

Numerical Fatigue Analysis of a PMMA/ZrO₂/nHA Composite Under Biaxial In-Phase Loading: Development of a Criterion Correlated with Monotonic Response

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Numerical Fatigue Analysis of a PMMA/ZrO₂/nHA Composite Under Biaxial In-Phase Loading: Development of a Criterion Correlated with Monotonic Response.

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Abstract

Widely used in engineering and bioengineering, composite materials offer attractive mechanical and physical properties. This study focuses on two PMMA-based nanocomposites reinforced with zirconium dioxide (ZrO₂) and nano-hydroxyapatite (nHA) particles. A numerical homogenization approach, based on representative volume elements (RVEs) and finite element simulations, was implemented to estimate the effective elastic properties. Subsequently, the durability of the composites was evaluated via a fatigue analysis under in-phase biaxial loading, using multiaxial fatigue criteria. To make use of this evaluation, a systematic correlation with the monotonic response was established, resulting in an original fatigue criterion linking quasi-static behavior to high-cycle fatigue performance. The results demonstrate that increasing the volume fraction of nanoparticles reduces fatigue damage and significantly improves the durability of PMMA-based nanocomposites.

Keywords: PMMA composite, ZrO₂ and nHA, Homogenization, In-phase loading, Biaxial fatigue, Fatigue criterion.

1. Introduction

Composite materials consist of a combination of at least two distinct materials, such as polymers, ceramics, or metals, which together provide improved properties compared with their individual constituents. Among these materials, polymer-based composites have attracted considerable interest due to their wide range of applications in various fields, including the automotive industry, construction, and biomedical engineering. In biomechanical applications, the combination of different materials aims to enhance key properties such as mechanical strength, durability, and thermal performance.

In clinical dentistry, various polymeric materials are used for different applications¹⁻⁴. Among these materials, polymethyl methacrylate (PMMA) has become, since its introduction, one of the most widely used polymers for dental bases and prosthetic devices⁵. For example, in the journal *Scientific Reports* alone, dozens of scientific articles on PMMA-based composites have been published over the past five years in the field of engineering. Some of these studies focus primarily on the use of this innovative material in several fields⁶⁻¹⁰. This popularity is mainly due to its excellent aesthetic results, ease of use, and reparability¹¹. PMMA is produced by the polymerization of methyl methacrylate and is characterized by its transparency, good mechanical properties such as stiffness and impact resistance, dielectric properties, and resistance to aging¹²⁻¹⁵.

To further improve the properties of PMMA-based materials, several types of reinforcements have been introduced into the polymer matrix. These include metallic fibers^{16,17}, natural and synthetic fibers¹⁸⁻²⁰, nanoparticles²¹⁻²³, and nanotubes²⁴⁻²⁷. The incorporation of such reinforcements significantly enhances the mechanical performance of PMMA composites, particularly their resistance to repeated loading and impact stresses.

The influence of nanoparticle size and morphology on the mechanical behavior of PMMA-based composites has been widely investigated in the literature. For example, Ben Hasan et al.²⁸ and Sura H.A. and Waffa M.S.²⁹ studied the mechanical properties of PMMA reinforced with different

nanoparticles commonly used in biomedical applications. Their results demonstrated that the tensile strength and hardness of PMMA composites increase with increasing nanoparticle content.

Zirconium dioxide (ZrO_2) and hydroxyapatite (nHA) nanoparticles have been widely used in several studies investigating the mechanical and thermal behavior of PMMA-based composites³⁰. These nanoparticles are particularly attractive for biomedical applications because they do not present biocompatibility issues. Moreover, their incorporation into the PMMA matrix has been shown to significantly improve the mechanical properties of the composite material^{23,31–34}, enhance thermal conductivity^{33,35}, increase dimensional stability³⁶, and provide antimicrobial activity^{37,38}. In addition, the presence of nanoparticles contributes to reducing water solubility and absorption in PMMA-based materials^{39,40}. These improvements are mainly attributed to the unique characteristics of nanoparticles, including their high specific surface area and their ability to achieve a more uniform dispersion within the polymer matrix⁴¹.

Understanding the effective physical and mechanical properties of composite materials is essential for analyzing the macroscopic behavior of engineering structures. Several techniques have been developed to evaluate these properties, among which homogenization methods are widely used within multiscale modeling approaches. This method relies on the concept of a representative volume element (*RVE*) to determine the effective behavior of heterogeneous materials. Several studies have employed this approach to evaluate the effective properties of composite materials^{42,43}. For example, the elastic behavior of aluminum matrix composites reinforced with ceramic particles (Al–SiC) has been analyzed using homogenization techniques in previous studies^{42,44}.

One of the major challenges faced by dental prostheses and implants is their ability to withstand complex loading conditions generated by mastication forces, friction between teeth, and the thermal variations caused by hot and cold food. Consequently, it is essential to investigate the durability of materials used in dental restorations by considering their resistance to fatigue loading, impact stresses, thermal effects, crack initiation, and thermal expansion⁴⁵. In this context, several studies have explored

the use of composite materials reinforced with additives possessing improved mechanical and physical properties ^{30,46,47}. In particular, PMMA-based composites with different fillers have demonstrated promising results in improving the durability of dental prosthetic materials ³⁰.

Previous studies have also investigated the influence of fiber reinforcement on the mechanical and fatigue performance of dental polymers. Manley ⁴⁸ reported that the fatigue strength of PMMA can be significantly improved by reinforcing the polymer with carbon fibers through heat treatment. Similarly, Hellman et al. ⁴⁹ demonstrated that reinforcing dental resins with carbon fibers can increase fatigue resistance by up to 83%. Nohrström et al. ⁵⁰ showed that the reinforcing effect of glass fibers in cold-cured acrylic resin prostheses becomes more significant in long-span dental bridges. Hamza et al. ⁵¹ concluded that the addition of glass fibers and polyethylene to temporary self-curing resins improves both fracture strength and flexural strength, while the placement of fibers in fixed partial dentures plays an important role in their mechanical performance. In addition, Kanie et al. ⁵² reported that the impact resistance of prosthetic base polymers reinforced with woven glass fibers is significantly higher than that of unreinforced polymers.

The present work is divided into two main parts. The first part focuses on the determination of the effective elastic properties of two PMMA-based nanocomposites reinforced with zirconium dioxide (ZrO₂) and nano-hydroxyapatite (nHA) particles. Five reinforcement volume fractions were considered for each composite material. A numerical approach based on finite element analysis was employed in combination with the numerical homogenization technique using representative volume elements (RVEs) in order to evaluate the effective mechanical behavior of the composites. The numerical results were validated using the analytical homogenization models of Voigt and Reuss ^{53–54}, as well as the Mori–Tanaka estimation model ⁵⁵.

The second part of the study is devoted to the investigation of the fatigue behavior and durability of the studied composites under complex loading conditions. In this context, fatigue analysis was performed using both experimental data available in the literature and numerical simulations under

multiaxial stress states. A fatigue criterion for PMMA-based composites is proposed by establishing a relationship between the mechanical response under monotonic loading and the fatigue response under cyclic loading. To achieve this objective, multiaxial fatigue criteria based on the critical plane concept were employed, which are widely used in fatigue analysis under complex loading conditions ^{56,57}. In particular, the fatigue criteria proposed by Dang Van, Mataké, and Findley were adopted in the present study ⁵⁸⁻⁶⁰.

2. Homogenization Methods and Multiscale Modeling

2.1 Generation of 3D RVE models

In order to find the elastic effective properties of two types of flat specimens that have a PMMA matrix and nanoparticle reinforcements (ZrO₂ or nHA), we applied the so-called numerical homogenization technique, by generating different representative volume element models RVEs (figure 1). The *RVE* is a cubic cell containing nanoparticles. Two types of nanoparticles were used, one is ZrO₂ and the second is nHA, with volume fractions varying from 1% up to 5% for each type.

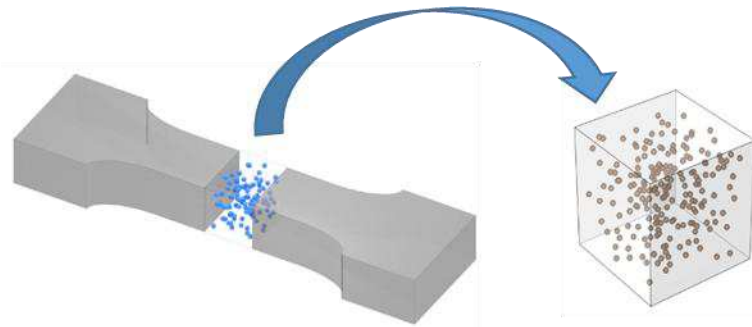


Figure 1. Specimen of a nanoparticle composite and the *RVE*.

The dimensions of the RVE are given according to the shape, the number of particles and the volume fraction.

$$V = \frac{4}{3} \frac{n \cdot \pi \cdot r^3}{f_p} \quad (1)$$

Or, V : Volume of the RVE ; n : number of particles ; r : radius of the particle ;

f_p : The volume fraction of particles in the *RVE*.

The volume fraction of the matrix f_m is defined by:

$$f_m = 1 - f_p \quad (2)$$

For the study of the behavior of considered composite materials, we relied mainly on the work of Sihama., et al ³⁰ and Sura and Waffa ²⁹, where the first work focused on the study of fatigue resistance and compression for a PMMA based composite, that is reinforced with different nanoparticles used in dental prostheses, while the second work concerned the development of the mechanical properties of acrylic resin by adding different types of nanoparticles. Before approaching the fatigue study of these composite materials, we will first determine the effective properties of these materials.

The models generated for the RVEs represent cubic cells containing 200 particles each one [see ^{42, 43, 61, 62}], the volume fractions of reinforcements ranging from 1 to 5%. These 3D models were generated by the Random Sequential Adsorption (RSA) method, where Matlab software was used for this purpose. (figure 2) represents three types of cells (RVEs) with their meshes for three different fractions of nanoparticles.

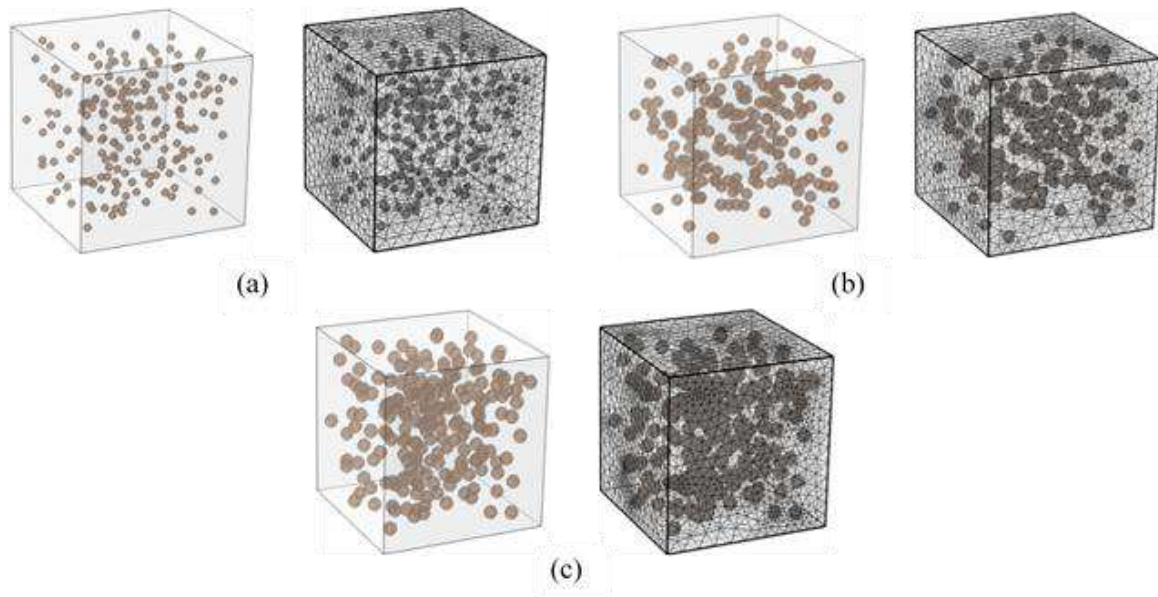


Figure 2. RVEs models for different types of inclusions (ZrO₂ or nHA); a) with 1% and its mesh, b) with 3% and its mesh, c) with 5% and its mesh

The mechanical characteristics of the two composites are defined in (Table 1), where it is noted that the nanoparticles have high mechanical properties compared to the PMMA matrix.

Table 1. Material properties for composite constituents

<i>Composite</i>	<i>Components</i>	<i>Young's moduli E (GPa)</i>	<i>Poisson ratio ν</i>
<i>Composite 1</i>	PMMA	3	0.40
	ZrO ₂	207	0.26
<i>Composite 2</i>	PMMA	3	0.40
	nHA	110	0.28

2.2 Numerical homogenization of PMMA-based nanocomposites

The homogenization method studies the effective elastic behavior (effective elastic properties) of the two composites, which depends on the equilibrium relation adopted in the continuum mechanics:

$$\nabla \cdot \underline{\sigma} + \underline{f} = 0 \quad (3)$$

where $\underline{\sigma}$ and $\underline{f}$ stand for the stress tensor and the body's force, respectively.

In the case of heterogeneous materials such as composites, a definition of generalized Hooke's law can be used to relate the average effective stiffness matrix (C^{eff}) with the stresses ($\langle \sigma \rangle$) and strains ($\langle \varepsilon \rangle$) of these materials, where the equation of this law is known as:

$$\langle \sigma \rangle = C^{eff} : \langle \varepsilon \rangle \quad (4)$$

In matrix form:

$$\underline{\sigma} = \begin{pmatrix} \underline{\underline{C}}_{11} & \underline{\underline{C}}_{12} & \underline{\underline{C}}_{13} & 0 & 0 & 0 \\ & \underline{\underline{C}}_{22} & \underline{\underline{C}}_{23} & 0 & 0 & 0 \\ & & \underline{\underline{C}}_{33} & 0 & 0 & 0 \\ & & & \frac{\underline{\underline{C}}_{11} - \underline{\underline{C}}_{12}}{2} & 0 & 0 \\ & \text{Symetry} & & & \frac{\underline{\underline{C}}_{11} - \underline{\underline{C}}_{12}}{2} & 0 \\ & & & & & \frac{\underline{\underline{C}}_{11} - \underline{\underline{C}}_{12}}{2} \end{pmatrix} \underline{\varepsilon}$$

Where, $\underline{C}^{eff}$ is the fourth-order stiffness tensor.

The effective coefficients can be characterized by a pseudo-isotropic behavior. For an isotropic material the number of independent constants is:

$$\underline{C}_{11} = \underline{C}_{22} = \underline{C}_{33}, \underline{C}_{12} = \underline{C}_{13} = \underline{C}_{23}, \underline{C}_{44} = \underline{C}_{55} = \underline{C}_{66} = \frac{1}{2}(\underline{C}_{11} - \underline{C}_{12}) \quad (5)$$

The average stresses and deformations are calculated from the average values of the finite elements V by the relation:

$$\underline{\sigma} = \frac{1}{V} \sum \sigma_{ij} V \quad (6)$$

$$\underline{\varepsilon} = \frac{1}{V} \sum \varepsilon_{ij} V$$

Where V is the volume of the finite element and v is the total volume of the *RVE*.

Boundary conditions, which are displacements imposed on the surfaces and their opposites on the *RVE*, have been prescribed on the boundaries of domain V (Figure 3). The results of the calculation on an *RVE* determined by these conditions converge better than with other boundary conditions⁴². These conditions are known as the uniform kinematic boundary conditions (*KUBC*). It should be noted that the minimum size needed to estimate the effective properties is lower for *KUBC* conditions^{42, 63}.

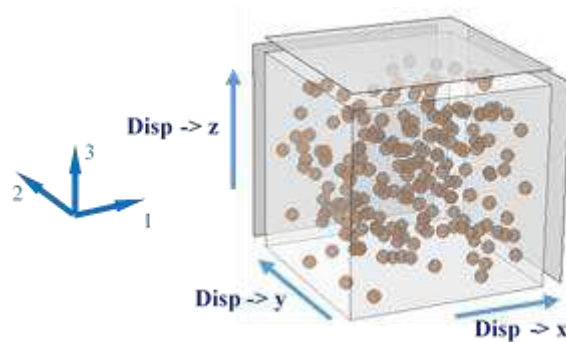


Figure 3. Boundary conditions: faces and opposite nodes in *KUBC*

A displacement $\underline{u}$ is imposed at the boundary point $\underline{x}$ of the boundary ∂V to specify the uniform kinematic boundary conditions (*KUBC*).

$$\underline{u} = E \cdot \underline{x}, \quad \forall \underline{x} \in \partial V \quad (7)$$

Or, E is a symmetric second-rank tensor independent on $\underline{x}$.

For heterogeneous materials, the macroscopic Young's modulus and shear modulus are determined by solving two fundamental boundary problems with imposed mean strain ^{42, 43}. The macroscopic strain tensors imposed, E_E and E_G , used to calculate the effective Young's modulus and effective shear modulus (E^{eff} , G^{eff}) are given by:

$$E_E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad E_G = \begin{pmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (8)$$

Where E_E et E_G are the macroscopic strain tensors.

We can then define the effective macroscopic Young's modulus E^{eff} and the effective macroscopic shear modulus G^{eff} as:

$$E^{\text{eff}} = \langle \frac{\sigma_{ii}}{\varepsilon_{ii}} \rangle; i = 1, 2, 3. \quad \text{And} \quad G^{\text{eff}} = \langle \sigma_{12} \rangle \quad (9)$$

The sign $\langle . \rangle$ means the mean value of the local stress.

To evaluate the numerical results, the mixing laws Voigt-Reuss model and the Mori-Tanaka model ⁵⁵ are used. To study the convergence of these results in relation to the effective properties of the two composite materials reinforced by nanoparticles, an upper and lower bounds of the Voigt-Reuss model related to the Young and shear moduli are defined by ⁶⁴ :

$$E^{\text{voigt}} = E_m f_m + E_p f_p$$

$$E^{\text{reuss}} = E_m E_p / (E_p f_m + E_m f_p) \quad (10)$$

$$G_c = G_m E G_p / (G_p f_m + G_m f_p)$$

Where,

E^{voigt} , E_m et E_p [GPa]: represent the Young's moduli of the composite, the matrix and the reinforcement respectively.

G_c, G_m et G_p [GPa]: represent the shear moduli of the composite, matrix and reinforcement respectively.

and, f_m, f_p (%) : are the volume fraction of the matrix and reinforcement respectively.

The Mori-Tanaka model ⁵⁵ is used to estimate the effective properties of a composite material from the properties of its constituents and their distribution, particularly in composites reinforced by particles or fibers ⁶⁵. When dealing with nanoparticle-reinforced composites, the Mori-Tanaka model is often used to estimate the effective mechanical properties of the material. However, size effects, increased interfacial interactions, and heterogeneity of the distribution at the nanoscale may require modifications or complementary models to ensure accurate prediction ⁶⁶.

In this case, the Mori-Tanaka model with spherical inclusions can be used to describe the effective elastic properties of a composite reinforced with nanoparticles. The Mori-Tanaka stiffness matrix with spherical inclusion can be expressed as:

$$C^{MT} = C_m + V_i(C_i - C_m).A \quad (11)$$

Where, C^{MT} represents the Mori-Tanaka effective stiffness tensor, C^m and C^i represent the stiffness tensors of the matrix and the inclusions respectively, V_i is the volume fraction of the inclusions, and A is the tensor of concentration factor. The latter is calculated using an approximation based on the dilute homogenization method, defined as:

$$A = [I + S.(C_m)^{-1} (C_i - C_m)]^{-1} \quad (12)$$

where I and S are respectively the Identity Tensor and the Eshelby Tensor.

This model thus provides a theoretical approach to characterize the overall mechanical behavior of a composite material accounting for the properties of the matrix and inclusions individually.

2.3 Meshing and converging

The mesh adapted to numerical homogenization methods is important on the basis of which the results converge towards the values close to the expected results, since the heterogeneous mesh between the matrix and the reinforcements is one of the problems that influence the study and the design of composites. The triangular type mesh (figure 2) was used in all the models studied in this work, this mesh was verified with different dimensions, where it converged with a number of 180908 elements for RVEs containing 200 spheres for the fraction of 1% reinforcement, and 160,000 elements for RVEs containing 200 spheres for the fraction of 5% of the reinforcement. (Figure 4) represents the variation of elastic properties as a function of the number of FE finite elements for different meshes.

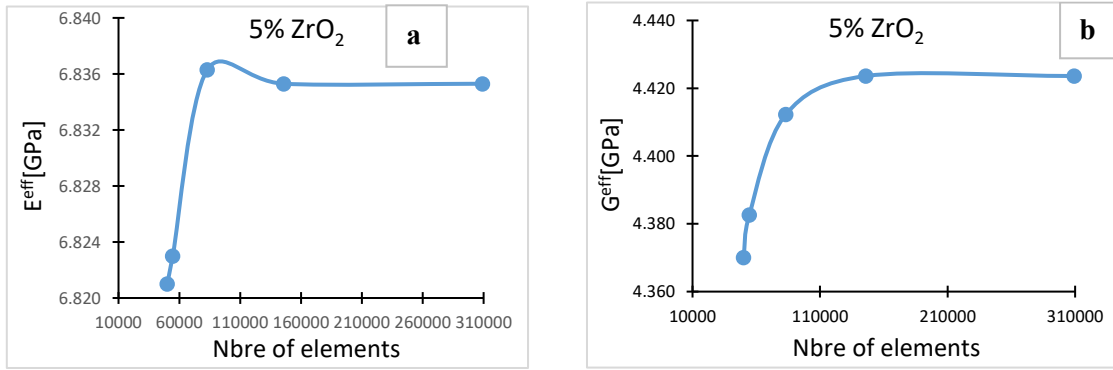


Figure 4. Variation of elastic moduli as a function of the number of EFs for different mesh grids, a) for modulus E, b) for modulus G.

3. Results and discussions

3.1. Numerical homogenization

In order to determine the effective mechanical properties of the studied composites, a numerical homogenization approach was employed using representative volume elements (RVEs). The simulations were performed under periodic boundary conditions (KUBC) for the different RVE

configurations corresponding to the two composite materials and the considered nanoparticle volume fractions.

The effective elastic properties obtained from the numerical simulations are presented in Figures 5 and 6. Figure 5 illustrates the variation of the effective Young's modulus E^{eff} (Figure 5a) and the effective shear modulus G^{eff} (Figure 5b) for the PMMA–ZrO₂ composite as a function of the nanoparticle volume fraction. Similarly, Figures 6a and 6b show the evolution of the effective Young's modulus and shear modulus for the PMMA–nHA composite with increasing nanoparticle content.

The results indicate that the incorporation of nanoparticles leads to an improvement in the effective elastic properties of the composites. In particular, both the effective Young's modulus and shear modulus increase with increasing nanoparticle volume fraction, which reflects the reinforcing effect of ZrO₂ and nHA particles within the PMMA matrix.

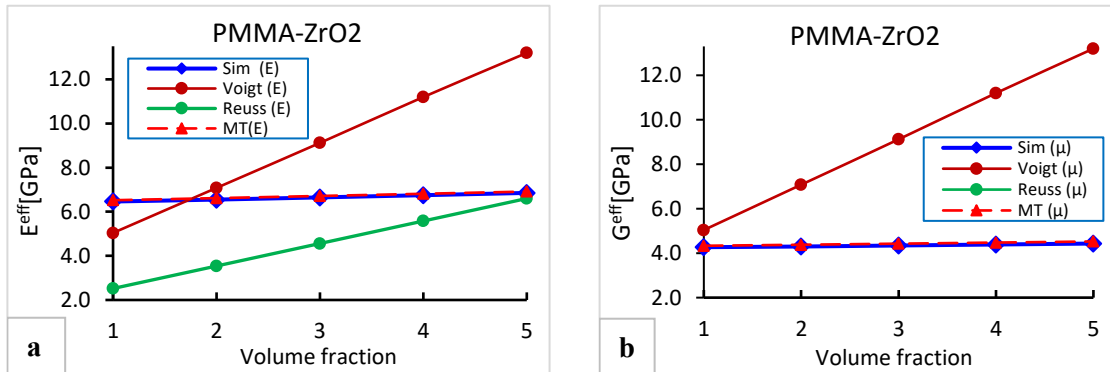


Figure 5. Effective elastic properties of PMMA-ZrO₂ as a function of nanoparticles volume fraction, a) Young's modulus and b) Shear modulus

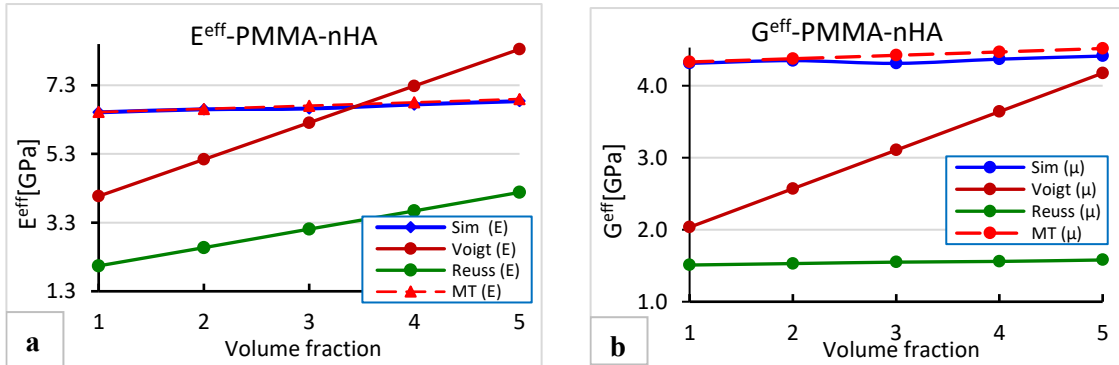


Figure 6. Effective elastic properties of PMMA-nHA as a function of nanoparticles volume fraction, a) Young's modulus and b) Shear modulus

These numerical results are compared with those obtained from analytical models, such as the Voigt-Reuss model and the Mori-Tanaka estimation model. The evaluation of these elastic properties shows the convergence of the two composite materials to the analytical models. It appears that these numerical results systematically lie between the analytical bounds and are closer to the Mori-Tanaka model.

Figures 7 and 8 illustrate the spatial distribution of the effective elastic properties within the RVE for the two composite materials reinforced with 5% nanoparticles. Figure 7 presents the distribution of the effective Young's modulus E^{eff} , while Figure 8 shows the distribution of the effective shear modulus G^{eff} .

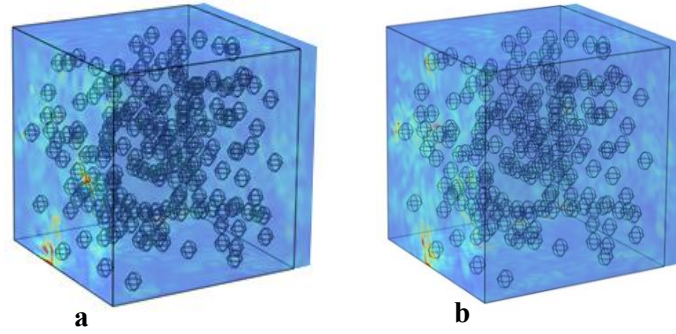


Figure 7. Local distribution of Young's modulus E in an RVE with 5% of, a) ZrO_2 , b) nHA

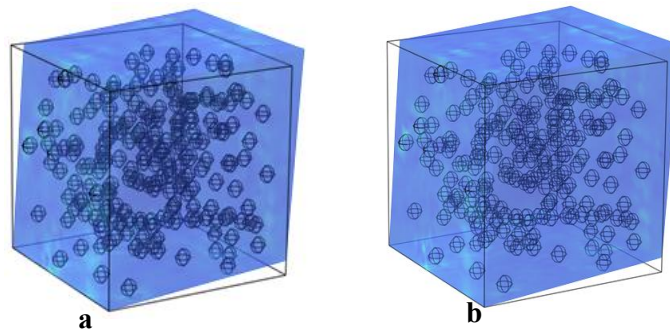


Figure 8. Local distribution of shear modulus G in an RVE with 5% of, a) ZrO_2 , b) nHA

The homogeneous stiffness matrix C^{eff} of the effective elastic moduli is symmetric with respect to isotropic characterization of the matrices and the reinforcements, and the application of the homogeneous boundary conditions to the contour of the RVE gives us the six components of the

macroscopic stress tensor Σ resulting from the stress averages then provide the six coefficients of the C^{eff} matrix. The results obtained for the different fractions for the two composite materials are given by:

$$\begin{pmatrix} 6.87 & 4.47 & 4.46 & 0 & 0 & 0 \\ & 6.87 & 4.47 & 0 & 0 & 0 \\ & & 6.87 & 0 & 0 & 0 \\ & & & 1.19 & 0 & 0 \\ & & & & 1.19 & 0 \\ & & & & & 1.19 \end{pmatrix} \text{ and } \begin{pmatrix} 6.85 & 4.45 & 4.45 & 0 & 0 & 0 \\ & 6.85 & 4.45 & 0 & 0 & 0 \\ & & 6.85 & 0 & 0 & 0 \\ & & & 1.19 & 0 & 0 \\ & & & & 1.19 & 0 \\ & & & & & 1.19 \end{pmatrix} \quad (13)$$

Where the first matrix represents the stiffness matrix of an *RVE* containing 200 particles of ZrO_2 and the second matrix represents the case of the PMMA-nHA material.

To study the durability of the nanoparticle-enhanced composites, fatigue resistance will be considered, based on the effective mechanical properties of the composites obtained in previous results (Table 2). This phase aims to understand the material's behavior under repeated loading that simulates real-world service conditions, as a prelude to a detailed study of the composites' fatigue behavior.

Table 2. Effective Young's moduli of the composites used according to the different reinforcement fractions.

Composite	PMMA - ZrO_2	PMMA-nHA
Volume fraction (%)	Module d'Young E^{eff} [GPa]	Module d'Young E^{eff} [GPa]
1	6.4676	6.4737
2	6.5517	6.5606
3	6.6230	6.6585
4	6.7196	6.7598
5	6.8080	6.8550

3. 2. Adopted fatigue model and study parameters

After analyzing the mechanical properties of the two studied composites, significant differences in their mechanical behavior were observed, particularly in terms of stiffness and strain resistance. The results indicate that the nature and volumetric fraction of the added particles, as well as their interaction with the matrix, directly influence the composite performance under static loading conditions. Based

on these observations, the second part of this work focuses on the fatigue behavior of the two composites in order to evaluate their response under repeated loading conditions representative of the real service environment of dental prosthetic structures during mastication.

The study of multiaxial fatigue at the critical plane is an essential step for estimating the service life of materials and structures subjected to complex loading conditions. Several models derived from the critical plane approach are widely used in this field. Local fatigue criteria such as the Matake, Findley, and Dang Van models have proven effective for analyzing fatigue behavior in many recent studies ^{67 - 72}. In order to analyze the degradation mechanisms of the materials and to identify the parameters influencing the expected service life of each component, numerical simulations were carried out to track the stress distribution within the matrix and to determine the zones of stress concentration that may constitute potential initiation sites for fatigue damage.

A multiaxial fatigue criterion makes it possible to determine a damage parameter, which depends on the stress trajectory applied to the material during cyclic loading. This parameter takes into account the mechanical characteristics of the studied composites, including their fatigue limits and elastic properties, as well as the influence of the applied multiaxial stress state.

The damage parameter E_D can be expressed as follows ⁷² :

$$E_D ([\sigma_{ij}(t)]_T, \sigma_{-1}, \sigma_0, \tau_{-1}, k, f) = 1 \quad (14)$$

Where: $[\sigma_{ij}(t)]_T$ represents the stress tensor at time t under the applied multiaxial, σ_{-1} , σ_0 and τ_{-1} represent respectively the endurance limits in fully reversed axial tension (load ratio $R = -1$), fully reversed rotary bending ($R = -1$), and fully reversed torsion ($R = -1$). In the present study, a fully reversed loading condition is considered.

In this study, it is necessary to determine the values of the parameters k and f . These parameters represent, respectively, the normal stress sensitivity coefficient and the limiting factor associated with each fatigue criterion.

In the analysis of composite materials subjected to multiaxial fatigue loading, fatigue criteria establish a relationship between the characteristics of the stress cycle and the endurance limits of the material. These quantities are evaluated over a loading period and include parameters such as stress amplitudes, mean stresses, maximum stresses, and alternating components.

Such parameters are defined according to different loading modes, including fully reversed axial tension, fully reversed bending, and torsion. These loading conditions are commonly used to characterize the fatigue resistance of materials and to determine the parameters required for multiaxial fatigue criteria.

Findley⁵⁸ defined the critical plane as the plane where the fatigue function reaches its maximum value, corresponding to the maximum linear combination of shear and normal stresses. The Findley criterion can be expressed as :

$$\tau_a + k \sigma_n = f \quad (16)$$

where k and f are material constants.

Matake's criterion⁵⁹ is very similar to the Findley criterion. The main difference lies in the definition of the critical plane. In Matake's approach, the critical plane corresponds to the plane where the **maximum shear stress range** is the largest, while the **maximum normal stress** is evaluated on this plane. The criterion can be expressed as:

$$\tau_a + k \sigma_{n,max} = f \quad (16)$$

where τ_a represents the maximum shear stress range on the considered plane, σ_n is the maximum normal stress on the same plane, and k and f are material parameters

When the stress state becomes compressive and the Matake relation becomes negative, the fatigue utilization factor is set to zero, similarly to the Findley criterion.

Dang Van's model combines the influence of **shear stress responsible for crack initiation** with the effect of **hydrostatic stress associated with crack propagation**. The Dang Van criterion is expressed as follows ⁶⁰ :

$$\tau_a + k\sigma_n = f \quad (17)$$

The use of these multiaxial fatigue criteria makes it possible to characterize the fatigue behavior of the studied composite materials while considering several influencing factors such as the stress state and the mechanical properties of the materials. This approach provides valuable insight into the fatigue resistance of the composite structure and contributes to improving its design in order to ensure durability and reliability. For the numerical simulation of fatigue behavior, several modeling approaches were considered. The specimens used in these simulations consist of homogenized composite plates with dimensions of $0.10 \times 0.04 \times 0.04 \text{ m}^3$ (Figure 9). The plate is fixed on one side, while the opposite side is subjected to a torque loading condition. The first load corresponds to a normal force denoted by F , whereas the second load represents a tangential force denoted by P .

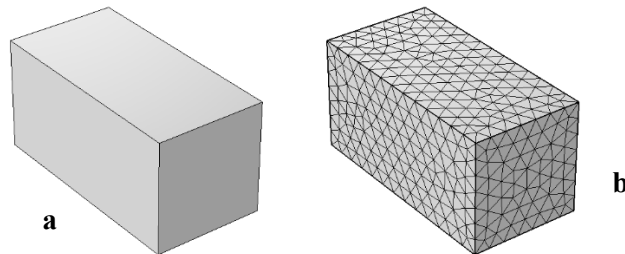


Figure 9. Flat specimen model, a) specimen flat, b) meshing of plat.

For the numerical simulation of fatigue behavior, several modeling approaches were considered. The specimens used in these simulations consist of homogenized composite plates, as illustrated in Figure 9. The plate is fixed on one side, while the opposite side is subjected to combined loading conditions.

The applied loading consists of two components. The first load corresponds to a normal force, denoted by P , while the second load represents a tangential force, denoted by F . These loads are applied on the free end of the specimen in order to reproduce the mechanical loading conditions experienced by dental prosthetic structures during mastication.

The analysis of the homogenized composite materials is performed in two main stages. The first stage corresponds to monotonic loading conditions, which are used to evaluate the mechanical response of the composite and to determine the safe operating domain of the structure. The second stage focuses on cyclic loading conditions, which are required to analyze the fatigue behavior of the studied materials.

Once the structural response of the composite has been validated under monotonic loading, the maximum admissible loads corresponding to the fatigue limit can be determined.

In practice, the continuous variation of the applied loads during the loading cycle produces a non-stable relationship between the normal and tangential stress components. This behavior reflects the cyclic and non-proportional nature of the loading conditions, as illustrated in Figure 10.

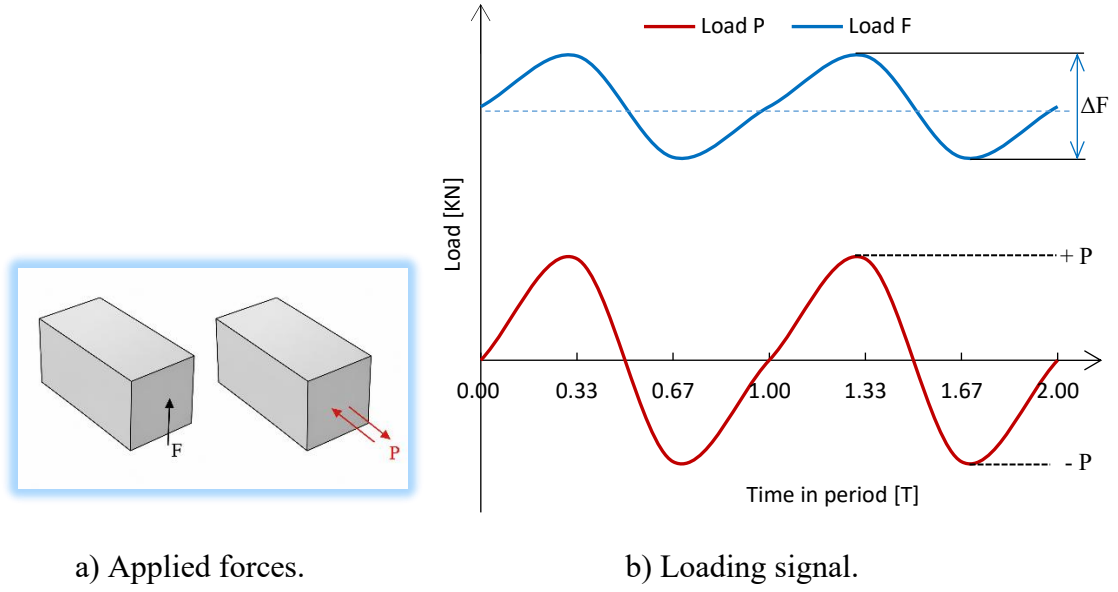


Figure 10. In-phase biaxial cyclic loading signal

When the selected damage parameter E_D reaches a critical value corresponding to the fatigue limit of the material, the fatigue failure condition is considered to be reached. This condition indicates that the applied cyclic loading has exceeded the resistance capacity of the composite structure.

This condition can also be expressed using the fatigue utilization factor, which represents the ratio between the calculated damage parameter and its critical value. The fatigue utilization factor can be written as:

$$\frac{E_D}{E_c} \geq 1 \quad (18)$$

The concept of equivalent Von Mises stress is frequently used to evaluate the fatigue limit of materials subjected to complex loading conditions. In another approach, as proposed by Liu and Yan⁶⁹, the equivalent stress can also be used to estimate the fatigue life of materials under cyclic loading.

Based on the understanding of the mechanical behavior of the composites obtained during the first stage of numerical homogenization, and considering the fatigue limits of the studied materials, the

parameters associated with the adopted fatigue criteria (Dang Van, Matake, and Findley) were implemented in the numerical model.

The mechanical characteristics of the studied composites under monotonic loading conditions as well as their fatigue limits are summarized in Table 3. These parameters include the elastic limit R_e , the tensile strength R_m , and the endurance limits obtained under different loading modes, namely fully reversed rotary bending, fully reversed axial tension, and fully reversed torsion.

For the two studied composites, namely PMMA reinforced with nHA particles and PMMA reinforced with ZrO_2 particles, the mechanical properties show a gradual improvement as the reinforcement volume fraction increases. This improvement reflects the contribution of the reinforcement particles to the load transfer mechanism within the polymer matrix, which enhances the stiffness and strength of the composite material.

The fatigue parameters associated with the adopted multiaxial fatigue criteria are presented in Table 4. These parameters include the coefficient k , representing the sensitivity of the material to normal stress on the critical plane, and the limiting factor f , which defines the fatigue threshold associated with each fatigue criterion.

The values of these parameters were determined based on the endurance limits of the studied composites and were used in the numerical simulations to evaluate the fatigue resistance of the structures subjected to multiaxial cyclic loading.

Table 3. Mechanical characteristics of PMMAs under monotonic loading and under fatigue, with the fatigue parameters.

Composite	Fraction of reinforcement %	Monotonous loading		Fatigue limits		
		Elastic strength: R_e [MPa]	Tensile strength: R_m [MPa]	Symmetric rotary bending ($R = -1$), σ_0 [MPa]	Symmetrical alternating traction ($R = -1$), σ_{-1} [MPa]	Symmetrical alternating twist ($R = -1$), τ_{-1} [MPa]
Composite 1: PMMA + ZrO ₂	1	70,00	84,00	18,00	16,00	13,00
	2	70,91	85,09	18,23	16,20	13,17
	3	71,68	86,23	18,48	16,43	13,35
	4	72,72	87,26	18,70	16,62	13,50
	5	73,68	88,42	18,95	16,84	13,68
Composite 2: PMMA + nHA	1	70,07	84,08	18,02	16,02	13,01
	2	71,07	85,28	18,27	16,24	13,20
	3	72,07	86,48	18,53	16,47	13,38
	4	73,16	87,80	18,81	16,72	13,59
	5	74,19	89,03	19,08	16,96	13,78

Table 4. Fatigue parameters for the different composites.

Fatigue Parameters							
Criteria		Dang Van criterion		Matake criterion		Findley criterion	
Composite	Volume %	Coefficient k	Limit factor f [MPa]	Coefficient k	Limit factor f [MPa]	Coefficient k	Limit factor f [MPa]
Composite 1: PMMA + ZrO ₂	1	0,94	13,00	0,63	13,00	0,80	16,65
	2	0,94	13,17	0,63	13,17	0,80	16,88
	3	0,94	13,35	0,63	13,35	0,80	17,10
	4	0,94	13,50	0,63	13,50	0,80	17,29
	5	0,94	13,68	0,63	13,68	0,80	16,67
Composite 2: PMMA + nHA	1	0,94	13,01	0,63	13,01	0,80	16,90
	2	0,94	13,20	0,63	13,20	0,80	17,12
	3	0,94	13,38	0,63	13,38	0,80	17,32
	4	0,94	13,59	0,63	13,59	0,80	16,70
	5	0,94	13,78	0,63	13,78	0,80	16,92

3.3. Mechanical behavior under fatigue

In this section, the mechanical behavior of the studied composites under cyclic loading conditions is investigated in order to evaluate their fatigue performance. Numerical simulations were carried out using a finite element analysis tool to analyze the response of the two composite materials under different loading conditions and reinforcement volume fractions.

The objective of this analysis is to identify the influence of the reinforcement particles on the fatigue behavior of the composites and to determine the stress distribution and fatigue risk within the structure under cyclic loading.

The numerical model consists of a beam with reduced dimensions and a rectangular cross-section representing the studied composite specimen. The beam is embedded at one end, while the opposite end remains free, reproducing a cantilever-type configuration.

The loading configuration consists of applying two loads at the free end of the beam. The first load corresponds to a normal compressive force, denoted by P , applied perpendicular to the cross-section. The second load corresponds to a bending moment, denoted by M , which generates additional stresses in the structure.

This loading configuration allows the simulation of the mechanical conditions experienced by dental prosthetic structures during mastication.

3. 3. 1. Influence of the volume fraction of reinforcements on the composites

To investigate the influence of the reinforcement volume fraction on the mechanical behavior of the composite materials, the free end of the composite beam was subjected simultaneously to the loads P and F . Figure 11 presents the maximum response obtained under monotonic loading conditions as well as the corresponding fatigue response for the two studied composites.

The Von Mises stress remains almost constant for reinforcement volume fractions ranging from 1% to 5%, with a value close to the reference stress level, as illustrated in Figures 11a and 11b. The highest stress levels are observed near the embedded section of the beam, particularly at the edges and corners of the structure. These regions correspond to stress concentration zones and therefore represent potential locations for the initiation of fatigue damage.

Figures 11c and 11d illustrate the fatigue response of the composite materials based on the Dang Van fatigue criterion. The results indicate that the fatigue limit is reached in the embedded region,

especially at the upper and lower edges of the beam, which correspond to the most highly stressed zones and therefore the most susceptible areas for fatigue damage.

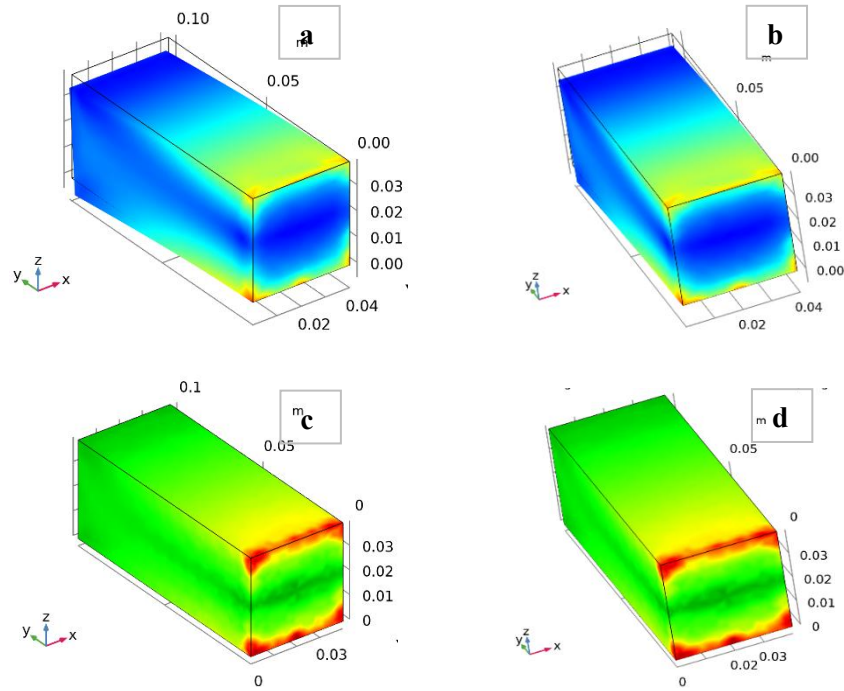


Figure 11. Monotonic loading and fatigue response of PMMA composites reinforced with 1% nHA and 5% nHA.

All fatigue results obtained using the three fatigue criteria are presented in Figure 12. As shown in Figure 12a, increasing the nHA reinforcement fraction from 1% to 5% leads to a gradual reduction in fatigue risk, ranging from 4.67% to 5.63% for approximately 87% of the studied cases. This reduction can be attributed to the improvement of the fatigue parameters associated with the composite material as the reinforcement fraction increases.

For the composite reinforced with ZrO_2 particles, Figure 12b shows the variation of fatigue risk for different reinforcement percentages. A consistent decrease in fatigue risk is observed as the reinforcement fraction increases. The reduction ranges between 2.44% and 4.95% when the reinforcement content increases from 1% to 5%, confirming the beneficial effect of particle reinforcement on the fatigue resistance of the composite.

In Figure 13a, a clear linear increase of approximately 58.32% is observed in the maximum Von Mises stress when the applied load varies from 0.5 to 1.2 times the initial load. This increase is accompanied by a similar rise in the fatigue risk, reaching approximately 58.50%, as illustrated in Figure 13b.

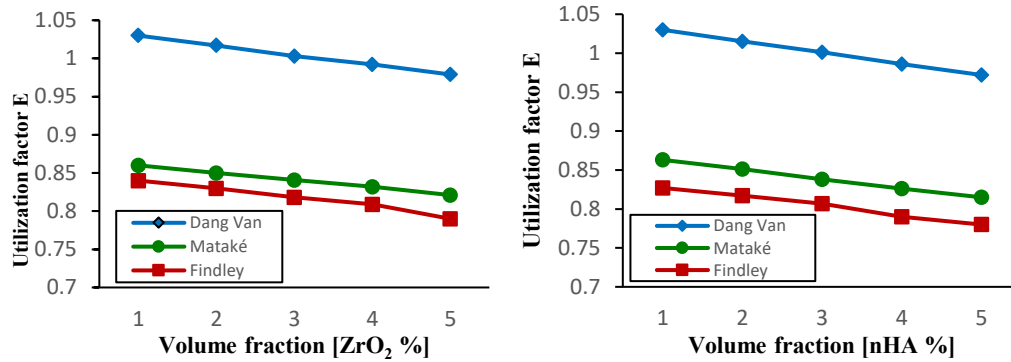


Figure 12. Evolution of the risk of fatigue according to the volume fractions of the reinforcements

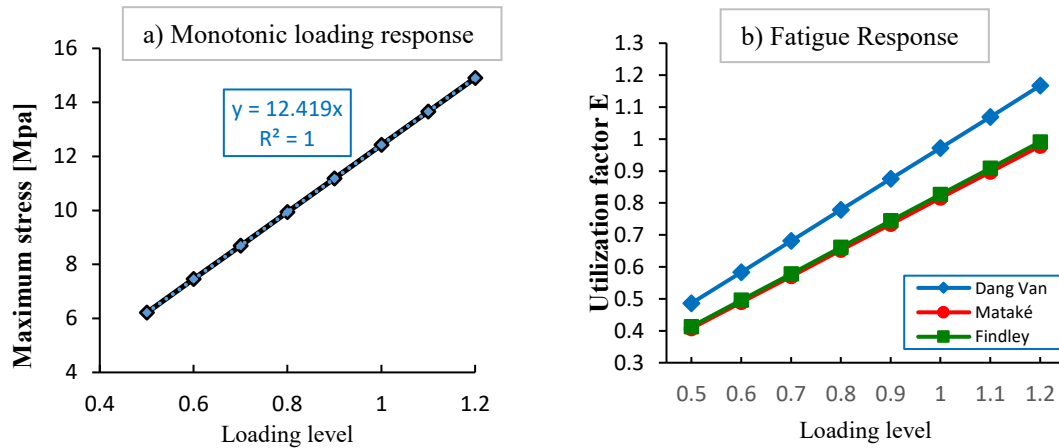


Figure 13. Mechanical behavior of PMMA with 5% nHA as a function of loading level

3. 3. 2. Influence of the specimen's length on mechanical behavior

In this section, the influence of the specimen length on the mechanical behavior and fatigue response of the composite is investigated. The composite composition is fixed at 5% ZrO_2 . The length of the composite specimen varies around the initial length $L = 10 \pm 2$ cm, while the applied loads remain constant ($F = P = 1$ kN)

Figure 14 shows the mechanical behavior of the composite containing 5% ZrO_2 for a variation of the plate length from $0.8L$ to $1.2L$. The results indicate an increase in fatigue risk from 11.84% to

17.91% as the specimen length increases. This increase in fatigue risk is directly related to the behavior observed under monotonic loading, where a corresponding increase of approximately 15.52% in stress is observed.

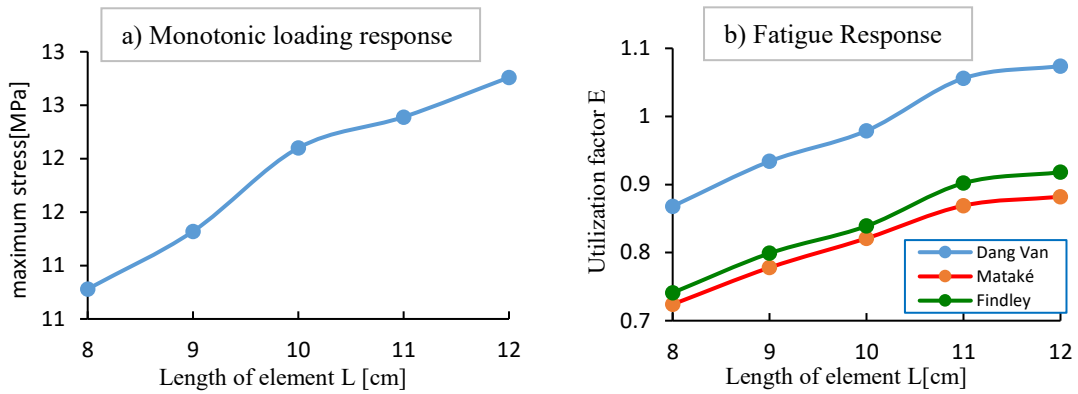


Figure 14. Mechanical behavior of PMMA reinforced with 5% ZrO_2 as a function of the specimen length.

Analysis of the previously studied cases reveals a strong correlation between fatigue risk and the mechanical response under monotonic loading conditions. Based on these observations, the following section proposes the formulation of a new fatigue criterion specifically adapted to the studied composites, derived from their response under monotonic loading.

3. 4. *Development of a fatigue criterion*

Based on the results obtained from the numerical simulations, a direct relationship between the fatigue risk and the loading level was observed in all previously studied cases. In order to quantify this relationship, a statistical analysis was performed using the results obtained from 48 numerical simulation cases. The outcomes of this analysis reveal a clear correlation between the applied loading level and the fatigue response of the studied composite materials. The statistical correlation results are illustrated in Figures 15 and 16, which highlight the relationship between the loading conditions and the corresponding fatigue behavior of the composite structure.

For a confidence interval of 95% and a tolerance of 0.0001, a statistical correlation model relating the loading level to the fatigue response of the composite material was obtained from the simulation results. The model shows a coefficient of determination R^2 very close to unity, indicating a strong correlation between the applied loading level and the corresponding fatigue risk. The resulting relationship is illustrated in Figure 17, which highlights the evolution of the fatigue response as a function of the loading level for the studied composite materials.

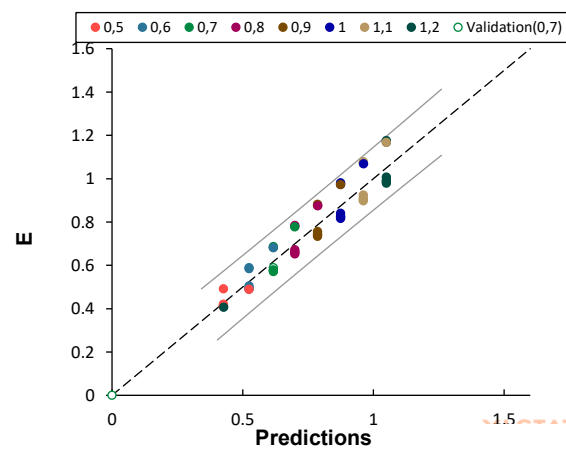


Figure 15. Predicted fatigue risk as a function of the applied loading level

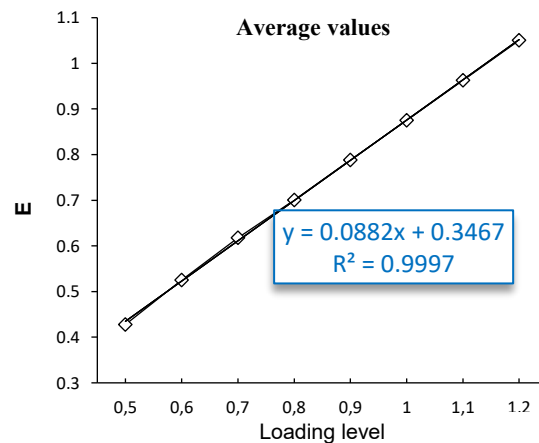


Figure 16. Statistical prediction of fatigue risk based on numerical simulation results

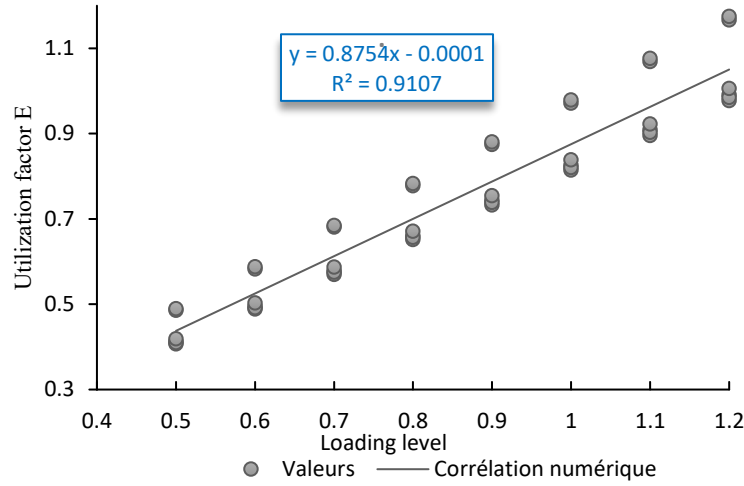


Figure 17. Numerical correlation between loading level and fatigue response

Based on this statistical correlation, a fatigue function relating the loading level to the fatigue risk of the studied PMMA-based composite materials can be established. The proposed fatigue criterion can therefore be expressed as follows

$$E_D = 0,8754 (N_C) - 0,0001 \quad (19)$$

where E_D represents the fatigue damage parameter and N_C corresponds to the level of monotonic loading associated with the reference Von Mises stress of 12.42 MPa. This parameter represents the normalized loading level used to describe the evolution of fatigue damage in the studied composite materials.

This fatigue criterion establishes a direct relationship between the monotonic loading level and the fatigue damage parameter of the studied composite materials. The obtained relationship highlights the strong correlation between the mechanical response under monotonic loading and the fatigue behavior predicted using the adopted multiaxial fatigue criteria. Such an approach provides a practical tool for estimating the fatigue performance of PMMA-based composites without requiring extensive fatigue testing.

The calibration of this fatigue criterion (Equation 19) with previously reported experimental models allows a comparison with available experimental data. Experimental results reported for PMMA-based composites indicate a fatigue limit in compression of approximately 15 MPa at 10^6 cycles³⁰. In the present study, a compression–shear fatigue limit of 12.42 MPa was obtained,

corresponding to a reduction of approximately 17% compared with the fatigue limit under pure compression loading

This difference can be explained by the fact that combined loading conditions are generally more severe in fatigue than uniaxial loading. Under pure compression, stress concentration mainly occurs along a single principal direction, whereas under compression–shear loading the stress distribution becomes more complex. This results in multiple stress concentration zones and consequently accelerates the initiation of fatigue damage.

The fatigue criterion defined by **Equation (19)** is applicable to PMMA composites reinforced with 1–5% of ZrO_2 and nHA particles. One of the main advantages of this approach is that the fatigue limit can be estimated without performing extensive experimental fatigue tests, providing a practical tool for evaluating the fatigue performance of composite materials during the design stage.

4. Conclusion

This study analyzed the fatigue behavior of PMMA-based composites reinforced with ZrO_2 and nHA nanoparticles, with volume fractions ranging from 1 to 5 percent, using models generated by the random sequential adsorption (RSA) method. The numerical homogenization results obtained corroborate the experimental trends reported in the literature, notably by Sihama et al., thereby validating the relevance of the proposed approach.

Furthermore, the analysis shows that multiaxial fatigue criteria are well suited to these materials. It appears that the fatigue risk decreases as the loading rate increases: for nHA, this reduction ranges from 4.67% to 5.63% (in 87% of cases), while for ZrO_2 , it ranges from 2.44% to 4.95% when the content increases from 1% to 5%. Furthermore, this risk is directly related to the behavior under monotonic loading. Based on this correlation, the proposed criterion (Equation 19) proves valid for predicting the service life of both composites based solely on their monotonic response, thus providing an effective and user-friendly predictive tool for biaxial fatigue.

However, although promising, this model requires further refinement. Future experimental investigations are essential to definitively validate and consolidate its applicability and to extend it to other types of loading as well as to various cycle levels.

Data availability

The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Funding and/or Conflicts of interests/Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Competing interests

The authors declare no competing interests.

Additional information

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