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A theory-constrained master-curve approach for the ultra-low-frequency dynamic modulus of steel slag asphalt mixtures with reduced reliance on extreme-temperature testing

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Dynamic modulus master curves of asphalt mixtures often require ultra-low-frequency extrapolation for mechanistic-empirical pavement analysis, yet direct calibration at extreme temperatures is difficult for steel slag asphalt mixtures (SSAMs). This study developed a theory-constrained framework that reduces reliance on extreme-temperature testing while preserving extrapolation stability. Five dense-graded SSAMs prepared with weathered converter slag and AH-70 or SBS-modified binders were tested in indirect tension using an optimized spherical-hinge fixture. Only dynamic-modulus data from -10 to 40 °C were used for calibration. A Hirsch-informed high-frequency envelope was combined with Global Aging System or Arrhenius shift factors and nonlinear optimization to construct master curves. Reserved -20 °C dynamic-modulus data and 60 °C creep-converted modulus data were used for independent validation. The optimized fixture reduced the coefficient of variation from 8.2% to 3.5%. The constrained curves achieved $R^2 \geq 0.990$ and $RMSE < 50$ MPa for all mixtures. Average extrapolation error relative to shifted -20 °C data was 6.3%, and average deviation from 60 °C creep-converted data was 8.7%. The proposed framework therefore offers a practical route to long-timescale modulus inputs for pavement analysis within the investigated material range.

Keywords dynamic modulus; indirect tensile test; master curve; steel slag asphalt mixture; time-temperature shift factor; ultra-low-frequency extrapolation

Introduction

Mechanistic-empirical pavement analysis uses dynamic modulus (E^*) as a key material input for calculating stress, strain, and asphalt pavement response under different temperatures and loading rates¹⁻⁴. After results measured at different temperatures and frequencies are shifted to the selected reference temperature, the resulting modulus-reduced-frequency curve provides the viscoelastic characterization required for long-term pavement response analysis. Even so, the master curve remains a material characterization tool; its implications for rutting, fatigue, and field durability still need to be interpreted together with damage models, repeated-load testing, and field evidence.

Under long-duration traffic loading, the equivalent reduced frequency in asphalt layers may fall far below the range directly covered by routine dynamic-modulus testing, making ultra-low-frequency extrapolation unavoidable in practice. In classical sigmoidal master-curve models, both asymptotes and shape parameters are identified from a limited dataset. When low-frequency information is insufficient, parameter interaction may absorb scatter in the high-temperature data and make ultra-low-frequency predictions overly sensitive to initial values and test range. This problem is particularly relevant for steel slag asphalt mixtures (SSAMs), whose aggregate skeleton is stiffer and more structurally heterogeneous than that of conventional natural-aggregate mixtures^{23,25}.

SSAMs offer an effective route for converter steel slag utilization and may also improve aggregate interlock and binder-aggregate interaction. Steel slag particles are characterized by high stiffness, rough surfaces, and strong angularity, all of which can enhance skeleton stability. At the same time, their aging state, surface chemistry, and contact structure may lead to modulus-response characteristics that differ from those of conventional mixtures made with natural aggregates^{11-18,22}. For this reason, conventional high-temperature and low-temperature indices alone are insufficient when the objective is to provide reliable material inputs for pavement structural analysis.

Among available test methods, uniaxial compression generally requires specimens about 150 mm in height and is therefore not well suited to the thin field cores commonly obtained from Chinese asphalt pavements⁶. The indirect tensile (IDT) configuration is more adaptable to both laboratory specimens and field cores and is convenient for multi-temperature, multi-frequency viscoelastic testing [7-10]. Its main limitation is contact sensitivity: fixed loading strips respond strongly to specimen end-face flatness, generatrix parallelism, and local contact conditions. For SSAMs, the high stiffness and irregular morphology of steel slag particles may further amplify such contact errors, making fixture reliability a prerequisite for robust master-curve identification.

A further practical difficulty is that direct coverage of the high-frequency glassy region often requires testing below $-10\text{ }^{\circ}\text{C}$, where temperature control, surface frosting, and loading-contact changes can all impair data quality^{4,19,20}. A framework that relies entirely on extreme-temperature testing is therefore costly, operationally demanding, and sensitive to both equipment condition and specimen quality. A more practical strategy is to retain routine temperature testing as the core dataset, introduce a physically meaningful high-frequency constraint and theoretical shift-factor relationships, and then assess the resulting extrapolation against data excluded from calibration.

Hirsch-based modulus estimation can provide a physically informed reference for the stiff end of the master curve, whereas the GAS and Arrhenius formulations provide two parameterized routes for time-temperature shifting based on binder rheology and activation-energy theory, respectively²⁷⁻³¹. However, the Hirsch model does not explicitly treat steel slag stiffness or three-dimensional aggregate contact structure as independent variables, and theoretical shift factors may still depend on binder type, aging state, and calibration temperature range. Their suitability for SSAM master-curve construction should therefore be judged by targeted validation rather than by goodness-of-fit metrics alone.

Against this background, the present study develops a theoretically constrained master-curve approach for the ultra-low-frequency dynamic modulus of SSAMs with reduced reliance on extreme-temperature testing. The test program included five dense-graded SSAMs containing weathered converter slag and either AH-70 or SBS-modified binder. Only dynamic-modulus data from -10 to $40\text{ }^{\circ}\text{C}$ were used for parameter calibration, whereas lower-temperature and longer-timescale data were reserved strictly for independent validation.

The study makes three main contributions:

An optimized spherical-hinge IDT fixture is evaluated to improve loading contact, reduce stress concentration, and enhance repeatability in SSAM modulus testing;

A master-curve framework combining a Hirsch-informed high-frequency envelope, GAS/Arrhenius theoretical shift factors, and numerical optimization is

established and shown to remain stable when extended toward 10^{-7} Hz within the investigated material range;

The resulting extrapolations are checked against shifted -20 °C dynamic-modulus data for all mixtures and 60 °C creep-converted data for the two AC-13 mixtures, while uncertainty and material-factor effects are further examined.

Raw Materials and Mixture Design

Converter steel slag sourced from a Guangdong steel plant and stockpiled outdoors for more than 6 months to control expansion associated with free CaO was used as the coarse aggregate (4.75-26.5 mm). The fine aggregate was granite manufactured sand (0-4.75 mm), and limestone powder was selected as filler. Hydrated lime, at 2% by mass of filler, was added to enhance moisture stability. Table 1 lists the aggregate properties; all measured indices met JTG F40-2004.

Performance Index	Unit	Converter Steel Slag Coarse Aggregate	Granite Fine Aggregate	JTG F40-2004 Requirement
Apparent Relative Density	-	3.24	2.68	≥ 2.50
Bulk Relative Density	-	3.02	2.61	≥ 2.45
Water Absorption	%	1.76	0.62	≤ 2.0
Crushing Value	%	19.8	22.5	≤ 26
Los Angeles Abrasion Value	%	21.3	25.7	≤ 28
Needle and Flake Particle Content	%	11.2	8.5	≤ 15
Soundness (5 cycles of sodium sulfate)	%	7.8	5.2	≤ 12
Free Calcium Oxide Content	%	1.2	-	≤ 3.0

Table 1 Basic properties of steel slag and granite aggregates.

AH-70 base asphalt and SBS-modified asphalt (SBSMA) were combined with the AC-13, AC-20, and AC-25 dense gradations to form five mixtures: AH-70 AC-13, AH-70 AC-20, AH-70 AC-25, SBSMA AC-13, and SBSMA AC-20. SBSMA AC-25 was not included because it was not used in the local expressway projects represented by this study. Specimens were compacted by the Marshall method ²⁴, and the design air-void content was controlled at approximately 4.0%. Table 2 gives the volumetric parameters adopted in the Hirsch calculation.

Mixture Type	Asphalt Binder	Voids in Mineral Aggregate (VMA) (%)	Effective Asphalt Content (Vbe) (%)	Air Void Content (%)
Steel Slag AC-13	AH-70 Base Asphalt	14.2	10.3	3.9
Steel Slag AC-13	SBS-Modified Asphalt	14.5	10.6	3.9
Steel Slag AC-20	AH-70 Base Asphalt	13.8	9.9	3.9
Steel Slag AC-20	SBS-Modified Asphalt	14.1	10.2	3.9
Steel Slag AC-25	AH-70 Base Asphalt	13.5	9.6	4.0

Table 2 Volume parameters of the tested SSAMs.

Figure 1 presents the designed gradations of the three AC mixtures. All gradations fall within the applicable JTG F40-2004 specification limits while retaining distinct nominal maximum aggregate sizes and clear engineering relevance.

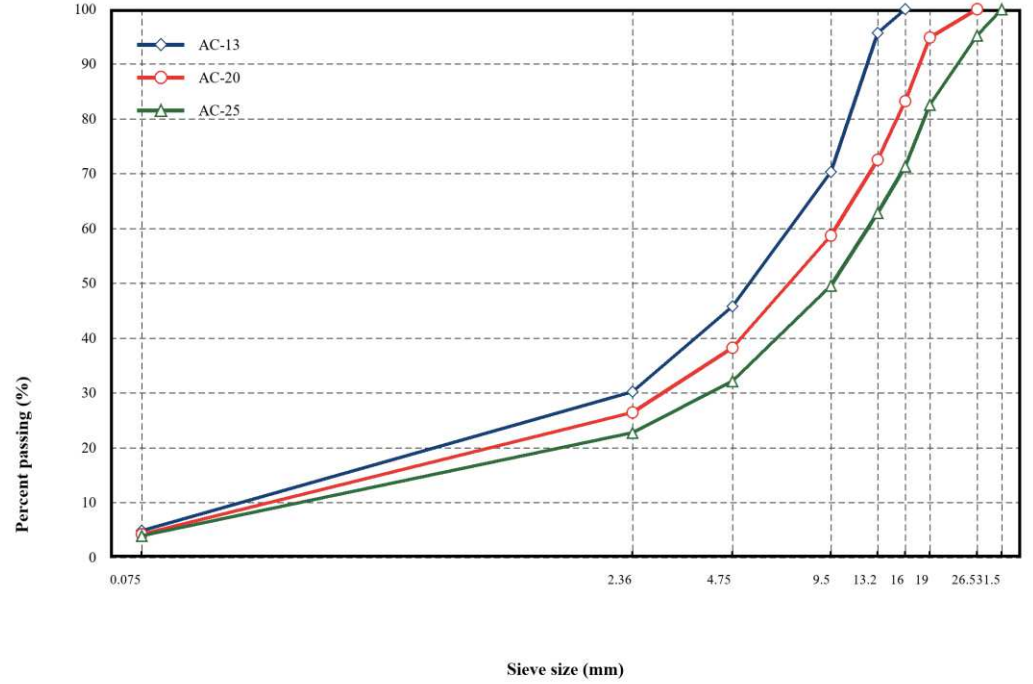


Figure 1. Aggregate gradation curves of the tested asphalt mixtures.

Note: All gradations conform to the mid-range requirements of JTG F40-2004 for dense-graded asphalt mixtures.

Theoretical Basis

Classical Sigmoidal Master Curve Model

Dynamic-modulus master curves of asphalt mixtures are commonly fitted with the sigmoidal equation given in Eq. (1) ²⁵:

$$\lg|E^*| = \delta + \frac{\alpha}{1 + e^{\beta + \gamma \cdot \lg f_r}} \quad (1)$$

In Eq. (1), E^* denotes SSAM dynamic modulus. The parameter δ corresponds to the logarithmic lower limit, while α gives the logarithmic interval between the upper and lower limits; thus, $\alpha + \delta$ is the logarithm of the upper limit. Parameters β and γ control curve shape. The reduced frequency f_r (Hz) is obtained from loading frequency f and the shift factor aT at temperature T .

The notation $\lg$ denotes the base-10 logarithm throughout this paper. Modified sigmoidal formulations developed for polymer-modified asphalt mixtures have also been reported in the literature ²⁶.

Improved Ultra-Low Frequency Extendable Master Curve Method

The classical approach has two core limitations for SSAMs: high-temperature data scatter prevents reliable extension into the ultra-low frequency domain, and testing below -10 °C for maximum modulus determination is both costly and prone to error. Furthermore, results from conventional mixtures cannot be directly transferred to SSAMs without calibration.

To address the limitations of the classical approach, the present study adopts the Hirsch model as a physically informed reference for the high-frequency modulus and defines a combined upper envelope that is not lower than the maximum modulus retained in the fitting dataset. The envelope is therefore used as a hybrid physically informed, data-retained constraint rather than as a fully independent material constant. Temperature shifts were calculated with the Global Aging System (GAS) model when binder viscosity data were available; otherwise, the Arrhenius

relation was adopted. Numerical optimization is then used to construct master curves that can be extended into the ultra-low-frequency domain for the investigated SSAMs.

High-frequency modulus estimation via the Hirsch model

In the Hirsch calculation, the stiff-end modulus is determined from mixture volumetrics together with the binder limiting shear modulus, thereby providing a theoretical reference without requiring tests below -10 °C^{27,28}. In this study, the industry-standard limiting shear modulus of 145,000 psi was adopted. Because the Hirsch estimate may be lower than the highest measured finite-frequency modulus, it was not used directly as the final upper asymptote; instead, the final bound was defined as the greater of the Hirsch estimate and the maximum measured modulus retained in the fitting dataset. The core equations are:

$$|E^*|_{\max} = P_c \cdot \left(4200000 \cdot \text{VMA} + \frac{|G^*|_{b,\max} \cdot V_{be}}{3} \right) + \frac{1 - P_c}{\frac{\text{VMA}}{4200000} + \frac{V_{be}}{3 \cdot |G^*|_{b,\max}}} \quad (2)$$

$$P_c = \frac{\left(\frac{V_{be} \cdot |G^*|_{b,\max}}{\text{VMA}} \right)^{0.58}}{650 + \left(\frac{V_{be} \cdot |G^*|_{b,\max}}{\text{VMA}} \right)^{0.58}} \quad (3)$$

Here, $E^*_{\max}$ is the limiting dynamic modulus in psi, and G_b is the binder shear modulus at the limiting state (145,000 psi). VMA and V_{be} denote voids in mineral aggregate and effective asphalt content, respectively, both expressed as percentages; P_c is the aggregate-contact term.

All results were converted to MPa (1 MPa = 145.038 psi) for direct comparison with IDT test data.

The standard Hirsch formulation uses mixture volumetrics and the binder limiting shear modulus, without introducing aggregate stiffness as a separate input^{27,28}. For the high-stiffness steel slag aggregates considered here, the model was checked against the measured modulus at the stiffest fitting condition (-10 °C, 25 Hz). As shown in Fig. 2, the Hirsch estimates are strongly correlated with the measured values ($R^2 = 0.884$) but are systematically 4.1%-5.8% lower than the corresponding measured stiff-state values. This comparison indicates that the volumetric model provides a reasonable order-of-magnitude estimate, but it also explains why a fixed Hirsch value cannot serve as a physically admissible upper bound for all fitting data. Accordingly, the final optimization bound was defined as $\max(E_{\text{Hirsch}}, E_{\text{measured,max}})$ for each mixture, where $E_{\text{measured,max}}$ is the largest modulus among the retained -10 to 40 °C fitting observations. This treatment preserves the theoretical reference while preventing the fitted curve from being capped below an observed modulus.

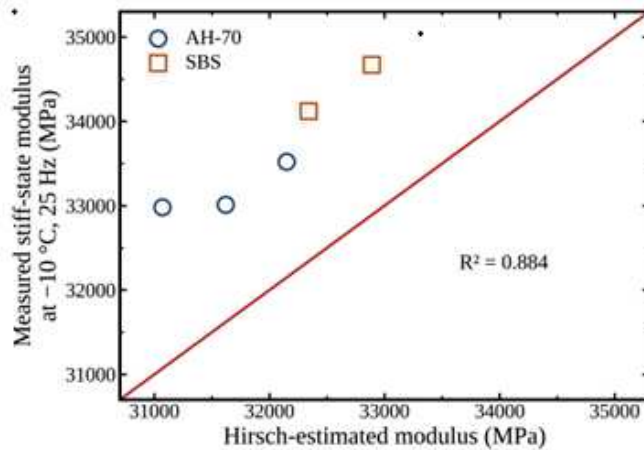


Figure 2. Correlation between Hirsch-predicted and measured maximum dynamic modulus for SSAM.

The Hirsch-informed upper envelope, defined as $\max(E_{\text{Hirsch}}, E_{\text{measured,max}})$, was imposed as a fixed high-frequency bound in the optimization routine. Because $E_{\text{measured,max}}$ is drawn from the retained fitting observations, the bound should be interpreted as a conservative, data-informed constraint rather than as an independently measured material constant. A sensitivity analysis based on three upper-bound scenarios, namely the Hirsch estimate, the fitting-data maximum, and the measured -20 °C value, produced relative modulus deviations below 7.2% at 10^{-7} Hz for the AC-13 (AH-70) mixture, indicating that the selected bound does not introduce substantial additional extrapolation variability within the examined range.

Shift Factor Calculation with Two Theoretical Models

Global Aging System (GAS) model

For binders with short-term-aged viscosity data at several temperatures, the shift factor was evaluated by the GAS (Mirza-Witczak) relation ²⁹:

$$\lg \alpha_T = C \cdot (\lg \eta_T - \lg \eta_{T_{\text{ref}}}) \quad (4)$$

In Eq. (4), a_T represents the shift at temperature T and C is obtained by fitting. The quantities η_T and $\eta_{T_{\text{ref}}}$ are the aged-binder viscosities (cP) at T and at the reference temperature, respectively; the latter corresponds to the RTFOT-aged condition.

The aging condition of SSAMs at 135 °C for 4 h was taken as equivalent to the RTFOT aging state of the neat binder ³⁰. The binder viscosity follows the relationship:

$$\lg \eta = A + VTS \cdot \lg T_R \quad (5)$$

In Eq. (5), T_R is absolute temperature in Rankine; A and VTS are the fitted intercept and viscosity-temperature slope, respectively.

Substituting Eq. (5) into Eq. (4) yields:

$$\lg \alpha_T = C \cdot VTS \cdot (\lg T_R - \lg T_{R,\text{ref}}) \quad (6)$$

where $T_{R,\text{ref}}$ is the Rankine temperature at the reference condition.

The viscosity-temperature parameters were obtained from Brookfield viscosity tests at 135, 150, 165, and 180 °C, while parameter C was fitted by least squares using measured shift factors within the test temperature range. The resulting parameters were $A = 8.72$, $VTS = -2.98$, and $C = 0.89$ for AH-70, and $A = 9.05$, $VTS = -3.21$, and $C = 0.92$ for SBSMA.

Arrhenius model

When binder viscosity data are unavailable, the Arrhenius model based on activation-energy theory is used ³¹:

$$\lg \alpha_T = \frac{\Delta E_a}{2.303R} \cdot \left(\frac{1}{T} - \frac{1}{T_{\text{ref}}} \right) \quad (7)$$

In Eq. (7), ΔE_a denotes binder activation energy and R is the universal gas constant; T and T_{ref} are the test and reference temperatures in Kelvin.

For the Arrhenius model, a standard activation energy of 200,000 J/mol was adopted as a practical engineering convention ³¹. It is recognized that SBS-modified binders may exhibit higher activation energies, typically in the range of 230,000-260,000 J/mol. For ΔE_a varied by $\pm 30,000$ J/mol, the calculated shift at 40 °C changed by less than 0.15 log units, and the corresponding modulus change at 10^{-6} Hz was 3%-5%. This level of deviation falls within the variation observed in the present validation results. Accordingly, the 200,000 J/mol value was considered acceptable for routine master-curve construction when binder-specific viscosity testing is unavailable.

The GAS model is empirically derived from extensive rheological data and accounts for aging effects, but it requires pre-measured binder viscosity curves and is therefore better suited to detailed laboratory studies and refined design analysis. By contrast, the Arrhenius model is rooted in chemical kinetics, uses a universal activation-energy parameter, and does not require additional binder testing, making it more suitable for rapid screening and preliminary engineering estimation. At low temperature and high frequency, the difference between the two calculations remained below 3%. At the high-temperature, low-frequency end, GAS gave modulus values 5%-8% lower, consistent with the nonlinear viscosity-temperature relation represented by that model.

Numerical Optimization Procedure

Parameters α , β , γ , and δ were solved by Levenberg-Marquardt nonlinear least squares. The fitting criterion was the residual sum of

squares for $\log_{10}(E^*)$, so modulus observations at different orders of magnitude contributed on a comparable scale. The Hirsch-informed upper envelope was imposed as the high-frequency constraint, and only test data at -10°C and above were used in fitting. No lower-temperature data were used for parameter calibration, consistent with the objective of avoiding routine testing below -10°C .

Parameter bounds were set within physically reasonable ranges reported for asphalt-mixture master-curve fitting. Convergence was defined as a relative change in the residual sum of squares below 10^{-6} . Sensitivity analysis showed that β and γ exerted the strongest influence on master-curve shape, with sensitivity coefficients of 0.82 and 0.76, whereas α and δ had smaller effects, with coefficients of 0.31 and 0.24. These results indicate that the optimization remained stable under the adopted constraint framework.

For reproducibility, the numerical workflow consisted of four steps: calculation of temperature shift factors, transformation of loading frequency into reduced frequency, nonlinear fitting of the logarithmic modulus data under the fixed upper-envelope constraint, and evaluation against data excluded from parameter calibration. Throughout the analysis, the fitting dataset, the validation dataset, and the role of each constraint were kept strictly separate.

Agreement of Calculated and Measured Shift Factors within the Test Range

Calculated shift factors were compared, as a within-range agreement check, with experimentally derived reference shift factors obtained by the classical WLF horizontal-shifting method for isothermal modulus curves for all five mixtures at -10 , 10 , 20 , and 40°C , with 20°C taken as the reference temperature. The AC-13 (AH-70) mixture is presented in Fig. 3 as a representative example.

For the five mixtures, the mean and maximum relative differences were 3.8% and 4.9% with GAS, compared with 5.7% and 7.3% with Arrhenius. These results indicate that the calculated shift factors reproduce the measured shift behavior without evident systematic bias within the tested temperature range and therefore provide a reasonable basis for subsequent extrapolation analysis.

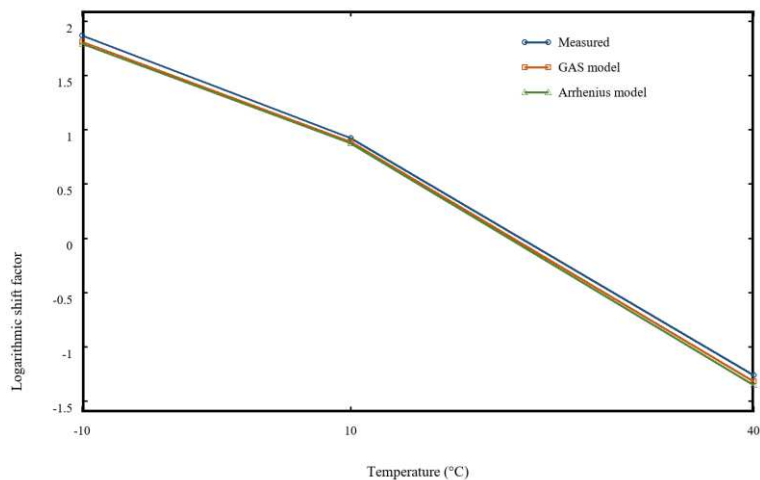


Figure 3. Comparison of measured and calculated shift factors.

Experimental Program

Test Equipment

IDT dynamic modulus was measured with an MTS 810 electro-hydraulic servo testing system fitted with a temperature-controlled chamber. The system applies sinusoidal loading with precise control of temperature and frequency and satisfies the accuracy requirements of AASHTO TP62-03 [2]. The test setup is shown in Fig. 4.



Figure 4. MTS 810 electro-hydraulic servo material testing system.

Test Fixture Optimization

The conventional double-limiting cylinder IDT fixture (Fig. 5), recommended by SHRP and AASHTO, reduces specimen vibration during loading but imposes strict dimensional-accuracy requirements because of its fixed loading strips. When the generatrices of a field core are not parallel, incomplete strip-specimen contact can produce stress concentration and a nonuniform stress field, reducing measurement accuracy³²⁻³⁴.



Figure 5. Double-limiting cylinder IDT test fixture.



Figure 6. Improved spherical-hinge IDT test fixture.

The optimized fixture uses fully separated loading strips: the upper strip is connected to the load cell through a freely rotating spherical hinge, whereas the lower strip is fixed (Fig. 6). The hinge allows the upper strip to adjust adaptively and maintain more uniform contact in the presence of minor specimen dimensional deviations, thereby reducing local stress concentration. The fixture is also compatible with standard gauge-point displacement measurement systems.

Comparative tests on three AC-13 (AH-70) replicates at 20 °C and 10 Hz, with five measurements per specimen, showed that the average CV of the measured modulus decreased from 8.2% with the conventional fixture to 3.5% with the optimized fixture. Additional verification using two-dimensional axisymmetric

finite-element models of the specimen-fixture interface indicated that the maximum stress-concentration factor decreased from 1.32 to 1.05. Across all five mixtures and four test temperatures, the average CV obtained with the modified fixture was 4.2%, about 45% lower than the 7.5%-9.5% range commonly reported in the literature. Taken together, the dedicated comparison and the full-program repeatability results indicate that the optimized fixture improved local contact conditions and sustained stable modulus acquisition throughout the broader test matrix. The reduction exceeded 40% at both -10 °C and 40 °C, decreasing from 7.9% to 4.1% at -10 °C and from 9.2% to 5.0% at 40 °C.

Test Condition Design

The master-curve test matrix comprised four temperatures and six frequencies selected with reference to AASHTO TP62-03 [2] and domestic engineering practice; neighboring temperature-frequency series were arranged to overlap. The complete loading matrix is provided in Table 3.

Test Temperature (°C)	Loading Frequencies (Hz)	Number of Replicates	Specimen Dimensions (mm)	Conditioning Time (h)	Quality Control Criterion	Data Purpose
-20	0.1, 0.5, 1, 5, 10, 25	4 per frequency	Φ 100 × 40	5	CV ≤ 15%	Independent validation of high-frequency extrapolation
-10	0.1, 0.5, 1, 5, 10, 25	4 per frequency	Φ 100 × 40	4	CV ≤ 15%	Master curve construction
10	0.1, 0.5, 1, 5, 10, 25	4 per frequency	Φ 100 × 40	4	CV ≤ 15%	Master curve construction
20	0.1, 0.5, 1, 5, 10, 25	4 per frequency	Φ 100 × 40	4	CV ≤ 15%	Master curve construction
40	0.1, 0.5, 1, 5, 10, 25	4 per frequency	Φ 100 × 40	4	CV ≤ 15%	Master curve construction

Table 3 Detailed test matrix for IDT dynamic modulus tests.

Each mixture was represented by four specimens, with four initial readings taken at every temperature-frequency combination. Before loading, specimens were held at the target temperature for 4 h to reduce the temperature gradient through the specimen. Grubbs' test was applied only when the replicate CV exceeded 15%; approximately 3%-5% of the individual replicate observations, mostly at 40 °C, were excluded. After quality control, each mixture retained mean values for all 24 temperature-frequency conditions, calculated from the remaining valid measurements. The overall average CV was 4.2%.

The -20 °C validation group was tested using the same spherical-hinge fixture. To limit surface frost, the specimens were enclosed in thin plastic film during conditioning, and the chamber was dehumidified before loading. The average CV of the -20 °C group was 7.8%, higher than that of the -10 °C group but still within an acceptable range. These data were not used in any fitting procedure.

IDT creep tests at 60 °C were carried out on AC-13 (AH-70) and AC-13 (SBS) specimens for ultra-low-frequency validation. Three replicates were tested for each mixture under a constant vertical compressive stress of 0.3 MPa, which pretesting confirmed to be within the linear viscoelastic range, for a duration of 3600 s. Horizontal deformation was used to calculate creep compliance $J(t)$, which was subsequently transformed to equivalent dynamic modulus at 10^{-5} and 10^{-6} Hz through the linear-viscoelastic correspondence used in this work.

The reference temperature was set at 20 °C, consistent with common pavement design practice in China. Changing the reference temperature within 15-25 °C altered R2 by less than 0.002 and RMSE by less than 5 MPa, confirming that the results are insensitive to reasonable changes in reference temperature.

Results and Discussion

Classical Sigmoidal Master Curves

The IDT modulus test followed the standard procedure; the loading arrangement is illustrated in Fig. 7. Measured horizontal shift factors were then used to fit the conventional sigmoidal curves at 20 °C (Figs. 8-12).

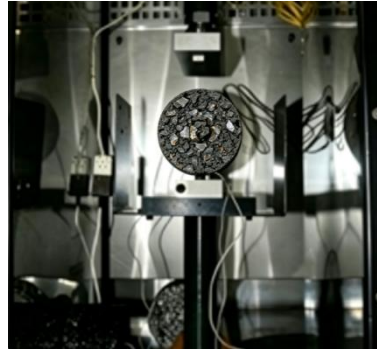


Figure 7. Test configuration of IDT test.

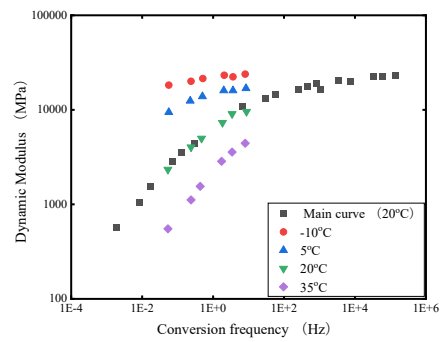


Figure 8. Master curve of AH-70 steel slag AC-13 mixture (20 °C reference).

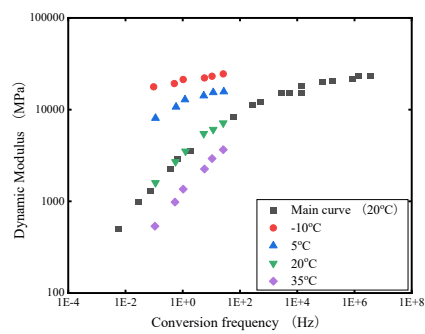


Figure 9. Master curve of SBSMA steel slag AC-13 mixture (20 °C reference).

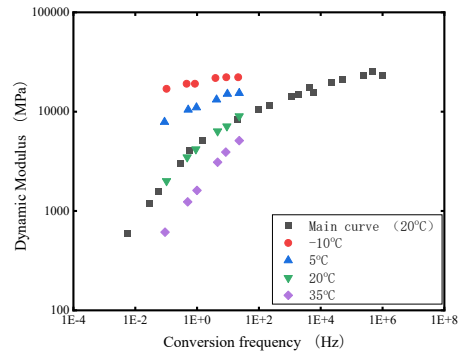


Figure 10. Master curve of AH-70 steel slag AC-20 mixture (20 °C reference).

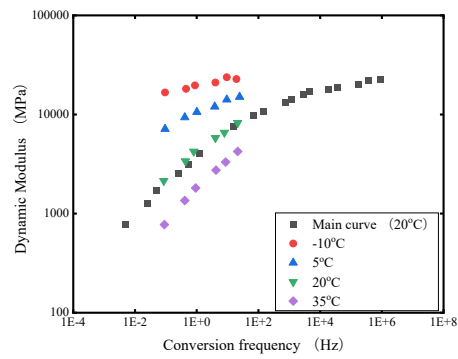


Figure 11. Master curve of SBSMA steel slag AC-20 mixture (20 °C reference).

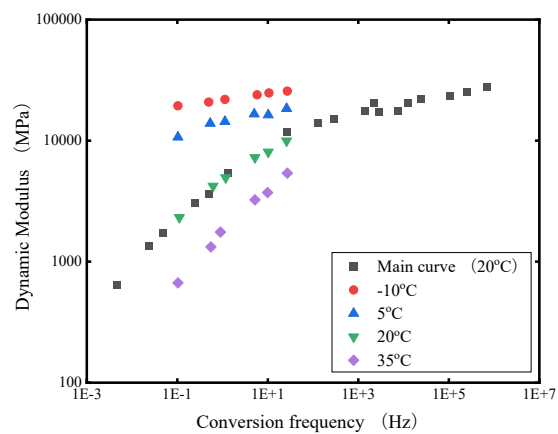


Figure 12. Master curve of AH-70 steel slag AC-25 mixture (20 °C reference).

$|E^*|$ As expected, all SSAMs showed pronounced temperature and frequency dependence, with modulus decreasing as frequency decreased or temperature

increased, consistent with the typical response of viscoelastic materials. At a given gradation, changing the binder produced little separation between curves in the low-temperature, high-frequency range. Differences became more apparent as frequency decreased: the SBSMA mixtures gave marginally lower modulus over the intermediate frequencies, and the two sets of curves gradually converged near the low-frequency end of the measured data, around 10⁻³ Hz. Beyond this range, however, the improved extrapolation methods indicated that the SBSMA mixtures retained distinctly higher modulus values. Because of the upper limit of the test temperature, the classical master curves extended only to about 10⁻³ Hz, with no clear inflection point and a continuously decreasing modulus trend. This result indicates that the classical method cannot reliably characterize ultra-low-frequency modulus behavior or define a stable minimum modulus.

Improved Master Curves from the Two Models

Figure 13 compares the improved master curves constructed using the GAS model together with the Hirsch-informed high-frequency upper-envelope constraint and numerical optimization. The two SBSMA mixtures exhibited flatter master curves and substantially higher moduli in the ultra-low-frequency domain than the three AH-70 mixtures. By contrast, the three AH-70 mixtures showed nearly overlapping master curves, indicating similar frequency dependence.

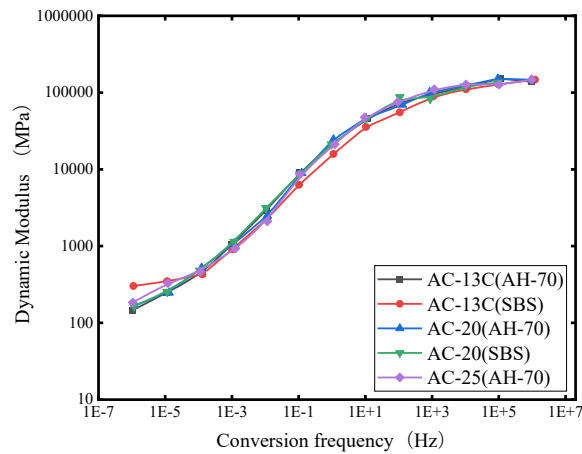


Figure 13. Comparison of master curves of five SSAMs (GAS model, 20 °C reference).

The improved master curves obtained from the Arrhenius model (Fig. 14) showed highly consistent trends: the SBSMA mixtures again showed less modulus reduction toward low frequency, whereas the three AH-70 mixtures showed no obvious difference from one another.

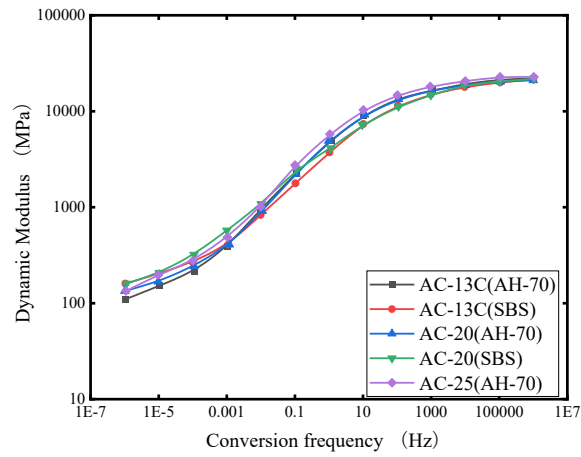


Figure 14. Comparison of master curves of five SSAMs (Arrhenius model, 20 °C reference).

At the high-frequency end, the difference between the two theoretical models was less than 3%. Toward low frequency, the modulus calculated with GAS was slightly lower, consistent with the viscosity-temperature analysis presented above.

Goodness-of-Fit Evaluation and Method Comparison

The three fitting methods were compared in terms of coefficient of determination (R^2) and root mean square error (RMSE), as summarized in Fig. 15.

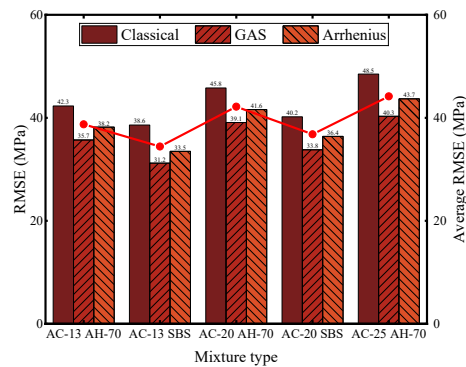
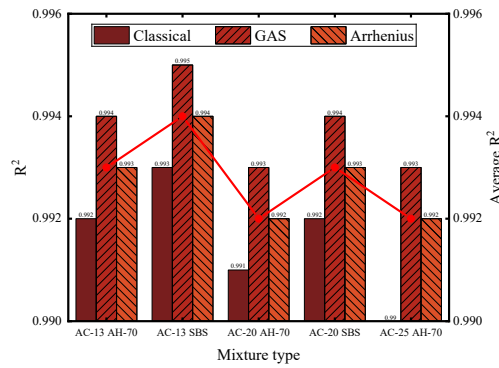


Figure 15. Goodness-of-fit comparison among the three master curve construction methods.

All three methods achieved $R^2 \geq 0.990$ and $RMSE < 50$ MPa, whereas the two constrained methods showed slightly better overall fit. More importantly, the constrained methods attained this accuracy while preserving a physically bounded high-frequency response and a stable low-frequency asymptote within the examined extension range, which is essential for credible long-range extrapolation.

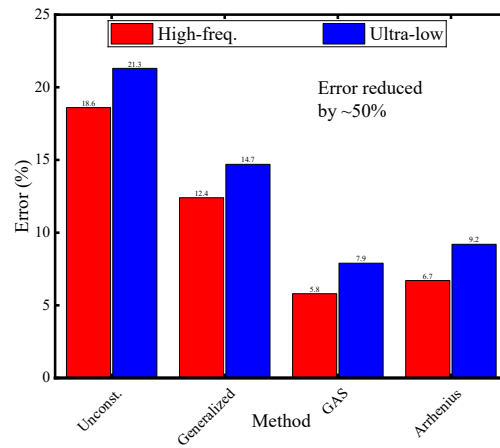


Figure 16. Comparison of extrapolation errors at high-frequency and ultra-low-frequency ends for four fitting methods.

Extrapolation was also checked against the unconstrained and generalized sigmoidal approaches. The measured -20 °C results were used at the high-frequency end, while the modulus converted from 60 °C creep served as the ultra-low-frequency reference. The -20 °C validation covered all five mixtures, whereas the 60 °C creep validation involved the two AC-13 mixtures only. As shown in Fig. 16, the proposed constrained framework provided the most stable long-range ultra-low-frequency extrapolation among the methods compared within this validation set.

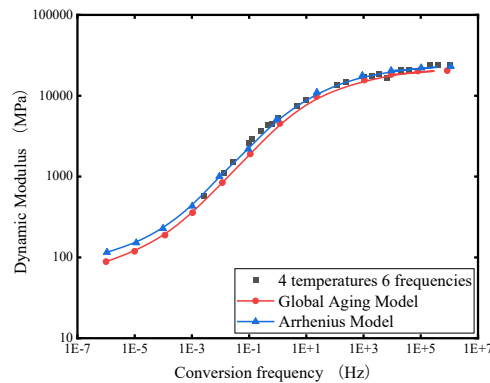


Figure 17. Comparison of master curves by three methods (AH-70 AC-13, 20 °C reference).

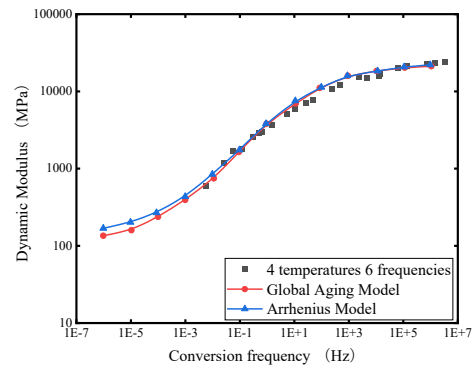


Figure 18. Comparison of master curves by three methods (SBS AC-13, 20 °C reference).

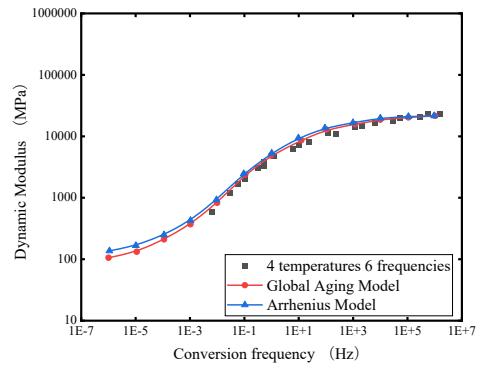


Figure 19. Comparison of master curves by three methods (AH-70 AC-20, 20 °C reference).

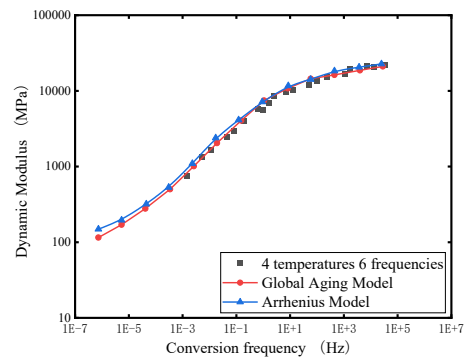


Figure 20. Comparison of master curves by three methods (SBS AC-20, 20 °C reference).

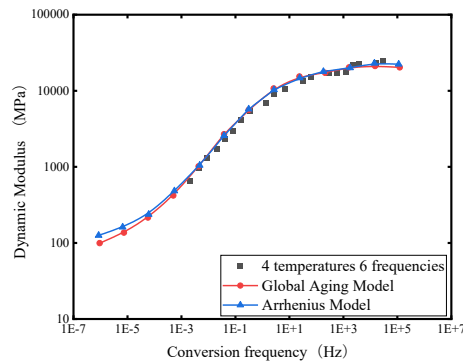


Figure 21. Comparison of master curves by three methods (AH-70 AC-25, 20 °C reference).

Figures 17-21 plot the curves from all three methods for each mixture.

Three consistent features can be identified across all mixtures:

(1) The classical sigmoidal curve continues to diverge at the low-frequency end and does not approach a clear asymptote. Below 10⁻³ Hz, its slope becomes steeper and the modulus decreases faster than would be physically reasonable, which would tend to overestimate permanent deformation if the curve were used directly in design.

(2) Both improved methods produced smooth S-shaped curves with a distinct inflection point and approached a stable low-frequency asymptote as the curves were extended toward approximately 10⁻⁷ Hz. Within the portion of this extension examined against reserved or converted data, they remained close to the fitting data and provided physically consistent extrapolation.

(3) At the high-frequency end, the classical curve lay slightly above the improved curves, with deviations within 6% for all mixtures, because it was fitted directly to the -10 °C data without an upper-bound constraint.

The gap between the classical and improved curves widened as frequency decreased and exceeded 15% at 10⁻⁶ Hz for all mixtures. This finding indicates that the choice of master-curve construction method substantially influences long-term performance predictions and suggests that the constrained framework provides a more reliable basis for ultra-low-frequency extrapolation within the tested material scope.

External Validation and Uncertainty Analysis

High-frequency extrapolation was checked against measured -20 °C data from all five mixtures after shifting to the 20 °C reference temperature. The average extrapolation error was 6.3%, and the maximum error was 8.1%, indicating that the constrained curves remained within a reasonable range of the measured high-frequency response for the materials investigated.

Ultra-low-frequency extrapolation was checked using 60 °C creep data from the two AC-13 mixtures converted to equivalent dynamic modulus. The average deviation at 10⁻⁵ Hz and 10⁻⁶ Hz was 8.7% for this partial validation set, providing material-specific support for the examined low-frequency decades rather than full confirmation for all mixtures to 10⁰ Hz. The remaining deviations can be attributed to three factors: accumulated error over long-range extrapolation, inherent differences between static creep and cyclic dynamic loading, and minor nonlinearity in the actual tests relative to the ideal linear-viscoelastic assumption.

V_{be} Monte Carlo simulation was used to provide an indicative uncertainty envelope for the extrapolated curves. The limiting shear modulus of the binder, VMA, effective asphalt content, and activation energy were treated as normally distributed input variables with coefficients of variation matched to the corresponding experimental error. The 95% prediction-interval width was less than 3% of the modulus value within the tested frequency range (10⁻³-25 Hz) and 5%-

8% in the ultra-low-frequency extrapolation range (10^{-7} - 10^{-3} Hz), suggesting that, under the adopted input-error assumptions, the extrapolated curves remained reasonably stable within the stated applicability boundaries.

Material Factor Effects via ANOVA

An unbalanced two-factor ANOVA was used to quantify the contributions of binder and gradation to dynamic modulus at 20 °C and 10 Hz, using Type III sums of squares to account for the missing SBSMA AC-25 treatment. Because of the missing group, the binder-gradation interaction term had only 1 degree of freedom rather than 2 for a complete 2×3 design, which reduced the statistical power for detecting interaction effects. To verify the conclusion regarding gradation, a separate one-way ANOVA was also performed for the three AH-70 mixtures, which formed a balanced design with adequate power.

The Shapiro-Wilk and Levene results showed no violation of the normality or equal-variance assumptions used in the ANOVA. Table 4 reports the complete two-factor ANOVA results.

Source of variation	Sum of squares	Degrees of freedom	Mean square	F value	P value	Partial η^2	Significance
Corrected model	1286542.3	4	321635.6	13.37	0.000	0.781	**
Intercept	3.286 ^{8} 1 3.286 $\times 10^8$	13651.2	0.000	0.999	**		
Asphalt type	1024768.9	1	1024768.9	42.59	0.000	0.740	**
Gradation type	85423.7	2	42711.85	1.78	0.212	0.191	—
Asphalt $\times$ Gradation interaction	37862.1	1	37862.1	1.57	0.229	0.095	—

Table 4. Two-way ANOVA results for dynamic modulus influencing factors.

Note: ** indicates $P < 0.01$ (highly significant effect); ns indicates $P > 0.05$ (not significant).

Binder type had a highly significant effect on dynamic modulus at 20 °C and 10 Hz ($P < 0.01$, partial eta squared = 0.740), making it the dominant factor at this design condition. The separate one-way ANOVA for the AH-70 mixtures also confirmed that gradation had no statistically significant effect within the AC-13 to AC-25 dense-graded range ($P = 0.235$), consistent with the two-way ANOVA result.

To further examine the gradation effect under the unbalanced design, separate one-way ANOVAs were conducted for the AH-70 and SBSMA binder systems. In the AH-70 system, which included three balanced gradations, gradation remained statistically insignificant. In the SBSMA system, only AC-13 and AC-20 were available; therefore, that comparison is descriptive rather than inferential.

Relative to the baseline modulus at 25 Hz, the AH-70 mixtures retained 12.1%-14.8% of their modulus at 10^{-6} Hz, whereas the SBS mixtures retained 15.7%-18.9%. The corresponding 25%-30% higher retention in the SBS mixtures indicates a greater resistance to modulus loss in the ultra-low-frequency range; this comparison concerns modulus retention only and cannot establish better long-term field durability without pavement-scale validation.

These modulus-retention results are relevant to the selection of material inputs for mechanistic-empirical pavement analysis, but they do not directly measure rutting, fatigue, or field durability. Such implications need to be confirmed through repeated-load testing, wheel-tracking experiments, or in-service pavement observations.

Limitations and Applicability Boundaries

The proposed framework should be interpreted within the scope of its present validation rather than as a universal replacement for all extreme-temperature testing.

(1) The experimental evidence in this study was obtained from naturally aged converter steel slag mixtures prepared with AH-70 and SBS binders in dense-graded AC systems; mixtures with different slag mineralogy, aging condition, binder chemistry, or aggregate structure may exhibit different shift behavior and limiting-modulus characteristics.

(2) In addition, the current calibration and validation window was bounded by dynamic-modulus data from -10 to 40 °C, together with independent checks using -

20 °C data and 60 °C creep-converted modulus. The reported reliability should therefore be understood within this tested temperature-frequency range.

(3) The present method is intended to improve viscoelastic material characterization for mechanistic-empirical pavement analysis. It provides a more defensible basis for long-timescale modulus input selection, but it does not by itself quantify rutting resistance, fatigue damage, or field durability, which still require structural response analysis and performance-based validation.

(4) Finally, although the constrained framework performed consistently for the five mixtures examined here, direct transfer to other steel slag sources, polymer systems, aging levels, or field-cored materials should be preceded by limited revalidation. Within these boundaries, the method remains useful as a practical alternative to unconstrained long-range extrapolation and as a means of reducing routine dependence on extreme-temperature measurements.

Conclusions

This study developed and validated a theoretically constrained procedure for extending the dynamic modulus master curves of dense-graded steel slag asphalt mixtures into the ultra-low-frequency domain while reducing routine dependence on extreme-temperature testing. Within the investigated material range, the main conclusions are as follows:

(1) The optimized spherical-hinge IDT fixture improved loading-contact stability and reduced fixture-induced scatter. In the dedicated head-to-head comparison at 20 °C and 10 Hz, the coefficient of variation decreased from 8.2% to 3.5%; across the full test matrix, the average CV obtained with the optimized fixture was 4.2%. These results show that the fixture improved local contact repeatability and supported stable modulus acquisition throughout the present test program.

(2) By combining a Hirsch-informed upper envelope with GAS- or Arrhenius-based shift factors and numerical optimization, the proposed framework generated smooth master curves that remained stable when extended toward 10^{-7} Hz without routine testing below -10 °C. For all mixtures, the constrained curves achieved $R^2 \geq 0.990$ and $RMSE < 50$ MPa. The main value of this framework is that it constrains long-range extrapolation with physically informed and data-retained bounds rather than allowing the low-frequency tail to drift freely under limited test data.

(3) External checks supported the credibility of the extrapolation within the investigated range. The average error relative to the shifted -20 °C dynamic-modulus data from all five mixtures was 6.3%, and the average deviation relative to the 60 °C creep-converted modulus data from the two AC-13 mixtures was 8.7%. Monte Carlo analysis further indicated a 95% prediction-interval width of 5%-8% in the ultra-low-frequency domain under the adopted input-error assumptions. These results suggest that the method is sufficiently stable for engineering-level material characterization within the tested scope.

(4) Both shift-factor routes were applicable, but they serve different levels of engineering use. The GAS model gave better agreement when binder viscosity-temperature data were available, whereas the Arrhenius formulation remained suitable for preliminary estimation when additional rheological input was not available. This distinction improves the practical usability of the framework under different data conditions.

(5) Within the tested dense-graded SSAM system, binder type had a significant effect on dynamic modulus at 20 °C and 10 Hz ($P < 0.01$), whereas gradation was not significant within the AH-70 system ($P = 0.235$). In the ultra-low-frequency range, SBS mixtures showed 25%-30% higher modulus retention than AH-70 mixtures at 10^{-6} Hz relative to the 25 Hz baseline, indicating a smaller loss of stiffness over long loading times. This difference is relevant to the selection of long-timescale viscoelastic inputs in mechanistic-empirical analysis, although it should not be interpreted directly as proof of superior rutting, fatigue, or field durability.

Overall, the proposed method should be regarded as a validated engineering procedure within the present material scope rather than as a universal substitute for all low-temperature or long-duration testing. Within that scope, it offers a more defensible route than unconstrained extrapolation for obtaining ultra-low-frequency modulus inputs for pavement analysis, especially when direct extreme-temperature or long-duration measurements are difficult to organize on a routine basis. Future work should extend the validation to other steel slag sources, aging levels, binder systems, and field-cored pavements.

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Author contributions

Cheng Wan: Conceptualization, Methodology, Writing original draft, Investigation, Formal analysis, Data curation, Validation. Shengming Zeng: Investigation, Resources, Validation. Jiankun Yang: Software, Visualization, Writing review & editing. Kan Huang: Supervision, Project administration, Funding acquisition.

Data availability

The datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.

Additional information

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