Table 1 Different models and equations used in data analysis

Names of various models and equations		Mathematical expression	Ref.
Adsorption kinetics	Pseudo-first-order	$\log(Q_{e,exp}-Q_t)=\log Q_{e,calc}-k_1t$	[23]
	Pseudo-second-order	$t/Q_t=1/(k_2Q_{e,calc}^2)+t/Q_{e,calc}$ (4)	[24]
	Intraparticle diffusion	$Q_t=K_{id}t^{0.5}+C$	[25]
Isothermal adsorption	Langmuir	$C_e/Q_e=C_e/Q_m+1/(Q_mK_L)$	[13]
	Freundlich	$\log Q_e=1/n\log C_e+\log K_F$	
	Dubinin-Radushkevich (D-R)	$\ln Q_e=\ln Q_m-\beta\varepsilon^2$ $\varepsilon=RT\ln(1+1/C_e)$ $E=1/\sqrt{-2\beta}$	
Thermodynamic study	Van't Hoff equation	$\Delta G°=-RT\ln K^0$ $\ln K^0=\Delta S°/R-\Delta H°/RT$	[26]

k_1 ($\min^{-1}$) and k_2 [g/(mg · min)]: the rate constants of pseudo-first-order and pseudo-second-order models; K_{id} [mg/(g·min$^{1/2}$)]: the intraparticle diffusion rate constant; C: the thickness of the boundary layer; T(K): the absolute temperature; R [8.314 J/(mol·K)]: the gas constant; Q_m (mg/g): the maximum adsorption capacity; K_L (L/mg): the Langmuir adsorption constant; K_F and n: the Freundlich empirical constants; E (kJ/mol): the mean adsorption energy; β(mol^2/kJ^2): a constant related to adsorption energy; ε(kJ/mol): the Polanyi potential; K^0 (=1000K_LM$_{dye}$ dimensionless): the adsorption standard equilibrium constant; $\Delta G°$ (kJ/mol): Gibbs free energy change; $\Delta H°$ (kJ/mol) and $\Delta S°$ (kJ/mol·K): the enthalpy change and entropy change.

Table 2 Kinetic fitting results of dye adsorption onto the gel composite by different models

Dyes	T (K)	$Q_{e,exp}$ (mg/g)	Pseudo-first-order			RE (%)
			k_1 (min^{-1})	$Q_{e,1}$ (mg/g)	R_1^2	
DR 13	298	2416	0.0161	244	0.745	90
	313	2398	0.0183	457	0.839	81
	328	2297	0.0192	278	0.840	88
DB 19	298	2856	0.0142	431	0.774	85
	313	2798	0.0145	585	0.803	79
	328	2683	0.0193	825	0.902	69
Dyes	**T (K)**	**$Q_{e,exp}$ (mg/g)**	**Pseudo-second-order**			**RE (%)**
			k_2 [g/(mg·min)]	$Q_{e,2}$ (mg/g)	R_2^2	
DR 13	298	2416	0.000423	2433	1.000	-6.9
	313	2398	0.000265	2421	1.000	-1.0
	328	2297	0.000413	2315	1.000	-0.78
DB 19	298	2856	0.000141	2907	1.000	-1.8
	313	2798	0.000112	2857	1.000	-2.1
	328	2683	0.000101	2747	0.999	-2.4

Dyes	T (K)	Intraparticle diffusion model				
		First stage		Second stage		Third stage
		K_{id}(mg/g·min$^{1/2}$)	R^2	K_{id}(mg/g·min$^{1/2}$)	R^2	Balance stage
DR 13	298	394	1.000	43.7	0.996	Upper platforms on fitting curves
	313	331	1.000	84.6	0.964	
	328	351	1.000	38.6	0.999	
DB 19	298	561	1.000	49.2	0.902	
	313	535	1.000	64.6	0.950	
	328	469	1.000	57.5	0.937	

Table 3 Fitting results of three isotherm models for equilibrium data and thermodynamic parameters of dye adsorption onto the gel composite

Dyes	T (K)	Langmuir model				Freundlich model			
		Q_m (mg/g)	K_L (L/mg)	R^2	RMSE	$1/n$	$\log K_F$	R^2	RMSE
DR 13	298	2353	0.0977	1.000	2.1×10^{-3}	0.089	3.13	0.893	1.9×10^{-2}
	313	2331	0.0622	1.000	2.6×10^{-3}	0.102	3.08	0.849	2.2×10^{-2}
	328	2232	0.0459	0.999	5.1×10^{-3}	0.105	3.04	0.874	1.8×10^{-2}
DB 19	298	2793	0.0745	0.999	1.8×10^{-3}	0.130	3.10	0.970	1.5×10^{-2}
	313	2755	0.0324	0.999	2.0×10^{-3}	0.170	2.96	0.948	1.8×10^{-2}
	328	2488	0.0211	0.998	5.9×10^{-3}	0.151	2.94	0.908	1.9×10^{-2}

Dyes	T (K)	Dubinin-Radushkevich (D-R) model					Thermodynamic parameters		
		Q_m (mg/g)	$\beta\times10^{-3}$ (mol^2/kJ^2)	E (kJ/mol)	R^2	RMSE	ΔG° (kJ/mol)	ΔH° (kJ/mol)	ΔS° (kJ/mol·K)
DR 13	298	3150	-0.913	23.4	0.962	5.2×10^{-2}	-27.6		
	313	3334	-1.04	21.9	0.929	2.9×10^{-2}	-27.9	-20.5	0.0237
	328	3212	-1.01	22.3	0.937	3.4×10^{-2}	-28.3		
DB 19	298	4352	-6.20	8.98	0.981	1.1×10^{-1}	-27.2		
	313	5021	-1.85	16.4	0.982	6.9×10^{-2}	-26.9	-34.3	-0.0238
	328	2261	-9.09	7.42	0.971	4.5×10^{-2}	-26.5		

Table 4 Comparison of the maximum adsorption capacities (Q_m) of direct dyes onto various adsorbents

Adsorbent	Dye	Q_m (mg/g)	Ref.
α-CDs-EPI	Direct Red 83:1	31.50	[35]
HP-α-CDs-EPI	Direct Red 83:1	23.41	
Bamboo sawdust	Direct red 81	6.43	[36]
Treated bamboo sawdust	Direct red 81	13.83	
Polyacrylamide/activated carbon	Direct red 80	162.0	[37]
Gelatin-CNT-MNPs	Direct red 80	380.7	[22]
Fe_3O_4-N-GO@SA	Direct red 28 (Congo red)	30.06	[12]
PS-CS	Direct red 28 (Congo red)	1081.1	[38]
N_U-UiO-67	Direct red 28 (Congo red)	2360	[39]
N_T-UiO-67	Direct red 28 (Congo red)	1986	
N_A-UiO-67	Direct red 28 (Congo red)	1493	
Magadiite-chitosan composite beads	Direct red 28 (Congo red)	200	[33]
APEADA (resin)	Direct red 28 (Congo red)	280	[40]
Fe_3O_4@ZTB-1	Direct red 28 (Congo red)	458	[5]
Fe(III)/Cs@Dia	Direct red 28 (Congo red)	1111	[13]
	Direct red 23 (Congo red)	1429	
	Direct yellow R	909.1	
	Direct orange 2GL	1250	
Graphene oxide (GO)	Direct red 23	15.3	[8]
Orange peel	Direct red 23	10.718	[41]
	Direct Red 80	21.052	
Biochar produced by the pyrolysis of pig manure	Direct red 23	17.32	[7]
Modified Gambir Adsorbent	Direct red 23	26.67	[42]
Powdered tourmaline (PT)	Direct red 23	153	[43]
CS/OSR/Silica hybrid membrane	Direct Red 31	94.4	[14]
	Direct Blue 71	67.2	
Y/CS/MFA	Direct green 6	627	[44]
Rice husk	Direct Red-31	25.63	[45]
	Direct Orange-26	19.96	
HCl treated rice husk	Direct Orange-26	24.22	[18]
	Direct Red-31	26.16	
	Direct Blue-67	19.85	
	Ever direct Orange-3GL	25.69	
Fe_3O_4@SA/Y	Direct Blue 86	370	[17]
	Direct Green 6	435	
	Direct Brown 2	1250	
CTAB–bentonite	Direct Red 2	153.84	[46]
ZAJHC ($ZnCl_2$ activated Jatropha husk carbon)	Direct Red 12B	39	[9]
Kaolinite	Direct Red 13	7.5	[15]
K-nZVI	Direct Fast Black G (Direct Black 19)	52.12	[47]
Fe-SA-Y@Fe_3O_4	**Direct Red 13**	**2487**	**This study**
	Direct Black 19	**2992**	