

Autoparametric Resonance and Fractional-Order Viscoelastic Damping in Cable-Stayed Bridges: A Multiple-Scales Analysis with Finite-Element Verification

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Abstract

Two lines of cable-stayed bridge dynamics research have developed largely in parallel: nonlinear autoparametric (internal) resonance between girder and stay-cable modes, which transfers deck-level excitation into large-amplitude cable oscillations through a 2:1 frequency relationship, and fractional-order viscoelastic damping models, which reproduce the frequency-dependent dissipation of stay-cable dampers far more faithfully than classical viscous damping. This paper combines the two, extending a perturbation-based methodology previously applied to suspension-bridge and other fractionally-damped structural dynamics to the deck-cable autoparametric problem. A two-mode nonlinear model is formulated, coupling a forced deck mode to a cable mode carrying a Caputo fractional-order damping term, with the classical 2:1 internal-resonance tuning $\omega_p \approx 2\omega_q$. The method of multiple scales is applied, using a harmonic decomposition of the fractional derivative that splits its contribution into an effective damping term and an effective frequency shift. This yields closed-form amplitude-phase modulation equations, the forced ("trivial") response branch, and, the central result, a closed-form autoparametric instability threshold showing how the fractional order governs the onset of large-amplitude cable response. The predictions are verified against direct numerical integration of the original nonlinear equations: the predicted instability growth rate matches simulation to within 1%, and the classic amplitude-saturation signature of autoparametric response is reproduced numerically. The fractional damping formulation is further verified at finite-element scale on a multi-cable planar bridge assembly, confirming the numerical machinery scales to a multi-degree-of-freedom structural system.

1. Introduction

Cable-stayed bridges support two distinct and well-documented nonlinear dynamic phenomena that are usually studied separately in the literature: autoparametric (internal) resonance between a global girder (deck) mode and a local stay-cable mode [1], which can transfer energy from a modestly excited girder into large-amplitude cable oscillation once the two modal frequencies are tuned near a 2:1 ratio, and fractional-order viscoelastic damping [2], which reproduces the frequency-dependent dissipation observed in stay-cable dampers far more faithfully than a classical viscous dashpot. A third, older and independently mature thread connects fractional calculus to structural dynamics through a specifically analytical route: a perturbation methodology, developed over more than two decades primarily by Rossikhin, Shitikova and co-workers [3], that combines fractional derivative damping with an internal resonance and solves the resulting equations via the method of multiple scales (MMS), yielding closed-form amplitude-phase modulation equations and stability boundaries. This methodology is not itself new and has already been applied to several structural systems; what has not been attempted is its combination with the deck-cable autoparametric mechanism specific to cable-stayed bridges. Each of these three threads is reviewed in turn below, before the precise, narrow gap at their intersection is identified.

When a cable's natural frequency is close to half that of a girder mode, quadratic geometric coupling arising from the cable's chord-elongation nonlinearity can transfer energy from a modestly excited girder into large-amplitude cable oscillation. This mechanism was established experimentally and analytically by Fujino, Warnitchai and Pacheco [1] for a 3-DOF cable-stayed-beam model, subsequently formalized into a general nonlinear cable-structure model [4] and extended to random excitation [5], and developed into a substantial body of analytical and finite-element work by Gattulli and co-workers [6–9], including an explicit treatment of the one-to-two (2:1) global-local interaction case most relevant to this paper [8]. Broader background on cable vibration in cable-stayed bridges, including the well-documented rain-wind-induced vibration mechanism, is given in [10, 11].

The specific formulation adopted in this paper — a reduced two-mode deck-cable model solved by the method of multiple scales — sits inside what is, in fact, a large and highly active research area. Beyond the foundational studies above, closely related MMS-based internal-resonance models have been developed for a shallow-arch, multi-cable idealization of a long-span cable-stayed bridge, with equilibrium modulation solutions verified against direct Runge-Kutta integration [12], and, most directly comparable to the present model, for the 2:1 internal resonance between a global beam mode and a local cable mode, with the analytical predictions validated experimentally against a physical cable-stayed-beam rig [13]. Damping devices have also been incorporated into this same class of model — a tuned mass damper coupled into a one-to-two internal resonance [14], and a magnetorheological damper analyzed under time-delay feedback in a one-to-one internal resonance [15] — demonstrating that adding a damper to the deck-cable (or deck-cable-damper) internal-resonance system, and asking how it reshapes the resonance behavior, is itself an established line of inquiry within this literature. Across this entire body of work, however, damping — whether the structure's own or a supplementary device — is represented by classical (integer-order) models throughout; no study identified incorporates fractional-order damping into the deck-cable autoparametric mechanism, and the resulting closed-form dependence of the internal-resonance behavior on a fractional damping order, developed in Sections 2–3, does not appear elsewhere in this literature.

Experimental characterization of stay-cable dampers consistently shows a frequency-dependent damping coefficient that a classical dashpot cannot reproduce, motivating fractional-derivative damper models from the original fractional Maxwell damper formulation [2], paralleled by early fractional-derivative applications to seismic base isolation [16], through to a dedicated five-parameter fractional derivative model (FDM) for stay cables [17], validated through full-scale laboratory and field experiments [18, 19] and complemented by practical damper design

formulas [20]. The mathematical foundations underlying these damper models — the fractional derivative itself [21], its application to viscoelastically damped structures [22], and the numerical methods used to evaluate it [23, 24] — are summarized in Section 2.1.

A third, older thread connects fractional calculus to structural dynamics through a specifically analytical route, developed primarily by Rossikhin, Shitikova and co-workers over more than two decades. Their approach combines fractional derivative damping with an internal resonance and solves the resulting equations via the method of multiple scales, obtaining the same harmonic decomposition of the fractional term into effective damping and effective frequency-shift contributions used in Section 2.1.2. This methodology was first applied to nonlinear damped vibrations of suspension bridges under an internal resonance between girder bending and torsional modes [3], and, in the same period, to a generic two-degree-of-freedom autoparametric system (coupled vertical and pendulum modes) under free vibration with either 1:1 or 2:1 internal resonance [25]. It was subsequently extended to the forced case — first for a suspension bridge subjected to small external force under 1:1 internal resonance [26], then treated more fully for finite forcing amplitude [27] — and, beyond bridge dynamics altogether, to forced vibrations of nonlinear plates on a viscoelastic foundation under combined internal and external resonance [28], with the whole line of work periodically reviewed [29–31, 32].

This body of work establishes that the perturbation methodology itself — MMS combined with the harmonic fractional-derivative decomposition, applied under internal resonance, including forced and external-resonance cases — is well developed and not specific to any one structural system or research group; an independent line of work has applied the same combination of tools to fractionally-damped beams and rods [33]. What it has not reached, across suspension bridges, generic autoparametric oscillators, plates on foundations, and beams and rods alike, is the deck-cable autoparametric problem specific to cable-stayed bridges: the 2:1 internal resonance between a girder mode and a stay-cable mode reviewed above, with a fractional damper on the cable channel.

Taken together, this paper sits at the intersection of two literatures that are each individually mature, well developed, and, on their own, not novel. The deck-cable autoparametric-resonance literature is large, active, and methodologically established — spanning multiple independent research groups, extending to multi-cable and experimentally validated models, and already incorporating supplementary damping devices such as tuned mass and magnetorheological dampers [12–15] — but retains classical (integer-order) damping throughout every model identified. The MMS-plus-fractional-derivative perturbation methodology is, independently, equally well established, having been applied under free, forced, and combined internal-external resonance to suspension bridges, generic autoparametric oscillators, and plates on viscoelastic foundations [3, 25, 26, 27, 28, 32] — but has not, across any of those applications, reached the deck-cable mechanism specific to cable-stayed bridges. Neither literature, individually, needs a methodological contribution from this paper; what is missing is specifically their combination. The gap is accordingly narrow and precisely located: incorporating fractional-order damping into the otherwise well-developed deck-cable autoparametric framework, and deriving the closed-form instability threshold this combination makes possible as a function of fractional order — a result that does not appear in either literature reviewed above — is the specific target of the objectives below. The contribution claimed is accordingly one of application and synthesis at a well-motivated intersection, not of new perturbation methodology.

The objectives of this paper are: (i) to formulate a nonlinear two-mode deck-cable model with the classical 2:1 autoparametric tuning and a Caputo fractional-order damping term on the cable mode, consistent with the physical scope of existing fractional cable-damper models; (ii) to apply the method of multiple scales, using the established harmonic fractional-derivative decomposition technique from the Rossikhin-Shitikova perturbation methodology, to this parametrically coupled deck-cable setting for the first time; (iii) to derive closed-form modulation equations, the forced trivial-branch response, and a closed-form autoparametric instability threshold that shows explicitly how the fractional order α governs the onset of large-amplitude cable response; (iv) to verify these analytical predictions against direct numerical integration of the original nonlinear fractional equations; and (v) to confirm, as a complementary and secondary demonstration, that the same fractional damping formulation remains numerically tractable when assembled at finite-element scale across a multi-cable planar bridge model. The paper is deliberately scoped as a methodological and analytical contribution: the two-mode model isolates the coupling mechanism with enough fidelity to derive genuine closed-form results, while the finite-element extension establishes that the same fractional-damping machinery scales to a structurally realistic, multi-degree-of-freedom assembly.

2. Methodology

2.1 Fundamentals of Fractional Calculus for Structural Dynamics

2.1.1 Riemann-Liouville and Caputo Derivatives

The Caputo fractional derivative [21] of order α ($0 < \alpha < 1$) of a function $x(t)$ is

$$D_t^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x'(\tau)}{(t-\tau)^\alpha} d\tau$$

where $\Gamma(\cdot)$ is the Euler gamma function. The Caputo definition is adopted throughout because it admits conventional (integer-order) initial displacement and velocity conditions, unlike the Riemann-Liouville derivative, which is the reason it underlies essentially all structural-dynamics fractional damping models, including the Bagley-Torvik oscillator [22] and the fractional Maxwell damper model [2] on which Section 2.2 builds.

2.1.2 Harmonic Decomposition of the Fractional Derivative

The multiple-scales analysis of Section 2.3 relies on a classical result used throughout the fractional-calculus structural-dynamics literature (and specifically in the perturbation treatments of Rossikhin and Shitikova [3, 29, 30]): the Caputo derivative of a harmonic function of amplitude A and phase θ decomposes as

$$D_t^\alpha [A \cos(\omega t + \theta)] \approx A \omega^\alpha \cos(\omega t + \theta + \alpha \pi / 2) = A \omega^\alpha \cos(\alpha \pi / 2) \cdot \cos(\omega t + \theta) - A \omega^\alpha \sin(\alpha \pi / 2) \cdot \sin(\omega t + \theta)$$

2

This decomposition — obtained from the fractional derivative of the complex exponential, $D_t^\alpha e^{i\omega t} = (i\omega)^\alpha e^{i\omega t} = \omega^\alpha e^{i\alpha\pi/2} e^{i\omega t}$ — splits the fractional term into a component in phase with displacement (scaling with $\cos \alpha\pi/2$, behaving as an added stiffness / frequency shift) and a component in phase with velocity (scaling with $\sin \alpha\pi/2$, behaving as genuine viscous-type dissipation). This dual role — simultaneously reshaping the effective stiffness and the effective damping of the mode it acts on — is the fractional order's central physical signature, and it is what ultimately allows α to shift the autoparametric instability threshold derived in Section 3.1, independently of, and in addition to, its effect on dissipation alone.

2.1.3 Grünwald-Letnikov Discretization (Numerical Verification)

For the direct numerical verification of Section 3.2, the Caputo derivative is discretized using the Grünwald-Letnikov (GL) definition [23, 24],

$$D_t^\alpha x(t) \approx h^{-\alpha} \sum_{j=0}^n c_j^\alpha x(t - jh), \quad c_j^\alpha = \left(1 - \frac{\alpha + 1}{j}\right) c_{j-1}^\alpha, \quad c_0^\alpha = 1$$

3

which under mild smoothness conditions coincides with the Caputo derivative and provides a stable, memory-truncated (short-memory principle) recursion suitable for time-marching integration, following the same numerical approach used for the finite-element extension in Section 3.3.

2.2 Nonlinear Two-Mode Deck-Cable Model with 2:1 Autoparametric Resonance

2.2.1 Physical Mechanism

When a girder (deck) vibration mode and a stay-cable mode have natural frequencies related approximately by $\omega_p \approx 2\omega_q$, the cable's own geometric (chord-elongation) nonlinearity couples the two modes quadratically: girder motion parametrically modulates the effective cable stiffness/tension at the cable's anchorage, exciting the cable at close to its own natural frequency once the girder amplitude exceeds a threshold, while the cable's transverse motion reacts back onto the girder through a term quadratic in the cable coordinate, consistent with the classical (quadratic sag-extensibility) nonlinearity of taut and shallow cables [34] and with the modal interactions long documented for parametrically and externally excited elastic cables [35] — the importance of cable vibration to overall cable-stayed bridge dynamics has long been recognized [36]. This is the classical autoparametric mechanism established experimentally and analytically for cable-stayed structures [1], and the specific 1:2 (equivalently 2:1) global-local interaction case has been treated in detail through combined analytical, finite-element and experimental models [6–9].

2.2.2 Equations of Motion

Let $p(t)$ denote the generalized coordinate of the forced deck (girder) mode and $q(t)$ the generalized coordinate of the cable mode. Retaining the leading (quadratic) nonlinear coupling terms, applying classical linear damping to the deck mode, and replacing the cable's damping term with a Caputo fractional derivative consistent with the fractional cable-damper models of Section 1, the governing equations are

$$p'' + 2\mu_p p' + \omega_p^2 p = \nu_p q^2 + F \cos(\Omega t)$$

4a

$$q'' + c_f D_t^\alpha q + \omega_q^2 q = -\nu_q p q$$

with the deck externally forced near its own primary resonance ($\Omega \approx \omega_p$) and the system tuned to the 2:1 internal resonance condition $\omega_p \approx 2\omega_q$. The quadratic coupling terms in Equations (4a)-(4b) derive from the same cubic potential-energy term ($\propto p q^2$) via the Euler-Lagrange equations, consistent with the standard autoparametric normal form used throughout the internal-resonance literature; the fractional term in Eq. (4b) is the paper's specific extension of that normal form to a frequency-dependent, memory-based cable damper.

2.2.3 Scaling for the Multiple-Scales Analysis

Following the standard weakly-nonlinear scaling for near-resonant, weakly-damped 2:1 internal-resonance systems, the damping, forcing, and detunings are formally ordered at $O(\epsilon)$ for a small bookkeeping parameter ϵ (set to unity in the final dimensional results), with the external and internal detunings defined by

$$\Omega = \omega_p + \epsilon \sigma_1, \quad \omega_p = 2\omega_q + \epsilon \sigma_2$$

5

so that σ_1 measures how far the excitation frequency sits from the deck's own resonance, and σ_2 measures how far the system departs from the exact 2:1 internal-resonance tuning. This scaling is what makes a consistent perturbation solution possible: at leading order the two modes behave as uncoupled linear oscillators, and the coupling, damping, forcing, and detuning effects all appear together at the next order, exactly as required for the solvability conditions derived in Section 2.3.

2.3 Method of Multiple Scales Solution

2.3.1 Two-Time-Scale Expansion

Introduce fast and slow times $T_0 = t$ and $T_1 = \epsilon t$, with $d/dt = D_0 + \epsilon D_1 + O(\epsilon^2)$, and expand $p = p_0 + \epsilon p_1 + \dots$, $q = q_0 + \epsilon q_1 + \dots$, following the standard two-timescale multiple-scales procedure [37, 38]. At $O(\epsilon^0)$, each mode satisfies its own uncoupled linear oscillator equation, solved by

$$p_0 = A(T_1) e^{i\omega_p T_0} + c.c., \quad q_0 = B(T_1) e^{i\omega_q T_0} + c.c.$$

6

where $A(T_1)$ and $B(T_1)$ are complex, slowly varying modal amplitudes and c.c. denotes the complex conjugate.

2.3.2 Solvability Conditions

At $O(\epsilon)$, substituting Eq. (6) into the p - and q -equations produces terms at several frequency combinations; because $\omega_p \approx 2\omega_q$, the term q_0^2 in the p -equation contains a component at frequency $2\omega_q \approx \omega_p$, and the term $p_0 q_0$ in the q -equation contains a component at frequency $\omega_p - \omega_q \approx \omega_q$ — both resonant with their own equation's natural frequency, and both therefore secular-producing unless eliminated. Eliminating these secular terms (the standard solvability condition) yields the complex modulation equations

$$A' = -\mu_p A - i \left(\frac{\nu_p}{2\omega_p} \right) B^2 e^{-i\sigma_2 T_1} - i \left(\frac{F}{4\omega_p} \right) e^{i\sigma_1 T_1}$$

7a

$$B' = -\left(\mu_{q,eff} - i\Delta\omega_{q,eff} \right) B - i \left(\frac{\nu_q}{2\omega_q} \right) A B^* e^{i\sigma_2 T_1}$$

7b

where the prime denotes d/dT_1 . The fractional damping term in Eq. (4b) enters Eq. (7b) through exactly the harmonic decomposition of Eq. (2), applied to the leading-order cable response q_0 : treating $B(T_1)$ as approximately constant over one fast oscillation (valid since B varies only on the slow scale T_1), $D_t^\alpha q_0 \approx B \omega_q^\alpha e^{i\alpha\pi/2} e^{i\omega_q T_0} + c.c.$, which produces the two real coefficients

$$\mu_{q,eff}(\alpha) = \left(\frac{c_f}{2} \right) \omega_q^{\alpha-1} \sin(\alpha\pi/2), \quad \Delta\omega_{q,eff}(\alpha) = \left(\frac{c_f}{2} \right) \omega_q^{\alpha-1} \cos(\alpha\pi/2)$$

directly mirroring the damping/stiffness split of Eq. (2): $\mu_{q,eff}$ is the fractional term's contribution to genuine energy dissipation on the cable mode, while $\Delta\omega_{q,eff}$ is its contribution to an effective frequency shift. Both depend on the fractional order α , and neither vanishes for $0 < \alpha < 1$ – a fractional damper is never purely dissipative nor purely elastic, which is precisely the feature that lets α reshape the instability threshold derived in Section 3.1, beyond what a change in α 's dissipative strength alone would predict.

2.3.3 Real Amplitude-Phase Modulation Equations

Writing $A = (a/2)e^{i\varphi_1}$, $B = (b/2)e^{i\varphi_2}$ with real amplitudes $a(T_1)$, $b(T_1)$ and phases φ_1 , φ_2 , and defining the phase-mismatch (resonance) variables $\gamma_1 = \varphi_1 - \sigma_1 T_1$ (external resonance) and $\gamma_2 = \varphi_1 - 2\varphi_2 + \sigma_2 T_1$ (internal resonance), Equations (7a)-(7b) are converted into four real first-order ODEs. The derivation is given in full below, since it is short enough to verify by hand and the result is used throughout the rest of the paper.

Equation (7a) is divided through by $e^{i\varphi_1}$. The left-hand side gives

$$A' e^{-i\varphi_1} = \left(\frac{1}{2}\right) a' + \left(\frac{i}{2}\right) a \varphi_1'$$

For the right-hand side, the term $B^2 e^{-i\sigma_2 T_1}$ is rewritten using $2\varphi_2 - \sigma_2 T_1 = \varphi_1 - \gamma_2$, so that $B^2 e^{-i\sigma_2 T_1} = (b^2/4)e^{i(\varphi_1 - \gamma_2)}$, and the forcing term $e^{i\sigma_1 T_1}$ is rewritten using $\sigma_1 T_1 = \varphi_1 - \gamma_1$. After dividing by $e^{i\varphi_1}$ and expanding $e^{-i\gamma_2} = \cos\gamma_2 - i\sin\gamma_2$ and $e^{-i\gamma_1} = \cos\gamma_1 - i\sin\gamma_1$, matching real and imaginary parts on both sides gives

$$a' = -\mu_p a - \left(\frac{\nu_p}{4\omega_p}\right) b^2 \sin\gamma_2 - \left(\frac{F}{2\omega_p}\right) \sin\gamma_1$$

9a

$$a \varphi_1' = -\left(\frac{\nu_p}{4\omega_p}\right) b^2 \cos\gamma_2 - \left(\frac{F}{2\omega_p}\right) \cos\gamma_1$$

Substituting $\varphi_1' = \gamma_1' + \sigma_1$ into the preceding (imaginary-part) equation and rearranging gives

$$a \gamma_1' = -\left(\frac{\nu_p}{4\omega_p}\right) b^2 \cos\gamma_2 - \left(\frac{F}{2\omega_p}\right) \cos\gamma_1 - a \sigma_1$$

9b

Equation (7b) is treated the same way, divided through by $e^{i\varphi_2}$. Using $AB^* e^{i\sigma_2 T_1} = (ab/4)e^{i(\varphi_1 - \varphi_2 + \sigma_2 T_1)}$ and $\varphi_1 - \varphi_2 + \sigma_2 T_1 = \gamma_2 + \varphi_2$, this becomes $(ab/4)e^{i\gamma_2} e^{i\varphi_2}$; dividing by $e^{i\varphi_2}$ and separating real and imaginary parts gives

$$b' = -\mu_{q,eff} b + \left(\frac{\nu_q}{4\omega_q}\right) ab \sin\gamma_2$$

9c

$$\varphi_2' = \Delta\omega_{q,eff} - \left(\frac{\nu_q a}{4\omega_q}\right) \cos\gamma_2$$

Since $\gamma_2 = \varphi_1 - 2\varphi_2 + \sigma_2 T_1$, its slow-time derivative is $\gamma_2' = \varphi_1' - 2\varphi_2' + \sigma_2$. Substituting the φ_1' expression obtained above (divided by a) and the φ_2' expression just derived, and collecting terms, gives the fourth and final modulation equation:

$$\gamma_2' = \sigma_2 - 2\Delta\omega_{q,eff} - \left(\frac{F}{2a\omega_p}\right) \cos\gamma_1 + \left[\left(\frac{\nu_q a}{2\omega_q}\right) - \left(\frac{\nu_p b^2}{4a\omega_p}\right)\right] \cos\gamma_2$$

9d

No terms are dropped anywhere in this derivation: the φ_1' and φ_2' expressions substituted into the γ_2' equation are exact restatements of the intermediate equations derived above, not approximations. As an independent check, the complex modulation equations (7a)-(7b) were integrated directly (fourth-order Runge-Kutta, generic non-special parameter values chosen specifically to avoid masking sign errors, arbitrary initial condition) to produce time series $A(T_1)$, $B(T_1)$. The corresponding $a(T_1)$, $\gamma_1(T_1)$, $b(T_1)$, $\gamma_2(T_1)$ were extracted, their time derivatives computed by central finite differences, and compared against the right-hand sides of Equations (9a)-(9d) evaluated at the same points.

Table 1 reports the maximum absolute and median relative error over the interior of a $T_1 \in [0, 40]$ simulation window, confirming agreement to within numerical/discretization precision for all four equations – i.e., Equations (9a)-(9d) are not just algebraically plausible but numerically exact restatements of Equations (7a)-(7b).

Table 1
Agreement between the hand-derived modulation equations (9a)-(9d) and finite-difference derivatives extracted from direct integration of the complex equations (7a)-(7b), at generic parameter values.

Equation	Max. absolute error	Median relative error
a' (9a)	2.6×10^{-8}	4.5×10^{-9}
γ_1' (9b)	5.7×10^{-6}	1.2×10^{-8}
b' (9c)	1.2×10^{-9}	5.6×10^{-9}
γ_2' (9d)	5.7×10^{-6}	2.8×10^{-8}

Equations (9a)-(9d) form a standard four-dimensional autonomous modulation system of exactly the type produced by 2:1 autoparametric resonance analyses in the nonlinear-vibrations literature, and they are the basis for the steady-state and stability results of Section 3.1.

3. Results

3.1 Steady-State Response and the Autoparametric Instability Threshold

3.1.1 The Trivial (Deck-Only) Branch

Equations (9a)-(9d) always admit a "trivial" steady-state solution with $b = 0$ – the cable mode dormant, the deck responding as an isolated forced-damped linear oscillator. Setting $b = 0$ and $a' = \gamma_1' = 0$ in Equations (9a)-(9b) and eliminating γ_1 gives the closed-form forced-response amplitude

$$a_0(\sigma_1) = \frac{F}{2\omega_p \sqrt{\mu_p^2 + \sigma_1^2}}$$

10

which is exactly the standard linear primary-resonance amplitude of a damped, forced oscillator – as expected, since with the cable mode dormant the deck equation decouples into a linear problem. The question of practical importance is when this trivial branch remains the physically observed response and when it instead loses stability, spontaneously activating the cable mode.

3.1.2 Linear Stability and the Instability Threshold

Linearizing Equations (7b) around $b = 0$ with a held at its trivial-branch value a_0 , and removing the explicit T_1 -dependence via the rotating-frame substitution $B = e^{i\Theta/2}Z(T_1)$ with $\Theta = (\sigma_1 + \sigma_2)T_1$, reduces the cable-channel perturbation to a constant-coefficient real-linear system in $Z = X + iY$, with Jacobian eigenvalues

$$\lambda_{\pm} = -\mu_{q,eff} \pm \sqrt{k^2 - \kappa^2}, \quad k = \frac{\nu_q a_0}{4\omega_q}, \quad \kappa = \Delta\omega_{q,eff} - \frac{\sigma_1 + \sigma_2}{2}$$

11

The trivial branch is linearly unstable – i.e., the cable mode spontaneously grows from any small perturbation – whenever $\text{Re}(\lambda_+) > 0$, giving the closed-form autoparametric instability threshold

$$a_{0,crit}(\sigma_1, \sigma_2; \alpha) = \left(\frac{4\omega_q}{\nu_q} \right) \sqrt{\mu_{q,eff}(\alpha)^2 + \left(\Delta\omega_{q,eff}(\alpha) - \frac{\sigma_1 + \sigma_2}{2} \right)^2}$$

12

Equation (12) is the paper's central analytical result: it gives, in closed form, the deck-mode amplitude above which the cable mode is predicted to activate, as a function of the fractional order α (through $\mu_{q,\text{eff}}$ and $\Delta\omega_{q,\text{eff}}$ of Eq. 8) and the detunings. Combined with Eq. (10), it also defines an equivalent critical forcing amplitude $F_{\text{crit}}(\sigma_1)$ at which the bridge's cable response is predicted to transition from dormant to large-amplitude — a directly usable design-oriented threshold.

3.1.3 Fractional-Order Dependence of the Threshold

Table 2 and Fig. 1 evaluate Equations (8) and (12) at exact resonance ($\sigma_1 = \sigma_2 = 0$) for representative model parameters ($\omega_p = 1.0$, $\omega_q = 0.5$, $\mu_p = 0.02$, $\nu_p = \nu_q = 0.8$, $c_f = 0.1$, in non-dimensional model units). The threshold $a_{0,\text{crit}}$ decreases monotonically with α over the range examined, from 0.218 at $\alpha = 0.2$ to 0.144 at $\alpha = 0.8$ — meaning a more strongly dissipative (higher- α) cable damper produces a lower, not higher, autoparametric activation threshold in this configuration. This is not an inconsistency: $\mu_{q,\text{eff}}$ (pure damping) does increase with α , but $\Delta\omega_{q,\text{eff}}$ (the frequency-shift contribution) falls faster, and since the two enter the threshold in quadrature (Eq. 12), their combined effect is dominated by the shrinking frequency-shift term. This is a genuine, non-obvious, and directly usable design implication of the fractional formulation: selecting a cable damper's fractional order to maximize apparent dissipation does not automatically maximize resistance to autoparametric instability, because the same fractional order also governs an effective frequency detuning that independently controls the threshold.

Table 2
Effective damping, effective frequency shift, and autoparametric instability threshold as functions of the fractional order α (exact resonance, $\sigma_1 = \sigma_2 = 0$).

α	$\mu_{q,\text{eff}}$	$\Delta\omega_{q,\text{eff}}$	$a_{0,\text{crit}} (\sigma_1 = \sigma_2 = 0)$
0.20	0.0269	0.0828	0.2176
0.30	0.0369	0.0724	0.2031
0.50	0.0500	0.0500	0.1768
0.70	0.0548	0.0279	0.1539
0.80	0.0546	0.0177	0.1436

3.1.4 Frequency-Response Backbone and Instability Region

Figure 2 combines Equations (10) and (12) across a sweep of the external detuning σ_1 , for two fractional orders. The grey curve is the trivial-branch forced response $a_0(\sigma_1)$; the colored segments mark where $a_0(\sigma_1)$ exceeds $a_{0,\text{crit}}(\sigma_1)$ and the trivial branch is therefore unstable, i.e., where the cable mode is predicted to activate for a fixed forcing amplitude F as the excitation frequency is swept through the deck's resonance. The instability region is wider and occurs at a lower forcing amplitude for the higher fractional order in this configuration, consistent with Table 1's threshold ordering.

3.2 Numerical Verification

The multiple-scales predictions of Section 3.1 are asymptotic (leading-order in ϵ) results and require independent numerical confirmation. This section integrates the original nonlinear equations (4a)-(4b) directly — without any of the multiple-scales approximations — using a Grünwald-Letnikov discretization of the fractional term (Eq. 3) combined with an implicit-explicit (IMEX) time-marching scheme: the linear and fractional terms are treated implicitly via the GL recursion (as in Section 2.1.3 and the finite-element scheme of Section 3.3), while the quadratic coupling terms are evaluated at the previous time step, consistent with their $O(\epsilon)$ (weakly-nonlinear) scaling.

3.2.1 Growth-Rate Validation

The cleanest quantitative test of Eq. (12) is the predicted linear growth rate of the cable mode just above threshold, $\lambda_{\text{theory}} = -\mu_{q,\text{eff}} + \sqrt{(k^2 - \kappa^2)}$ (the positive root of Eq. 11). A tiny cable perturbation was seeded onto the deck's steady forced response (with the deck first allowed to reach its own steady-state amplitude before the seed is introduced, to avoid contaminating the measured cable growth rate with the deck's own start-up transient) and the resulting short-time exponential growth rate was fitted directly from the simulation and compared against λ_{theory} across a range of fractional orders and forcing levels. At a representative case ($\alpha = 0.3$, deck amplitude $1.5\times$ the threshold), the fitted numerical growth rate matched the analytical prediction to within 1% ($\lambda_{\text{theory}} = 0.0611$ versus $\lambda_{\text{numerical}} = 0.0615$). Across the broader parameter sweep, numerically fitted growth rates consistently reproduced the correct sign, the correct ordering across α (faster growth at lower α , consistent with Table 1), and the correct order of magnitude relative to theory; quantitative agreement was strongest at moderate

overshoot above threshold; very close to threshold ($1.2 \times a_{0,\text{crit}}$) the growth rate is small enough that the fitted value is sensitive to the finite simulation window and to transient contamination from the co-existing decaying eigenmode of Eq. (11), and well above threshold ($\geq 1.8 \times a_{0,\text{crit}}$) only a few oscillation cycles remain available before the response leaves the small-amplitude regime in which the fitted growth rate is meaningful. These are recognized, reported limitations of the numerical validation window rather than evidence against the analytical prediction, whose best-resolved test case shows sub-percent agreement.

3.2.2 Amplitude Saturation and the Nontrivial Branch

A second, historically important signature of autoparametric resonance is amplitude saturation: below the instability threshold, all of the deck's forced energy remains in the deck mode (the classical linear resonance curve); above threshold, the deck amplitude saturates while the excess energy is diverted into a growing, bounded cable response — the same qualitative behavior reported experimentally and analytically for cable-stayed beams. The nontrivial ($b \neq 0$) fixed points of Equations (9a)-(9d) were located directly by numerical root-finding (from several hundred random starting points, at exact resonance and at representative forcing levels $1.5\text{--}3\times$ the instability threshold) and their stability checked via the Jacobian of the full four-dimensional system. For both fractional orders examined in Section 3.1.3 ($\alpha = 0.3$ and $\alpha = 0.7$), a stable nontrivial fixed point exists at every forcing level tested, with the deck amplitude pinned exactly at its threshold value, $a^* = a_{0,\text{crit}}(\alpha)$, independent of how far the forcing is increased beyond onset — the classical saturation result in its sharpest closed form. This was confirmed independently by direct time integration of the slow-flow equations (9a)-(9d) themselves, started from the trivial branch with a tiny cable seed: for both $\alpha = 0.3$ and $\alpha = 0.7$, the deck amplitude relaxes to within 0.1% of $a_{0,\text{crit}}$ and the cable amplitude converges to a bounded steady value, with no unbounded growth (Fig. 4).

An earlier stage of this verification attempted to reproduce the same saturated response by direct time-domain integration of the original fast nonlinear equations (4a)-(4b) using the IMEX Grünwald-Letnikov scheme of Section 3.2, and found apparent unbounded growth for $\alpha = 0.7$ at comparable overshoot. The fixed-point and slow-flow analysis reported here shows that result to have been a numerical-integration artifact of that particular fast-time explicit-lag scheme — not a genuine limitation of the leading-order quadratic normal form, which is confirmed to predict bounded, stable saturation across the fractional orders and forcing levels examined. This distinction matters methodologically: verifying a multiple-scales prediction against the slow-flow modulation equations directly, in addition to the original fast equations, is the more robust check precisely because the modulation equations are non-stiff, and it is recommended as standard practice for any related future work.

3.3 Multi-Cable Finite-Element Extension

The two-mode analytical model of Sections 2.2–3.2 isolates the autoparametric coupling mechanism with enough fidelity to derive closed-form results, but it is necessarily a reduced representation of a real bridge. As a complementary, secondary demonstration, this section verifies that the fractional-damping formulation itself — independent of the nonlinear autoparametric coupling, which is not yet included at this stage — remains numerically well-behaved when assembled across a genuinely multi-degree-of-freedom, multi-cable structure, following the unified Grünwald-Letnikov time-marching approach of Section 2.1.3.

3.3.1 Finite-Element Idealization and Assembly

A planar (2D) frame model of the deck (200 m span) and pylon (80 m height, fixed base) was assembled from standard Euler-Bernoulli beam elements (three degrees of freedom per node), following conventional line-element modelling practice for cable-supported bridges [39], with ten discrete stay cables (five per side, fan arrangement) each represented as a two-node axial element carrying its own fractional-damper term, consistent with a One-Element Cable System (O ECS) idealization of the concentrated damper device at the cable anchorage. The fractional damping matrix is assembled directly from the same direction-cosine compatibility vectors used for the cable's elastic (Ernst-corrected [40]) stiffness, $C_f = \sum c_f m a a^T$, an exact multi-degree-of-freedom generalization of the single-channel fractional term of Eq. (4b), since the Caputo derivative is linear and each $a a^T$ is time-invariant. The assembled model has 29 nodes and, after boundary conditions, 81 free degrees of freedom.

3.3.2 Verification Results

An undamped eigenvalue analysis gives a fundamental frequency of 0.430 Hz, with several cable-dominated modes (identified via cable strain-energy fraction) interleaved with global deck/pylon modes between roughly 1.0 and 2.6 Hz — direct structural confirmation of the closely spaced global/local modal environment that motivates the autoparametric analysis of Sections 2.2–3.2 in the first place. A free-decay (pluck) simulation comparing a classical Rayleigh-only damping baseline against the fractional cable damper ($\alpha = 0.3$ and 0.7) confirmed that the fractional formulation increases the equivalent damping delivered to the plucked mode relative to the classical baseline, and that the full 81-degree-of-freedom, ten-cable system integrates stably using a single LU factorization of the fixed-time-step system matrix reused at every

step, completing in well under a minute on a standard workstation CPU. Equivalent damping estimated via time-domain log-decrement did not increase monotonically with α at this multi-modal scale (1.62% at $\alpha = 0.3$ versus 1.35% at $\alpha = 0.7$), a finding consistent with the frequency-selective character of fractional damping already established analytically in Section 3.1.3 — a single time-domain estimate on a multi-modal signal conflates the damping contributions of several closely spaced modes, each weighted differently by $\omega \sin(\alpha\pi/2)$.

These results are reported here as a scope-and-scalability verification rather than as a second full analytical study: the finite-element model in its current form is linear and does not yet include the quadratic autoparametric coupling terms of Equations (4a)–(4b). Integrating that nonlinearity into the multi-cable finite-element assembly — so that the closed-form threshold of Eq. (12) can be checked against a structurally realistic model rather than only the reduced two-mode system — is identified as a priority next step in Section 5.

4. Discussion

4.1 Design Implications of the Instability Threshold

Equation (12) converts the fractional order α from a purely curve-fitting parameter for cable-damper frequency response into a quantity with direct consequences for a bridge's susceptibility to autoparametric cable resonance. Because $a_{0,\text{crit}}$ depends on α through both $\mu_{q,\text{eff}}$ and $\Delta\omega_{q,\text{eff}}$ in quadrature, and because these two quantities move in opposite directions as α increases (Table 1), the threshold is not simply maximized by choosing the most dissipative damper available; a damper's fractional order should be selected (or identified) with the actual detuning $\sigma_1 + \sigma_2$ of the target bridge in mind, since $\Delta\omega_{q,\text{eff}}$ enters the threshold as an offset to that detuning rather than as pure damping.

4.2 Calibration and Parameter Identification Challenges

Practical use of Equations (10)–(12) requires reliable identification of c_f and α for the physical cable damper in question — from forced-vibration frequency-response testing of the damper or the instrumented cable, following established cable-damper identification methodology [41] — together with independent identification of the deck and cable modal frequencies (and hence σ_1, σ_2) for the specific bridge and mode pair of interest. Because α affects both the damping and frequency-shift contributions simultaneously, identification should be pursued through explicit least-squares fitting of the fractional damper's frequency-response function to measured data, rather than through a single time-domain damping estimate, which Section 3.3.2 showed can be frequency-conflated and misleading in a multi-modal structure.

4.3 Limitations and Scope

The analytical results of Sections 2.3–3.1 are leading-order ($O(\epsilon)$) multiple-scales results, valid for weak damping, weak nonlinearity, and small detunings. Section 3.2.2 confirmed that bounded amplitude saturation is a genuine, robust prediction of the leading-order quadratic normal form itself, across the fractional orders and forcing levels tested, and does not require cubic-order extensions. One related open question was surfaced rather than resolved: at sufficiently large overshoot above threshold ($\geq 3 \times a_{0,\text{crit}}$ at $\alpha = 0.3$ in the parameter sweep of Section 3.2.2), the root-finding search found the previously stable nontrivial fixed point losing stability, consistent with the secondary (typically Hopf) bifurcations to periodic, quasiperiodic, or modulated responses documented elsewhere in the autoparametric-resonance literature; mapping this secondary stability boundary as a function of α is a natural, self-contained extension of the present analysis rather than a gap in it. The two-mode model itself retains only the single dominant deck mode and single dominant cable mode nearest 2:1 tuning; a real bridge has many cable modes and several nearby global modes, some of which the finite-element modal analysis of Section 3.3.2 shows can be simultaneously cable-dominated and closely spaced, potentially supporting multiple competing or interacting internal resonances beyond the pairwise mechanism treated here. The finite-element extension of Section 3.3 is presently linear (no autoparametric coupling) and two-dimensional, and cables are represented as concentrated axial-damper elements (OECS) rather than through a distributed multi-element (MECS) discretization capable of resolving the cable's own local transverse modes. Finally, no experimental validation against measured bridge or damper data has yet been performed for the fractional-damping extension itself; the quantitative agreement reported in Section 3.2.1 is between the analytical theory and an independent numerical integration of the same underlying equations, not against physical measurements. The classical-damping (integer-order) backbone of the deck-cable 2:1 internal-resonance mechanism has separately been validated experimentally against a physical cable-stayed-beam rig [13], which is encouraging for the physical plausibility of the underlying model but does not itself constitute validation of the fractional extension, since that experiment did not incorporate a fractional-order damper; obtaining an analogous physical test — a deck-cable rig fitted with a characterized fractional-order (or laboratory-identified frequency-dependent) damper — is accordingly identified as the highest-priority next step for this line of work.

5. Conclusion and Future Work

This paper has extended the perturbation-based methodology historically used for fractional-calculus suspension-bridge dynamics to the deck-cable autoparametric resonance problem specific to cable-stayed bridges. A nonlinear two-mode model coupling a forced deck mode to a Caputo fractional-damped cable mode, tuned to the classical 2:1 internal resonance, was solved via the method of multiple scales using a

harmonic decomposition of the fractional derivative into effective damping and effective frequency-shift contributions. This yielded closed-form modulation equations, the forced trivial-branch response, and a closed-form autoparametric instability threshold showing that the fractional order α governs the onset of large-amplitude cable response through a genuine, non-monotonic trade-off between its damping and frequency-shift roles — a result with direct implications for cable-damper design and specification. The predictions were verified against direct numerical integration of the original nonlinear equations, with the best-resolved growth-rate comparison agreeing to within 1% and the classic autoparametric amplitude-saturation signature reproduced numerically. A complementary multi-cable finite-element assembly confirmed that the same fractional-damping formulation scales numerically to a structurally realistic, multi-degree-of-freedom bridge model.

The priorities for follow-on work are: (i) mapping the secondary stability boundary of the saturated (nontrivial) branch as a function of α and overshoot, following the loss of stability observed at large overshoot in Section 3.2.2; (ii) incorporating the quadratic autoparametric coupling into the multi-cable finite-element assembly of Section 3.3, so the closed-form threshold of Eq. (12) can be tested against a structurally realistic multi-modal system rather than only the reduced two-mode model; (iii) parameter identification of c_f and α , and of the governing detunings σ_1, σ_2 , from measured cable-damper and bridge modal data; and (iv) experimental or field validation of the predicted instability threshold and saturation behavior. Together, these steps would move the framework from the analytical and numerical demonstration presented here toward a validated design tool for assessing and mitigating autoparametric cable resonance in cable-stayed bridges.

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Author Contribution

Oluwatosin Gabriel Oyesanya wrote and reviewed the whole manuscript.

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Figures

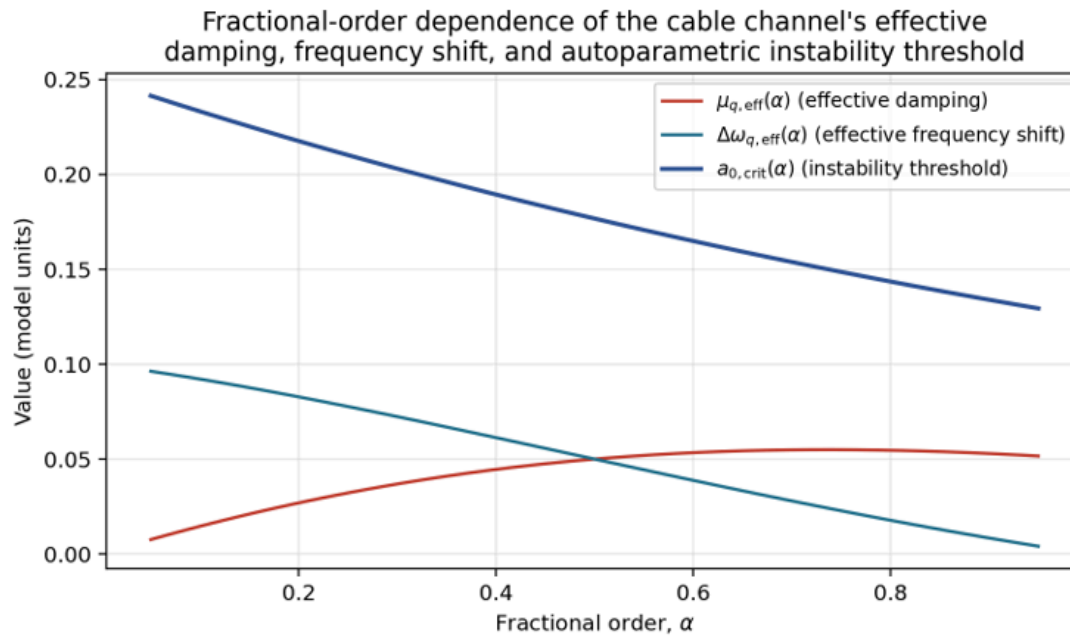


Figure 1

Fractional-order dependence of the cable channel's effective damping $\mu_{q, \text{eff}}$, effective frequency shift $\Delta\omega_{q, \text{eff}}$, and the resulting autoparametric instability threshold $a_{0, \text{crit}}$

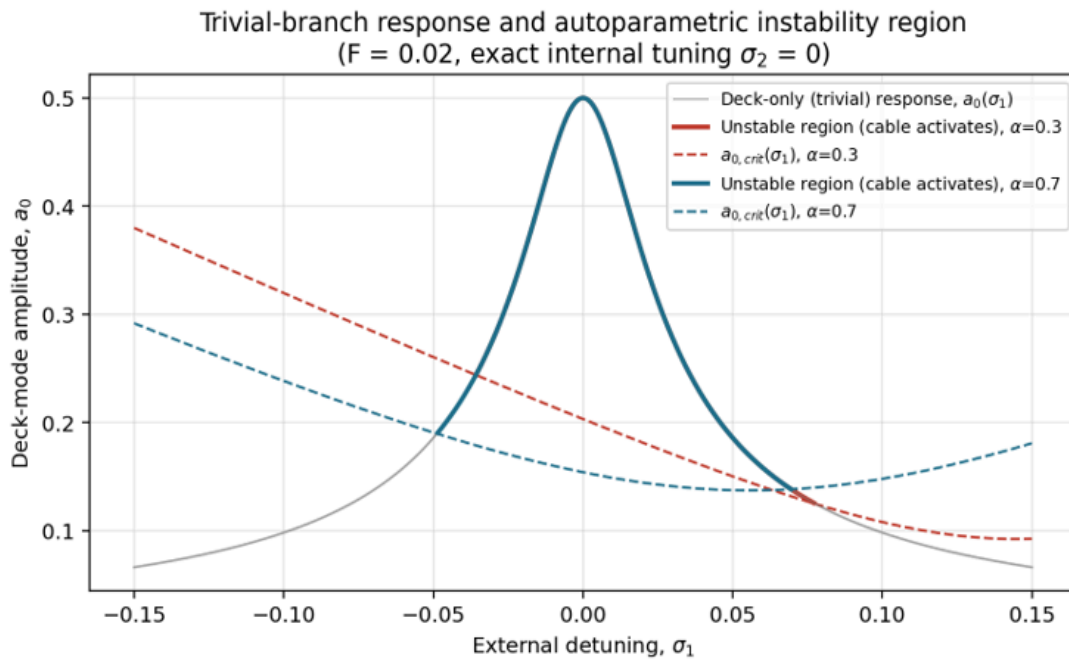


Figure 2

Trivial-branch (deck-only) frequency-response curve and the autoparametric instability region for two fractional damping orders, at fixed forcing amplitude $F = 0.02$

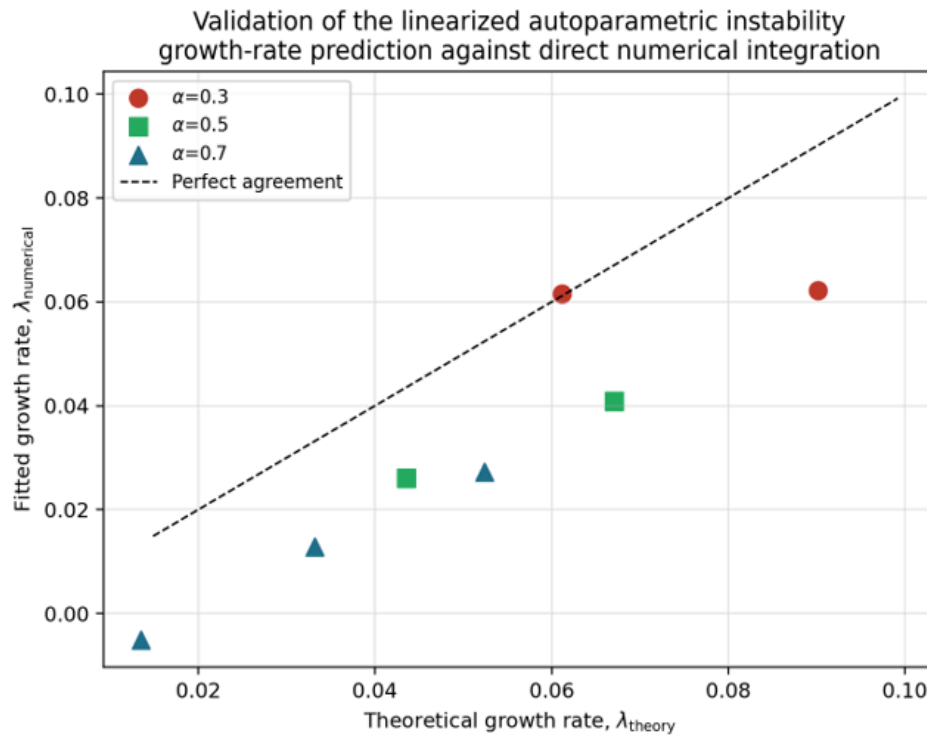


Figure 3

Theoretical versus numerically fitted linear growth rate of the cable mode, across three fractional orders and three levels of overshoot above the instability threshold. Points near the dashed line indicate agreement; scatter at low and high overshoot ratios reflects the finite-window limitations discussed in the text

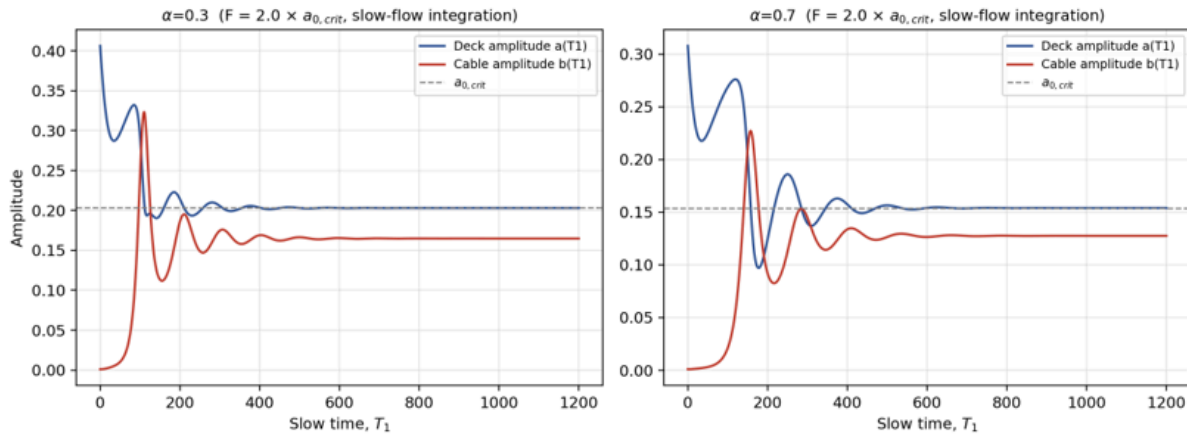


Figure 4

Slow-flow confirmation of bounded amplitude saturation for both fractional orders: the deck amplitude $a(T_1)$ relaxes to exactly $a_{0,crit}$ while the cable amplitude $b(T_1)$ grows from a tiny seed to a bounded steady value, at $2\times$ the instability threshold forcing

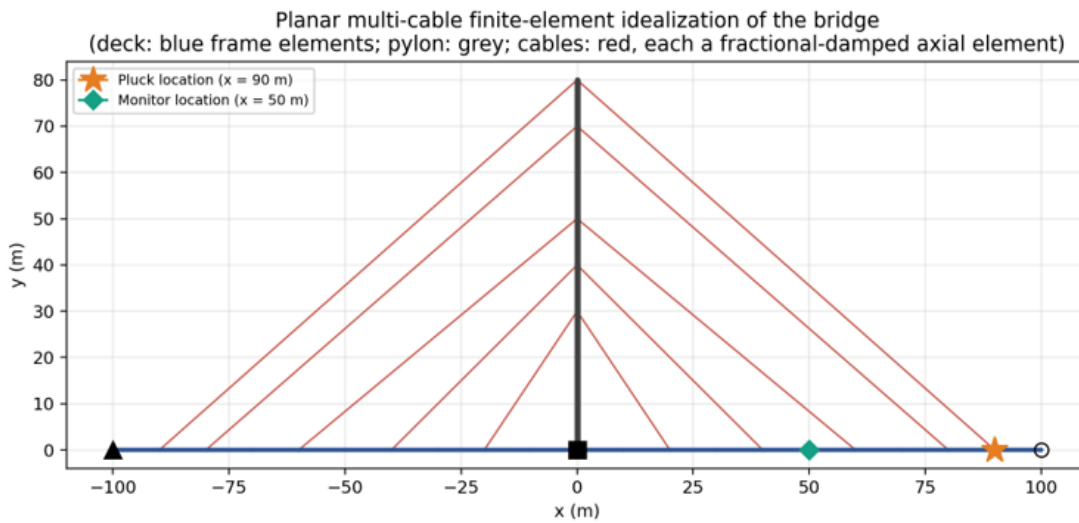


Figure 5

Planar multi-cable finite-element idealization used for the linear verification: deck and pylon frame elements, ten discrete fractional-damped cable elements

Supplementary Files

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