

A Hybrid Binary–Ternary Encoding Strategy for High-Capacity and Robust Phase Unwrapping in Fringe Projection Profilometry

QIAN ZHU

zhuqian@xatu.edu.cn

Xi'an Technological University

HUBING DU

Xi'an Technological University

BO TANG

Xi'an Technological University

YUEYANG MA

Xi'an Technological University

Research Article

Keywords: fringe projection profilometry, phase unwrapping, ternary Gray code, fringe order correction, complementary code

Posted Date: August 10th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10530191/v1>

License:   This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

A Hybrid Binary–Ternary Encoding Strategy for High-Capacity and Robust Phase Unwrapping in Fringe Projection Profilometry

QIAN ZHU^{1*}, HUBING DU¹, BO TANG¹, YUEYANG MA²

¹*Xi'an Technological University, School of Mechanical Engineering, Xi'an, Shaanxi, 710021, China*

²*Xi'an Technological University, School of Optoelectronic Engineering, Xi'an, Shaanxi, 710039, China*

*zhuqian@xatu.edu.cn

Abstract: This paper proposes a novel phase unwrapping method for fringe projection profilometry based on a hybrid binary–ternary Gray code and a dynamic threshold segmentation strategy. The method employs a sequence consisting of four-step phase-shifting patterns, three-step phase-shifting patterns, and three ternary Gray code patterns. By directly extracting the background and modulation intensities from the phase-shifting patterns, dynamic thresholds are computed point-wise, eliminating the need for additional calibration patterns such as all-black, all-gray, or all-white images, thereby enabling robust ternary decoding. Furthermore, an auxiliary binary codeword is generated through adaptive thresholding of the background intensity, which is then combined with the ternary codewords to form a hybrid encoding scheme. This scheme significantly increases the coding capacity of a single pattern set, allowing the unwrapping of up to 108 fringe periods using only six projected patterns. To enhance decoding accuracy, a phase-consistency-based pre-correction algorithm is introduced. This algorithm identifies error-prone boundary regions by analyzing the parity of the preliminarily decoded fringe order and the sign of an auxiliary wrapped phase, and proactively corrects boundary errors by selectively replacing fringe orders in these regions with complementary candidate orders. Experimental results demonstrate that the proposed method reduces the overall fringe order error by approximately 21-fold and decreases the root-mean-square error by about 43-fold. The method exhibits reliable performance under strong noise conditions and on complex curved surfaces, offering a robust solution for high-precision, high-speed 3D shape measurement.

Keywords: fringe projection profilometry, phase unwrapping, ternary Gray code, fringe order correction, complementary code.

1. Introduction

Non-contact measurement techniques provide an effective means of acquiring object information without physical interaction, making them highly valuable in a wide range of applications [1, 2]. Among these techniques, fringe projection profilometry (FPP) has become a prominent non-contact optical method due to its high measurement accuracy, simplicity, and broad applicability across diverse fields [3–5]. However, the computational process in FPP typically results in phase values confined to the range $[-\pi, \pi]$, requiring a subsequent phase unwrapping step to recover the absolute phase [6, 7]. Phase unwrapping methods can be broadly categorized into spatial and temporal approaches [8–13]. While spatial methods evaluate adjacent pixel values to determine the unwrapped phase, they are susceptible to error propagation, which can lead to significant measurement inaccuracies. Temporal methods, in contrast, mitigate such issues by encoding each pixel independently through the projection of multiple patterns, thus enhancing the accuracy of the measurement. Notable temporal approaches include multi-frequency [14, 15], multi-wavelength [16–18], and Gray code techniques [19, 20]. Among these, Gray code-assisted phase unwrapping has been widely researched and applied due to its strong ability to minimize errors and improve accuracy.

The Gray code is characterized by the unique property that adjacent codewords differ by only one bit, which limits error propagation [21]. The application of Gray codes to three-dimensional measurement systems was first demonstrated by Inokuchi et al. [22], who associated each codeword with a projected stripe, extracting stripe centers and boundaries. However, accurate boundary extraction became a critical factor influencing decoding precision. In subsequent research, Sansoni et al. integrated phase-shifting with Gray codes to improve the phase unwrapping process, significantly enhancing measurement accuracy [23]. Despite these advancements, several challenges remain in the practical implementation of Gray code-assisted methods, particularly with regard to the impact of ambient lighting, surface reflectivity variations, system noise, and projector defocus. These factors can blur the boundaries between adjacent binary code regions, leading to segmentation errors, especially during black-white transitions. Several solutions have been proposed to address these issues, such as projecting dedicated all-white and all-black binary images for segmentation [24] or using defocusing methods to generate intermediate grayscale patterns [25]. More recently, Wu et al. introduced a pixel-wise threshold calculation method, utilizing the background intensity of sinusoidal fringe patterns to improve segmentation robustness and reduce unwrapping errors caused by environmental variations [26]. Additionally, Zhang et al. proposed a complementary Gray code encoding method, using an additional Gray code pattern to create interleaved transition boundaries, further enhancing decoding reliability [27].

A fundamental limitation of binary Gray codes is their restricted information-carrying capacity, where each bit represents only two states ('0' or '1'). This constraint necessitates a larger number of patterns to encode higher fringe orders (K), which increases the total projection count and consequently reduces the system's acquisition speed. To achieve greater fringe order capacity, researchers have begun exploring higher-base Gray code schemes.

Wei *et al.* proposed a ternary complementary Gray code phase unwrapping method [28]. Building upon this, Li *et al.* subsequently introduced an improved quaternary complementary Gray code method [29]. These developments in higher-base Gray codes provide promising avenues for achieving increased encoding capacity and enhanced measurement robustness using a limited number of projected patterns.

Recent advances have further explored hybrid strategies that combine Gray codes with multi-frequency phase-shifting or complementary encoding. He *et al.* developed a quaternary Gray-code approach specifically for binary fringe projection profilometry, demonstrating the feasibility of multi-level coding with defocused projection [30]. Han *et al.* further refined complementary Gray code methods through phase adjustment techniques to enhance boundary robustness [31]. Yu *et al.* combined Gray code with dual-frequency phase-shifting to improve unwrapping reliability [32], while Song *et al.* integrated dual-frequency heterodyne with dual complementary phase encoding [33]. These hybrid approaches have advanced the field, yet achieving an optimal balance among encoding capacity, noise immunity, and computational efficiency remains challenging. To overcome these challenges, this paper proposes a novel phase unwrapping method based on a hybrid binary-ternary Gray code. The key contributions of this work are as follows:

This paper introduces a hybrid encoding and dynamic threshold segmentation strategy. The approach employs a sequence of four-step and three-step phase-shifting patterns, along with three ternary Gray code patterns. By extracting the background and modulation intensity directly from the phase-shifting patterns, we calculate dynamic thresholds point-wise, without the need for additional calibration patterns such as all-black, all-gray, or all-white images. This strategy enables robust ternary decoding. Additionally, we generate an auxiliary binary codeword by adaptively thresholding the background intensity, combining it with ternary codewords to form a hybrid encoding scheme that increases the coding capacity of a single pattern set. To further enhance accuracy, a novel phase-consistency-based pre-correction algorithm is proposed. It identifies error-prone boundary regions by analyzing the parity of the preliminarily decoded fringe order and the sign of an auxiliary wrapped phase. The algorithm proactively addresses boundary errors by selectively replacing fringe orders in these regions with complementary candidate orders, thereby enhancing the accuracy of fringe orders.

The remainder of this paper is organized as follows: Section 2 elaborates on the principles of proposed method, including the encoding strategy, decoding process, and the order correction algorithm. Section 3 presents comparative experiments to validate the performance of the proposed method. Finally, Section 4 concludes the paper.

2. Principle

2.1. Fringe Projection Profilometry

In fringe projection profilometry (FPP), the computer generates multi-step phase-shifting sinusoidal fringe patterns, which are projected onto the surface of the target object. A camera then captures the deformed fringe patterns, which are modulated by the variations in the object's surface height.

Two primary fringe encoding strategies are utilized in this work. The first strategy employs standard multi-step phase-shifting patterns, where the intensity of the deformed fringes can be expressed as:

$$I_n(x, y) = A(x, y) + B(x, y) \cos \left[\phi(x, y) - \frac{2\pi n}{N} \right], \quad n = 0, 1, \dots, N-1 \quad (1)$$

where (x, y) represents the pixel coordinates, I_n denotes the intensity of the deformed fringes, A represents the background intensity related to background illumination, B signifies the intensity modulation related to surface reflectivity, N is the number of phase-shifting steps, n is the phase-shifting index, and $\phi = 2\pi x/T$ is the corresponding wrapped phase, T is the period length of sinusoidal patterns.

The second encoding strategy adopts a composite sinusoidal pattern designed for enhanced robustness. As illustrated in Fig. 1(a), the pattern spatially multiplexes two sets of sinusoidal fringes with different background intensities in two adjacent horizontal regions. The intensity distribution of this pattern is given by:

$$I_n(x, y) = \begin{cases} A_1 + B_1 \cos \left[\phi(x, y) - \frac{2\pi n}{N} \right], & \text{if } 1 \leq x < x_m \\ A_2 + B_2 \cos \left[\phi(x, y) - \frac{2\pi n}{N} \right], & \text{if } x_m \leq x \leq x_e \end{cases} \quad (2)$$

where x_m and x_e denote the boundary and the end of the composite pattern, respectively. The first region contains M fringe periods, thus $x_m = \frac{M \cdot T}{2}$, and the second region also contains M periods, so $x_e = M \cdot T$. Here T is the fringe period (in pixels), and M is the number of periods in each region, which is independent of the phase-shifting step number N . To maximize contrast while avoiding saturation in 8-bit grayscale representation, the background intensities are set to $A_1 = 255/3$ and $A_2 = 255/2$. The modulation amplitudes are both set to $B_1 = B_2 = 255/3$ to ensure adequate fringe contrast without exceeding the maximum intensity. A cross-sectional view of the composite pattern is shown in Fig. 1(b).

For both encoding strategies, the wrapped phase $\phi(x, y)$ can be calculated using the phase-shifting algorithm. For the standard patterns described by Eq. (1), the phase is obtained through:

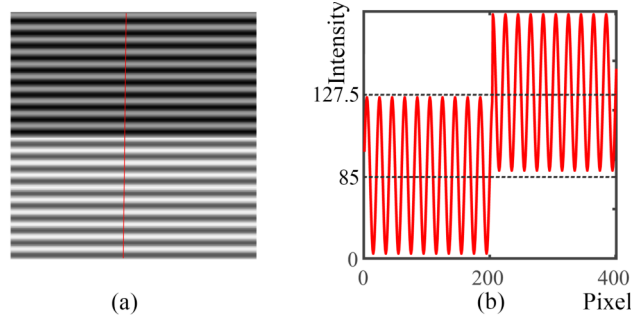


Fig. 1. (a) Composite pattern; (b) cross section of red line in (a).

$$\phi(x, y) = \tan^{-1} \left[\frac{\sum_{n=0}^{N-1} I_n(x, y) \sin \left(\frac{2\pi n}{N} \right)}{\sum_{n=0}^{N-1} I_n(x, y) \cos \left(\frac{2\pi n}{N} \right)} \right] \quad (3)$$

Similarly, for the composite patterns in Eq. (2), an appropriate phase-shifting algorithm is applied to extract the phase information based on the specific encoding scheme.

Importantly, both encoding strategies enable the simultaneous extraction of two crucial parameters: the background intensity and the modulation intensity. These parameters can be derived from the captured fringe sequences using the following general formulas:

$$A(x, y) = \frac{1}{N} \sum_{n=0}^{N-1} I_n(x, y) \quad (4)$$

$$B(x, y) = \frac{2}{N} \sqrt{\left[\sum_{n=0}^{N-1} I_n(x, y) \sin \left(\frac{2\pi n}{N} \right) \right]^2 + \left[\sum_{n=0}^{N-1} I_n(x, y) \cos \left(\frac{2\pi n}{N} \right) \right]^2} \quad (5)$$

The background intensity $A(x, y)$, as defined in Eq. (4), represents the DC component of the fringe pattern, which encompasses information about the ambient illumination and surface reflectivity. The modulation intensity $B(x, y)$, calculated using Eq. (5), characterizes the amplitude of the sinusoidal component and serves as an indicator of measurement quality. These parameters are particularly valuable for subsequent processing steps, including threshold determination in Gray code decoding and reliability assessment of the phase data. The wrapped phase ϕ obtained from these calculations is constrained to the range $[-\pi, \pi]$, thus requiring phase unwrapping to obtain the continuous phase distribution.

2.2. Proposed Hybrid Binary-Ternary Gray Code Decoding Method

2.2.1. Encoding Strategy

The proposed method employs a temporal phase unwrapping framework utilizing three distinct sets of patterns, projected sequentially. The complete set comprises ten patterns, logically divided into three groups, as summarized in Tab. 1.

Table 1. Projected Pattern Groups and Their Functions

Group	Patterns	Primary Function
Group 1	I_1-I_4 (High-frequency f_1)	Four-step phase-shifting for calculating the primary wrapped phase ϕ_1 .
Group 2	I_5-I_7 (Sub-high-frequency f_2)	Three-step phase-shifting for calculating the auxiliary wrapped phase ϕ_2 .
Group 3	I_8-I_{10}	Ternary Gray code patterns for auxiliary fringe order determination.

*Group 1 employs the four-step algorithm to achieve the highest accuracy for the final unwrapped phase, as it determines the overall measurement precision.

Group 1 consists of conventional four-step phase-shifting sinusoidal fringe patterns with a high frequency, generated using the standard encoding method described in Section 2.1. Group 2 consists of composite three-step phase-shifting sinusoidal fringe patterns with a sub-high frequency, where the composite encoding embeds additional

information to facilitate auxiliary phase calculation and background intensity extraction. Group 3 consists of ternary Gray code patterns, designed to determine the auxiliary fringe order. The key innovation of this approach lies in the synergistic use of Group 2. Specifically, the background intensity $A_2(x, y)$, obtained through Eq. (4), is not only utilized for phase calculation but is also dynamically binarized to generate a supplementary binary codeword. This process effectively enhances the coding capacity of the subsequent Gray code patterns.

2.2.2. Decoding and Fringe Order Determination

The decoding process begins with the independent processing of each group. From Group 1 (I_1-I_4), the primary wrapped phase $\phi_1(x, y)$ is obtained using a four-step phase-shifting algorithm, along with the background intensity $A_1(x, y)$.

A. Codeword Extraction

The encoding strategy and the intermediate results of the codeword extraction process are illustrated in Fig. 2.

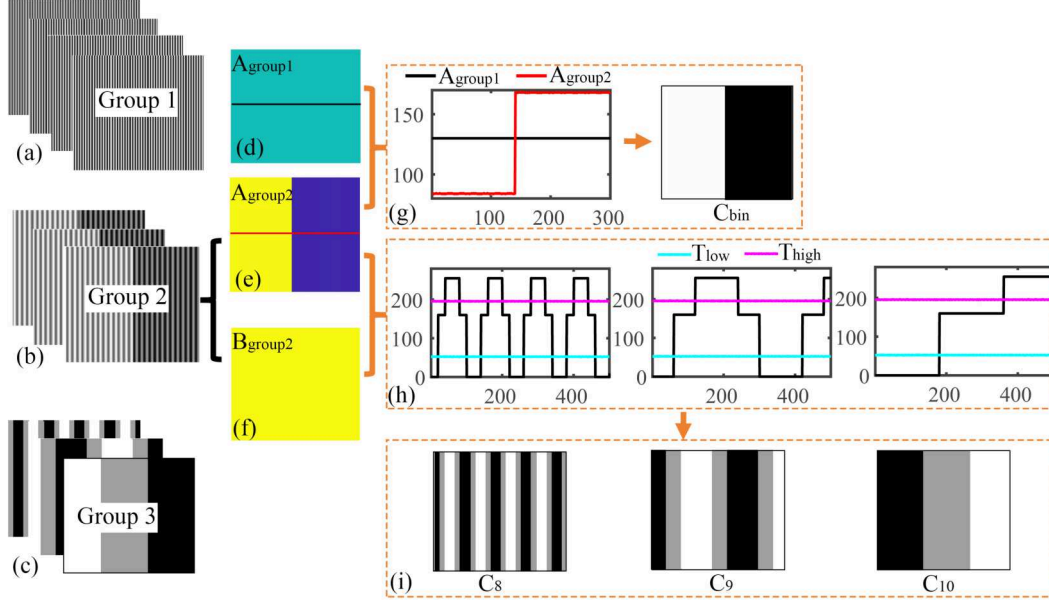


Fig. 2. Schematic diagram of the encoding strategy. (a)-(c) Captured images of Group 1 fringe patterns, Group 2 composite fringe patterns, and Group 3 ternary Gray code patterns, (d)-(e) background intensity solved from Group 1 and Group 2, (f) modulation intensity solved from Group 2, (g) Cross-sections of the black and red lines in (d) and (e), along with the generated C_{bin} map, (h) Intermediate process for solving the ternary Gray code patterns using T_{low} and T_{high} of Eq. 7, (i) ternary Gray code map obtained from (c) and (h).

From Group 2 (I_5-I_7), the auxiliary wrapped phase $\phi_2(x, y)$ is calculated using the three-step phase-shifting algorithm (Eq. 3). Simultaneously, the background intensity $A_{group2}(x, y)$ and modulation $B_{group2}(x, y)$ are obtained as: $A_{group2}(x, y) = \frac{I_5+I_6+I_7}{3}$, $B_{group2}(x, y) = \frac{\sqrt{2}}{3} \sqrt{(I_5 - I_6)^2 + (I_6 - I_7)^2 + (I_7 - I_5)^2}$.

Following the general formulation in Eqs. (4) and (5), the background intensity $A_{group1}(x, y)$ and modulation amplitude $B_{group1}(x, y)$ for Group 1 can be directly computed from the four-step phase-shifting patterns (I_1 to I_4).

Because the composite pattern in Group 2 (Eq. 2) introduces two distinct background levels A_1 and A_2 , the resulting $A_{group2}(x, y)$ exhibits two clusters corresponding to the two horizontal regions. The uniformly illuminated Group 1 provides a reference background $A_{group1}(x, y)$ that lies between these two clusters. Therefore, we generate the auxiliary binary code by comparing $A_{group2}(x, y)$ with $A_{group1}(x, y)$:

$$C_{bin}(x, y) = \begin{cases} 1, & \text{if } A_{group2}(x, y) > A_{group1}(x, y), \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

This yields a binary map that complements the Gray code information.

For the ternary Gray code patterns (Group 3), dynamic thresholds are computed using $A_{group2}(x, y)$ and $B_{group2}(x, y)$:

$$\begin{aligned} T_{high}(x, y) &= A_{group2}(x, y) + \beta \cdot B_{group2}(x, y), \\ T_{low}(x, y) &= A_{group2}(x, y) - \beta \cdot B_{group2}(x, y), \end{aligned} \quad (7)$$

where β is an empirically determined coefficient (typically ranging from 0.8 to 1.2) that optimizes the segmentation performance, and can be appropriately adjusted according to environmental variations and camera parameters.

These adaptive thresholds enable robust classification of the ternary Gray code patterns:

$$C_i(x, y) = \begin{cases} 0, & \text{if } I_i(x, y) < T_{\text{low}}(x, y), \\ 1, & \text{if } T_{\text{low}}(x, y) \leq I_i(x, y) \leq T_{\text{high}}(x, y), \\ 2, & \text{if } I_i(x, y) > T_{\text{high}}(x, y), \end{cases} \quad \text{for } i = 8, 9, 10. \quad (8)$$

The intermediate segmentation results are visualized in Fig. 2(h), and the final ternary Gray code map is shown in Fig. 2(i).

B. Composite Codeword to Fringe Order Conversion

The extracted codeword components form the composite vector $\mathbf{C} = [C_{\text{bin}}, C_8, C_9, C_{10}]$. This hybrid encoding structure provides $2 \times 3^3 = 54$ unique codewords, significantly expanding the coding capacity compared to conventional methods.

The complete coding sequence is detailed in Fig. 3, which illustrates the specific Gray code progression where adjacent codewords differ in only one digit, effectively limiting error propagation across fringe boundaries. The composite codeword is transformed to an intermediate decimal value through weighted summation:

C_{bin}																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
C_8																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
C_9																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					
C_{10}																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																					

Fig. 3. Ternary complementary Gray code (3,3) and the process of solving order.

$$V(x, y) = C_{\text{binary}}(x, y) \times 27 + C_8(x, y) \times 9 + C_9(x, y) \times 3 + C_{10}(x, y) \quad (9)$$

However, due to the non-monotonic nature of the Gray code sequence, the intermediate value V does not directly correspond to the physical fringe order. A lookup table mapping based on Fig. 3 is therefore employed to establish the correct correspondence:

$$K_1(x, y) = \mathbf{LUT}[V(x, y)] \quad (10)$$

where the lookup table $\mathbf{LUT}$ implements the specific Gray code sequencing shown in the figure.

While seven binary Gray code patterns could theoretically encode up to 128 fringe periods, such an approach faces severe practical limitations. The highest-frequency binary patterns contain alternating black and white stripes at the pixel level, which become heavily blurred when projected through a defocused projector. This blurring leads to decoding errors at stripe boundaries, particularly in the presence of ambient noise and varying surface reflectivity.

Our hybrid encoding strategy circumvents these limitations through a gradual coding approach. By combining a single robust binary bit (derived from the composite background) with three ternary Gray code patterns, we distribute the coding burden across multiple levels. Furthermore, the three composite patterns in Group 2 serve a dual function: they simultaneously provide the auxiliary binary codeword and the sub-high-frequency wrapped phase essential for reliable high-frequency unwrapping. This integrated design, while using ten patterns total, achieves robust performance that would be unattainable with pure binary coding in practical measurement scenarios.

2.2.3. Fringe Order Correction Algorithm

To mitigate decoding errors at codeword boundaries, a sophisticated fringe order correction algorithm is implemented. The process begins with the generation of three candidate fringe orders from the fringe order K_1 obtained through the composite Gray code decoding:

$$\begin{aligned} K_2(x, y) &= \text{round} \left(\frac{K_1(x, y) - 1}{2} \right) \times 2 \\ K_3(x, y) &= \text{floor} \left(\frac{K_1(x, y) - 1}{2} \right) \times 2 \end{aligned} \quad (11)$$

where K_2 and K_3 represent shifted candidate orders that provide alternative phase order near boundary regions.

This candidate generation strategy is inspired by our previous work on triple fringe order correction [34], where three complementary orders were constructed from hierarchical Gray-code patterns. In this paper, we extend that

concept by introducing a phase-consistency check based on the auxiliary wrapped phase $\phi_2(x, y)$, which enables more precise identification of error-prone boundary regions without relying solely on the geometric constraints of the coding pattern. The core of the correction strategy employs a phase-consistency check based on the auxiliary wrapped phase $\phi_2(x, y)$. A binary discrimination scheme is established through the following steps:

First, the parity of the fringe order and the phase sign are computed:

$$\begin{aligned} D_0(x, y) &= \text{mod}(K_1(x, y), 2), \\ D_1(x, y) &= \begin{cases} 0, & \text{if } \phi_2(x, y) \leq 0, \\ 1, & \text{if } \phi_2(x, y) > 0. \end{cases} \end{aligned} \quad (12)$$

where D_0 represents the parity of the fringe order, and D_1 indicates the sign of the wrapped phase. An exclusive OR (XOR) operation is then applied to identify inconsistent regions as shown in Fig. 4:

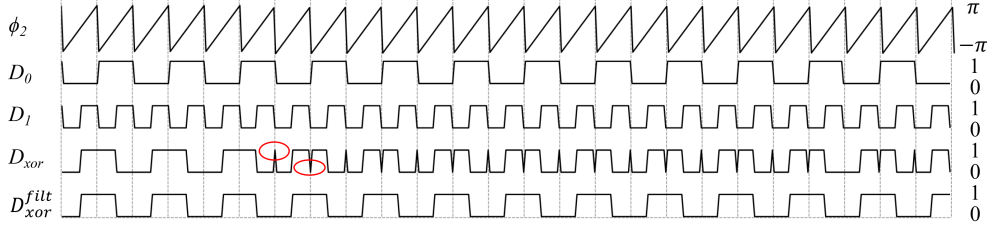


Fig. 4. Inconsistent region identification based on phase parity and sign.

$$D_{xor}(x, y) = \text{bitxor}(D_0(x, y), D_1(x, y)) \quad (13)$$

The XOR result D_{xor} is subsequently filtered using a median filter with a $p \times q$ kernel to eliminate spurious noise as shown in Fig. 4:

$$D_{xor}^{filt}(x, y) = \text{medfilt2}(D_{xor}(x, y), [p, q]) \quad (14)$$

A 5×5 kernel size is empirically selected as it provides an optimal balance between removing isolated noise pixels (typically 1-3 pixels wide) and preserving the contiguous error regions that require correction. Smaller kernels (e.g., 3×3) may leave residual noise, while larger kernels (e.g., 7×7 or above) tend to erode the boundaries of error regions, reducing correction effectiveness.

The corrected fringe order $K_1^c(x, y)$ is determined through a region-based selection method. Defining a phase threshold $\theta_{th} = \pi/3$, the correction logic is implemented as follows:

$$K_1^c(x, y) = \begin{cases} K_1(x, y), & \text{if } |\phi_2(x, y)| < \theta_{th}, \\ K_3(x, y), & \text{if } D_{xor}^{filt} = 0 \text{ and } \phi_2(x, y) \geq \theta_{th}, \\ K_3(x, y) + 1, & \text{if } D_{xor}^{filt} = 0 \text{ and } \phi_2(x, y) \leq -\theta_{th}, \\ K_2(x, y) + 1, & \text{if } D_{xor}^{filt} = 1 \text{ and } \phi_2(x, y) \geq \theta_{th}, \\ K_2(x, y) + 2, & \text{if } D_{xor}^{filt} = 1 \text{ and } \phi_2(x, y) \leq -\theta_{th}. \end{cases} \quad (15)$$

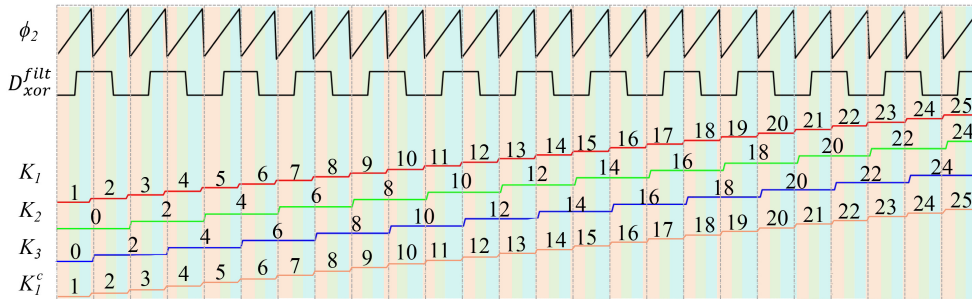


Fig. 5. Segment selection strategy of the corrected fringe order K_1^c .

This comprehensive correction strategy ensures that the fringe order K_1^c aligns optimally with the actual phase distribution as shown in Fig. 5, particularly in vulnerable boundary regions where traditional decoding methods are prone to errors.

2.2.4. Absolute Phase Retrieval

The absolute phase is retrieved through a cascaded unwrapping strategy based on the corrected fringe order K_1^c , as illustrated in Fig. 6. First, the auxiliary phase is unwrapped using both the preliminary and the corrected fringe orders for comparative analysis:

$$\Phi_2'(x, y) = \phi_2(x, y) + 2\pi \cdot K_1(x, y) \quad (16)$$

$$\Phi_2(x, y) = \phi_2(x, y) + 2\pi \cdot K_1^c(x, y) \quad (17)$$

Here, $\Phi_2'(x, y)$ is the unwrapped phase obtained directly from the fringe order K_1 , while $\Phi_2(x, y)$ is the refined result based on the corrected order K_1^c . As shown in Fig. 6, the phase $\Phi_2(x, y)$ successfully suppresses order-jump errors present in $\Phi_2'(x, y)$.

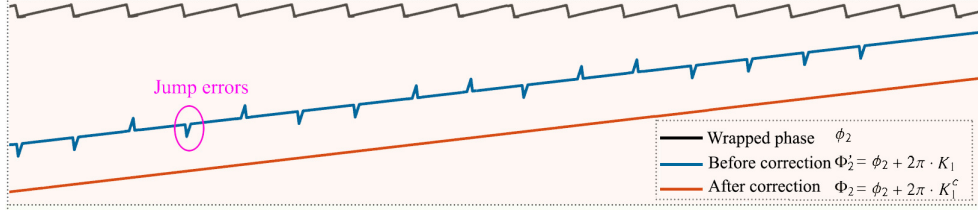


Fig. 6. Comparison of unwrapped auxiliary phase maps.

Subsequently, the high-frequency primary wrapped phase $\phi_1(x, y)$ is unwrapped using a frequency-ratio-based method. Given the spatial frequency ratio ($N_R = f_2/f_1$) between the sub-high-frequency and high-frequency fringe patterns, the final fringe order for the primary absolute phase is determined by

$$K^{final}(x, y) = \text{round} \left(\frac{N_R \cdot \Phi_2(x, y) - \phi_1(x, y)}{2\pi} \right) \quad (18)$$

The final high-precision primary absolute phase used for three-dimensional reconstruction is then obtained as:

$$\Phi_1(x, y) = \phi_1(x, y) + 2\pi \cdot K^{final}(x, y) \quad (19)$$

This cascaded phase unwrapping approach leverages the robustness of the corrected auxiliary absolute phase to guide primary absolute phase, ensuring measurement accuracy throughout the entire field of view. The workflow is illustrated in Fig. 7.

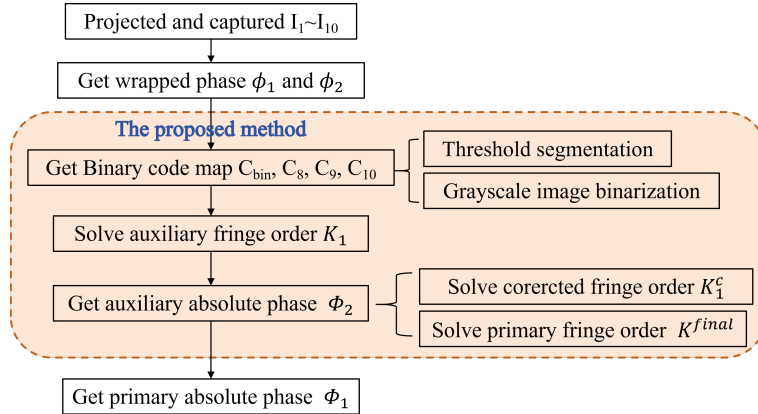


Fig. 7. Work flow.

3. Simulation and Experiment

3.1. Simulation

To evaluate the effectiveness of the proposed method, numerical simulations were conducted under various levels of measurement noise. A Peaks-type surface morphology, serving as a model for a complex object, was used to generate standard phase-shifted fringe patterns. The error analysis focuses on two distinct stages of the unwrapping process: the auxiliary fringe order (K_1) and the primary fringe order (K^{final}). As shown in Fig. 8, before applying the correction, the surface deformation resulted in an auxiliary fringe order (K_1) error rate of 6.68%, and a primary fringe order (K^{final}) error rate of 0.88%, with noticeable streak-like artifacts appearing in the unwrapped phase.

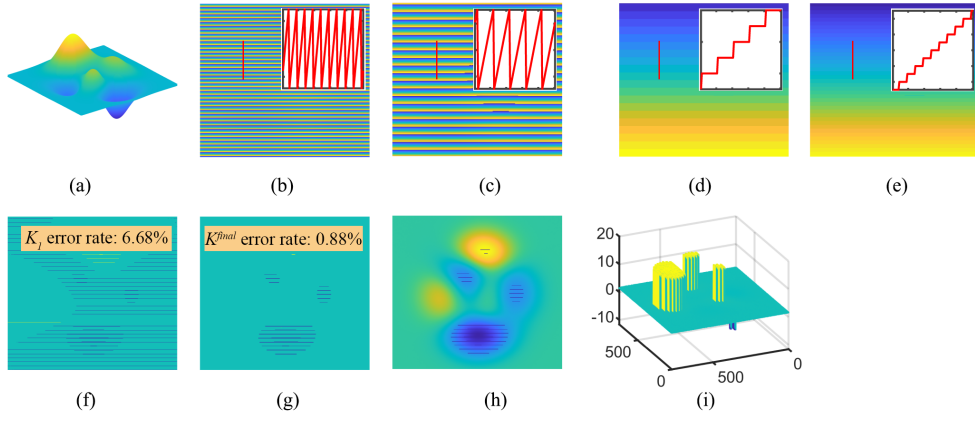


Fig. 8. Simulation results before correction. (a) Object under test, (b) high-frequency wrapped phase, (c) sub-high-frequency wrapped phase, (d) auxiliary fringe order K_1 (uncorrected), (e) primary fringe order K^{final} (uncorrected), (f) error rate of K_1 , (g) error rate of K^{final} , (h) reconstructed 3D result (uncorrected), (i) error of the unwrapped phase (uncorrected).

3.1.1. Simulations under different noise levels

To simulate real measurement conditions, Gaussian white noise was added to the fringe patterns at signal-to-noise ratios (SNRs) of 20 dB, 40 dB, 60 dB, and 80 dB. For each noise level, the proposed method was applied to compute the corrected auxiliary fringe order (K_1^c) and the primary fringe order (K^{final}), followed by phase unwrapping. Statistical analysis was performed on the error rates of K_1^c and K^{final} , as well as the root mean square error (RMSE) of the unwrapped phase.

The uncorrected results in Fig. 8 reveal the inherent challenges in fringe order decoding. To address these issues, we apply the correction algorithm described in Section 3.4, and the improved results under varying noise levels are presented in Fig. 9.

The simulation results are presented in Fig. 9. As the SNR decreases, the fringe order errors and phase RMSE increase, as expected. Importantly, the proposed method significantly reduces the error rate during the initial fringe order estimation, demonstrating robust fringe order discrimination even under high noise conditions. Furthermore, the integration of a multi-frequency phase unwrapping strategy effectively suppresses the final fringe order error and enhances the reliability of the unwrapped phase.

Quantitatively, even under severe noise interference (SNR = 20 dB), the K_1^c error rate remains below 5.5%, and the K^{final} error rate stays below 0.9%. Additionally, the phase RMSE remains within 0.9 radians. At higher SNR levels (80 dB), both the K_1^c and K^{final} error rates decrease to below 0.5%, with the phase RMSE reducing to just 0.045 radians. These results indicate that the proposed method consistently maintains low error rates and high phase unwrapping accuracy across a broad range of noise levels, confirming its robustness to noise and practical reliability.

3.1.2. Simulations under different N_R

In our encoding scheme, the sub-high frequency (Group 2) is inherently fixed because the ternary Gray code patterns, together with the auxiliary binary bit, encode exactly $2 \times 3^3 = 54$ unique fringe orders. Consequently, the number of periods for Group 2 is $N_2 = 54$, and the corresponding fringe period in pixels is $T_2 = W/N_2$, where W is the projector width. For a typical projector with $W = 1080$ pixels, this yields $T_2 = 20$ pixels. The high frequency (Group 1) is then designed with a period $T_1 = T_2/N_R$, where N_R is the frequency ratio. The number of high-frequency periods becomes $N_1 = W/T_1 = 54 \times N_R$. As N_R increases from 1 to 2.2, T_1 decreases from 20 pixels to approximately 9.1 pixels, and N_1 grows from 54 to about 119 periods.

The feasible range of N_R is bounded by practical considerations: When $N_R \approx 1$, the two frequencies are nearly equal, producing an excessively large synthetic wavelength that amplifies noise sensitivity. When $N_R > 2.2$, the high-frequency period becomes too small (< 9 pixels), causing severe fringe degradation due to projector defocus and camera sampling limitations, which leads to unacceptable decoding errors. Thus, the optimal operating range for the proposed method is $1 < N_R \leq 2.2$. Simulation results in Fig. 10 confirm that within this range, the phase error remains below 0.06 rad. Beyond $N_R = 2.2$, significant interlayer jump errors occur, rendering the phase unwrapping unreliable.

3.1.3. Comparative Simulations

To evaluate the performance of the proposed method, we conducted comparative simulations against three representative approaches: (i) the dual-frequency heterodyne with complementary phase encoding method (Song et al. [4]), which eliminates phase jump errors via dual-frequency heterodyne and complementary phase encoding; (ii) the combined Gray code and dual-frequency phase-shifting method (Yu et al. [3]), which adds low-frequency

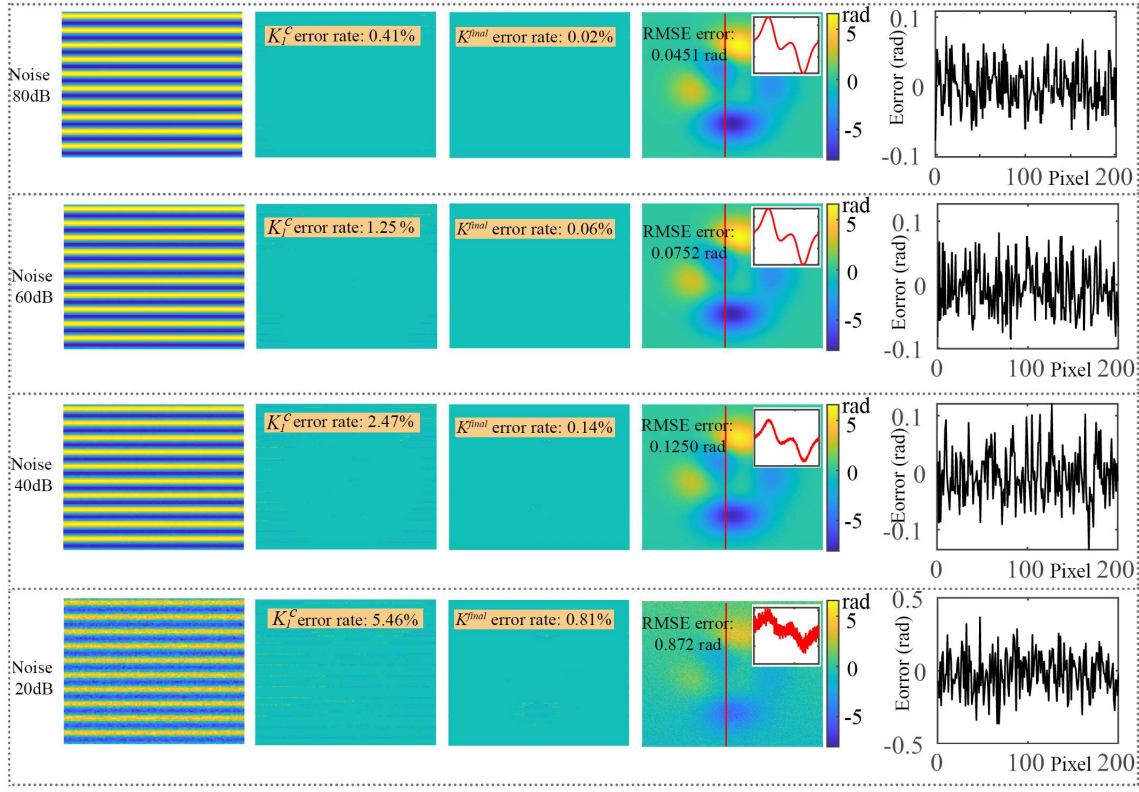


Fig. 9. Comparison of simulation results under different noise levels.

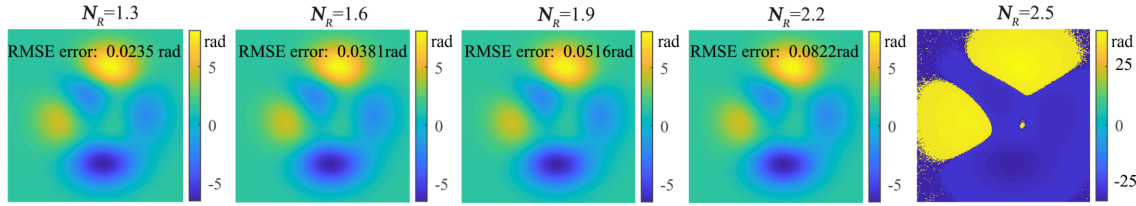


Fig. 10. Comparison of simulation results under different N_R values.

phase-shifted fringes to correct period jump errors; and (iii) the improved complementary Gray code method (Han et al. [1]), which uses phase adjustment to resolve mismatches between wrapped phase and Gray code arrays. To ensure a fair comparison, all methods were implemented following their core principles as originally proposed, with consistent experimental conditions. In particular, the high-frequency phase-shifting patterns were uniformly implemented using the four-step algorithm for all methods, as it offers superior noise immunity and harmonic suppression over three-step methods. All approaches were evaluated on the Peaks test surface under Gaussian noise at SNR levels of 20, 40, 60, and 80 dB.

Table 2 summarizes the phase reconstruction RMSE for each method across different noise levels. The error types observed during decoding are categorized as follows:

- **Error Type 1:** Fringe order mismatch due to noise-induced decoding errors in the coding patterns.
- **Error Type 2:** Boundary decoding errors occurring near critical phase thresholds (e.g., $\pm\pi$).
- **Error Type 3:** General decoding failures caused by severe noise corruption.

As shown in Table 2, the proposed method consistently yields the lowest RMSE across all noise levels. At 80 dB and 60 dB, all four methods perform similarly, with the proposed method showing a slight edge. At 40 dB, the three comparison methods begin to exhibit Type 1 errors, with RMSE values ranging from 0.1994 rad to 0.2684 rad. In contrast, the proposed method maintains error-free decoding with an RMSE of 0.1250 rad. This improvement is attributed to its hybrid encoding strategy and adaptive thresholding, which together provide more reliable code discrimination under moderate noise.

The advantage becomes particularly evident at 20 dB. The Song method suffers severe degradation (RMSE=1.8698 rad) with both Type 1 and Type 2 errors. The Yu method retains moderate performance (RMSE = 0.4652 rad) but exhibits Type 1 and Type 3 errors, while the Han method fails completely, producing invalid data. Remarkably,

Table 2. Comparative simulation results under different noise levels.

Method	SNR = 80 dB	SNR = 60 dB	SNR = 40 dB	SNR = 20 dB
Song et al. [4]	0.0585 rad	0.0866 rad	0.2320 rad (Type 1)	1.8698 rad (Types 1 & 2)
Yu et al. [3]	0.0482 rad	0.0798 rad	0.1994 rad (Type 1)	0.4652 rad (Types 1 & 3)
Han et al. [1]	0.0542 rad	0.0875 rad (Type 1)	0.2684 rad (Type 1)	Invalid data
Proposed method	0.0451 rad	0.0752 rad	0.1250 rad	0.0872 rad (Type 3)

the proposed method maintains an RMSE of only 0.0872 rad—even lower than its 40 dB result—with only minor Type 3 errors. This robustness stems from three complementary features: the hybrid binary-ternary encoding that enhances code discrimination, modulation-based adaptive thresholds that adjust to local intensity variations, and a phase-consistency pre-correction algorithm that proactively identifies and rectifies boundary errors. Moreover, all these capabilities are achieved with only ten projected patterns demonstrating an effective balance between measurement efficiency and noise resilience. These comparative results confirm that the proposed method not only delivers higher accuracy under low-noise conditions but also remains reliable where conventional approaches break down.

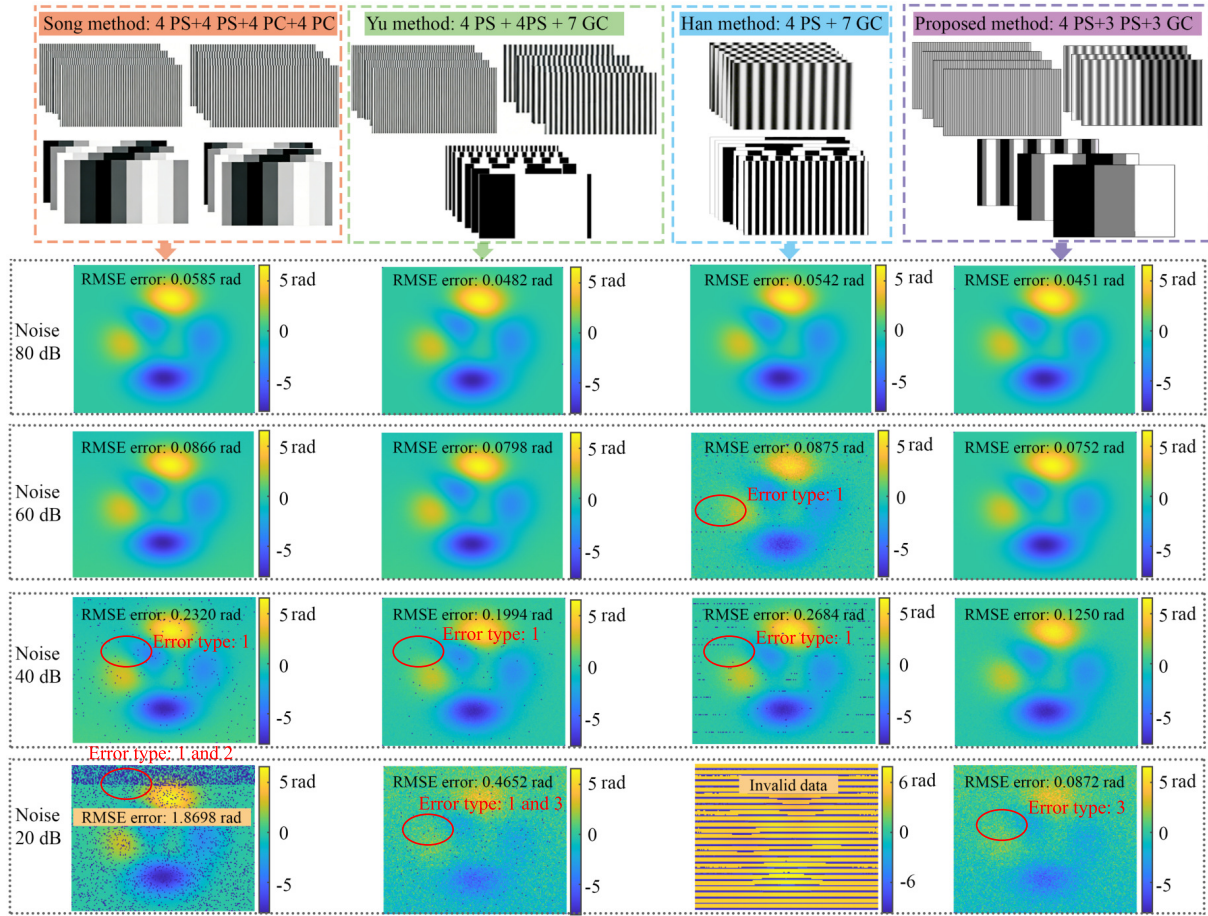


Fig. 11. Comparison of different methods under different noise levels.

3.2. Experiment

To validate the feasibility and superiority of the proposed method, an FPP measurement system was developed. The architecture of this system is shown in Fig. 12, consisting of a DLP LightCrafter 4500 projector and a 3-CMOS prism-based color camera (JAI AP-1600T). The projector has a resolution of 1280×800 pixels and supports structured light mode with pre-loaded pattern sequences. In our implementation, the ten patterns (four 8-bit phase-shifting patterns, three 8-bit composite patterns, and three ternary Gray code patterns) were pre-loaded into the projector's internal buffer, achieved through optimized sequence configuration and hardware trigger synchronization. The camera, equipped with a 16 mm lens ($f/1.4$ aperture), captures images at a resolution of 1456×1044 pixels with a maximum frame rate of 80 fps and is synchronized with the projector via an external trigger at the same effective

rate.

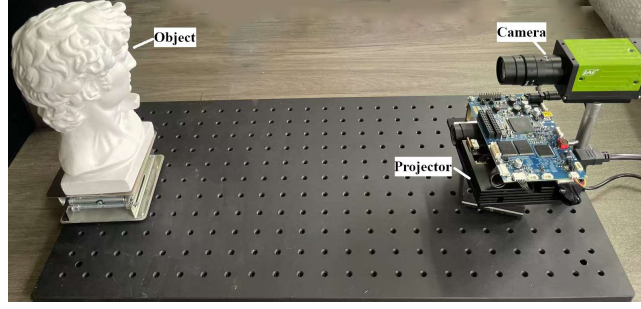


Fig. 12. Measurement System.

3.2.1. Facial Morphology Measurement

The performance of the proposed hybrid ternary-binary phase coding method was first evaluated through facial morphology measurement. Fig. 13 illustrates the codeword extraction process. Fig. 13(a) shows the background intensity map $A_2(x, y)$ calculated from the three-step phase-shifting images and Figs. 13(b)-(d) display the captured deformed ternary Gray code images (I_8, I_9, I_{10}). The corresponding decoded codewords are presented in Figs. 13(e)-(h): Fig. 13(e) shows the binary codeword C_{binary} obtained by adaptively thresholding $A_2(x, y)$ against $A_1(x, y)$, while Figs. 13(f)-(h) present the ternary codewords C_8, C_9, C_{10} decoded using dynamic thresholds derived from $A_2(x, y)$ and $B_2(x, y)$. The sharp transitions and spatial continuity in these codeword maps confirm the effectiveness of the adaptive thresholding strategy.

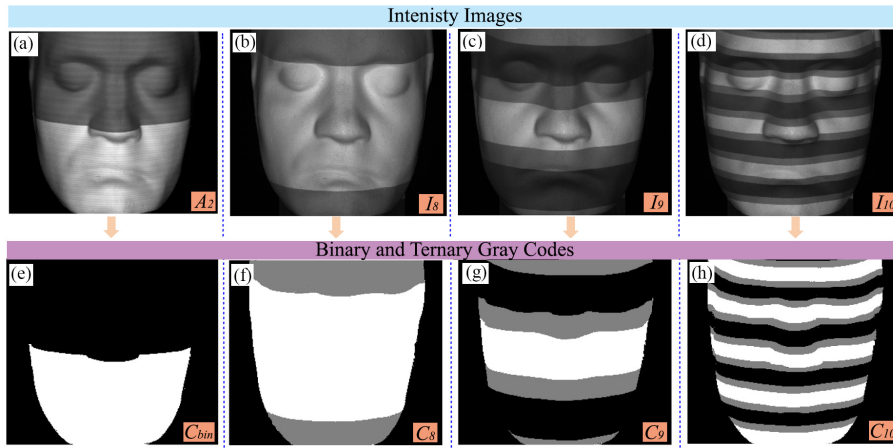


Fig. 13. Codeword extraction process for facial measurement. (a) Background intensity map A_2 , (b)-(d) captured ternary Gray code patterns I_8, I_9, I_{10} , (e)-(h) corresponding decoded binary and ternary codewords.

For detailed analysis, Fig. 14 presents cross-sectional profiles of the middle columns from Figs. 13(a)-(h). The intensity profiles of A_2, I_8, I_9 , and I_{10} in Figs. 14(a)-(d) clearly show modulation due to facial curvature, while the corresponding decoded codeword profiles in Figs. 14(e)-(h) confirm successful and stable binarization/ternarization with well-defined transitions at code boundaries.

The complete phase unwrapping process is visualized in Fig. 15. The auxiliary wrapped phase ϕ_2 calculated from Group 2 patterns is shown in Fig. 15(a), appearing continuous and relatively noise-free despite the complex facial surface. The corrected fringe order K_1^c , obtained after applying the phase-consistency based correction algorithm, is displayed in Fig. 15(b), showing consistent stair-like transitions with clean boundaries. Figure 15(c) shows the low-frequency absolute phase Φ_2 obtained by combining ϕ_2 and K_1^c . The primary wrapped phase ϕ_1 calculated from the high-frequency patterns of Group 1 is presented in Fig. 15(d), containing more detailed surface information but with rapid 2π discontinuities. The final fringe order K^{final} for the primary phase, determined using the frequency-ratio-based method, is shown in Fig. 15(e). The resulting high-precision absolute phase Φ_1 , smooth and continuous while retaining fine facial details, is displayed in Fig. 15(f). Finally, the high-quality 3D reconstruction result in Fig. 15(g) accurately captures both the overall facial contour and fine features such as the nose and eyes.

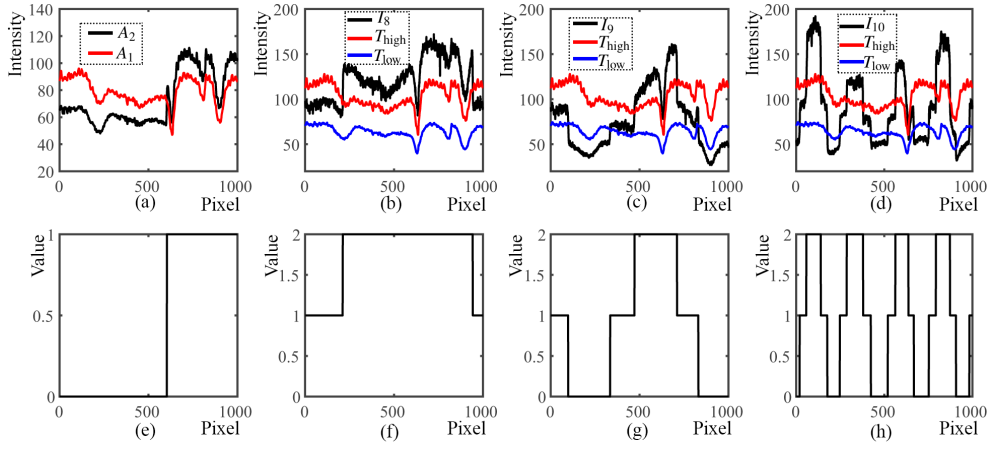


Fig. 14. Cross-sectional Profiles. (a)-(h) Corresponding cross-sections of the middle columns from Figure 13(a)-(h).

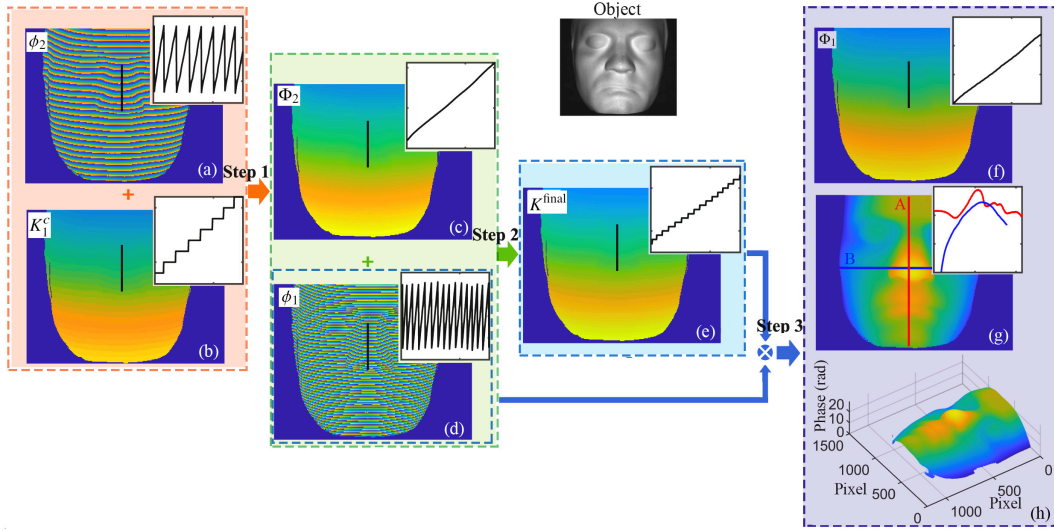


Fig. 15. Facial Morphology Reconstruction Flowchart. (a) Wrapped phase ϕ_2 , (b) corrected auxiliary fringe order K_1^c , (c) unwrapped phase Φ_2 obtained from (a) and (b), (d) wrapped phase ϕ_1 , (e) primary fringe order K^{final} obtained from (c) and (d), (f) final unwrapped phase Φ_1 obtained from (d) and (e), (g) and (h) 3D morphology.

3.2.2. Multi-Object Measurement and Error Suppression

The method's effectiveness in suppressing fringe order errors was validated using a multi-object scene. Fig. 16(c) shows the phase Φ_2 unwrapped directly using the preliminary fringe order K_1 , where line artifacts from order jumps are visible. Applying the proposed correction algorithm to K_1 yielded the corrected order K_1^c , reducing the fringe order error rate from 4.282% to 0.371% (an ~11.5-fold reduction). Fig. 16(b) shows the corresponding unwrapped phase Φ_2 . Subsequently, a dual-frequency approach was employed: based on Φ_2 , the final fringe order K^{final} was derived, leading to the high-precision absolute phase Φ_1 shown in Fig. 16(a). This step further reduced the error by approximately seven times. Overall, the proposed framework reduced the fringe order error by approximately 21 times and the RMSE by approximately 43 times compared to the uncorrected baseline, using a 12-step multi-frequency heterodyne phase unwrapping result as the reference [35].

Fig. 17 provides a visual comparison of projection patterns and corresponding 3D results for different methods. Subfigures 17(a)-(d) show the pattern sets for the binary complementary, ternary complementary, traditional ternary, and proposed methods, respectively. The corresponding reconstruction results and error maps are shown in Figs. 17(e)-(h) and (i)-(l). The traditional ternary method exhibits the largest error. While the binary and ternary complementary methods reduce errors, significant residual errors remain. In contrast, the proposed method achieves the lowest final RMSE of 0.068 rad, demonstrating its superior effectiveness.

A comparative summary of key performance indicators is provided in Table 3. The proposed method achieves a balanced trade-off among pattern efficiency, encoding capacity, measurement accuracy, and operational robustness.

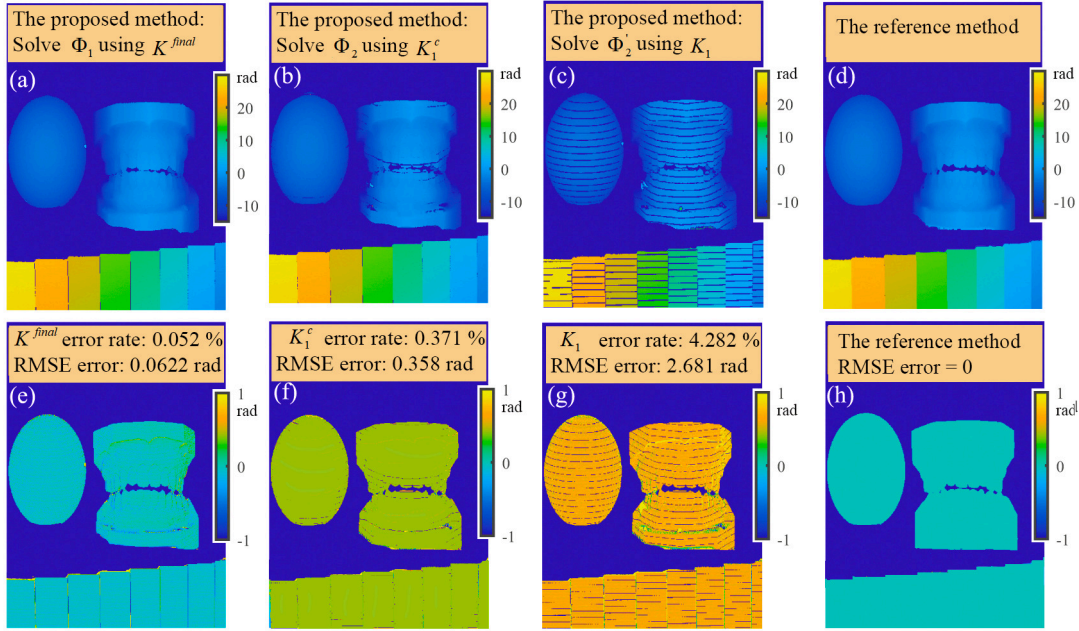


Fig. 16. Multi-object phase unwrapping results. (a) Unwrapped phase Φ_1 using ϕ_1 and K^{final} , (b) unwrapped phase Φ_2 using ϕ_2 and K_1^c , (c) unwrapped phase Φ_2' using ϕ_2 and K_1 , (d) reference phase from the multi-frequency method, (e)-(h) corresponding phase error distributions for (a)-(d).

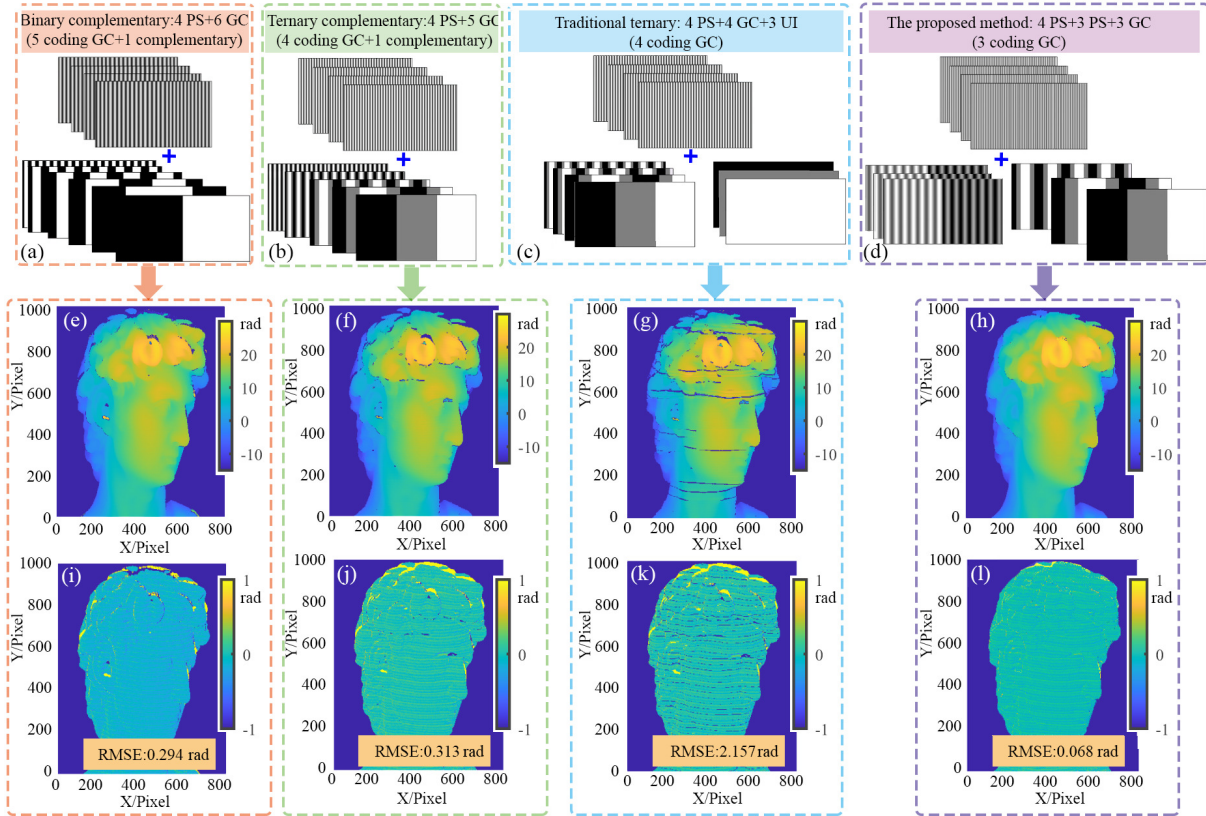


Fig. 17. Comparative evaluation of different methods. (a)-(d) Projected pattern sets (PS: phase-shifting, GC: Gray code), (e)-(h) corresponding 3D reconstruction results, (i)-(l) corresponding phase error maps, with a 12-step phase-shifting result as reference.

3.2.3. Experiment on Defocusing Degree

The robustness against optical defocus—a common practical disturbance—was evaluated on a spherical object. Defocus was systematically introduced by adjusting the projector's focus across six discrete levels while the camera remained in focus [38]. Level 1 is the in-focus condition; Levels 2–6 represent increasing projector defocus. The captured sinusoidal and ternary Gray code patterns are shown in Fig. 18(a)-(l). High contrast is observed at Level 1, with progressive blurring at higher levels due to optical low-pass filtering.

Table 3. Comprehensive performance comparison of phase unwrapping methods.

Criterion	Binary Complementary Gray Code [36]	Ternary Complementary Gray Code [28]	Traditional Ternary Gray Code [37]	Multi-frequency Phase-shifting	Proposed Method
Pattern Count	10 (4 PS + 6 GC)	9 (4 PS + 5 GC)	11 (4 PS + 4 GC + 3 UI)	36 (12 PS $\times$ 3 Freq)	10 (4 PS + 3 PS + 3 GC)
Theoretical Encoding Capacity	$2^5 = 32$	$3^4 = 81$	$3^4 = 81$	Limited by frequency selection	$2 \times 3^3 = 54$ (single) $\rightarrow$ 108 (dual)
Phase Error (RMSE)	Moderate	Moderate	High	Very Low (Reference)	Very Low (0.068 rad)
Noise Robustness	Moderate	Moderate (intensity sensitive)	Low (requires uniform images)	High	High (adaptive threshold)

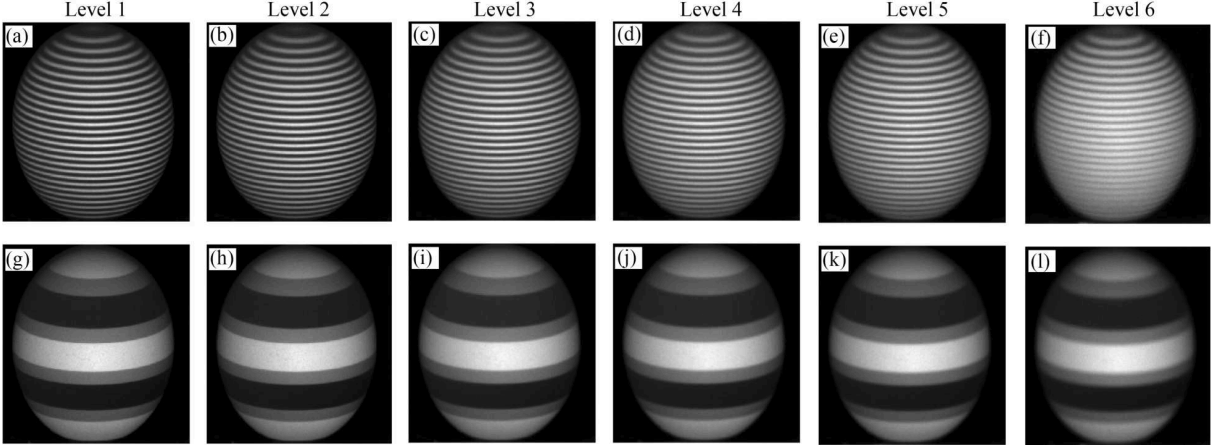


Fig. 18. Images captured under different defocus levels. (a)-(f) Sinusoidal fringe patterns, (g)-(l) ternary Gray code patterns. Levels 1 to 6 correspond to increasing defocus.

The corresponding 3D reconstruction results and error maps are shown in Fig. 19. This experiment demonstrates that the method effectively decouples the robust, continuous phase-retrieval process from the more noise-sensitive, discrete order-identification step. Consequently, the fundamental phase information retains its accuracy despite significant optical degradation, while any failures are confined to—and can be directly traced to—the ternary code segmentation process.

Quantitative results are summarized in Table 4. The phase RMSE increases moderately with progressive defocus. In contrast, the fringe order error remains below 0.03% for Levels 1–3 but exhibits a sharp, non-linear increase at Level 4 (1.822%) and Level 5 (2.586%), with Level 6 yielding invalid measurements. This divergence originates from the fundamentally different nature of the two algorithmic steps. Phase retrieval relies on continuous sinusoidal fringe images and is inherently robust to moderate low-pass filtering. In contrast, decoding the ternary Gray code is a discrete classification task that requires well-defined intensity transitions. Defocusing blurs these critical intensity boundaries, leading to code misclassification and the observed sharp increase in order errors.

Table 4. Quantitative phase and fringe order errors under progressive projector defocus.

Error Type	Level 1 (In-focus)	Level 2	Level 3	Level 4	Level 5	Level 6
Fringe Order Error (%)	0.019	0.023	0.021	1.822	2.586	Invalid
RMSE (rad)	0.0580	0.0755	0.0847	0.1623	0.1684	Invalid

These comprehensive experiments demonstrate that the proposed hybrid method successfully handles complex surfaces, provides high coding capacity with minimal patterns, and maintains robustness through adaptive thresholding processing. The cascaded unwrapping framework effectively combines the reliability of low-frequency phase unwrapping with the precision of high-frequency measurement.

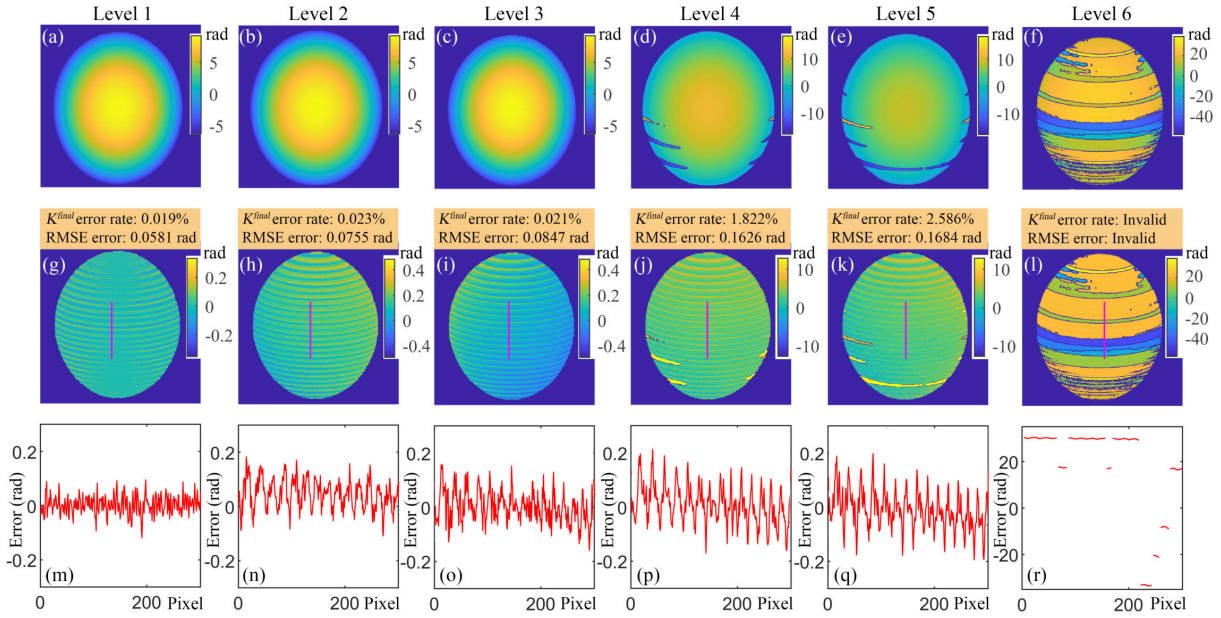


Fig. 19. Results for a spherical object under defocus. (a)-(f) Reconstructed absolute phase maps, (g)-(l) corresponding RMSE error maps, (m)-(r) cross-sectional RMSE profiles along the purple lines in (g)-(l).

4. Conclusions

This paper presents a novel phase unwrapping method based on a hybrid binary-ternary Gray code, developed to address key limitations of conventional coding strategies in fringe projection profilometry. The proposed technique effectively suppresses unwrapping errors, thereby enhancing the reliability of high-precision 3D reconstruction for complex surfaces. Its effectiveness originates from the synergistic integration of an adaptive hybrid encoding strategy, a phase-consistency-based pre-correction algorithm, and a cascaded unwrapping architecture, which collectively ensure high accuracy while maintaining computational efficiency.

Hybrid Encoding Strategy: The core innovation lies in a hybrid binary-ternary Gray code structure. By deriving an auxiliary binary codeword from the background intensity of phase-shifting patterns and integrating it with three ternary Gray codes, the method achieves a base encoding capacity of 108 fringe orders. This represents a substantial improvement in coding efficiency over conventional binary or ternary schemes, enabling the use of higher-frequency fringes for detailed measurement without a proportional increase in the number of projected patterns.

Phase-Consistency Pre-Correction: A key algorithmic contribution is a robust pre-correction mechanism that proactively mitigates fringe order jumps at codeword boundaries. This mechanism identifies and corrects error-prone regions by analyzing the logical relationship between the sign of an auxiliary wrapped phase and the parity of a preliminary fringe order, significantly reducing boundary-induced artifacts prior to final unwrapping. Experimental results demonstrate that this approach reduces the fringe order error rate from 4.282% to 0.371% in multi-object measurements.

Cascaded Unwrapping: Furthermore, the method employs a dual-frequency, cascaded unwrapping framework. It leverages the inherent robustness of low-frequency phase unwrapping, guided by the corrected hybrid Gray code orders, to reliably determine the fringe orders for a high-frequency phase map. This architecture effectively decouples the stable retrieval of continuous phase information from the more noise-sensitive discrete code decoding, ensuring both global consistency and the preservation of intricate surface details.

Limitations and Future Work: Despite its advantages, the proposed method has certain limitations. It exhibits sensitivity to severe projector defocus, as the ternary Gray code patterns become excessively blurred under such conditions, compromising the reliability of fringe order. This is reflected in the experimental results, where the fringe order error sharply increases from below 0.03% under moderate defocus to 1.822% at Level 4 and 2.586% at Level 5. This limitation is inherent to discrete coding schemes that rely on distinguishable intensity levels, highlighting a trade-off between coding efficiency and defocus tolerance. Future work will focus on developing more defocus-robust encoding patterns to mitigate this issue.

Experimental validation confirms the method maintains robust performance under challenging conditions, including measurements of complex surfaces such as faces and scenarios with moderate optical defocus, demonstrating its practical utility for high-accuracy 3D shape measurement. Despite the identified limitations, the method offers a valuable approach to balancing coding efficiency, measurement accuracy, and practical robustness in fringe projection profilometry.

Acknowledgements

This work was supported by Shaanxi Province Natural Science Basic Research Program (2025JC-YBQN-640). Additional support was provided by Shaanxi Provincial Department of Education Special Scientific Research Project

Disclosures

The authors declare no conflicts of interest.

References

1. F. Wang, J. Liu, P. Song, J. Gong, W. Peng, G. Liu, M. Chen, and Y. Wang, "Multimodal optical excitation pulsed thermography: Enhanced recognize debonding defects of the solid propellant rocket motor cladding layer," *Mech. Syst. Signal Proc.* **163**, 108164 (2022).
2. F. Wang, J. Liu, L. Liu, L. Xu, Y. Wang, M. Chen, and Y. Wang, "Quantitative non-destructive evaluation of cfrp delamination defect using laser induced chirp-pulsed radar photothermal tomography," *Optics Lasers Engineering* **149**, 106873 (2022).
3. S. Zhang, "High-speed 3d shape measurement with structured light methods: A review," *Opt. Lasers Eng.* **106**, 119–131 (2018).
4. A. G. Marrugo, F. Gao, and S. Zhang, "State-of-the-art active optical techniques for three-dimensional surface metrology: A review," *J. Opt. Soc. Am. A* **37**, B60–B77 (2020).
5. J. Xu and S. Zhang, "Status, challenges, and future perspectives of fringe projection profilometry," *Opt. Lasers Eng.* **135**, 106193 (2020).
6. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe pattern analysis for computer-based topography and interferometry," *J. Opt. Soc. Am. A* **72**, 156–160 (1982).
7. M. Zhong, W. Chen, and X. Su, "A comparison of one and two-dimensional s-transform in fringe pattern demodulation," *Optics Lasers Engineering* **55**, 212–220 (2014).
8. X. Su and W. Chen, "Reliability-guided phase unwrapping algorithm: A review," *Opt. Lasers Eng.* **42**, 245–261 (2004).
9. M. Zhao, L. Huang, Q. Zhang, X. Su, A. Asundi, and Q. Kemao, "Quality-guided phase unwrapping technique: Comparison of quality maps and guiding strategies," *Appl. Opt.* **50**, 6214–6224 (2011).
10. K. Chen, J. Xi, and Y. Yu, "Quality-guided spatial phase unwrapping algorithm for fast three-dimensional measurement," *Opt. Commun.* **294**, 139–147 (2013).
11. C. Zuo, Q. Chen, G. Gu, S. Feng, and F. Feng, "High-speed three-dimensional profilometry for multiple objects with complex shapes," *Opt. Express* **20**, 19493–19510 (2012).
12. X. Meng, F. Wang, J. Liu, M. Chen, and Y. Wang, "Phase shifting profilometry based on hilbert transform: An efficient phase unwrapping algorithm," *J. Appl. Phys.* **131**, 193104 (2022).
13. Y. Xing, C. Quan, and C. J. Tay, "A generalized multi-sensitivity temporal phase unwrapping method for absolute phase retrieval," *Opt. Lasers Eng.* **96**, 290–298 (2017).
14. J. M. Huntley and H. Saldner, "Temporal phase-unwrapping algorithm for automated interferogram analysis," *Appl. Opt.* **32**, 3047–3052 (1993).
15. H. Zhao, W. Chen, and Y. Tan, "Phase-unwrapping algorithm for the measurement of three-dimensional object shapes," *Appl. Opt.* **33**, 4497–4500 (1994).
16. R. Carsten, R. Reinhold, and T. Jan, "3-d shape measurement of complex objects by combining photogrammetry and fringe projection," *Opt. Eng.* **39**, 224–231 (2000).
17. J. S. Hyun and S. Zhang, "Enhanced two-frequency phase-shifting method," *Appl. Opt.* **55**, 4395–4401 (2016).
18. W. Deng, "Absolute phase recovery based on two-wavelength fringes using a defocusing technique in fringe projection profilometry," *Optical Engineering* **59**, 1002–1014 (2020).
19. L. Lu and et al., "High-efficiency dynamic three-dimensional shape measurement based on misaligned gray-code light," *Optics Lasers Engineering* **150**, 106873 (2022).
20. H. Wu, Y. Cao, H. An, C. Xu, H. Li, and Y. Li, "A general phase ambiguity suppression algorithm combining geometric constraints and temporal phase unwrapping," *Opt. Laser Technol.* **150**, 107955 (2022).
21. Z. Wu, W. Guo, and Q. Zhang, "Two-frequency phase-shifting method vs. gray-coded-based method in dynamic fringe projection profilometry: A comparative review," *Optics Lasers Engineering* **153**, 106995 (2022).
22. S. Inokuchi, "Range imaging system for 3-d object recognition," *Proc. ICPR* **8**, 806–808 (1984).
23. G. Sansoni, M. Carocci, and R. Rodella, "Three-dimensional vision based on a combination of gray-code and phase-shift light projection: Analysis and compensation of the systematic errors," *Appl. Opt.* **38**, 6565–6573 (1999).
24. J. I. Laughner, S. Zhang, H. Li, C. C. Shao, and I. R. Efimov, "Mapping cardiac surface mechanics with structured light imaging," *Am. J. Physiol. Heart Circ. Physiol.* **303**, H712–H720 (2012).
25. D. Zheng, F. Da, and H. Huang, "Phase unwrapping for fringe projection three-dimensional measurement with projector defocusing," *Opt. Eng.* **55**, 034107 (2016).
26. Z. Wu, W. Guo, and Q. Zhang, "High-speed three-dimensional shape measurement based on shifting gray-code light," *Opt. Express* **27**, 22631–22644 (2019).
27. Q. Zhang, X. Su, L. Xiang, and X. Sun, "3-d shape measurement based on complementary gray-code light," *Opt. Lasers Eng.* **50**, 574–579 (2012).
28. H. Wei, H. Li, and J. Liu, "A ternary complementary gray code phase unwrapping method in fringe projection profilometry," *Journal Applied Physics* **135**, 15 (2024).
29. X. Li, H. Li, and H. Wei, "An improved quaternary complementary gray code phase unwrapping method in fringe projection profilometry," *Optics Lasers Engineering* **184**, 108680 (2025).
30. X. He, D. Zheng, and Q. Kemao, "Quaternary gray-code phase unwrapping for binary fringe projection profilometry," *Optics Lasers Engineering* **121**, 358–368 (2019).
31. S. Han, Y. Yang, X. Zhang, X. Zhao, and X. Li, "An improved complementary gray code combined with phase-shifting profilometry based on phase adjustment," *Meas. Sci. Technol.* **36**, 035012 (2025).
32. S. Yu, J. Zhang, X. Yu, X. Sun, H. Wu, and X. Liu, "3d measurement using combined gray code and dual-frequency phase-shifting approach," *Opt. Commun.* **413**, 283–290 (2018).
33. L. Song, X. Zhang, H. Huang, Q. Guo, Y. Yang, and X. Zhu, "Phase unwrapping method combining dual-frequency heterodyne with dual complementary phase encoding," *Opt. Eng.* **60**, 085106 (2021).
34. Q. Zhu, H. Du, B. Tang, and X. Yan, "A robust phase unwrapping approach using hierarchical gray-code and triple fringe order correction," *Optics Laser Technology* **192**, 113992 (2025).
35. S. Zhang, "Absolute phase retrieval methods for digital fringe projection profilometry: A review," *Optics Lasers Engineering* **107**, 28–37 (2018).
36. Z. Wu, W. Guo, Y. Li, Y. Liu, and Q. Zhang, "High-speed and high-efficiency three-dimensional shape measurement based on gray-coded light," *Optics Express* **8**, 819–829 (2020).
37. D. Zheng, Q. Kemao, F. Da, and H. S. Seah, "Ternary gray code-based phase unwrapping for 3d measurement using binary patterns with projector defocusing," *Appl. Opt.* **56**, 3660–3665 (2017).
38. S. Lei and S. Zhang, "Digital sinusoidal fringe pattern generation: Defocusing binary patterns vs focusing sinusoidal patterns," *Optics Lasers Engineering* **48**, 561–569 (2010).