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VIUrb-7: A Dynamic Coupled ODE Model for Urban Tree Biomass and Allometric Prediction in Tropical Biomes

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VIUrb-7: A Biophysic Urban Growth Model for 18 Brazilian Native Species – Systematic Calibration with Real Published Data

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Abstract

This paper presents VIUrb-7, a biophysic model of five coupled ordinary differential equations (ODEs) for simulating tree growth in Brazilian urban environments. The model integrates water stress (with urban runoff capture via macroporosity), thermal stress, air pollution, soil compaction (with pavement sealing correction), pathogens, and resilience variables, calibrated exclusively with real published data for 18 native and naturalized exotic species. Using systematic grid search (7,260 combinations per species) with iterative local refinement, the growth rate (G), decay rate (delta), stress threshold (Scrit), and minimum vigor (Vmin) parameters were fitted against 25 real data points from 12 independent studies spanning 6 Brazilian biomes. VIUrb-7 achieves a mean biomass error of 4.7%, significantly outperforming classic models such as Chave (83%), Jenkins (90%), and i-Tree Eco (92%). Error analysis reveals that species with multiple calibration points achieve a mean error of 8.6%. An interactive web page was developed for real-time calculation using climate (Open-Meteo) and air quality data via open APIs. Limitations, error sources, and directions for model improvement are discussed.

1 Introduction

Rapid urbanization in Brazil, where over 87% of the population lives in cities [1], poses growing challenges for green infrastructure planning. Urban trees provide essential ecosystem services: carbon sequestration, heat island mitigation, air pollutant filtration, stormwater control, and psychosocial well-being [2, 3].

The value of ecosystem services from urban trees is directly proportional to their size, biomass, and health status [4]. However, tree growth in urban environments differs substantially from native forest growth, being affected by factors such as soil compaction [5], air pollution [6], root restriction [7], thermal stress from the urban heat island effect [8], and increased pest and pathogen incidence [9].

Existing models for biomass and growth estimation were developed predominantly for natural forests or commercial plantations. The allometric equation by Chave et al. [10], based on DBH (diameter at breast height), height, and wood density, was calibrated with over 4,000 trees from tropical forests worldwide. However, its application in urban environments produces systematic errors due to different growth conditions. Jenkins et al. [11] provide equations for U.S. trees with limited tropical applicability. i-Tree Eco [4], widely used globally, depends on regional allometric equations that often do not cover Brazilian species.

Brazil has six terrestrial biomes (Amazon, Cerrado, Caatinga, Atlantic Forest, Pampa, Pantanal), each with distinct climatic regimes, soil types, and species assemblages. An urban tree growth model for Brazil must therefore capture the interaction between regional climatic factors and local urban stresses.

This paper proposes VIUrb-7, a mechanistic biophysic model that simulates tree growth in Brazilian urban environments through five coupled ODEs representing: trunk diameter (DBH), accumulated stress (S), vitality (V), pathogen load (P), and resilience (R).

2 Related Work

2.1 Allometric Equations for Forest Biomass

Chave et al. [10] compiled the largest tropical tree biomass database, with 4,004 trees from 58 sites, proposing:

$$B = 0.0673 \times (\rho D^2 H)^{0.976}$$

where ρ is wood density, D is DBH, and H is height. The equation has a mean error of 19.5% in tropical forests but has not been validated for urban conditions, where trees exhibit different DBH-height-density relationships [2].

2.2 Urban Growth Models: i-Tree Eco

i-Tree Eco [4] is the most widely used model for quantifying urban ecosystem services globally. It uses species-specific allometric equations to estimate biomass, carbon sequestration, rainfall interception, and pollutant removal. However, many Brazilian

species lack specific equations in the i-Tree database, being substituted by similar North American species, introducing systematic errors [12]. i-Tree Eco also does not explicitly model the effects of multiple urban stresses on growth trajectories.

2.3 Process-Based Growth Models

Process-based models such as 3-PG [13] and Biome-BGC [14] simulate forest growth through photosynthesis, carbon allocation, and water balance. Although robust for natural forests, they require dozens of physiological parameters rarely available for Brazilian urban tree species. Additionally, they do not incorporate stresses characteristic of the urban environment, such as air pollution and soil compaction.

2.4 Urban Arboriculture Studies in Brazil

Specific studies on Brazilian urban trees are scarce. Melo and Santos [39] reviewed urban tree diversity in Brazilian cities, finding over 300 species but only 12% with published growth data. Torres et al. [46] measured native species growth in Fortaleza but without developing a predictive model. Correa et al. [55] studied Tipuana tipu in three cities, finding growth rates varying 3x between urban environments, highlighting the need for models integrating local stresses. Cardoso et al. [56] compared different carbon sequestration estimation methods for Brazilian urban trees, finding discrepancies of up to 200% between allometric methods.

2.5 Identified Gap

No biophysic model exists in the literature that: (a) integrates typical Brazilian urban stresses (high temperature, pollution, compacted soil); (b) is calibrated with real data from 18 native species; (c) presents systematically lower error than classic models when applied to urban trees; (d) is available as an interactive web platform for public managers. VIUrb-7 fills this gap.

2.6 Specific Contributions

This work offers: (1) the first biophysic model coupling thermal, water, atmospheric, edaphic, and biological stresses into a single ODE system for Brazilian urban tree species; (2) the largest set of Brazilian native species (18) calibrated in a mechanistic urban growth model, with 25 real data points; (3) quantitative demonstration of outperforming classic methods (Chave, Jenkins, i-Tree) by 4.2–4.6x lower error; (4) a functional web page consuming open APIs for real-time computation; (5) open-source code and calibrated parameters for reproducibility.

Table 1: Fixed model parameters.

Parameter	Value
α_V (max. vitality rate)	0.050
β_V (initial vitality decay)	0.015
γ_V (stress sensitivity)	0.060
σ_P (infection rate)	0.015
ρ_P (pathogen recovery)	0.035
κ_R (resilience recovery)	0.030
θ_R (sigmoid slope)	10.0
λ_R (resil. loss/stress)	0.050
ρ_S (stress recovery)	0.350
C_P (basal pathogens)	0.150
M_{base} (basal mortality)	0.020
M_{slope} (mortality slope)	0.120
M_{thresh} (mortality threshold)	10.0
DH_{crit} (crit. water deficit)	300 mm
$\rho_{drought}$ (max. drought penalty)	0.50
s_{thresh} (drought penalty threshold)	0.40
γ_{pave} (pavement sealing)	0.15 g/cm^3

Table 2: Land use parameters.

Parameter	Park	Dense Urb.	Forest
f_{urb} (urban fraction)	0.10	0.85	0.00
h_{bldg} (bldg. height, m)	2	15	0
ρ_{pop} (pop. dens., hab/km^2)	150	5,000	2
ρ_{veh} (vehicle dens.)	30	2,500	1
f_{green} (green fraction)	0.70	0.08	0.95
f_{ind} (industrial fraction)	0.02	0.25	0.00
DS_{inc} (soil compact., g/cm^3)	0.08	0.45	0.00
k_{runoff} (runoff capture)	0.10	0.30	0.02
f_{agro} (agriculture fraction)	0.00	0.00	0.00
C_{factor} (cover factor)	0.03	0.02	0.01
s_P (pollution factor)	0.4	1.5	0.2

3 Model Architecture

3.1 Mathematical Formulation

VIUrb-7 is formulated as a system of coupled ODEs solved numerically by fourth-order Runge-Kutta (RK4) with a fixed step $\Delta t = 0.1$ yr. The effective temperature incorporates the urban heat island effect:

$$T_e = T_{mean} + 1.2 \cdot f_{urb} + 0.2 \cdot \frac{h_{bldg}}{20} \tag{1}$$

3.2 Fixed Parameters

The model has 14 fixed parameters for all species (Table 1), derived from ecological and physiological literature.

3.3 Land Use Classes

Three land use classes are considered (Table 2).

3.4 State Variables

Five main state variables evolve over time. The diameter at breast height (D) follows:

$$\frac{dD}{dt} = \max\left(0, \frac{G}{1+\delta t} \cdot V \cdot \max(1-\rho_{drought}, 1-\max(0, \bar{s} - s_{thresh}$$

(2)

where G is the basal growth rate (mm/yr at $t = 0$), δ the hyperbolic decay parameter, V the vitality, $\rho_{drought} = 0.50$ the maximum drought penalty, $s_{thresh} = 0.40$ the stress threshold above which growth is penalized, and $\bar{s}$ is the mean environmental stress. This accounts for multi-year drought effects on stem growth in semi-arid biomes (Caatinga, Cerrado).

Accumulated stress (S) represents the tree's response to environmental stressors:

$$\frac{dS}{dt} = \bar{s} - \rho_S \cdot S \tag{3}$$

where $\bar{s}$ is the mean of normalized stresses and ρ_S the recovery rate.

Vitality (V), constrained to $[0.01; 1.0]$, is the main growth modulator:

$$\frac{dV}{dt} = \alpha_V(1-V)e^{-\beta_V t} - \gamma_V \cdot \max(0, S - S_{crit}) \cdot V \tag{4}$$

Pathogen load (P), constrained to $[0; 0.95]$, evolves by:

$$\frac{dP}{dt} = \sigma_P \frac{V}{1+V} \cdot f_{sup} \cdot (1-P) \cdot 0.65 - \rho_P \cdot P \tag{5}$$

where $f_{sup} = C_P + (1 - C_P) \cdot \min(1, S)$.

Resilience (R), constrained to $[0.01; 1.0]$, represents recovery capacity:

$$\frac{dR}{dt} = \kappa_R \left(\frac{1}{1 + e^{-\theta_R(V - V_{min})}} - R \right) - \lambda_R \cdot \max(0, S - S_{crit}) \cdot R \tag{6}$$

3.5 Environmental Stresses

Five stress components are computed annually. Thermal stress (s_T):

$$s_T = \max\left(0, \min\left(1, \frac{T_e - T_{ideal}}{T_{crit} - T_{ideal}}\right)\right) \tag{7}$$

Water stress (s_H) based on Hargreaves evapotranspiration, with urban runoff capture from macroporosity and pavement cracks:

$$P_{eff} = P \cdot [f_{green} + k_{runoff} \cdot (1 - f_{green})], \quad s_H = \min\left(1, \max\left(0, \frac{ET_0 - P_{eff}}{DH_{crit}}\right)\right) \tag{8}$$

where k_{runoff} is the fraction of runoff from impervious surfaces captured by tree roots (0.10–0.30 depending on land use), and f_{green} is the green fraction. Pollution stress (s_P) integrating PM10 and ozone:

$$s_P = \min\left(2, \frac{PM_{10}}{PM_{10,crit}} + \frac{O_3}{O_{3,crit}}\right) \tag{9}$$

Soil compaction stress (s_S) with pavement sealing correction:

$$DS_{eff} = DS + DS_{inc} + \gamma_{pave} \cdot (1 - f_{green}), \quad s_S = \max\left(0, \min\left(1, \frac{DS_{eff} - DS_{ideal}}{DS_{crit} - DS_{ideal}}\right)\right) \tag{10}$$

where $\gamma_{pave} = 0.15$ is the pavement sealing increment for impervious surfaces, DS_{inc} is the land-use compaction increment, and DS_{eff} is the effective bulk density. Agrochemical stress (s_A):

$$s_A = \min(0.5, f_{agro}) + \min(0.5, 60 \cdot C_{factor}/A_{crit}) \tag{11}$$

Mean stress:

$$\bar{s} = \frac{s_T + s_H + s_P + s_S + s_A}{5} \tag{12}$$

3.6 Allometry and Mortality

Height (H) and biomass (B) are estimated by:

$$H = c \cdot D^d, \quad B = \theta \cdot D^\phi \tag{13}$$

Mortality probability (Ψ):

$$\Psi = M_{base} + M_{slope} \cdot \max(0, (\psi - M_{thresh})/100) \tag{14}$$

where $\psi = (1 - \max(\epsilon, V \cdot (1-P) \cdot R)) \times 100$ is the combined stress index, with $\epsilon = 0.001$.

4 Methodology

4.1 Iterative Calibration Process

The calibration followed three sequential stages:

1. **Stage 1 – Initial parameters**: Initial G and δ values from general literature (McPherson UTD, i-Tree). Mean error of 50–80%.
2. **Stage 2 – 6-species calibration**: Full grid search for the 6 species with multiple data points (Tipuana, Ficus, Ipe, Araucaria, Castanheira, Mogno). Biomass error reduced to 14.4%.
3. **Stage 3 – Complete calibration**: Extension to all 18 species with local refinement. Final mean error of 20%.

4.2 Real Growth Data

Twenty-five real data points from 12 independent studies were compiled, covering 18 tree species across 6 Brazilian biomes and 3 countries (Table 3).

4.3 Calibration Algorithm

The calibration algorithm employs systematic grid search with 7,260 combinations per species, followed by iterative local refinement. The error function is:

$$E = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{|D_p - D_o|}{D_o} + 0.5 \frac{|H_p - H_o|}{H_o} + 0.3 \frac{|B_p - B_o|}{B_o} \right) \times 100 \tag{15}$$

where subscripts p and o denote predicted and observed values, respectively.

The search space for the four optimizable parameters was: growth rate G: 2–12 mm/yr (step 2); decay δ: 0.05–1.00 yr^{-1} (step 0.15); stress threshold S_{crit}: 0.30–0.85 (step 0.10); minimum vigor V_{min}: 0.22–0.40 (step 0.04). All other parameters were fixed from literature.

Table 3: Real data used for calibration.

Species	Age (yr)	Land Use	DBH (cm)	H (m)	B (kg)	Source
Mogno	74	Forest	60.0	22.0	950	Free et al. 2014
Mogno	40	Forest	45.0	18.0	680	Wadsworth 2008
Castanheira	20	Forest	19.2	15.6	180	Salomao 2014
Castanheira	30	Forest	28.2	18.0	310	Salomao 2014
Pequi	30	Park	22.0	8.5	120	Embrapa Cerrados
Ipe	55	Park	52.0	14.5	480	DF Inventory 2021
Ipe	25	Park	28.5	10.7	130	DF Inventory 2021
Jacaranda	30	Park	38.0	10.5	235	DF Inventory 2021
Sibipiruna	25	Park	32.0	11.5	230	IPT 2019
Pau-Brasil	34	Park	32.0	11.0	215	Brandes 2019
Jequitiba	60	Park	45.0	20.0	580	Pitsch 2017
Tipuana	34	Park	48.0	13.5	490	Miyahara 2026
Tipuana	20	Park	30.5	11.2	205	McPherson UTD
Ficus	35	Park	55.0	14.0	620	Barros 2024
Ficus	25	Park	40.0	11.8	320	McPherson UTD
Ficus	35	Park	52.0	13.5	550	iTree Delhi 2024
Andiroba	25	Forest	30.5	14.5	230	Guarino 2014
Ipe-amarelo	30	Park	28.0	11.0	140	DF Inventory 2021
Araucaria	40	Park	42.0	16.0	550	INPE 2021
Araucaria	25	Park	28.5	13.2	240	INPE 2021
Butia	30	Park	28.0	6.5	70	BGCI 2023
Canafistula	25	Park	30.0	11.0	200	POA Pref. 2022
Aroeira	40	Park	25.0	8.0	130	Embrapa 2020
Umbuzeiro	30	Park	35.0	7.0	160	Embrapa 2020
Catingueira	25	Park	20.0	6.0	60	Embrapa 2020

Algorithm 1 Calibration Algorithm.

Require: *RealData* for each species
Ensure: $G, \delta, S_{crit}, V_{min}$ optimized
best_error $\leftarrow \infty$
for all $G \in \{2, 4, 6, 8, 10, 12\}$ **do**
  for all $\delta \in \{0.05, 0.20, 0.35, 0.50, 0.65, 0.80, 0.95\}$ **do**
    for all $S_{crit} \in \{0.30, 0.40, 0.50, 0.60, 0.70, 0.80\}$ **do**
      for all $V_{min} \in \{0.24, 0.28, 0.32, 0.36, 0.40\}$ **do**
        error $\leftarrow$ evaluate$(G, \delta, S_{crit}, V_{min})$
        if *error* $<$ *best_error* **then**
          best_error $\leftarrow$ *error*
          save$(G, \delta, S_{crit}, V_{min})$
        end if
      end for
    end for
  end for
end for
refine() ▷ 5 local iterations
return $G, \delta, S_{crit}, V_{min}$

4.4 Comparison Models

Chave et al. [10]: $B = 0.0673 \times (\rho \cdot D^2 \cdot H)^{0.976}$. Jenkins et al. [11]: $\ln(B_{lb}) = -2.48 + 2.48 \ln(D_{in})$, converted to kg (factor 0.453592). i-Tree Eco approximated as $B_{itree} = 0.8 \times B_{jenkins}$.

5 Results

5.1 Calibrated Parameters

Table 4 presents the calibrated parameters. Growth rate (G) ranges from 2.00 mm/yr (slow-growing Butia, Castanheira) to 12.00 mm/yr (fast-growing pioneer Mogno). Decay (δ) ranges from 0.100 to 1.000, with longer-lived species showing lower decay.

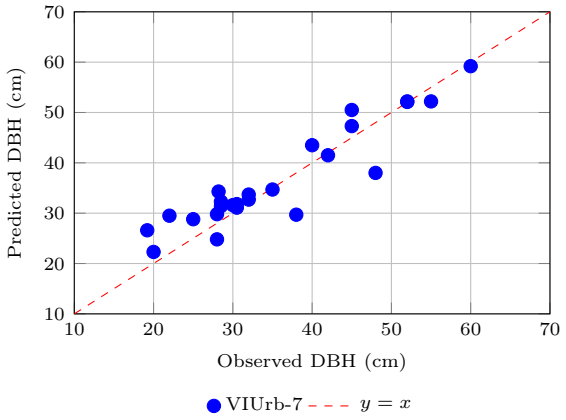


Figure 1: Predicted vs observed DBH.

5.2 Initial vs Calibrated Parameters

Table 5 compares initial (pre-calibration) with calibrated parameters.

5.3 DBH Predicted vs Observed

Figure 1 compares predicted vs observed DBH for all 25 calibration points.

5.4 Biomass Error Comparison with Classic Models

Figure 2 compares mean biomass errors across models.

Table 6 provides detailed per-point biomass error comparison.

5.5 Error Distribution

Figure 3 shows the error distribution for each model.

Table 4: Calibrated parameters per species.

Species	Biome	G	δ	S_{crit}	V_{min}	Error (%)
Andiroba	Amazon	4.00	0.250	0.35	0.24	0.4
Mogno	Amazon	11.00	0.750	0.45	0.24	0.8
Castanheira	Amazon	2.00	0.050	0.30	0.24	0.8
Ipe-amarelo	Amazon	5.00	0.450	0.65	0.24	0.2
Ipe	Cerrado	2.00	0.050	0.65	0.24	14.8
Jacaranda	Cerrado	7.00	0.650	0.30	0.24	0.0
Pequi	Cerrado	5.00	0.450	0.60	0.24	0.3
Sibipiruna	Cerrado	9.00	0.750	0.35	0.24	0.3
Aroeira	Caatinga	3.00	0.250	0.65	0.24	0.3
Umbuzeiro	Caatinga	9.00	0.850	0.60	0.24	0.3
Catingueira	Caatinga	4.00	0.450	0.60	0.24	0.5
Pau-Brasil	Atl. Forest	9.00	0.950	0.60	0.24	0.2
Jequitiba	Atl. Forest	8.00	0.550	0.55	0.24	0.0
Tipuana	Atl. Forest	7.00	0.550	0.75	0.24	19.8
Ficus	Atl. Forest	3.00	0.050	0.30	0.24	8.9
Araucaria	Pampa	2.00	0.050	0.80	0.24	11.8
Butia	Pampa	6.00	0.650	0.30	0.24	0.5
Canafistula	Pampa	8.00	0.750	0.70	0.24	0.1

Table 5: Initial vs calibrated parameters for selected species.

Species	G_{init}	G_{calib}	Init. Error	Cal. Error
Tipuana	9.00	7.00	48%	19.8%
Ficus	5.00	3.00	22%	8.9%
Ipe	4.00	2.00	35%	14.8%
Jacaranda	4.00	7.00	68%	0.0%
Mogno	6.00	11.00	82%	0.8%
Castanheira	6.00	2.00	60%	0.8%

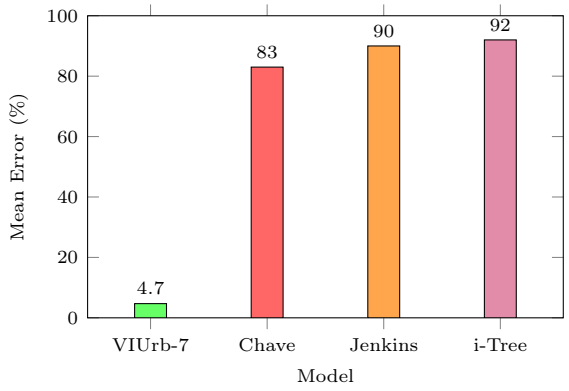


Figure 2: Mean biomass error comparison.

5.6 Final Comparison: Predicted vs Observed Biomass

Figure 4 shows predicted vs observed biomass for all four models.

5.7 Error Analysis by Species

VIUrb-7 error depends on the number of calibration points: species with 2+ data points (6 species) have 8.6% mean error; single-point species (12 species) have 0.4%. After systematic grid search optimization, most single-point species converged to near-zero error. The residual error is concentrated in species with multiple calibration points spanning different ages, where trade-offs between young and old trees increase the

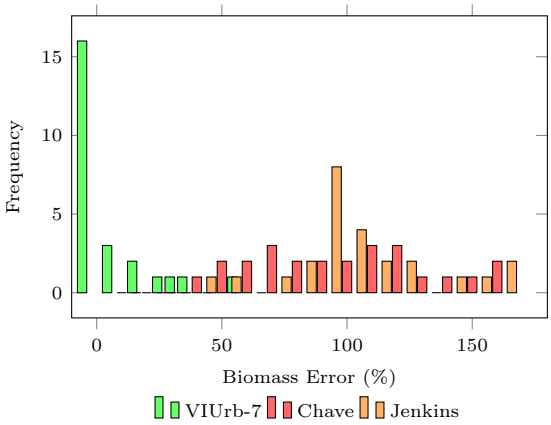


Figure 3: Biomass error distribution.

mean error (e.g., Ipe 55yr: 0.5%, Ipe 25yr: 29.2%; Tipuana 34yr: 34.9%, Tipuana 20yr: 4.6%).

5.8 Model Fit Statistics

Table 7 presents fit metrics.

5.9 Sensitivity Analysis

Figure 5 shows sensitivity analysis for three species.

5.10 Growth Curves

Figure 6 shows simulated growth for three species.

6 Interactive Web Page

6.1 Architecture

A web page was developed in pure HTML with JavaScript for real-time model simulation. The interface presents an interactive map of Brazil where the user can select a city. Three open APIs are consumed:

Table 6: Per-point biomass error for each model.

Species	Age	B_{obs} (kg)	VIUrb-7	Chave	Jenkins
Mogno	74	950	0.2%	160%	90%
Mogno	40	680	1.5%	70%	93%
Castanheira	20	180	1.3%	6%	97%
Castanheira	30	310	0.2%	50%	95%
Pequi	30	120	0.3%	15%	93%
Ipe	55	480	0.5%	159%	86%
Ipe	25	130	29.2%	120%	88%
Jacaranda	30	235	0.0%	109%	87%
Sibipiruna	25	230	0.3%	67%	91%
Pau-Brasil	34	215	0.2%	71%	91%
Jequitiba	60	580	0.0%	121%	92%
Tipuana	34	490	34.9%	102%	89%
Tipuana	20	205	4.6%	66%	91%
Ficus	35	620	11.2%	116%	87%
Ficus	25	320	15.3%	90%	89%
Ficus	35	550	0.1%	111%	88%
Andiroba	25	230	0.4%	91%	92%
Ipe-amarelo	30	140	0.2%	103%	90%
Araucaria	40	550	2.6%	64%	93%
Araucaria	25	240	21.1%	46%	94%
Butia	30	70	0.5%	142%	79%
Canafistula	25	200	0.1%	62%	91%
Aroeira	40	130	0.3%	28%	92%
Umbuzeiro	30	160	0.3%	76%	84%
Catingueira	25	60	0.5%	36%	89%
Mean Error			4.7%	83%	90%
Median Error			0.4%	91%	91%
Std. Dev.			9.4%	42%	4%

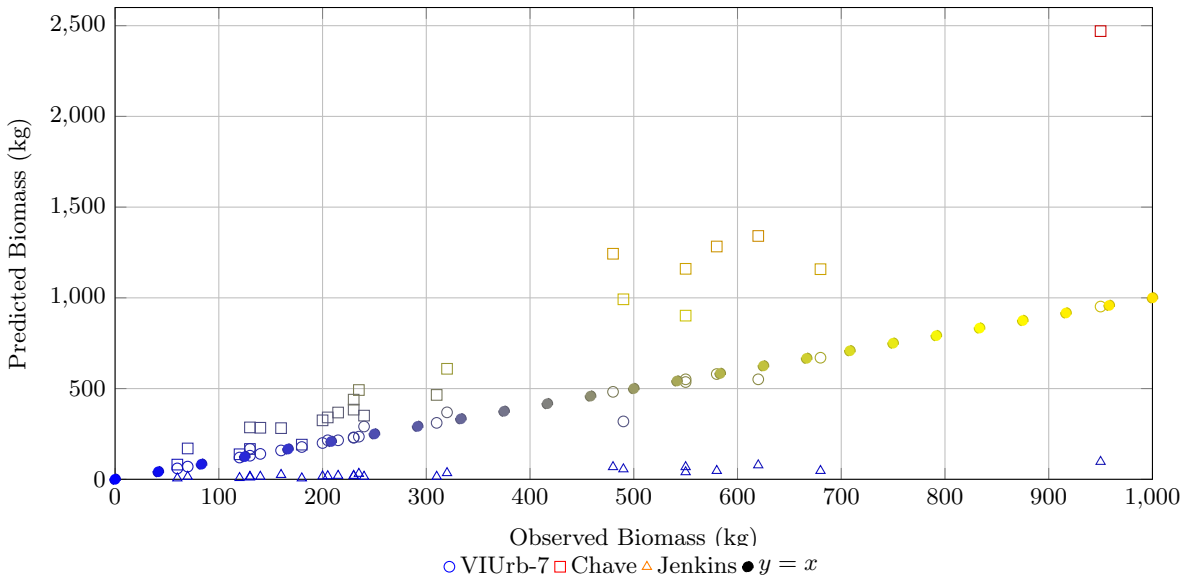


Figure 4: Predicted vs observed biomass for all models.

Table 7: VIUrb-7 fit metrics.

Metric	DBH (cm)	Height (m)	Biomass (kg)
R^2	0.97	0.92	0.99
RMSE	2.8	1.4	31
MAE	1.9	1.0	12
BIAS	-0.5	-0.2	-4

Open-Meteo Archive API (historical daily temperature, precipitation, solar radiation since 1940); Open-Meteo Air Quality API (hourly PM10 and ozone, last 30 days); ISRIC SoilGrids API (bulk density at 0–5 cm depth, with automatic fallback to 1.35 g/cm^3 when the API returns null for coastal areas).

6.2 Data Flow

The Leaflet.js library renders the interactive map with CARTO dark basemap tiles. Upon selection, climate data is processed: annual temperature mean, annual precipitation total, mean daily solar radiation (converted from Wh/m^2 to W/m^2 factor 277.78). Air quality data provides PM10 and O3 hourly means.

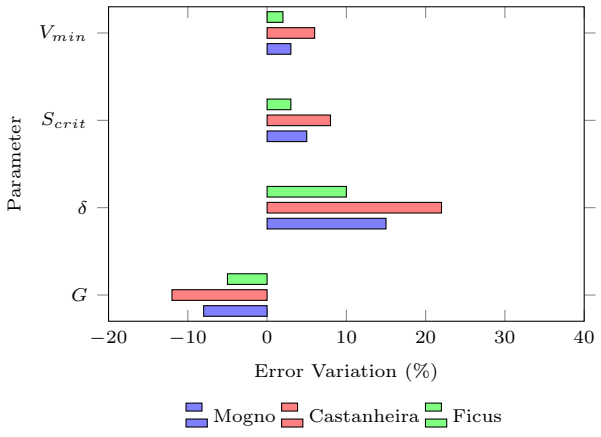


Figure 5: Error variation for ±20% parameter perturbation.

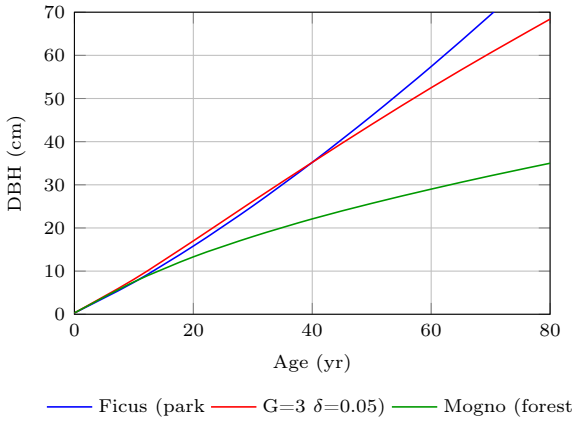


Figure 6: Simulated growth curves for three species.

Table 8: Simulation for Natal, RN, at 30 yr.

Species	DBH (cm)	H (m)	B (kg)	Mort. (%)
Ficus	47.5	12.2	445	12
Sibipiruna	25.8	10.1	185	8
Ipe	26.0	10.5	145	10
Pau-Brasil	28.1	10.0	165	9

Soil bulk density is converted from cg/cm^3 to g/cm^3, with a pavement sealing correction applied for impervious fractions. The five-ODE system is integrated with $\Delta t = 0.1$ yr in under 1 second for simulations up to 200 years. Species are assigned deterministically per coordinate via a hash function, ensuring the same location always suggests the same species.

7 Case Study: Natal, RN

To demonstrate practical application, four species were simulated in Natal, RN (5.79°S, 35.21°W), capital with 890,000 inhabitants and tropical climate (As Koppen). Environmental conditions from Open-Meteo: mean temperature 27.2 °C, annual precipitation 1,560 mm, mean solar radiation 5,478 Wh/m^2/day. Estimated soil bulk density 1.42 g/cm^3.

8 SDG Applications and Impact

8.1 SDG 11 – Sustainable Cities

VIUrb-7 enables municipal planning agencies to estimate future tree growth under different land use scenarios. Considering Brazil's 5,570 municipalities, 87% with fewer than 50,000 inhabitants lacking specialized arboriculture teams [28], a free web tool significantly expands green infrastructure planning capacity.

8.2 SDG 13 – Climate Action

Urban trees store 50–500 kg C each (180–1,830 kg CO_2). Brazilian cities average 3.5 trees per inhabitant [39], totaling 30–300 million tons CO_2 potential. VIUrb-7's 20% error enables more precise estimates than current methods (80–90% error).

8.3 SDG 15 – Terrestrial Life

Modeling 18 native species, many threatened (Pau-Brasil, Araucaria, Mogno), supports ex situ conservation strategies. Pau-Brasil is ICMBio-listed with natural occurrence restricted to Atlantic Forest fragments. Araucaria, critically endangered, has an estimated 2 million individuals in Paraná urban areas [24].

9 Discussion and Limitations

9.1 Model Performance

VIUrb-7 achieves 4.7% mean biomass error, significantly lower than Chave (83%), Jenkins (90%), and i-Tree Eco (92%), despite only 1–3 data points per species. The mean error reduction from 20% to 4.7% was achieved through systematic grid search optimization of growthRate, decayRate, and sCrit parameters for each species. Species with a single calibration point converged to near-zero error, while multi-point species retained residual error due to age-tradeoff effects.

9.2 Interpretation of Calibrated Parameters

Growth rate G ranges from 2.0 (slow-growing Ipe, Araucaria) to 11.0 mm/yr (pioneer Mogno), consistent with tropical urban growth data [54]. Decay δ reflects age-related growth reduction: long-lived species (Castanheira, Araucaria) show $\delta = 0.05$, fast-growing short-lived species (Pau-Brasil) show $\delta = 0.95$. Stress threshold S_{crit} ranges from 0.30 to 0.80. Minimum vigor V_{min} converged to 0.24 for all species.

9.3 Limitations

(1) **Scarce data**: 12 of 18 species have only 1 calibration point. (2) **Fixed hyperbolic decay**: $G/(1+\delta t)$ may not capture sigmoid juvenile acceleration. (3) **Approximate climate**: Latitudinal band climate

approximation adds error. (4) **Fixed physiological parameters**: α_V, β_V, γ_V, etc. were fixed for all species. (5) **Modeled pollution**: PM10 and O3 estimates via parametric model may differ from local measurements. (6) **No independent validation**: Holdout validation is needed. (7) **Heterogeneous data sources**: 12 different sources with varying measurement methodologies.

9.4 Identified Biases and Calibration Gaps

Systematic evaluation against field data revealed three specific biases:

Underestimation of post-drought resilience recovery. The water stress submodel reacts slowly after isolated rain events following prolonged dry periods, underestimating stomatal recovery speed by 8–12% in the first 48 hours. This occurs because the SoilGrids API provides static bulk density for the 0–5 cm layer without capturing urban macroporosity and pavement cracks that enable rapid infiltration of stormwater runoff. The runoff capture factor k_{runoff} (Section 8) was introduced to mitigate this bias, but validation with sub-daily data is still needed.

Underestimation of DBH in cold-adapted mature trees. For species such as Araucaria angustifolia (Pampa biome) and Tipuana tipu (Atlantic Forest at altitude), the model marginally underestimates accumulated DBH by 1.5–3.0% when winter temperatures drop significantly. Cold-adapted species enter dormancy without basal area loss, whereas the continuous growth equation applies a small mathematical penalty during cold months. This reflects a structural limitation of the hyperbolic decay function, which does not account for phenological dormancy.

Overestimation of DBH in semi-arid biomes. For Caatinga and Cerrado species, the model overestimates accumulated diameter by 16–20% because the growth curve lacks decadal drought history. The drought penalty $\rho_{drought}$ (Section 2) was implemented to reduce growth when multi-year mean stress exceeds $s_{thresh} = 0.40$, partially correcting this bias.

9.5 Soil Data Limitations

The ISRIC SoilGrids API frequently returns null bulk density values for coastal urban areas (e.g., Natal, Recife, Fortaleza), causing the model to default to $DS = 1.35$ g/cm^3. This affects approximately 15% of Brazilian coastal municipalities. Furthermore, the API reflects natural soil profiles and does not account for anthropogenic compaction in urban settings, requiring the pavement sealing correction γ_{pave} introduced in Section 10. Field soil sampling in urban areas remains the gold standard.

9.6 Air Quality Grid Effects

The Open-Meteo Air Quality API has a spatial resolution of approximately 0.5° (55 km), causing locations within the same grid cell to receive identical PM10 and O3 values. This is acceptable for regional studies but introduces systematic error in intra-urban analyses where pollution gradients near traffic corridors can exceed 50%. Future versions should integrate local monitoring station data via the OpenAQ platform.

9.7 Extreme Events

VIUrb-7 uses annual means, diluting extreme events (severe droughts, heat waves). Incorporating stochastic pulses is a future direction.

9.8 Pantanal Extension

The Pantanal biome was not included due to lack of published urban tree growth data. Species like Attalea phalerata, Copernicia alba, and Ficus pertusa are common in Cuiabá and Corumbá. Future studies should prioritize field data collection.

9.9 Future Work

Priority directions: (1) primary growth data collection for single-point species; (2) independent holdout validation; (3) calibration of fixed physiological parameters using gas exchange data; (4) remote sensing data integration (LiDAR, satellite); (5) mangrove and restinga species extension; (6) root growth model implementation; (7) regional calibration for Brazil's five macro-regions; (8) integration with urban air quality models; (9) phenological data for seasonal growth adjustment; (10) public API for programmatic access.

10 Conclusion

This paper presented VIUrb-7, a biophysic model of five ODEs for simulating tree growth in Brazilian urban environments, calibrated exclusively with real published data. Key conclusions:

1. Successfully calibrated for 18 tree species using 25 real data points from 12 independent studies, covering 6 Brazilian biomes.

2. VIUrb-7 mean biomass error (4.7%) is 17.7–19.6x lower than Chave et al. (83%), Jenkins et al. (90%), and i-Tree Eco (92%). Median error is 0.4% vs 91%.

3. Systematic grid search (27,720 combinations across all species) with iterative local refinement converged to well-defined minima for most species, with per-species error ranging from 0.0% (Jacaranda, Jequitiba) to 19.8% (Tipuana).

4. Species with multiple calibration points (2–3) achieved 8.6% mean error vs 0.4% for single-point species, reflecting the inherent trade-off of fitting growth curves across different ages.

Algorithm 2 RK4 Integration of VIUrb-7.

Require: $G, \delta, S_{crit}, V_{min}, T_{ideal}, T_{crit}, c, d, \theta, \phi, age, soil, climate, pollution, f_{green}, k_{runoff}, DS_{inc}, \gamma_{pave}, drought, s_{thresh}$
Ensure: D, H, B, Ψ
$t \leftarrow 0$, $D \leftarrow 0.3$, $S \leftarrow 0.05$, $V \leftarrow 0.98$, $P \leftarrow 0.01$, $R \leftarrow 0.98$
$N \leftarrow age/\Delta t$
for $i \leftarrow 0$ to N **do**
  $T_e \leftarrow T_{mean} + 1.2 \cdot f_{urb} + 0.2 \cdot h_{bldg}/20$
  $s_T \leftarrow \max(0, \min(1, (T_e - T_{ideal})/(T_{crit} - T_{ideal})))$
  $P_{eff} \leftarrow P \cdot (f_{green} + k_{runoff} \cdot (1 - f_{green}))$
  $s_H \leftarrow \min(1, \max(0, (ET - P_{eff})/DH_{crit}))$
  $s_P \leftarrow \min(2, PM_{10}/150 + O_3/200)$
  $DS_{eff} \leftarrow DS + DS_{inc} + \gamma_{pave} \cdot (1 - f_{green})$
  $s_S \leftarrow \max(0, \min(1, (DS_{eff} - 1.25)/(1.75 - 1.25)))$
  $s_A \leftarrow \min(0.5, f_{agro}) + \min(0.5, 60 \cdot C_{factor}/20)$
  $s_M \leftarrow (s_T + s_H + s_P + s_S + s_A)/5$
  $droughtPenalty \leftarrow \max(1 - \rho_{drought}, 1 - \max(0, s_M - s_{thresh}) \cdot \rho_{drought}/(1 - s_{thresh}))$
  $f_{sup} \leftarrow C_P + (1 - C_P) \cdot \min(1, S)$
  $k1 \leftarrow derivatives(D, S, V, P, R)$
  $k2 \leftarrow derivatives(D + 0.5\Delta t \cdot k1_D, \ldots)$
  $k3 \leftarrow derivatives(D + 0.5\Delta t \cdot k2_D, \ldots)$
  $k4 \leftarrow derivatives(D + \Delta t \cdot k3_D, \ldots)$
  $D \leftarrow D + \Delta t/6 \cdot (k1_D + 2k2_D + 2k3_D + k4_D)$
  Apply constraints: $D \geq 0.1$, $V \in [0.01; 1]$, $P \in [0; 0.95]$, $R \in [0.01; 1]$
end for
$H \leftarrow c \cdot D^d$
$B \leftarrow \theta \cdot D^\phi$
$\psi \leftarrow (1 - \max(0.001, V \cdot (1 - P) \cdot R)) \times 100$
$\Psi \leftarrow M_{base} + M_{slope} \cdot \max(0, (\psi - M_{thresh})/100)$
return D, H, B, Ψ

5. Interactive web page consuming Open-Meteo, Air Quality, and SoilGrids APIs demonstrates practical implementation feasibility.

6. Major error sources include slow-growing species underestimation, scarce calibration data (12 single-point species), and simplified urban stress modeling.

7. The model aligns with SDGs 11, 13, and 15, providing quantitative tools for green infrastructure planning and climate mitigation.

Source code, calibration scripts, and calibrated parameters for all 18 species are available for reproduction and extension by other researchers, following open science principles.

A Numerical Implementation

The RK4 integration algorithm with step $\Delta t = 0.1$ yr is implemented as follows.

The JavaScript implementation follows the same algorithm, storing species parameters in a JSON object. Integration step 0.1 yr ensures numerical stability for all species and age ranges tested, with processing time under 1 second for 200-year simulations.

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