized free-form surface model demonstrates the effectiveness of the proposed method and provides a basis for designing similar free-form surface structures.

Key words: Feature layering modeling method; Forward and reverse modeling; Free-form surface; Bulge-formed joint; Optimization

0 List of symbols

Symbol	Definition	Symbol	Definition
L_B	Length of bulge plate	$d_i/d_{i,j}$	Control vertex of the NURBS curve/surface
R_B	Radius of bulge plate	$R_{i,k}(u) / R_{i,k;j,l}(u,v)$	k -degree NURBS rational base function for curve/surface
θ_B	Spread angle of the semi-cylinder	$w_i/w_{i,j}$	Weight factor for curve/surface

H_b	Height of the bulge	$N_{i,k}(u) /$ $N_{j,l}(v)$	k -degree B-spline basis function
L_b	Length of the bulge	$\mathbf{U} / \mathbf{V}$	Node vector
L_T	Length of the transition segment	$\Delta_{i,j}^k$	Error between the surface and the contour points
τ_1	$\tau_1 = L_b / L$	ε	Convergence threshold
τ_2	$\tau_2 = L_T / L_b$	e^k	Sum of the norm values of the error $\Delta_{i,j}^k$
τ_3	Proportion of bulge width	$\mathbf{R}$	Matrix of $R_{i,j}(u, v)$
α	Chord angle	$y_K(\mathbf{x})$	Function of Kriging model
β	Brace angle	$f_K(\mathbf{x})$	Basis function of Kriging model
f_y	Yielding stress	β_K	Coefficient of the basis function $f_K(\mathbf{x})$
f_u	Ultimate strength	$Z_K(\mathbf{x})$	Stationary random function
ε_1	Yield strain	$\mathbf{R}_K$	Spatial correlation matrix
ε_2	Strain at the onset of strain hardening	$\hat{\sigma}_K^2$	Estimated variance
$\varepsilon_3 / \varepsilon_4$	Strain at ultimate strength	θ_K	Modified parameter in Kriging model

SS	Sum of the squared deviation	R^2	Negative correlation coefficient
df	Degree of the freedom of SS		
MS	Mean square		

1. Introduction

The surface shapes of industrial products can be broadly categorized into two types: the first consists of elementary analytical surfaces, such as spheres, cylinders, and cones, while the second encompasses free-form surfaces, which are complex, varied, and challenging to fully represent using traditional drawing board geometry and mechanical drafting techniques. In recent years, free-form surface structures have gained widespread application in engineering fields, including machinery and architecture [1-2], due to their flexible and versatile shapes, lightweight structures, high smoothness, and the integration of artistic and engineering principles. These structures are not only aesthetically appealing but also exhibit excellent mechanical properties, as illustrated in Fig. 1.



(a) Fan blades



(b) Pearl-shaped building

Fig. 1 Free-form surface structures

Free-form surfaces are generally complex in shape and exhibit a high degree of surface freedom, making it difficult to directly characterize their shape information using conventional analytical functions. With advancements in Computer Aided Geometric Design (CAGD) technology, numerous methods for describing free-form surfaces have been continuously developed, including B-spline surfaces, implicit surfaces, subdivision surfaces, Non-Uniform Rational B-Splines (NURBS), etc. The NURBS method, proposed and studied by Piegl and Tiller et al. [3-4], builds upon B-spline curves. To address the challenges of representing elementary analytic surfaces with B-spline curves, the NURBS method enables a unified approach to describe both elementary analytic and free-form surfaces through the design of control points, node vectors, and weight factors. This method allows for accurate and consistent representation of various curves and surfaces while facilitating efficient shaping and modification. Consequently, it has become a widely adopted mainstream method for surface description.

However, these methods overlook the direct connection between surface shape parameters and mathematical models of surface. Due to the extremely complex mathematical forms of NURBS, designers often find it difficult to efficiently modify and remodel surface models based on the requirements of surface shape parameters. Therefore, a design approach that on the basis of the shape features has been proposed, which combines free-form surface features under different parameters and type templates to obtain free-form surface models. Nyirenda et al. [5] summarized the conditions that feature-based surface modeling methods should meet: (1) The features should be easy to edit with complete information retention; (2) The features should be easily parameterized and encompass a wide variety with a minimal number of parameters; (3) The features should allow for easy control and possess a high degree of shape freedom; (4) The operations related to features

should be simple and diverse. Vergeest et al. established a point-based feature representation scheme for free-form surfaces, utilizing feature points to represent surface features and providing a parameterized form for analysis [6]. Pernot utilized the properties of B-spline surfaces to define the range of surface variation through boundary lines and target lines, proposing a user-driven surface feature deformation method while offering predefined template features [7]. He Kunjin [8] proposed a surface feature representation method centered on feature lines to address the challenges of parameterized design of surface features. The feature lines are defined, represented, and parameterized as key elements in surface modeling, with the feature line information used to control both the overall shape and the detailed shape of surface features. Additionally, Reference [9] proposed a feature-based method to establish parameterized templates of the typical anatomical features of the femur and obtain a personalized plate by optimizing the initial plate. However, the article does not clarify how to solve curves based on parameters. Currently, the concept of feature modeling is predominantly concentrated in the theoretical stage, with limited application in engineering.

Additionally, in engineering, reverse expression often entails addressing challenges such as acquiring data point sets and reconstructing surface. In forward design, a common NURBS parametric model is derived from an actual physical model using a limited number of shape parameters. Consequently, it is essential to extract obvious features and provide convenient methods for modifying these features. The application of the NURBS method based on feature modeling in forward design is relatively limited, and its modeling techniques typically lack universality. In contrast to conventional reverse engineering methods, forward design is driven by product requirements. There are primarily three approaches: (1) Determine the cross-sectional shape based on design specifications, and subsequently employ cross-sectional or skin methods to derive the overall surface shape, as exemplified in wind turbine blade and ship design. [10]; (2) Establish partial differential equations or matrix equations according to design requirements to directly compute the spatial positions of surface points, thereby obtaining a point cloud dataset, which can then be fitted to create NURBS surface models, as seen in gear design [11]; (3) For relatively simple surface shapes, the parameters of NURBS surfaces can be directly designed to establish the corresponding surface shapes [12]. Currently, these methods are contingent upon prior empirical theories in related fields, resulting in a lack of universality in engineering surface design.

The establishment of parameterized surface model is merely the initial step in addressing related challenge. It is essential to optimize and adjust this model to achieve an aesthetically pleasing surface. Cui et al. proposed a structural morphology method using Bezier surfaces, which optimizes the mechanical properties of the surface [13]. Yang et al. employed the NURBS surface method along with an isogeometric approach to enhance the structural strain energy of free-form surface[14]. However, due to the complexity and diversity of free-form surface shapes as well as the non-linear relationship between shape features and mechanical properties, approximate models are required to accurately reflect these relationships. Additionally, a comprehensive evaluation of both mechanical and smoothness attributes is necessary.

Therefore, this paper proposes a layered forward and reverse hybrid modeling scheme for the free-form surface structure of bulge-formed joints, addressing the engineering requirements for surface shape design. The approach is based on feature modeling and surface modeling techniques in reverse engineering. The effectiveness of this method is demonstrated through the surface modeling and optimization design process for bulge-formed joint. Section 2 introduces the engineering background of bulge-formed joint and the overall scheme for surface feature layering modeling specific to bulge-formed joints, and then illustrates the acquisition of the initial feature surface of the bulge-formed joint and discusses the related theory of surface reconstruction, culminating in the development of the NURBS model for the bulge-formed joint. Section 3 presents the optimization design of the bulge-formed joint based on the Kriging model, focusing on shape features. Section 4 presents the optimization design of the bulge-formed joint based on the Kriging model, focusing on shape features. The conclusion is located in Section 5. Fig. 2 provides an overview of the framework of this article.

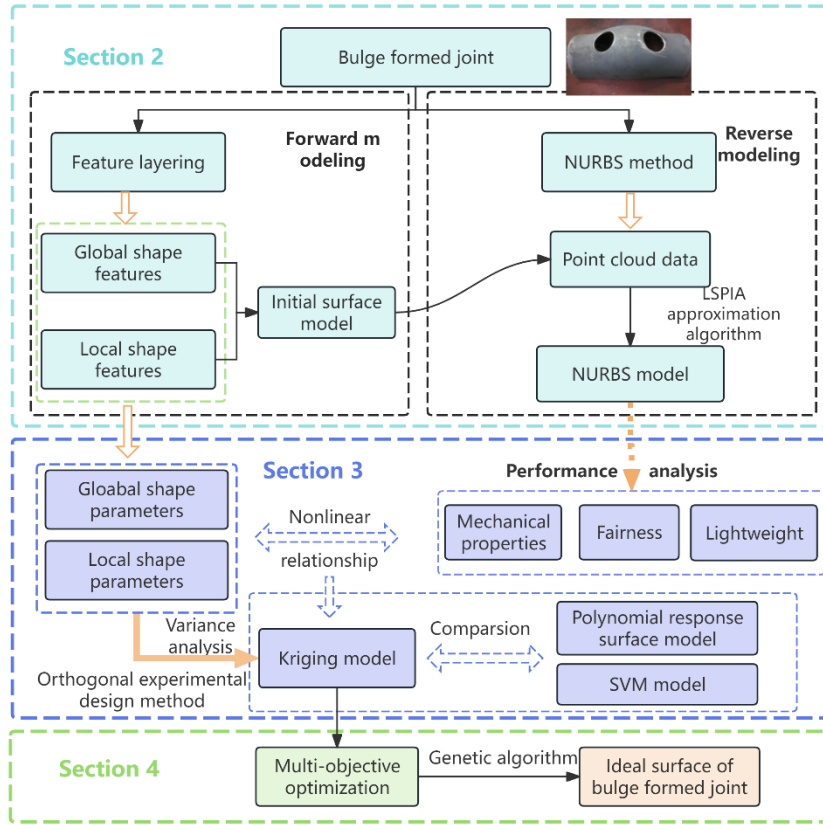


Fig. 2 Framework of the article

2. Overview of bulge-formed joint and surface modeling scheme

2.1 Free-form surface features of bulge-formed joint

Due to their simple structure, strong load-bearing capacity, ease of processing, and light weight nature, truss structures have found widespread application in engineering, particularly in the fields of crane and construction machinery. Practical engineering experience indicates that failures in truss structures are primarily attributed to the joints. Consequently, the joints of truss structures play a crucial role in determining both the load-bearing capacity and the fatigue life of the structures. Traditional joint types include tubular joints, plate joints, etc. While these joints are straightforward in design and easy to manufacture, they often exhibit several shortcomings:

- (1) The volume of the joint limits the number of braces that can be connected at the same joint. For certain truss structures that require multiple braces to intersect, conventional joints are inadequate to meet these requirements.
- (2) In traditional intersecting joints, the length of the intersecting line is primarily determined by the section size of the brace, and the length of the weld cannot be increased. This limitation significantly restricts the bearing capacity of the joint, ultimately leading to a constrained working capacity for the entire structure.
- (3) The Z-direction force at the joint is substantial, necessitating the use of a Z-direction plate, which further increases costs.

To further enhance the mechanical properties of products and the aesthetic appeal of surface structures, the morphological characteristics of joints have been continuously explored and refined, with a focus on their coordination with the truss structure system. As illustrated in Fig. 3, the joint surface transitions from an initially uneven and 'ugly' shape to a progressively smooth and 'beautiful' form. However, this evolutionary process relies on engineering experience and intuitive

understanding, lacking robust scientific theoretical guidance.



Fig. 3 Morphological evolution process of bulge-formed joint surface

The bulge-formed joint offers several advantages:

(1) The bulging design significantly increase the area available for welding brace members. When braces of the same section size are employed, the length of the intersecting line is extended, thereby enhancing the bearing capacity of the joint and improving the overall safety of the truss structure;

(2) The internal cavity created by the bulging joint allows the brace to be inserted and welded within it, effectively eliminating the Z-direction force on the joint;

(3) The joint is processed separately before being welded to the chord, which not only ensures the quality of the joint itself but also prevents damage to the chord.

At present, the bulge-formed joints have been preliminarily applied in engineering practice, as shown in Fig. 4:



Fig. 4 Practical application of bulge-formed joint and its model

However, as a new type of joint, the design of bulge-formed joints remains in the empirical design stage. Its initial conceptual design is largely arbitrary. Although its shape has been optimized, further development is hampered by the absence of a mathematical model. Therefore, establishing a surface model is crucial for advancing scientific and systematic research. Unlike research fields such as airfoil surface design, ship surface design or gear profile modeling, bulge-formed joints lack a relatively mature and systematic foundation for surface modeling and mathematical description. Sun et al. directly designed the control points and weight functions of the NURBS model to obtain the bulge plate of the bulge-formed joint, which were then integrated with the finite element method to optimize the overall truss structure [15]. However, the initial analytical surface model used to describe the bulge-formed joint is not accurate, and the direct modeling of NURBS surfaces lacks repeatability. In summary, given the varying shape feature requirements and mechanical property demands, bulge-formed joints often require redesign based on the parameters of NURBS surfaces, which is both time-consuming and labor-intensive. As it is challenging to directly correlate the NURBS surface with parameters in physical space, such as length, height, etc., rapid modeling and preliminary optimization research on bulge-formed joints must be conducted based on a feature hierarchical modeling scheme.

2.2 Forward and reverse modeling method for free-form surfaces based on feature layering

Surfaces and curves be mathematically represented as combinations of function spaces and their basis functions. However, for complex surfaces, while possessing a fundamental topological morphology, they often exhibit local intricacies

such as subtle undulations, variations in local curvature. These detailed characteristics significantly influence properties such as local resistance, surface stress distribution. Using a single high-degree NURBS surface to forcibly fit the global shape can lead to numerical instability. Therefore, this paper proposes a forward and reverse modeling method based on feature layering. This method is specifically divided into two steps: (1) construction of an initial model based on feature layering, and (2) reverse reconstruction of the surface based on NURBS principles.

2.2.1 Forward feature layering modeling

This step allows for the generation of an initial model with few shape parameters, tailored to the designed complex surface. These shape parameters can either originate from the designer's original input or be derived from the extraction of geometric features of existing structures. During this process, free-form surface shapes are divided into global shapes and local shapes. The global shape is represented by a relatively simple analytical surface form, while the local shape is expressed through a polynomial that facilitates matrix operation and solution. This approach primarily transforms the designed parameters into precise geometric shapes, which serve as the initial model for subsequent analyses.

Specifically, analytical surface forms such as spherical, cylindrical, conical surfaces, etc. are employed to describe the global shape of the designed surface. These parameters are defined as global shape parameters, which govern the overall size and shape variations of the surface.

Local shape features are used to describe the difference between the target surface and the global shape features. Based on three types of feature lines: shape lines, drive lines, and limit lines, the global analytic surface and local parametric surface are combined to construct the desired free-form surface model. The positions of local shape features are determined by the parameter coordinates corresponding to their shape centers. The closed area formed by the limit lines in the parameter coordinate system delineates the action area of the local shape features on the global surface. The specific local shape is defined by shape lines that express the overall shape and drive lines that illustrate the evolution trend of the shape. The shape parameters controlling the shape feature lines and driving the feature lines are termed local shape parameters.

Taking the container shown in Figure 5 as an example, its global feature is a cylindrical surface, and its characteristic parameters can be defined as length L_0 and radius R_0 . The local feature lies in the curved surface portion at the lower end, which is thus defined as the limiting curve, i.e., the red curve in Fig. 5. The formation of the local feature surface can be regarded as the shape line (i.e., the blue curve in Figure 5) sweeping along the drive line (i.e., the cyan curve).



Fig. 5 Example of feature layering

The relationship between parameters and shapes in this scheme is shown in the Fig. 6:

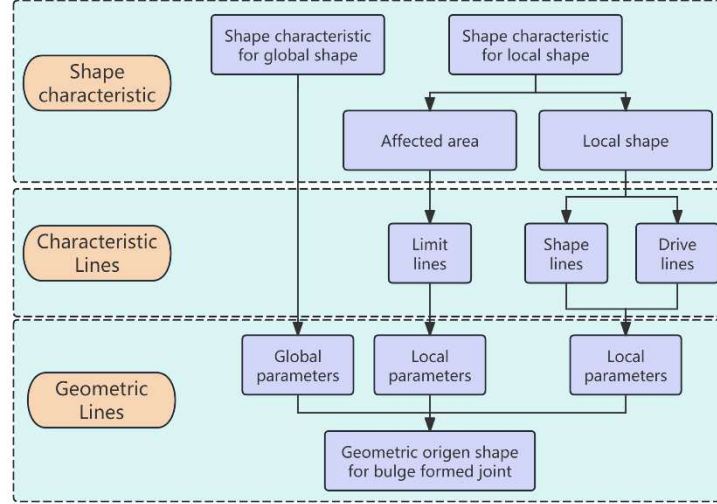


Fig. 6 Mapping relation in Feature Modeling

2.2.2 Reverse surface reconstruction

The initial model is mainly represented by analytical parameter models and polynomial parameter models that exhibit low degrees of freedom. By discretizing and approximating these models, a NURBS representation of the free-form surface is generated, utilizing a specified number of control points. This representation is employed to accurately depict the surface shape. The key to this step lies in the NURBS method and the approximation method, as detailed below.

2.2.2.1 NURBS method

A k -degree NURBS curve can be described as [2]:

$$\begin{cases} \mathbf{p}(u) = \sum_{i=0}^n R_{i,k}(u) \mathbf{d}_i \\ R_{i,k}(u) = \frac{w_i N_{i,k}(u)}{\sum_{i=0}^n w_i N_{i,k}(u)}, 0 \leq u \leq 1 \end{cases} \quad (1)$$

where $\mathbf{d}_i (i = 0, 1, \dots, n)$ is the control vertex of the NURBS curve and $R_{i,k}(u)$ is the k -degree NURBS rational base function. $w_i (i = 0, 1, \dots, n)$ is the weight factor corresponding to the control point, and $N_{i,k}(u) (i = 0, 1, \dots, n)$ is the k -degree B-spline basis function. The specific value can be determined via the de-door recursion formula according to the node vector $\mathbf{U} = [u_0, u_1, \dots, u_{n+k}, u_{n+k+1}]$ corresponding to the NURBS curve. The de-door recursion formula is as follows:

$$\begin{cases}
N_{i,0}(u) = \begin{cases} 1, & u_i \leq u \leq u_{i+1} \\ 0, & else \end{cases} \\
N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u) \\
\text{define } \frac{0}{0} = 0
\end{cases} \quad (2)$$

Similarly, a $k \times l$ -degree NURBS surface is defined as:

$$\begin{cases}
\mathbf{p}(u, v) = \sum_{i=0}^m \sum_{j=0}^n R_{i,k;j,l}(u, v) \mathbf{d}_{i,j} \\
R_{i,k;j,l}(u, v) = \frac{w_{i,j} N_{i,k}(u) N_{j,l}(v)}{\sum_{i=0}^m \sum_{j=0}^n w_{ij} N_{i,k}(u) N_{j,l}(v)}, 0 \leq u \leq 1, 0 \leq v \leq 1
\end{cases} \quad (3)$$

where $\mathbf{d}_{i,j} (i = 0, 1, \dots, m; j = 0, 1, \dots, n)$ is the control vertex of the NURBS surface, and $R_{i,k;j,l}(u, v)$ is the binary NURBS rational base function. $w_{i,j}$ is the weight factor corresponding to the control point, and $N_{i,k}(u)$ and $N_{j,l}(v)$ are the B-spline basis functions, which can be determined via de-door recursion formula according to the node vectors $\mathbf{U} = [u_0, u_1, \dots, u_{m+k}, u_{m+k+1}]$ and $\mathbf{V} = [v_0, v_1, \dots, v_{n+l}, v_{n+l+1}]$ respectively.

2.2.2.2 LSPIA approximation algorithm

The Least Square Progressive and Iterative Approximation (LSPIA) algorithm facilitates the convergence of the asymptotic iterative approximation solution of a given dataset to the least squares solution. Additionally, it allows the direct application of the calculation results from the previous step during the iteration process, thereby reducing computational complexity [16-18].

For a given set of type value points $\mathbf{p}_{i,j} (i = 0, 1, 2 \dots m, j = 0, 1, 2 \dots n)$ in a topologically rectangular array, the specific process of approximation is as follows:

(1) Calculate the node vector of each side upward. The set of type value points is divided into m groups and n groups along the direction of parameter u and v , respectively. Each group of curves is parameterized to derive the initial parameter coordinates $(u_{i,j}^0, v_{i,j}^0)$ corresponding to each type of value point $\mathbf{p}_{i,j}$. The initial parameters obtained from the curve are treated as intermediate parameter values in both directions using the average method. Subsequently, the node vectors in both directions are obtained using average and the knot placement technique (AVG+KTP) [3]. The specific formula is as follows:

$$\begin{cases} u_0 = u_1 = u_2 = u_3 = 0, \\ u_i = u_{i-1} + \frac{|p_{i-3} - p_{i-4}|}{\sum_{i=1}^m |p_i - p_{i-1}|} & i = 4, 5, \dots, n \\ u_{n+1} = u_{n+2} = u_{n+3} = u_{n+4} = 1 \end{cases} \quad (4)$$

This method is suitable for situations where the distribution of data points is uneven, and can reflect the distribution of chord lengths of shape points, resulting in smoother curves.

(2) Calculate the control points. The control points of the approximating NURBS surface are denoted as $\mathbf{d}_{i,j}$ ($i = 0, 1, \dots, s; j = 0, 1, 2, \dots, t$), and points are uniformly selected from the set of shape value points to serve as the control points, as shown in Eq. (5):

$$\begin{cases} \mathbf{d}_{0,j} = \mathbf{p}_{0,j}, \mathbf{d}_{s,j} = \mathbf{p}_m, \mathbf{d}_{i,0} = \mathbf{p}_{i,0}, \mathbf{d}_{i,t} = \mathbf{p}_{i,n} \\ \mathbf{d}_{i,j}^0 = \mathbf{p}_{\lfloor \frac{m-i}{s} \rfloor, \lfloor \frac{m-j}{s} \rfloor}, i = 1, 2, \dots, s-1, j = 1, 2, \dots, t-1 \end{cases} \quad (5)$$

(3) Update the parameter coordinates corresponding to each type value point. The type value point $\mathbf{p}_{i,j}$ is iteratively solved in the interval $[u_{i-1}, u_{i+1}] \times [v_{j-1}, v_{j+1}]$.

(4) Set convergence threshold ε and calculate the error $\mathbf{A}_{i,j}^k$ between the surface and the contour points in the k -th iteration. The vector difference between the type value point and the nearest point on the surface is defined as the error $\mathbf{A}_{i,j}^k$:

$$\mathbf{A}_{i,j}^k = \mathbf{p}_{i,j} - \mathbf{p}_{i,j}^k \quad (6)$$

(5) Calculate e^k . e^k is the sum of the norm values of the error $\mathbf{A}_{i,j}^k$, namely $e^k = \sum \|\mathbf{A}_{i,j}^k\|$. If $(e^k - e^{k-1}) / e^k \leq \varepsilon$, it proves that the algorithm has converged and the iteration is terminated. Otherwise, proceed to the next step.

(6) Adjust the control points based on the error of the type value point, as illustrated in Eq. (7).

$$\begin{aligned} \mathbf{d}_{i,j}^{k+1} &= \mathbf{d}_{i,j}^k + \mu \sum_{g=0}^m \sum_{h=0}^n R_{i,j}(\hat{u}_{g,h}^k, \hat{v}_{g,h}^k) \mathbf{A}_{g,h}^k \\ i &= 1, 2, \dots, s-1; j = 1, 2, \dots, t-1 \end{aligned} \quad (7)$$

Where, $R_{i,j}(u, v)$ is the rational basis function value of the i -th control point in u -direction and j -th control point in v -direction at parameter (u, v) . μ is the weight, and its value should satisfy $0 \leq \mu \leq 2/\lambda_0$, while λ_i is the non decreasing

sequence of eigenvalues of the matrix $\mathbf{R}^T \mathbf{R}$. The following formula is applied in actual computation:

$$\mu = \frac{2}{\max\{c_i\}} \quad (8)$$

where, c_i is defined as the sum of the elements in each row of matrix $\mathbf{R}^T \mathbf{R}$.

Moreover, in engineering practice, the first and last control points of a NURBS curve are generally made to coincide with the data points. Thus, when performing control point iterations in this algorithm, the first and last control points must be excluded from the iteration.

Through the above process, we can obtain the approximation calculation of $\mathbf{p}_{i,j}$. The specific flow chart is shown in

Fig. 7.

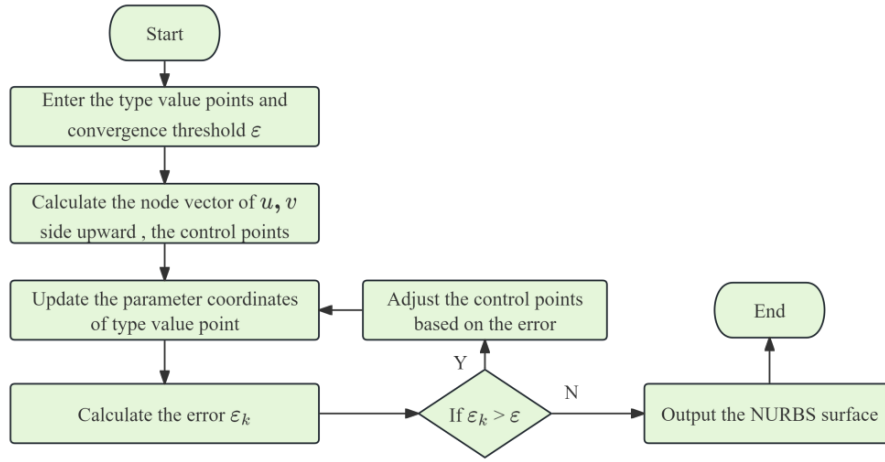


Fig. 7 LSPIA algorithm flowchart

2.3 Establishment of NURBS model of bulge-formed joint based on feature modeling

The modeling process for the bulge-formed joint is as follows: First, the surface of the bulge-formed joint is divided into 3 global features and 3 local features through feature modeling method. These features are then described using polynomial functions and combined to form an initial model. The initial model is subsequently discretized into a point cloud, and a NURBS surface model is approximately obtained via the LSPIA (Least Squares Progressive Iterative Approximation) algorithm.

The overall flowchart is shown in Fig. 8:

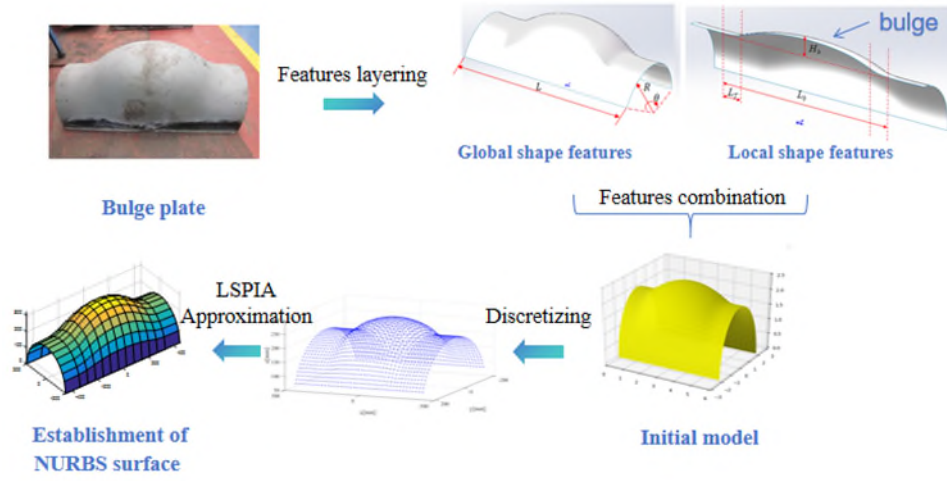


Fig. 8 Feature model flow chart

2.3.1 Initial surface acquisition scheme of bulge-formed joint

As previously mentioned, the bulge-formed joint is an enhanced variant of the tubular joint, modeled according to a specific feature modeling scheme, as follows:

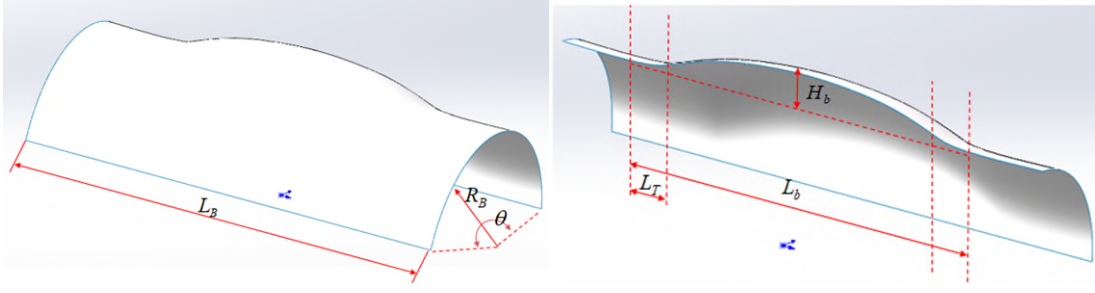
(1) Surface features and layering

Numerous variables influence the shape of the free-form surface of the bulge plate and the relationships among these variables are not immediately apparent. The surface of the bulge plate can be decomposed into a global analytic surface and a local parametric surface which represent the global and local morphological information, respectively.

The shape of the bulge plate surface consists of a semi-cylindrical surface and a bulge generated by deformation in its central region. The semi-cylindrical surface is used to align with the chord, and its shape is fixed, while the bulge serves to connect the braces, thereby enhancing the mechanical properties of the joint, as illustrated in Fig. 9. Since the semi-cylinder dictates the global size and shape of the bulge-formed joint, it is considered a global shape feature, characterized by the length L_B , radius R , and spread angle of the semi-cylinder θ . The design parameters related to the local shape features include the height of the bulge H , the length of the bulge L_b , the length of the transition segment in the direction of the bulge L_T , and the proportion of the bulge width τ_3 , as detailed in Table 1.

Table 1 Definition of global and local features

Global feature		Local feature	
Length	L_B	Height of the bulge	H_b
Radius	R_B	Length of the bulge	L_b
Spread angle of the semi-cylinder	θ	Length of the transition segment	L_T



(a) Global characteristic parameters of bulge plate (b) Local feature parameters of bulge plate

Fig.9 Shape parameters of bulge plate

(2) Establishment of Global Shape

The global shape is a semi cylindrical surface, and the parameter equation for its global shape can be directly established:

$$\mathbf{p}_g(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} = \begin{bmatrix} L_B u \\ R_B \cos(0.5\pi + \theta(v - 0.5)) \\ R_B \sin(0.5\pi + \theta(v - 0.5)) \end{bmatrix} \quad (9)$$

$$\mathbf{n}(u, v) = \begin{bmatrix} 0 \\ \cos(0.5\pi + \theta(v - 0.5)) \\ \sin(0.5\pi + \theta(v - 0.5)) \end{bmatrix} \quad (10)$$

where, (u, v) is the center of the local feature.

(3) Establishment of local shape

Polynomial surfaces are advantageous due to their ease of solution, making them suitable for representing the local shape features of bulge-formed joints. Given that the convex hull of a bulge-formed joint is symmetrical around its center and exhibits at least first-order geometric continuity, a quadratic curve is used to represent both its shape line and drive line.

Shape line:

$$h(u) = \begin{cases} 1 - u^2, & -1 \leq u \leq 1 \\ 0, & \text{else} \end{cases} \quad (11)$$

Drive line:

$$w(v) = \begin{cases} 1 - v^2, & -1 \leq v \leq 1 \\ 0, & \text{else} \end{cases} \quad (12)$$

There exists a transition region between the local shape and the global shape, where at least one order of geometric continuity must be maintained, accompanied by a total of four boundary conditions. Consequently, a third-order polynomial employed to refine the shape line, yielding the following modified shape line:

$$h(u) = H_b \begin{cases} -au^3 + bu^2 - cu + d, & -1 \leq u < -1 + 2\tau_2 \\ (-u^2 + 1), & -1 + 2\tau_2 \leq u \leq 1 - 2\tau_2 \\ au^3 + bu^2 + cu + d, & 1 - 2\tau_2 < u \leq 1 \\ 0, & \text{else} \end{cases} \quad (13)$$

where: $\tau_2 = \frac{L_r}{L}$, $a = \frac{1}{2\tau_2^2}$, $b = -1 + \frac{2}{\tau_2} - \frac{3}{2\tau_2^2}$, $c = 2 - \frac{4}{\tau_2} + \frac{3}{2\tau_2^2}$, $d = -1 + \frac{2}{\tau_2} - \frac{1}{2\tau_2^2}$

The equation of local features in a local coordinate system is:

$$s(u, v) = \begin{cases} h(u)w(v), & -1 \leq u \leq 1, -1 \leq v \leq 1 \\ 0, & \text{else} \end{cases} \quad (14)$$

It can be observed from the actual bulge-formed at the joint that the influence range of the local features within the global parameter coordinate system is rectangular. Therefore, the limit lines within the global parameter domain can be derived as follows:

$$\begin{cases} u_s(v) = u_1 - 0.5\tau_1, v_s \leq v \leq v_e \\ u_e(v) = u_1 + 0.5\tau_1, v_s \leq v \leq v_e \\ v_s(u) = v_1 - 0.5\tau_3, u_s \leq u \leq u_e \\ v_e(u) = v_1 + 0.5\tau_3, u_s \leq u \leq u_e \end{cases} \quad \text{where } \tau_1 = \frac{L_b}{L} \quad (15)$$

(4) Combination of local and global shape features

After determining the semi-cylindrical surface as a global shape feature and defining the local shape feature as a convex hull, the local shape features are combined along the normal direction relative to the position of the global shape. The overall shape is as follows:

$$\begin{aligned} \mathbf{p}(u, v) &= \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix} \\ &= \mathbf{p}_g(u, v) + \mathbf{n}(u, v) \times H_B \times s(u_p, v_p) \\ &= \begin{bmatrix} L_B u \\ R_B \cos(0.5\pi + \theta(v - 0.5)) \\ R_B \sin(0.5\pi + \theta(v - 0.5)) \end{bmatrix} + \begin{bmatrix} 0 \\ \cos(0.5\pi + \theta(v - 0.5)) \\ \sin(0.5\pi + \theta(v - 0.5)) \end{bmatrix} \times H_b \times s(u_p, v_p) \\ u_p &= \frac{2u - u_s(v) - u_e(v)}{u_e(v) - u_s(v)}, v_p = \frac{2v - v_s(u) - v_e(u)}{v_s(u) - v_e(u)} \end{aligned} \quad (16)$$

where $0 \leq u, v \leq 1$.

So far, the initial parametric surface model of bulge-formed joint surface has been completely established.

2.3.2 NURBS surface reconstruction based on LSPIA approximation algorithm

As a coarse model, the purpose of initial model is to highlight key morphological features of free-form surface and provide a foundation for subsequent surface reconstruction. Its limitations lie in the difficulty of modifying local features

and its inability to reflect the influence of local characteristics on mechanical performance. Since NURBS models are compatible with CAD/CAE software and allow easy local modifications, the LSPIA method is adopted to reconstruct the NURBS model.

First, the obtained initial model of the bulge-formed joint is discretized and converted into point cloud data. In this process, 40 points are sampled in each direction of the u and v coordinates, as shown in the Fig. 10.

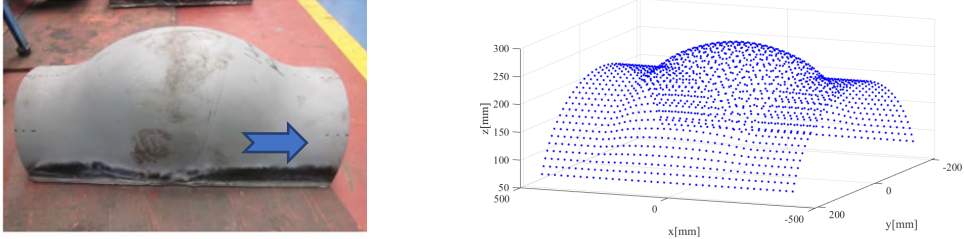


Fig. 10 Point cloud data from actual bulge-formed joint

Take 19 control points in the direction of parameter u and in the direction of parameter v , respectively. The LSPIA fitting was conducted on the point cloud data. The iterative process and the resulting outcomes are illustrated in Fig. 11.

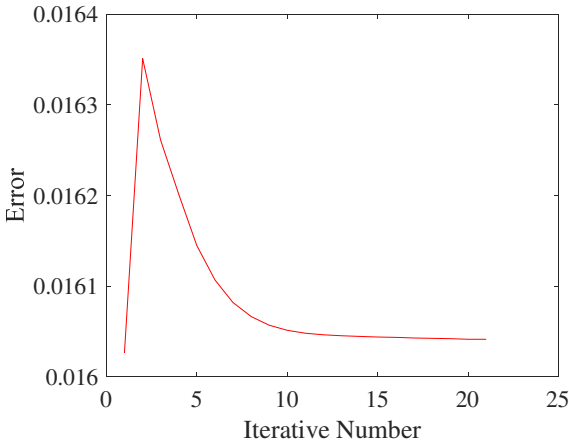


Fig. 11 Error in iterative process

As shown in Fig. 11, when iterating to step 15, the surface fitting basically converges, resulting in a fitting error of less than 0.0161. The specific fitting effect is shown in the Fig. 12:

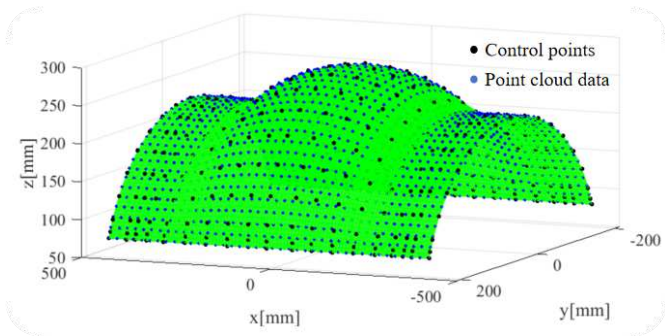


Fig. 12 Fitting effect

The blue points in the figure represent the point cloud data, while the green surface illustrates the fitted NURBS surface, and the black points denote the control points. As depicted in Fig. 12, the LSPIA algorithm successfully facilitates the transformation from the initial model to the NURBS model. The acquisition of free-form surfaces based on feature layering has now been completed.

3. Optimal design of bulge-formed joint surface based on NURBS parametric modeling method

The free-form surface forward and backward modeling based on feature layering is utilized to obtain surfaces in current engineering applications. To enhance product quality, further optimization processes are essential. The aforementioned modeling approach begins with the actual physical model of the bulge-formed joint, deriving a common NURBS parametric model from a limited set of shape parameters. In the subsequent optimization phase, various joint models can be generated simply by manipulating these shape parameters.

For bulge-formed joints, critical considerations include mechanical properties, fairness characteristics and lightweight design. The relationship between these performance metrics and shape parameters is nonlinear; therefore, the Kriging approximate model, recognized for its unbiased estimation with minimal variance, is employed to develop a response surface model that facilitates optimization design.

In this section, the orthogonal experimental design method is applied to gather sample points, allowing for an analysis of how morphological characteristics influence joint performance. Subsequently, the Kriging model is fitted and its accuracy is evaluated.

3.1 The principle of Kriging approximate model

For design variables and design responses, the general mathematical expression of the model is as follows[19]:

$$y_K(\mathbf{x}) = f_K(\mathbf{x}) + Z_K(\mathbf{x}) = \mathbf{f}_K^T(\mathbf{x})\boldsymbol{\beta}_K + Z_K(\mathbf{x}) \quad (17)$$

Among above, $f_K(\mathbf{x}) = \mathbf{f}_K^T(\mathbf{x})\boldsymbol{\beta}_K$ is a polynomial regression model; $Z_K(\mathbf{x})$ is a stationary random function, which can correct the local deviation of the model, and has the following statistical characteristics:

$$\begin{cases} E[Z_K(\mathbf{x})] = 0 \\ Var[Z_K(\mathbf{x})] = \sigma^2 \\ Cov[Z_K(\mathbf{x}_i), Z_K(\mathbf{x}_j)] = \sigma_K^2 \mathbf{R}_K[\mathbf{R}_K(\mathbf{x}_i, \mathbf{x}_j)] \end{cases} \quad (18)$$

where $\mathbf{R}_K$ is expressed as a spatial correlation matrix and belongs to a symmetric matrix. $R_K(\mathbf{x}_i, \mathbf{x}_j)$ is a cross-correlation function of any two sample points and can usually be used as a Gauss cross-correlation function. Its mathematical expression is as follows:

$$R_K(\mathbf{x}_i, \mathbf{x}_j) = e^{\left(-\sum_{k=1}^n \theta_k |x_i^k - x_j^k|\right)^2} \quad (19)$$

where $\boldsymbol{\theta}_K = [\theta_1, \theta_2, \dots, \theta_n]^T$ is the parameter to be modified and n is the dimension of the design variable.

When n sample points $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ and corresponding response $\mathbf{Y}_K = [y_{K1}, y_{K2}, \dots, y_{Kn}]^T$ are known, the predicted response $\hat{y}_K(\mathbf{x})$ based on Kriging model at unknown point $\mathbf{x}$ is expressed as follows :

$$\hat{y}_k(\mathbf{x}) = \mathbf{f}_k^T(\mathbf{x})\hat{\boldsymbol{\beta}}_k + \mathbf{r}_k^T(\mathbf{x})\mathbf{R}_k^{-1}(\mathbf{Y}_k - \mathbf{F}_k\hat{\boldsymbol{\beta}}_k) \quad (20)$$

where $\mathbf{r}_k(\mathbf{x}) = [R_k(\mathbf{x}, \mathbf{x}_1), R_k(\mathbf{x}, \mathbf{x}_2), \dots, R_k(\mathbf{x}, \mathbf{x}_n)]$

and $\mathbf{F}_k = [\mathbf{f}_k^T(\mathbf{x}_1), \mathbf{f}_k^T(\mathbf{x}_2), \dots, \mathbf{f}_k^T(\mathbf{x}_n)]^T$

$\hat{\boldsymbol{\beta}}_k$ is the unknown coefficient of the basis function, which can be obtained by the weighted least squares method:

$$\hat{\boldsymbol{\beta}}_k = (\mathbf{F}_k^T \mathbf{R}_k^{-1} \mathbf{F}_k)^{-1} \mathbf{F}_k^T \mathbf{R}_k^{-1} \mathbf{Y}_k \quad (21)$$

The estimated variance $\hat{\sigma}_k^2$ of the global model can be represented by $\hat{\boldsymbol{\beta}}_k$ and $\mathbf{Y}_k$ as follows:

$$\hat{\sigma}_k^2 = \frac{1}{n} (\mathbf{Y}_k - \mathbf{F}_k \hat{\boldsymbol{\beta}}_k)^T \mathbf{R}_k^{-1} (\mathbf{Y}_k - \mathbf{F}_k \hat{\boldsymbol{\beta}}_k) \quad (22)$$

The modified parameter θ_k in correlation function $R_k(\mathbf{x}_i, \mathbf{x}_j)$ of the above can be obtained by maximum likelihood estimation, that is, when $\theta_k > 0$, the following expressions are maximized:

$$\max_{\theta_k > 0} -\frac{1}{2} [n \ln(\hat{\sigma}_k^2) + \ln(|\mathbf{R}_k|)] \quad (23)$$

where $\hat{\sigma}_k^2$ and $\mathbf{R}_k$ are functions of θ_k , and each parameter θ_k will construct a corresponding Kriging interpolation model. Therefore, θ_k can be obtained by solving the optimization problem of Eq. (23).

Finally, the expression of Kriging model is obtained as follows:

$$\hat{y}_k(\mathbf{x}) = \mathbf{f}_k^T(\mathbf{x})\hat{\boldsymbol{\beta}}_k + \mathbf{r}_k^T(\mathbf{x})\mathbf{R}_k^{-1}(\mathbf{Y}_k - \mathbf{F}_k\hat{\boldsymbol{\beta}}_k) \quad (24)$$

3.2 Orthogonal experimental design based on morphological characteristics of bulge-formed joint

3.2.1 Orthogonal experimental design method

The sampling methods employed in Kriging modeling include Latin Hypercube Sampling, Box-Behnken Design, and Orthogonal Experimental Design. Among these, the Box-Behnken Design is tailored for three-level experiments, and when the number of levels exceeds three, both of these design methods tend to become complex. On the other hand, Latin Hypercube Design lacks orthogonality, which presents limitations in studying the significance and patterns of influence of different factors. Therefore, to minimize the time and cost required for experiments while effectively analyzing the degree and patterns of influence of different factors, this paper will adopt Orthogonal Experimental Design to construct sample points.

In addition to the geometric characteristics of the bulging plate surface mentioned earlier, the mechanical properties of the bulge-formed joint are closely related to the morphological characteristics, specifically the relative position between the

braces and the bulging plate. As shown in Fig. 13, the angle between the axis of the brace and the longitudinal section of the node is defined as the chord angle α , while the angle at which the brace deviates from the longitudinal section is referred to as the brace angle β .

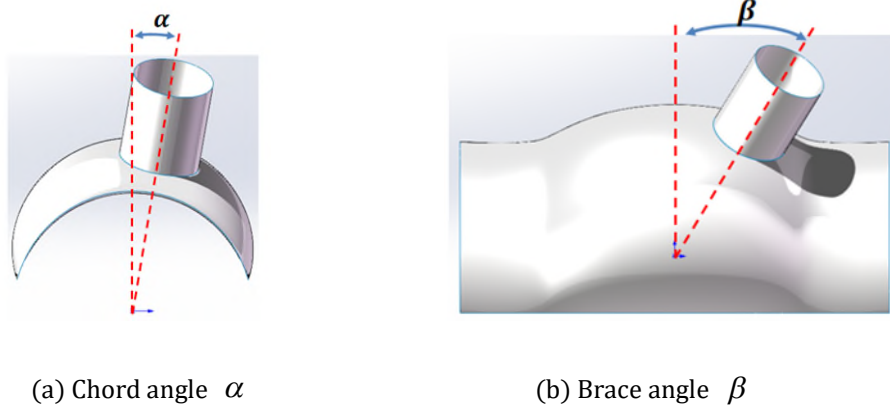


Fig. 13 Position between the braces and the bulge plate

According to the morphological characteristics of bulge-formed joint, three radian parameters, θ , α , and β , and three dimensionless parameters, $\tau_1 = L_b / L$, $\tau_2 = L_T / L_b$, and $\gamma = H_b / R$, are used as variables to study the effects of morphological characteristics on the mechanical properties, fairness and lightweight characteristics of bulge-formed joint. To improve efficiency, in this paper the orthogonal experimental design method is adopted, treating the six aforementioned morphological characteristic variables as factors. To accommodate the necessity for blank columns in subsequent data processing and in accordance with the principle of orthogonal experimental design, the $L_{32}(4^9)$ orthogonal array is chosen, as shown in Table 2, with each factor having 4 levels. According to the orthogonal array, factor values corresponding to different levels were randomly set, as shown in Table 3. In addition, 6 factors are randomly arranged in 9 column headers, with the remaining 3 columns as blank columns. The specific headers are detailed in Table 4.

Table 2 $L_{32}(4^9)$ orthogonal array

Col. No.	1	2	3	4	5	6	7	8	9	Col. No.	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1	17	1	1	4	1	4	2	3	2	3
2	1	2	2	2	2	2	2	2	2	18	1	2	3	2	3	1	4	1	4
3	1	3	3	3	3	3	3	3	3	19	1	3	2	3	2	4	1	4	1
4	1	4	4	4	4	4	4	4	4	20	1	4	1	4	1	3	2	3	2
5	2	1	1	2	2	3	3	4	4	21	2	1	4	2	3	4	1	3	2

6	2	2	2	1	1	4	4	3	3	22	2	2	3	1	4	3	2	4	1
7	2	3	3	4	4	1	1	2	2	23	2	3	2	4	1	2	3	1	4
8	2	4	4	3	3	2	2	1	1	24	2	4	1	3	2	1	4	2	3
9	3	1	2	3	4	1	2	3	4	25	3	1	3	3	1	2	4	4	2
10	3	2	1	4	3	2	1	4	3	26	3	2	4	4	2	1	3	3	1
11	3	3	4	1	2	3	4	1	2	27	3	3	1	1	3	4	2	2	4
12	3	4	3	2	1	4	3	2	1	28	3	4	2	2	4	3	1	1	3
13	4	1	2	4	3	3	4	2	1	29	4	1	3	4	2	4	2	1	3
14	4	2	1	3	4	4	3	1	2	30	4	2	4	3	1	3	1	2	4
15	4	3	4	2	1	1	2	4	3	31	4	3	1	2	4	2	4	3	1
16	4	4	3	1	2	2	1	3	4	32	4	4	2	1	3	1	3	4	2

Table 3 Levels of the 6 factors

Factor Levels	θ	α	β	γ	τ_1	τ_2
1	2.909	0.262	0.273	0.442	0.641	0.220
2	3.142	0.000	0.820	0.111	0.599	0.193
3	2.676	0.785	0.547	0.332	0.558	0.207
4	2.443	0.524	0.000	0.221	0.682	0.180

Table 4 Headers of the orthogonal array

Column	1	2	3	4	5	6	7	8	9
Factor	τ_2	γ		β	α		τ_1		θ

3.2.2 Finite element method

According to Tables 2, 3, and 4, the parameters of the joints corresponding to each test can be obtained, as presented in Appendix Table A1. Based on these data, parameterized NURBS models of the bulge plate are carried out, allowing for the determination of fairness characteristics and weights. Subsequently, three-dimensional models of the bulge-formed joint are established, followed by finite element analysis to ascertain the ultimate bearing capacity and fatigue characteristic of the joints. This process is shown in Fig. 14.

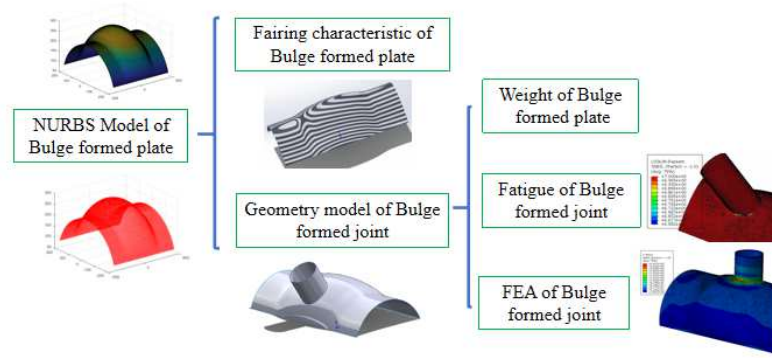


Fig. 14 Research process of bulge-formed joint

Performance analysis is conducted using the finite element method. The primary finite element parameter settings include: (1) material parameters, (2) boundary conditions, and (3) mesh generation.

(1) Material parameters

The material used in the engineering application is Q355B. The mechanical properties of this material is modeled using a bilinear stress-strain material model based on the experimental stress-strain data provided by Liu Qiao [20], as illustrated in Fig. 15 and Table 5. In Table 5, f_y represents the nominal yield stress, and f_u represents the ultimate strength. The yield strain (ε_1) and strain at the onset of strain hardening (ε_2) are 0.172% and 2%, respectively. When the strain reaches the latter value, the material attains its ultimate strength with no further strengthening thereafter.

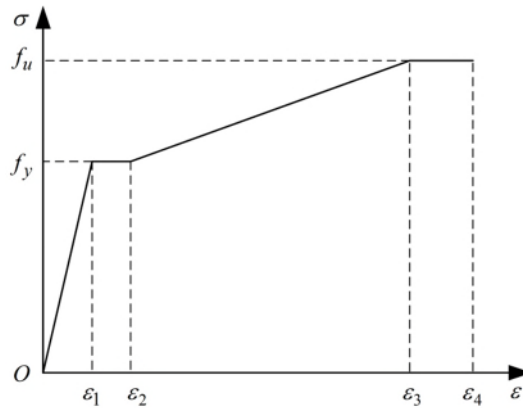


Fig. 15 Bilinear stress-strain material model of Q355B

Table 5 Material parameters in elastic stage

Parameters	Model [MPa]	Poisson's ratio ν	Density [t/mm ³]	f_y [MPa]
Value	2.06×10^5	0.29	7.85×10^{-9}	355

Parameters	ε_1 [%]	ε_2 [%]	ε_3 [%]	ε_4 [%]	f_u [MPa]
Value	0.172	2	20	25	510

(2) Boundary conditions

The displacement boundary condition is set as fully fixed, applied on the four edges. The force boundary condition is a concentrated force of 200 kN acting along the axial direction of the brace, applied at the reference point on the end of the web member. This reference point is tied to the end nodes of the brace via an MPC Tie constraint, ensuring that the load is evenly applied to the end of the brace, as shown in Fig. 16.

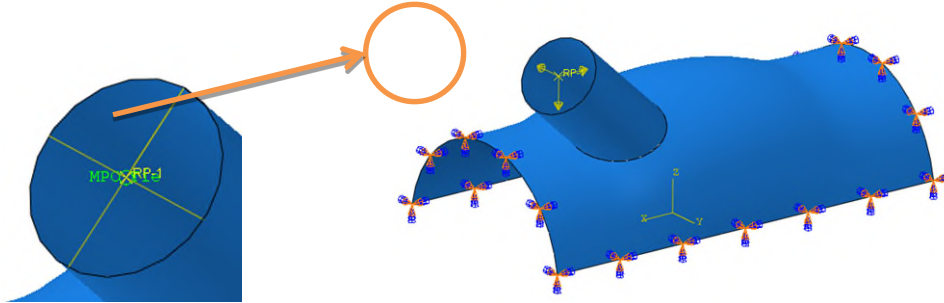
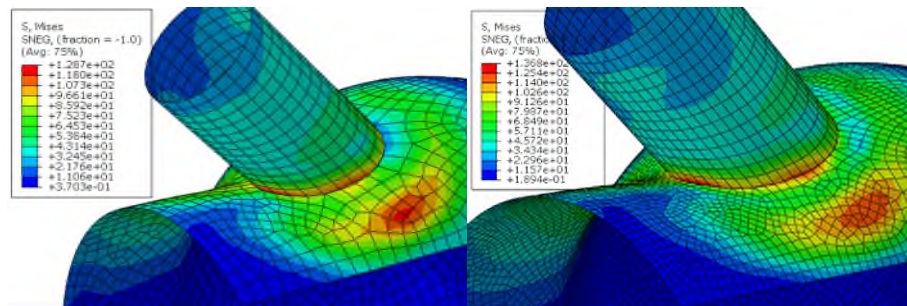


Fig. 16 Boundary conditions

(3) Element settings and mesh generation

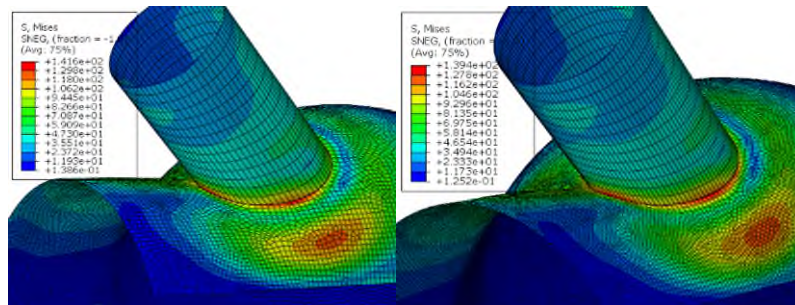
The bulge-formed joint is a shell structure, and therefore the standard S4R shell element in ABAQUS is adopted. This element uses reduced integration, offering high computational efficiency. Based on the Mindlin-Reissner plate and shell theory, it also accounts for the effects of transverse shear deformation. As a result, it can simulate both thin shells (where shear deformation is negligible) and thick shells (where shear effects are significant).

The mesh is primarily composed of quadrilateral elements. To investigate the sensitivity of the simulation results to the mesh, element sizes of 20 mm, 10 mm, 7 mm, and 5 mm were tested. The results are shown in Fig. 17.



(a) Element size 20 mm

(a) Element size 10 mm



(c) Element size 7 mm

(d) Element size 5 mm

Fig. 17 The stress contour with different size of elements

It can be observed from Fig. 17 that as the mesh is refined, the von Mises stress stabilizes around 140 MPa. Considering computational efficiency, a mesh size of 5 mm was selected for the model. The corresponding finite element model is illustrated in Fig. 18.

In addition, as shown in Fig. 17, significant stress concentration occurs in the weld region, while areas of high stress also appear near the bulge. This is related to the geometric characteristics of the bulge, underscoring the necessity of studying the influence of different parameters on the joint's performance.

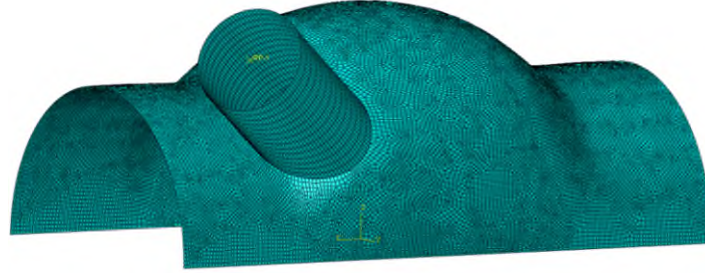


Fig. 18 The finite element model

3.2.3 Results of orthogonal experimental

This paper investigates the mechanical properties, including the ultimate bearing capacity (UBC) and fatigue characteristic (logarithm of the fatigue life), as well as fairness characteristics, which encompass the maximum Gaussian curvature and the variance of Gaussian curvature, in addition to lightweight characteristics. The results of each test are summarized in Table 6.

Table 6 The results of orthogonal test

	UBC [kN]	Fatigue life [log]	Max Gaussian curvature [10^{-5}]	Gaussian curvature variance [10^{-11}]	Weight [kg]
1	1031.624	6.352	7.900	25.399	69.934
2	1177.416	6.872	2.449	2.791	65.771
3	1074.178	5.594	7.621	24.338	60.605
4	984.437	6.909	3.751	7.782	53.599
5	1183.237	7	11.035	40.917	57.960
6	890.190	5.296	2.145	2.271	56.423
7	936.688	5.713	6.803	19.382	72.162
8	1021.147	5.761	5.533	12.216	63.516
9	1040.188	5.7	9.276	33.199	58.412
10	930.352	4.757	2.282	2.515	56.367
11	943.832	6.63	5.896	15.653	72.496
12	1160.051	7	6.065	13.786	63.074

13	1033.550	7	8.867	26.660	70.409
14	957.099	6.544	3.406	4.199	66.069
15	1120.141	7	7.921	25.049	60.857
16	954.738	6.67	5.128	11.351	53.431
17	1105.655	7	9.816	35.392	63.534
18	1582.847	5.703	1.893	2.003	50.629
19	933.323	6.785	6.020	17.323	66.678
20	897.270	6.361	4.924	10.502	68.867
21	1149.045	7	8.975	27.893	75.414
22	909.640	5.467	2.775	3.230	61.287
23	1035.504	7	8.360	28.213	55.399
24	954.336	6.725	4.225	8.733	58.762
25	932.145	5.919	7.725	23.001	76.040
26	850.770	5.431	2.993	3.670	61.198
27	1173.755	5.021	7.129	22.625	55.525
28	1212.474	6.589	4.534	9.634	58.228
29	1034.668	6.713	10.542	35.818	64.037
30	994.951	5.978	2.601	2.942	51.104
31	1116.520	6.904	6.823	18.296	66.852
32	983.768	6.969	6.820	15.735	68.621

3.3 Kriging model of bulge-formed joint

3.3.1 Analysis of the influence of morphology characteristics on performance of joint

This paper investigate the impact of various parameters on the performance of bulge-formed joints by conducting a variance analysis based on the results of orthogonal tests [21]. The variance analysis addresses the limitations of trend analysis, while the sensitivity analysis method serves to mutually validate the findings of the variance analysis.

(1) Steps of variance analysis

For the orthogonal design method, where the factor level is r , the number of orthogonal table columns is m , and the total number of experiments is n , the results are $y_i (i = 1, 2, \dots, n)$. The steps of the specific variance analysis are as follows:

(A) Calculation of the sum of squared deviations

(a) Total sum of squared deviations (SST)

Let:

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad (25)$$

$$T = \sum_{i=1}^n y_i \quad (26)$$

$$Q = \sum_{i=1}^n y_i^2 \quad (27)$$

$$P = \frac{1}{n} \left(\sum_{i=1}^n y_i \right)^2 = \frac{T^2}{n} \quad (28)$$

The total sum of the squared deviations is as follows:

$$SS_{Total} = \sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - \frac{1}{n} \left(\sum_{i=1}^n y_i \right)^2 = Q - P \quad (29)$$

The SST reflects the overall difference in the results of the orthogonal test. A larger SST indicates a greater disparity among the results, suggesting that variations in factor levels have a more pronounced impact on the outcomes.

(b) The sum of the squared deviations of each factor

The sum of the squared deviations caused by factor A being arranged in a column of the orthogonal array is as follows:

$$SS_A = \frac{n}{r} \sum_{i=1}^n (k_i - \bar{y})^2 = \frac{r}{n} \sum_{i=1}^n K_i^2 - \frac{T}{n} = \frac{r}{n} \sum_{i=1}^n K_i^2 - P \quad (30)$$

The sum of the squared deviations caused by error (blank columns) is as follows:

$$SS_e = \sum SS_{blank} \quad (31)$$

(B) Calculation of degrees of freedom

The degree of the freedom of SST is as follows:

$$df_T = n - 1 \quad (32)$$

The degree of freedom of factor A is as follows:

$$df_A = r - 1 \quad (33)$$

The degree of freedom of error is as follows:

$$df_e = \sum df_{blank} \quad (34)$$

(C) Calculation of the mean square

The mean square of factor A is as follows:

$$MS_A = \frac{SS_A}{df_A} \quad (35)$$

The mean square of the error is as follows:

$$MS_e = \frac{SS_e}{df_e} \quad (36)$$

(D) Calculation of the F-value

The mean square of each factor is divided by the mean square of the error to obtain the F-value:

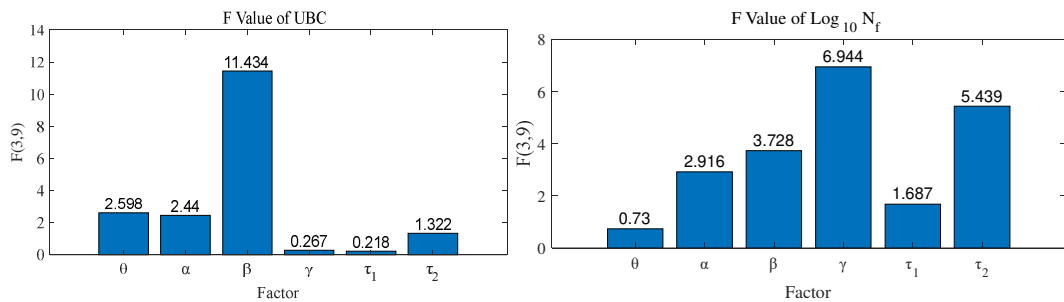
$$F_A = \frac{MS_A}{MS_e} \quad (37)$$

(E) Significance test

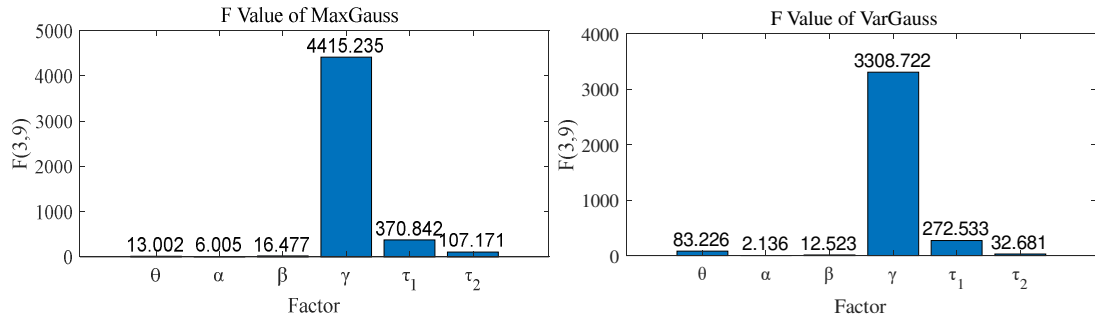
For a given F-value, find the critical value $F_{\alpha}(df_A, df_e)$ from the F distribution table; then, compare the F-value with the critical value. If $F_A > F_{\alpha}(df_A, df_e)$, the factor has a significant effect on the results; otherwise, it has no significant effect.

(2) The results of variance analysis

In this paper, the $L_{32}(4^9)$ orthogonal array is selected, with each factor comprising 4 levels, which results in 3 degrees of freedom for each column. The blank column contains a total of 3 columns, yielding a total error degree of freedom of 9. According to the F distribution table in Reference [21], $F_{0.10}(3, 9) = 2.81$. The results of variance analysis are shown in Fig. 19.

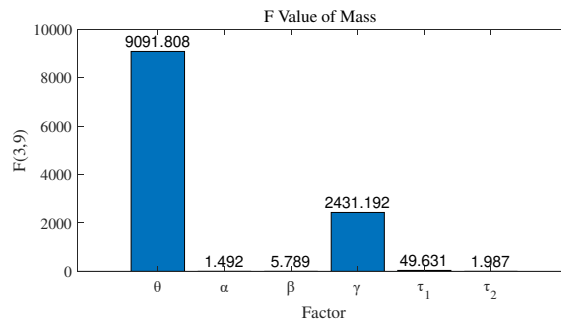


(a) Variance analysis of ultimate bearing capacity (b) Variance analysis of fatigue characteristic



(c) Variance analysis of global fairness

(d) Variance analysis of local fairness



(e) Variance analysis of lightweight characteristic

Fig. 19 F-values of factors in variance analysis

As illustrated in Fig. 19(a), the variation in the factor significantly influences the ultimate bearing capacity, with the most pronounced effect observed, followed by the factors θ and τ_2 . Fig. 19(b) demonstrates that regarding the fatigue characteristic, the factor γ exerts the most substantial impact, succeeded by the factors τ_2 , α , and β .

Furthermore, as depicted in Fig 19(c) and Fig. 19(d), both global and local fairness characteristics, the change in the factor γ has the most significant effect on the results, with F-values of 4415.235 and 3651.812, respectively. In terms of global fairness characteristics, factors τ_1 and τ_2 also exhibit significant effects; similarly, for local fairness characteristics, factors θ and τ_1 are also impactful.

As shown in Fig. 19(e), factors θ and γ significantly influence the lightweight characteristics, exhibiting F-values of 9091.808 and 2431.192, respectively. Factor τ_1 shows an F-value approaching 50, while factor β has an F-value near to 6, whereas the other factors have almost no significant effect on the lightweight characteristics, with F-values less than 2.

According to the results of the F-test, for the ultimate bearing capacity, the factors that have a significant impact are θ , α , β and γ . Therefore, the function expression for defining ultimate bearing capacity is :

$$f_{UBC} = y_K(\mathbf{x}_{UBC}); \mathbf{x}_{UBC} = [\theta \quad \alpha \quad \beta \quad \gamma]^T \quad (38)$$

Where, y_K represents the Kriging approximation model, and $\mathbf{x}_{UBC}$ is the normalized value of the shape parameters corresponding to the sampling points, the same applies below.

For the fatigue characteristic, the factors that have a significant impact are α , β and γ and τ_1 . Therefore, the function expression for defining the ultimate bearing capacity is :

$$f_{LogN_f} = y_K(\mathbf{x}_{LogN_f}); \mathbf{x}_{LogN_f} = [\alpha \quad \beta \quad \gamma \quad \tau_2]^T \quad (39)$$

For the lightweight characteristic, the factors that have a significant impact are θ , β and γ and τ_1 . Therefore, the function expression for defining the ultimate bearing capacity is :

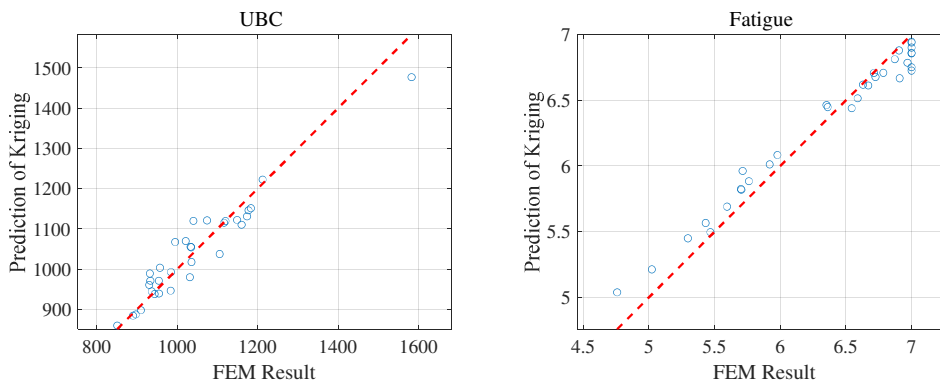
$$f_{Mass} = y_K(\mathbf{x}_{Mass}); \mathbf{x}_{Mass} = [\theta \quad \beta \quad \gamma \quad \tau_1]^T \quad (40)$$

For the fairness characteristics, the factors that have a significant impact are θ , β , τ_1 and τ_2 . Therefore, the function expression for defining the ultimate bearing capacity is :

$$\begin{cases} f_{MaxGauss} = y_K(\mathbf{x}_{Fairness}) \\ f_{VarGauss} = y_K(\mathbf{x}_{Fairness}) \end{cases}; \mathbf{x}_{Fairness} = [\theta \quad \gamma \quad \tau_1 \quad \tau_2]^T \quad (41)$$

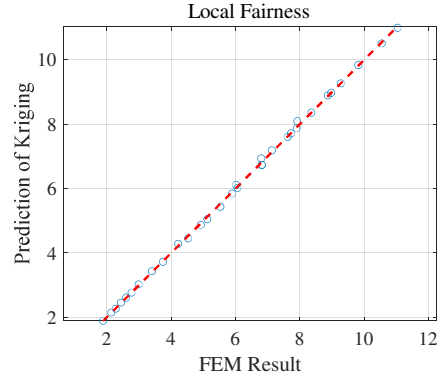
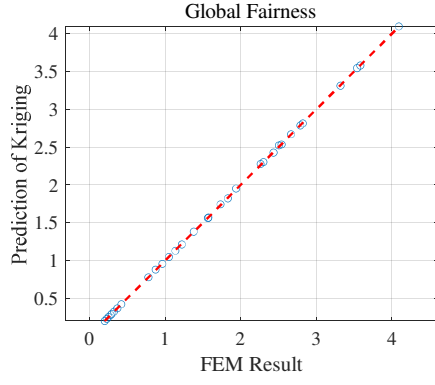
3.3.2 Fitting accuracy of the Kriging model

The Kriging model was employed to fit the sampled data, with the fitting results for each performance prediction presented in Fig. 20, where the red dashed line represents the straight line $y=x$. The closer the data points are to this red line, the more accurate the predictions of Kriging model. It can be observed from the Fig. 20 that the predictions for fairness characteristic and lightweight characteristic are highly accurate, while certain errors exist in the predictions of ultimate bearing capacity and fatigue life, which is consistent with the subsequent model accuracy verification results.



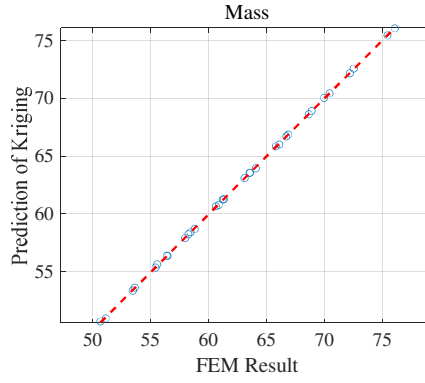
(a) UBC [kN]

(b) Fatigue [Log N_f]



(c) Global fairness [10^{-11}]

(d) Local fairness [10^{-5}]



(e) Mass [kg]

Fig. 20 Fitting result of Kriging model

The primary indices used to assess the fitting accuracy of the Kriging model against the experimental data include the negative correlation coefficient R^2 , the Normalized Root Mean Square Error NRMSE[22], the C_p statistic, and the sum of squares of prediction error, etc. In this paper, negative correlation coefficient R^2 and NRMSE are adopted, which are defined as follows:

$$R^2 = 1 - \frac{S_{SE}}{S_{SY}} \quad (42)$$

$$NRMSE = 100\% \times \sqrt{S_{SE} / N} / \bar{y} \quad (43)$$

where N is the number of experiments, n is the number of terms of the response surface model, S_{SE} is the sum of squares of the difference between the test values and the mean test value, and S_{SY} is the sum of squares of the difference between the response value and the mean value of the response:

$$S_{SY} = \sum_{i=1}^N (y_i - \bar{y})^2 \quad (44)$$

$$S_{SE} = \sum_{i=1}^N (y_i - \tilde{y})^2 \quad (45)$$

where the subscript i represents the i -th experiment, $\bar{y}$ is the mean value of orthogonal test results, and $\tilde{y}$ is the mean value of response surface function.

The values of R^2 and NRMSE are presented in Table 7. All R^2 values exceed 0.9, while all NRMSE values are all below 5%. This indicates that the Kriging model demonstrates high precision and is suitable for optimization purposes.

Table 7 The fitting accuracy of response surface model

	UBC	Fatigue life	Max Gaussian curvature	Gaussian curvature variance	Weight
R^2	0.9123	0.9566	0.9996	0.9999	1.0000
NRMSE[%]	3.9169	2.2161	0.4473	0.1435	0.0876

3.3.3 Comparison with other method

In addition to the Kriging model, in this paper fitting using polynomial response surface models and Support Vector Machine (SVM) models are also conducted. Their accuracies are presented in the Table 8 and Table 9. Comparing the data from the Table 7, 8 and 9, in terms of R^2 , the prediction accuracy of the polynomial response surface model for fatigue life is lower than that of the Kriging model, while the prediction accuracies for other metrics are comparable. On the other hand, the prediction accuracy of the SVM model for all metrics is inferior to that of the Kriging model, with a particularly significant discrepancy in ultimate load-carrying capacity.

Table 8 The fitting accuracy of polynomial response surface model

	UBC	Fatigue life	Max Gaussian curvature	Gaussian curvature variance	Weight
R^2	0.9135	0.7034	0.9980	0.9997	0.9999
NRMSE[%]	3.8910	5.7926	1.9326	1.1624	0.20873

Table 9 The fitting accuracy of SVM model

	UBC	Fatigue life	Max Gaussian curvature	Gaussian curvature variance	Weight
R^2	0.5175	0.9447	0.9662	0.9610	0.9550
NRMSE[%]	9.1887	2.5004	7.9914	13.3079	2.3107

4. Optimal design of bulge-formed joint surface based on NURBS parametric modeling method

4.1 Optimization model

As mentioned earlier, the main performance indicators of bulge-formed joints are categorized into mechanical property indicators, fairness characteristic indicators and lightweight indicators. The mechanical properties encompass ultimate bearing capacity and fatigue life, while fairness characteristics include both global and local fairness. Consequently, a multi-objective optimization method is employed and the optimization model is as follows:

$$\begin{aligned} \mathbf{F}(\mathbf{x}) &= \left[\min(-f_{UBC} / 200 - f_{LogN_f}) \quad \min(f_{MaxG} + f_{VarG}) \quad \min(f_{Mass}) \right]^T \\ \text{s.t. } \mathbf{x} &= [\theta \quad \alpha \quad \beta \quad \gamma \quad \tau_1 \quad \tau_2] \end{aligned} \quad (46)$$

where, f_{UBC} , f_{LogN_f} , f_{MaxG} , f_{VarG} , and f_{Mass} are the fitted function values of the Kriging model, calculated according to Eqs (38), (39), (40), and (41). Since the fitted values from the Kriging model are not normalized, f_{UBC} is replaced by $f_{UBC} / 200$ to balance the influence of ultimate bearing capacity and fatigue life on the mechanical performance. f_{MaxG} and f_{VarG} represent the fairness and f_{Mass} represent the lightweight characteristic.

4.2 Results of optimization

The genetic algorithm (GA) is used to address the multi-objective optimization model. The initial population is configured to 100, with a mutation probability of 0.8, and a maximum iteration count of 200. The Pareto front is shown in Fig. 21, and the resulting series of Pareto solution sets are presented in Table A2 in Appendix A.

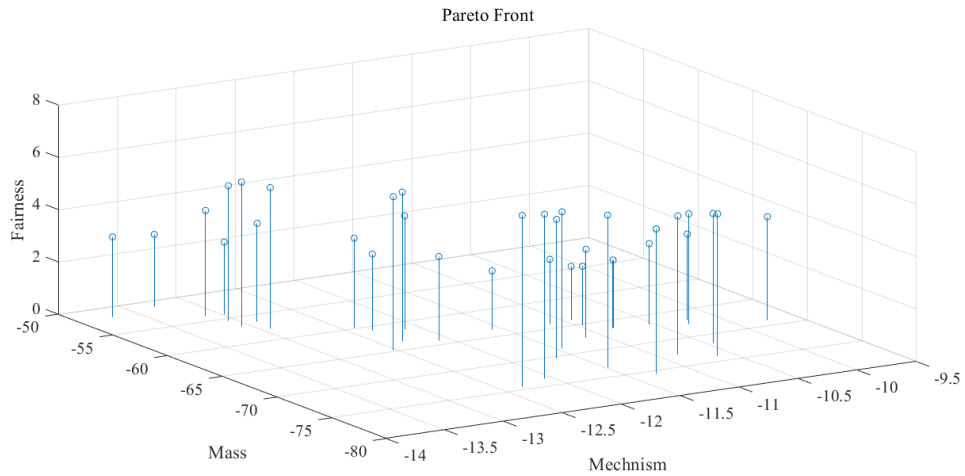


Fig. 21 Pareto front

In the Pareto solution set, the performance of joint corresponding to different solutions varies. Utilizing the predicted values from the Kriging model, Pareto solutions are filtered by comparing them with the performance of the original model, as shown in Fig. 22, where the blue points represent the predicted values, while the black line indicates the performance of initial joint. Based on the Fig. 22, among the 35 solutions in the Pareto solution set, three solutions exhibit performance

superior to the initial model, as listed in Table 10.

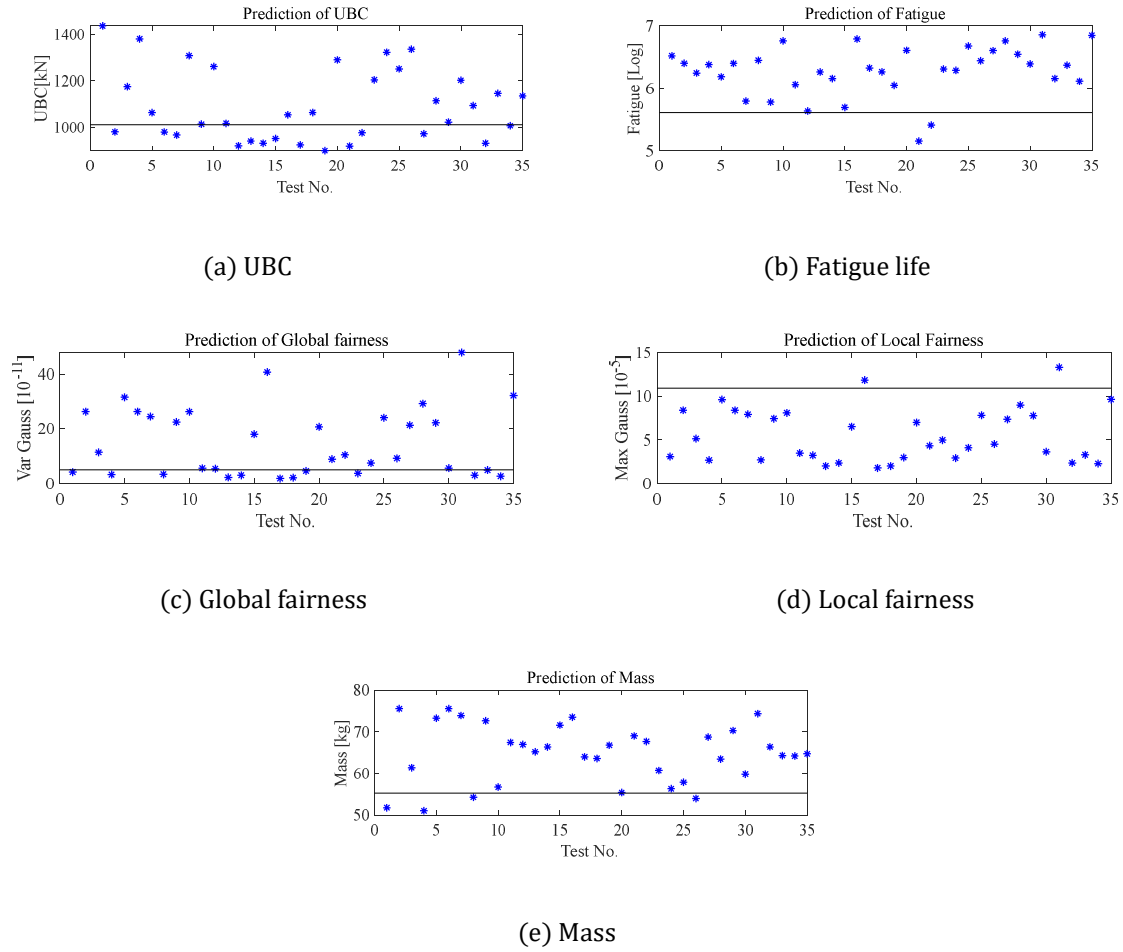


Fig. 22 The prediction of the Pareto solutions

Table 10 Comparison of performance of Kriging prediction and numerical simulation

	UBC [kN]	Fatigue life [log]	Max Gaussian curvature [10 ⁻⁵]	Gaussian curvature variance [10 ⁻¹¹]	Weight [kg]
Solution 1	1436.951	6.520	3.082	4.141	51.783
Solution 4	1381.028	6.378	2.673	3.250	51.049
Solution 8	1308.485	6.447	2.687	3.349	54.289

Considering the predicted performance of bulge-formed joint, especially the ultimate bearing capacity and the mass, Solution 4 is selected as the final outcome, as presented in Table 11. The model of the joint corresponding to this solution is constructed, and a performance analysis of the joint is conducted, as illustrated in Fig. 23 Fig. 24 and Fig. 25(c) (d). The performance of the optimized joint, in comparison with the predictions from the Kriging model and the performance of the initial model, is detailed in Tables 12 and 13, respectively.

Tabel 11 Screened solution in Pareto solution set

Factors	θ	α	β	γ	τ_1	τ_2
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Solution 4	2.455	0.604	0.810	0.117	0.622	0.194
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Fig. 23 Optimized bulge-formed joint

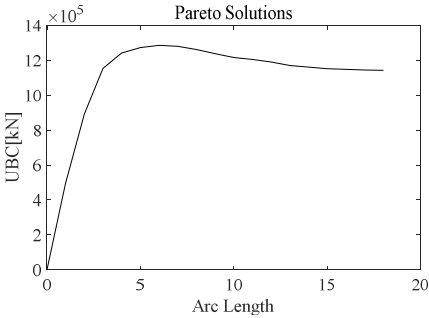
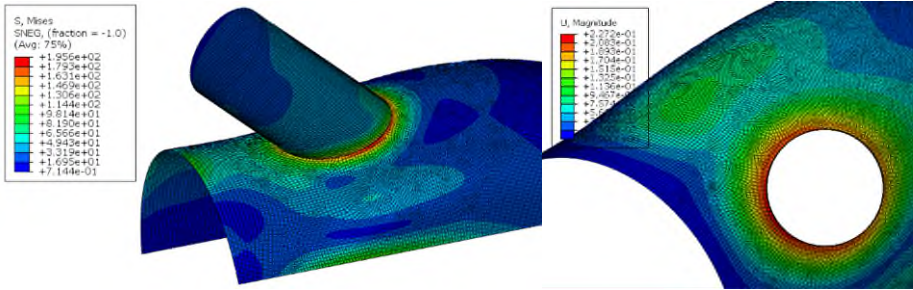
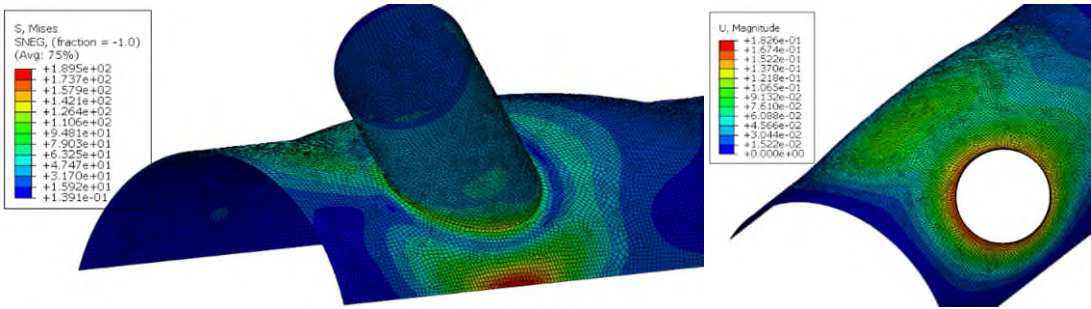


Fig. 24 Ultimate bearing capacity



(a) Stress contour of optimized model(b) Displacement contour of optimized model



(c) Stress contour of optimized model(d) Displacement contour of optimized model

Fig. 25 FE results of bulge-formed joint

Table 12 Comparison of performance of Kriging prediction and numerical simulation

	UBC [kN]	Fatigue life [log]	Max Gaussian curvature [10^{-5}]	Gaussian curvature variance [10^{-11}]	Weight [kg]
Kriging prediction	1381.028	6.378	2.673	3.250	51.049
Actual performance	1286.623	6.591	2.631	3.247	51.388
Error (%)	7.34	3.23	1.60	0.09	0.66

Table 13 Comparison of performance of initial model and optimized model

	UBC [kN]	Fatigue life [log]	Max Gaussian curvature [10^{-5}]	Gaussian curvature variance [10^{-11}]	Weight [kg]
Initial model	1011.100	5.609	10.926	4.964	55.326
Optimized model	1286.623	6.591	2.631	3.247	51.388
Optimal extent (%)	27.25	17.51	75.92	34.59	7.12

As presented in Table 12, the prediction errors of the Kriging model are 7.34% for ultimate bearing capacity, 1.60% for local fairness characteristic, 0.09% for global fairness characteristic, and 0.66% for weight, demonstrating the accuracy of the Kriging model. The error associated with the ultimate bearing capacity presented in the table is higher than that of the other data due to the stress concentration on the boundary, as shown in Fig. 25.

As illustrated in Table 13, the optimized joint exhibits improvements across all its characteristics: (1) The ultimate bearing capacity has increased by 27.25%, which is attributed to the enhanced stiffness of the joint. As shown in Fig. 25(b) and 25(d), the displacement at the weld decreased from 0.223mm to 0.172mm; (2) The logarithmic fatigue life has increased by 17.51%, translating to a 9.594-fold increase in fatigue life because the stress concentration in the weld has decreased. As shown in Fig. 25(a) and 25(c), the von Mises stress in weld has decreased from 189 MPa to 140 MPa; (3) Additionally, there is a 75.92% decrease in maximum Gaussian curvature, and a 34.59% decrease in variance of Gaussian curvature, which means a significant improvement in fairness characteristics; (4) The weight of bulge plate has decreased by 7.12%.

5. Conclusion

Based on feature modeling theory and the practical requirements of engineering design, this paper proposes a forward and reverse modeling method of free-form surfaces. The physical model of the bulge-formed joint is analyzed, from which shape parameters are extracted and converted into limit lines, shape lines and drive lines, thereby establishing the initial mathematical model of the joint. Subsequently, the LSPIA approximation method is employed to transform the initial model into a NURBS surface with enhanced degrees of freedom. Utilizing the orthogonal experimental design method, a Kriging model is developed and applied to optimize the joint design, resulting in a more satisfactory surface shape. The specific conclusions are as follows:

(1) Through the feature layering modeling method, the actual physical shape features of the joint are decomposed into fewer global and local shape parameters, which are mathematically expressed using limit lines, shape lines and drive lines. An initial model of the bulge-formed joint is established, allowing for control over the joint's shape by adjusting a limited number of shape parameters. This facilitates a convenient and rapid forward design of joint model.

(2) Building upon the initial model obtained through forward modeling, the LSPIA approximation method, in

conjunction with the principles of NBRBS, is employed to reconstruct the initial model, resulting in the NURBS surface of the bulge-formed joint. The example demonstrates that the LSPIA algorithm effectively fits the NURBS surface, enabling the reverse reconstruction of the bulge-formed joint. Consequently, a forward and reverse modeling method for the bulge-formed joint is established.

(3) Utilizing the orthogonal experimental design method, the shape parameters that significantly influence the joint's performance are identified through variance analysis, leading to the development of a Kriging model that captures the nonlinear relationship between shape parameters and performance. The optimized NURBS surface is achieved using a multi-objective genetic algorithm. A comparison of the indicators before and after optimization reveals that the ultimate bearing capacity has increased by 27.25%, while the logarithmic fatigue life has improved by 17.51%, equivalent to a 9.594-fold increase in fatigue life. Additionally, the maximum Gaussian curvature has decreased by 75.92%, and the variance of Gaussian curvature has decreased by 34.59%, which indicates a significant improvement of fairness characteristics while the lightweight characteristic has improved by 7.12%.

6. Acknowledgements

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Declaration and statement

Conflict of Interest

We declare that we have **no** financial and personal relationships with other people or organizations that can inappropriately influence our work, there is **no** professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled.

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715 **Data availability statement**

716 Data available on request from the authors.

717 **Author contributions**

718 Yuantao Sun, Jinjin Zhai and Chen Yang wrote the main manuscript text. Chen Yang and Ziyi Yang provided the guidance
719 on software usage. All authors reviewed the manuscript.

720 **Ethics approval and Consent to participate**

721 Not applicable.

722 **Replication of Results**

723 The proposed framework is built on NURBS method, FEM, Kriging method and GA multi-objective optimization method,
724 which has been fully expounded in this work, so the results can be easily reproduced. Moreover, the opening the source code
725 of the proposed method is banned by a project.

726 **Appendix A**

727

Table A1 Parameters of joints corresponding to each test						
	θ	α	β	γ	τ_1	τ_2
1	2.909	0.262	0.273	0.442	0.641	0.220
2	3.142	0.000	0.820	0.111	0.599	0.220
3	2.676	0.785	0.547	0.332	0.558	0.220
4	2.443	0.524	0.000	0.221	0.682	0.220
5	2.443	0.000	0.820	0.442	0.558	0.193
6	2.676	0.262	0.273	0.111	0.682	0.193
7	3.142	0.524	0.000	0.332	0.641	0.193
8	2.909	0.785	0.547	0.221	0.599	0.193
9	2.443	0.524	0.547	0.442	0.599	0.207
10	2.676	0.785	0.000	0.111	0.641	0.207
11	3.142	0.000	0.273	0.332	0.682	0.207
12	2.909	0.262	0.820	0.221	0.558	0.207
13	2.909	0.785	0.000	0.442	0.682	0.180
14	3.142	0.524	0.547	0.111	0.558	0.180
15	2.676	0.262	0.820	0.332	0.599	0.180
16	2.443	0.000	0.273	0.221	0.641	0.180

17	2.676	0.524	0.273	0.442	0.558	0.220
18	2.443	0.785	0.820	0.111	0.682	0.220
19	2.909	0.000	0.547	0.332	0.641	0.220
20	3.142	0.262	0.000	0.221	0.599	0.220
21	3.142	0.785	0.820	0.442	0.641	0.193
22	2.909	0.524	0.273	0.111	0.599	0.193
23	2.443	0.262	0.000	0.332	0.558	0.193
24	2.676	0.000	0.547	0.221	0.682	0.193
25	3.142	0.262	0.547	0.442	0.682	0.207
26	2.909	0.000	0.000	0.111	0.558	0.207
27	2.443	0.785	0.273	0.332	0.599	0.207
28	2.676	0.524	0.820	0.221	0.641	0.207
29	2.676	0.000	0.000	0.442	0.599	0.180
30	2.443	0.262	0.547	0.111	0.641	0.180
31	2.909	0.524	0.820	0.332	0.682	0.180
32	3.142	0.785	0.273	0.221	0.558	0.180

Table A2 Pareto solution sets

	θ	α	β	γ	τ_1	τ_2
1	2.480	0.764	0.808	0.128	0.605	0.180
2	3.141	0.304	0.185	0.442	0.645	0.210
3	2.816	0.678	0.743	0.222	0.613	0.195
4	2.455	0.604	0.810	0.117	0.622	0.194
5	3.103	0.631	0.725	0.415	0.605	0.203
6	3.141	0.304	0.186	0.442	0.645	0.210

7	3.138	0.531	0.379	0.396	0.637	0.208
8	2.594	0.567	0.807	0.124	0.647	0.190
9	3.080	0.581	0.596	0.394	0.653	0.209
10	2.492	0.731	0.781	0.338	0.603	0.184
11	3.139	0.592	0.586	0.166	0.635	0.194
12	3.087	0.596	0.346	0.181	0.659	0.212
13	3.076	0.312	0.463	0.120	0.673	0.216
14	3.115	0.397	0.431	0.135	0.659	0.216
15	3.140	0.525	0.452	0.317	0.635	0.208
16	3.099	0.631	0.685	0.436	0.578	0.182
17	3.027	0.252	0.431	0.112	0.681	0.219
18	3.022	0.462	0.659	0.112	0.662	0.212
19	3.099	0.399	0.354	0.165	0.653	0.213
20	2.480	0.720	0.780	0.296	0.605	0.187
21	3.133	0.675	0.256	0.223	0.648	0.204
22	3.094	0.705	0.451	0.216	0.613	0.200
23	2.892	0.609	0.785	0.122	0.607	0.197
24	2.641	0.750	0.790	0.176	0.606	0.192

25	2.587	0.779	0.750	0.304	0.572	0.181
26	2.513	0.719	0.767	0.197	0.618	0.187
27	2.985	0.385	0.557	0.342	0.639	0.187
28	2.758	0.667	0.691	0.359	0.591	0.183
29	3.121	0.444	0.689	0.303	0.596	0.194
30	2.824	0.708	0.722	0.149	0.599	0.188
31	3.142	0.780	0.731	0.440	0.558	0.182
32	3.115	0.397	0.431	0.135	0.659	0.216
33	3.021	0.594	0.774	0.152	0.630	0.195
34	3.028	0.359	0.595	0.126	0.675	0.196
35	2.768	0.644	0.784	0.397	0.596	0.187

730