

Detection of Anomalies in the Yellow River Embankment via Seismic CT and Mixture Theory

Chaoyang Song

scyncwu@163.com

Henan Geology Mineral College

Zhihui Kuang

Henan Geology Mineral College

Wenxin Feng

Henan Geology Mineral College

Article

Keywords: Three-phase porous medium, Unsaturated soil, Porosity, Seismic CT

Posted Date: August 26th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10502423/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

Detection of Anomalies in the Yellow River Embankment via Seismic CT and Mixture Theory

Chaoyang Song*, Zhihui Kuang, Wenxin Feng

Henan Geology Mineral College, Zhengzhou, 451464, China

Emails: scyncwu@163.com (Chaoyang Song); 1057234299@qq.com (Zhihui Kuang);
346317465@qq.com (Wenxin Feng)

*** Corresponding author:**

Intended for publication in *Scientific Reports*

Abstract

Embankment safety is critical for flood control and disaster reduction. Current methods for detecting embankment anomalies on the basis of seismic wave velocity anomalies lack quantitative coupling analysis between seismic wave propagation characteristics and internal structural parameters of the soil. To solve this problem, this paper establishes governing equations for elastic waves in unsaturated soil on the basis of mixture theory. The propagation velocity of compression waves in soil is calculated, and the influences of the seismic wave frequency and soil porosity on the P-wave velocity are quantitatively analyzed. Combined with the algebraic reconstruction technique (ART), cross-hole seismic CT tomography is adopted to carry out numerical simulations of porosity inversion for unsaturated embankment soil. The results show that under constant saturation, the velocity of the P1 compression wave in unsaturated soil has a significant negative correlation with porosity. The wave velocity clearly exhibits dispersion characteristics. Seismic CT inversion can accurately locate zones with anomalous porosity in embankments and quantitatively retrieve the porosity of anomalous bodies. However, false iterative anomalies tend to occur near the boundaries of anomalous regions, which must be distinguished by geological conditions in engineering detection.

Keywords: Three-phase porous medium; Unsaturated soil; Porosity; Seismic CT

1. Introduction

The Yellow River embankment projects are widely distributed and span large construction periods. Most embankment bodies have been in long-term operation, with anomalies such as internal cracks and seepage cavities developing. At present, the identification of embankment anomalies via seismic exploration relies mostly on qualitative judgments of hazard locations and scales on the basis of profile wave velocity anomalies. However, this method does not quantitatively reveal the intrinsic control mechanism of wave velocity anomalies, cannot establish quantitative relationships between wave velocity and soil pore structure parameters, and it is difficult to achieve a quantitative evaluation of the degree of pore development in hazardous soil.

Driven by experimental and engineering demands, Terzaghi first proposed a one-dimensional consolidation theory applicable to saturated soils¹. In addition, Gassmann established a constitutive relationship between volumetric deformation and applied

stress². However, these early models did not systematically account for the dynamic coupling effects between the solid skeleton and the pore fluid. This critical breakthrough was achieved by Biot, completing the pivotal transition from elastic to viscoelastic poroelastic theory³⁻⁶. Virtually all subsequent theoretical variants developed over the following decades have been built upon this framework.

Vardoulakis and Beskos systematically investigated the dynamic behavior of nearly saturated porous media⁷. The French scholar Coussy carried out a modern reformulation of Biot theory within the framework of continuum thermodynamics⁸. During the same period, Dvorkin and Nur proposed a dynamic poroelastic model that unified the microscopic squirt flow mechanism with the macroscopic Biot flow mechanism⁹. Lo et al., on the basis of mixture theory, derived generalized Biot constitutive equations for porous media saturated with two immiscible fluids¹⁰. Xu and Wei conducted a detailed analysis of elastic wave propagation characteristics in unsaturated porous media¹¹.

Huang et al. enabled the simulation of the full evolution of waves in porous viscoelastic media within a unified system¹². Tong et al. explained the coexistence of positive and negative dispersion—a phenomenon that classical Biot theory cannot predict¹³.

These research efforts have established the relationships between seismic wave velocity and the parameters of three-phase porous media, providing a theoretical foundation for inverting the porosity of unsaturated soils in Yellow River embankments via seismic exploration. Seismic tomography uses travel-time data of direct seismic waves to compute the velocity structure of the surveyed region. The theoretical basis of image reconstruction is the Radon transform, and the algebraic reconstruction technique (ART) is a discrete image reconstruction method.

The mathematical roots of algebraic reconstruction techniques can be traced back to the projection iterative method for solving systems of linear equations proposed by Kaczmarz in 1937¹⁴. Backus and Gilbert provided an important methodological foundation for the subsequent application of ART in seismic tomography¹⁵. The scudder provided a systematic introduction to computed tomography, serving as a technical bridge for the migration of ART from medical imaging to geophysics¹⁶.

Dines and Lytle published a seminal paper on computerized geophysical tomography, presenting the first systematic investigation of the application of iterative reconstruction techniques such as ART to subsurface medium imaging¹⁷. Ustaoglu investigated the inversion of the seismic generalized Radon transform via ART and

SIRT¹⁸. Chang et al. proposed the anisotropic algebraic reconstruction technique (AART) and anisotropic SIRT (ASIRT)¹⁹. Yang et al. proposed a three-dimensional velocity imaging method that combines the fast marching method (FMM) with the algebraic reconstruction technique (ART). The core innovation of this method lies in the first introduction of FMM-based curved-ray tracing into mine-scale seismic tomography²⁰. This study is based on the straight ray assumption and employs the ART algorithm for seismic CT inversion.

2. Solution of Elastic Wave Velocity in Three-phase Unsaturated Soil

The unsaturated fill soil of the embankment is regarded as a statistically isotropic three-phase porous medium in which solid soil particles, liquid pore water, and gaseous pore air jointly constitute the medium system. By combining the equilibrium equations of medium motion, fluid continuity equations, and constitutive relations of the three-phase medium, the coupled governing equations of elastic waves in the three-phase system are obtained.

$$\begin{cases} -\nabla \cdot \mathbf{u}^l = a_{11} \nabla \cdot \mathbf{u}^l + a_{12} \nabla \cdot \mathbf{u}^l + a_{13} \nabla \cdot \mathbf{u}^g \\ -\nabla \cdot \mathbf{u}^g = a_{21} \nabla \cdot \mathbf{u}^l + a_{22} \nabla \cdot \mathbf{u}^l + a_{23} \nabla \cdot \mathbf{u}^g \end{cases}, \quad (1)$$

$$\begin{cases} \mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} - a \gamma \nabla p^l - a(1 - \gamma) \nabla p^g \\ = \bar{\rho}_s \mathbf{u}^l + \bar{\rho}_l \mathbf{u}^l + \bar{\rho}_g \mathbf{u}^g \\ -\nabla p^l = b^l (\mathbf{u}^l - \mathbf{u}^g) + \bar{\rho}_l \mathbf{u}^l \\ -\nabla p^g = b^g (\mathbf{u}^g - \mathbf{u}^l) + \bar{\rho}_g \mathbf{u}^g \end{cases}, \quad (2)$$

Here, $a_{11} = \frac{A_{22}A_{13} - A_{13}A_{23}}{A_{11}A_{22} - A_{12}A_{21}}$, $a_{12} = \frac{A_{22}A_{14} - A_{12}A_{24}}{A_{11}A_{22} - A_{12}A_{21}}$, $a_{13} = \frac{A_{22}A_{15} - A_{12}A_{25}}{A_{11}A_{22} - A_{12}A_{21}}$,
 $a_{21} = \frac{A_{11}A_{23} - A_{21}A_{13}}{A_{11}A_{22} - A_{12}A_{21}}$, $a_{22} = \frac{A_{11}A_{24} - A_{21}A_{14}}{A_{11}A_{22} - A_{12}A_{21}}$, $a_{23} = \frac{A_{11}A_{25} - A_{21}A_{15}}{A_{11}A_{22} - A_{12}A_{21}}$, $A_{11} = \frac{\gamma}{K_s}(a - n) + \frac{n}{K_l}$,
 $A_{12} = \frac{1 - \gamma}{K_s}(a - n)$, $A_{13} = a - n$, $A_{14} = n(2 - S_r)$, $A_{15} = -n(1 - S_r)$, $A_{21} = \frac{\gamma}{K_s}(a - n)$,
 $A_{22} = \frac{1 - \gamma}{K_s}(a - n) + \frac{n}{P^a}$, $A_{23} = A_{13}$, $A_{24} = -nS_r$, $A_{25} = n(1 + S_r)$, $b^l = \frac{nS_r\eta_l}{k_l}$,
 $b^g = \frac{n(1 - S_r)\eta_g}{k_g}$, $\bar{\rho}_s = (1 - n)\rho_s$, $\bar{\rho}_l = nS_r\rho_l$, and $\bar{\rho}_g = n(1 - S_r)\rho_g$.

In the formula, $\mathbf{u}$, $\mathbf{u}^l$, and $\mathbf{u}^g$ are the displacements of the solid, liquid, and gas phases,

respectively; $\bar{\rho}_s$, $\bar{\rho}_l$, and $\bar{\rho}_g$ are the relative densities of the solid, liquid, and gas phases, respectively; K_s , K_l , and P^a are the bulk moduli of the solid, liquid, and gas phases, respectively; n is the porosity; S_r is the saturation; η_l , and η_g are the dynamic viscosity coefficients of the pore water and gas; and k_l , and k_g are the permeability coefficients of the pore water and gas.

By calculating the divergence of Formula 2 and substituting it into Formula 1, the longitudinal wave velocity in the soil can be obtained. The solution of the equation can be obtained.

$$\varphi^\alpha = A^\alpha \exp\left(ik_p(l_1x + l_2z - c_pt)\right), \quad (3)$$

where k_p is the wavenumber of the longitudinal wave; c_p is the wave velocity of the longitudinal wave; l_1 and l_2 are the direction vectors of the corresponding waves; and A^α ($\alpha = S, L, G$, respectively, for the solid, liquid, and gas phases) are the amplitudes of the longitudinal wave in the α phase medium. The wave velocities of three forms of longitudinal waves (P1 wave, P2 wave, P3 wave) in unsaturated soil can be obtained.

$$c_{pj} = \omega / (k_{pj})(j=1,2,3). \quad (4)$$

Among the three independent compression waves (P1, P2, and P3) in the three-phase medium, the P1 wave propagates with strong energy and low attenuation, making it effective for cross-hole seismic CT detection of embankments. In contrast, the P2 and P3 waves attenuate rapidly and have weak energy, so their first-arrival signals cannot be reliably picked up in engineering detection. Therefore, this paper focuses on quantitative analysis of the P1 wave only.

3. Relationships among the wave velocity, frequency, and porosity

3.1 Physical Parameters of the Calculation Model

To obtain the seismic wave velocity of unsaturated soil, numerical calculations are used to analyze the correlation between the longitudinal wave velocity and porosity of unsaturated soil. Typical silty unsaturated soil from embankments is selected as the research object, and the basic physical parameters of the three-phase medium are shown in Table 1, with the benchmark saturation set to 0.5.

Table 1 Calculation parameters of unsaturated soil

n	$\rho_s/\text{kg}\cdot\text{m}^{-3}$	$\rho_l/\text{kg}\cdot\text{m}^{-3}$	$\rho_g/\text{kg}\cdot\text{m}^{-3}$	K_s/Pa	K_l/Pa^{-1}	P^a/Pa^{-1}	k_l/m^2	k_g/m^2
				1				
0.3	3060	1000	1.29	3.6×10^{10}	2×10^9	0.13×10^6	1×10^{-3}	6.38×10^{-6}

3.2 Relationship between the wave velocity and frequency

Keeping the porosity of unsaturated soil at 0.3 and constant saturation, with frequencies ranging from 10^{-2} to 10^{10} Hz, the curve of the P1 wave velocity varying with frequency is calculated (Fig. 1).

The results show that the P1 wave has obvious dispersion effects, which can be divided into three intervals overall:

- 1) Low-frequency band (<1 Hz): The wave velocity is stable at 1173.1 m/s, and frequency changes have no effect on the wave velocity;
- 2) Intermediate-frequency band ($1\text{--}10^6$ Hz): The wave velocity continues to increase with frequency, showing significant dispersion effects;
- 3) High-frequency band ($>10^6$ Hz): The wave velocity tends to be constant at 1882.0 m/s, and the frequency no longer changes the propagation velocity.

The dominant excitation frequency of seismic exploration at embankment engineering sites is mostly at the hundred-hertz level, falling within the intermediate-frequency dispersion range. The wave velocity is easily affected by the excitation frequency, so the dominant frequency of the seismic source should be fixed during field detection to eliminate errors caused by dispersion.

3.3 Relationship between wave velocity and soil porosity

Fixing the commonly used excitation dominant frequency of 100 Hz in engineering and keeping the saturation constant, with the porosity ranging from 0.1 to 0.5, the P1 wave velocity variation curve is calculated (Fig. 2). The calculation results show that the P1 longitudinal wave velocity has a monotonically negative correlation with the soil porosity. The greater the porosity is, the greater the proportion of gas and water inside the pores, the lower the overall equivalent stiffness of the medium, and the slower the elastic wave propagation velocity.

When the porosity $n=0.1$, the P1 wave velocity is 1705.6 m/s; when the porosity $n=0.5$, the P1 wave velocity decreases to 1053.2 m/s.

The greater the degree of soil pore development is, the greater the degree of wave velocity attenuation. This quantitative law provides the core theoretical relationship for inverting porosity through wave velocity and identifying cavity and seepage anomalies.

4. Principle of Seismic CT

Seismic CT tomography uses a dual-borehole cross-hole observation system: seismic sources are arranged in one borehole to generate elastic waves, and geophones are arranged in another borehole to receive waveforms, with multiple crossing rays covering the embankment section to be tested. The first-arrival travel times of P1 waves for each ray are selected, and iterative algorithms are used to solve the wave velocity distribution of grid cells in the section.

Imaging methods based on first-arrival ray theory can be reduced to solving a large, sparse system of linear equations.

$$\begin{bmatrix} l_{11} & l_{12} & L & l_{1M} \\ l_{21} & l_{22} & L & l_{2M} \\ L & L & L & L \\ l_{N1} & l_{N2} & L & l_{NM} \end{bmatrix} \cdot \begin{bmatrix} S_1 \\ S_2 \\ L \\ S_4 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ L \\ T_N \end{bmatrix} \quad (5)$$

where a_{ij} is the path length of the i -th ray within the j -th cell, $f_j = 1/V_j$ is the average slowness of the j -th cell, and T_i is the measured travel time of the i -th ray.

In this study, the algebraic reconstruction technique (ART) is employed to perform wave velocity field inversion. The wave slowness is modified according to the slowness correction equation.

$$f_j^{q,i+1} = f_j^{q,i} + \mu \frac{a_{ij}}{\sum_j a_{ij}^2} (T_i - T_i^q) \quad (6)$$

where μ is the relaxation parameter, with $0 < \mu \leq 1$. When $\mu = 1$, the formula reduces to the standard correction form. The relaxation parameter is introduced to enhance the stability of the computation.

After the wave velocity distribution of the entire section is obtained, combined with the quantitative relationship between the wave velocity and porosity, the porosity distribution of the soil in each grid of the section is converted, resulting in visual imaging of the porosity.

5. Numerical Simulation and Inversion Analysis of Embankment Porosity

5.1 Construction of the Theoretical Numerical Model

Two sets of two-dimensional embankment section numerical models are

constructed. The benchmark porosities of the intact embankment soil are set to 0.2 and 0.3. Areas with higher porosity anomalies are set inside the models to simulate anomalies such as cavities and seepage channels:

1) Model 1: The porosities of the two anomalous zones are 0.4 and 0.5, and the benchmark porosity is 0.2.

2) Model 2: The porosities of the three anomalous zones are 0.5, 0.5, and 0.4, and the benchmark porosity is 0.3.

The model has a horizontal distance of 0–12 m and depth of 0–10 m, with uniform grid division (Fig. 3). The physical parameters are adopted from the three-phase medium parameters listed in Table 1.

5.2 Analysis of the ART Inversion Results

The ART algorithm is used to perform travel time inversion on the two sets of theoretical models, and the porosity imaging results are obtained (Fig. 4). Comparing the theoretical models with the inversion results yields the following patterns:

1) Seismic CT can accurately capture the planar positions of anomalous porosity zones. The inverted anomalous porosity values basically match the theoretical set values and can be used to quantitatively identify high-porosity anomalies areas inside the embankment body.

2) Local false anomaly phenomena exist at abrupt porosity boundaries: scattered false high/low porosity grids appear at the edges of high-porosity anomalous bodies. These false anomalies do not exhibit a continuous distribution and are clearly distinguishable from the continuous zones of real anomalies.

3) The greater the benchmark porosity is, the greater the proportion of gas and liquid phases inside the medium, the more severe the degree of ray energy attenuation, and the slightly greater the degree of false boundary anomalies.

In engineering applications, real anomalies can be comprehensively discriminated through continuous anomalies, anomaly scales, and porosity differences, eliminating discrete false anomalies generated by iteration.

6 Conclusions

In this study, the P1 wave velocity of unsaturated soil was determined on the basis of mixture theory, the coupling laws among the wave velocity, frequency, and soil porosity were quantitatively analyzed, and numerical simulations of embankment porosity inversion combined with cross-hole seismic CT and the ART iterative algorithm were conducted. The main conclusions are as follows:

1) Under constant saturation conditions, the P1 longitudinal wave velocity of

unsaturated soil has a stable negative correlation with soil porosity. The P1 wave exhibits dispersion characteristics: the wave velocity does not change with frequency in the low-frequency and high-frequency ranges, whereas the wave velocity increases with frequency in the intermediate-frequency range. Field detection should fix the dominant frequency of the seismic source to reduce dispersion errors;

2) Cross-hole seismic CT imaging combined with the ART iterative algorithm can be used to quantitatively invert the porosity of unsaturated soil in embankment sections, accurately locate high-porosity anomalous areas such as embankment cavities and seepage channels, and simultaneously quantify the degree of pore development in anomalous zones.

3) Abrupt porosity boundaries are prone to produce discrete iterative false anomalies without continuous distribution characteristics. When discriminating embankment anomalies, comprehensive identification should be performed in combination with geological profiles and anomaly morphology to avoid misjudgement;

This research establishes a quantitative analysis system of "elastic wave velocity–porosity–seismic CT imaging," making up for the deficiency of traditional seismic geophysical prospecting, which only qualitatively judges anomalies and provides theoretical and numerical support for refined quantitative detection of hidden damage in embankment engineering.

Acknowledgments

The authors appreciate the support provided by the Henan Provincial Science and Technology Research Project (Grant No. 262102321110). We acknowledge their assistance with the project.

Author contributions

C. S. designed the study. Z.K. wrote the manuscript. W. F. contributed to algorithm development and data interpretation. All the authors reviewed and approved the final manuscript.

Funding

This work was supported by the Henan Provincial Science and Technology Research Project (Grant No. 262102321110).

Declarations

Competing interests

The authors declare that they have no competing financial interests.

Additional Information

Correspondence and requests for materials should be addressed to C. S.

Availability of Data

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

References

- 1 Terzaghi, K. *Theoretical Soil Mechanics* (Wiley, 1943).
- 2 Gassmann, F. Über die Elastizität poröser Medien. *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*. **96**: 1-23 (1951).
- 3 Biot, M. A. Theory of elasticity and consolidation for a porous anisotropic solid. *Journal of Applied Physics*. **26**(2): 182-185 (1955).
- 4 Biot, M. A. The theory of propagation of elastic waves in a fluid saturated porous solid (I). Low-frequency range. *Journal of the Acoustical Society of America*. **28**(2): 168-178 (1956).
- 5 Biot, M. A. Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range. *The Journal of the Acoustical Society of America*. **28**(2): 179-191 (1956).
- 6 Biot, M. A. Mechanics of deformation and acoustic propagation in porous dissipative media. *Journal of Applied Physics*. **33**(4): 1482-1498 (1962).
- 7 Vardoulakis, I., Beskos, D. E. Dynamic behavior of nearly saturated porous media. *Mechanics of Materials*. **5**(1): 87-108 (1986).
- 8 Coussy, O. *Mechanics of Porous Continua* (Wiley, 1995).
- 9 Dvorkin, J., Nur, A. Dynamic poroelasticity: A unified model with the squirt and Biot mechanism. *Geophysics*. **58**(4): 524-533 (1993).
- 10 Lo, W. C., Sposito, G., Majer, E. Immiscible two-phase fluid flows in deformable porous media. *Advances in Water Resources*. **25**(8-12): 1105-1117 (2002).

- 11 Xu, M. J., Wei, D. M. Characteristics of wave propagation in partially saturated poroelastic media. *Science Technology and Engineering*. **9**(18): 5403-5409 (2009).
- 12 Huang, X., Greenhalgh, S., Han, L., Liu, X. Generalized effective Biot theory and seismic wave propagation in anisotropic, poroviscoelastic media. *Journal of Geophysical Research: Solid Earth*. **127**(3): e2021JB023590 (2022).
- 13 Tong, L. H., Ding, H. B., Yan, J. W., Xu, C. J., Lei, Z. Strain gradient nonlocal Biot poromechanics. *International Journal of Engineering Science*. **156**: 103372 (2020).
- 14 Kaczmarz, S. Angenäherte auflösung von systemen linearer gleichungen. *Bulletin International de l'Académie Polonaise des Sciences et des Lettres*. **35**: 355-357 (1937).
- 15 Backus, G., Gilbert, F. The resolving power of gross Earth data. *Geophysical Journal of the Royal Astronomical Society*. **16**(2): 169-205 (1968).
- 16 Gilbert, P. Iterative methods for the three-dimensional reconstruction of an object from projections. *Journal of Theoretical Biology*. **36**(1): 105-117 (1972).
- 17 Dines, K. A., Lytle, R. J. Computerized geophysical tomography. *Proceedings of the IEEE*. **67**(7): 1065-1073 (1979).
- 18 Ustaoglu, Z. Inversion of a generalized Radon transform by algebraic iterative methods. *Mathematical Methods in the Applied Sciences*. **46**(1): 167-179 (2023).
- 19 Chang, Y. F., Chang, W. Z., Tong, L. T. Detect multiple-set fractures by crosshole seismic tomography for use in environmental and engineering geophysics. *Journal of Environmental and Engineering Geophysics*. **29**(2): 61-74 (2024).
- 20 Yang, J., Shang, X., Liu, L., Wang, Y., Li, X. Fast marching method-based bent-ray tracing for three-dimensional velocity imaging in mines. *Rock Mechanics Bulletin*. **5**(2): 100262 (2026).
- 21 Yang, W. C. *Theory and Methods in Geophysical Inversion* (Geological Publishing House, 1997).

Figures

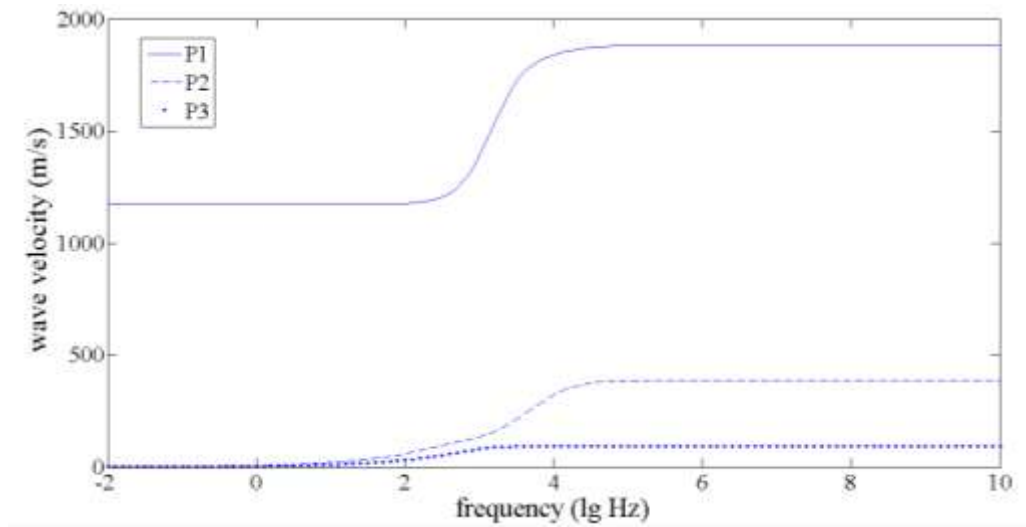


Figure 1 Relationships between wave velocity and frequency

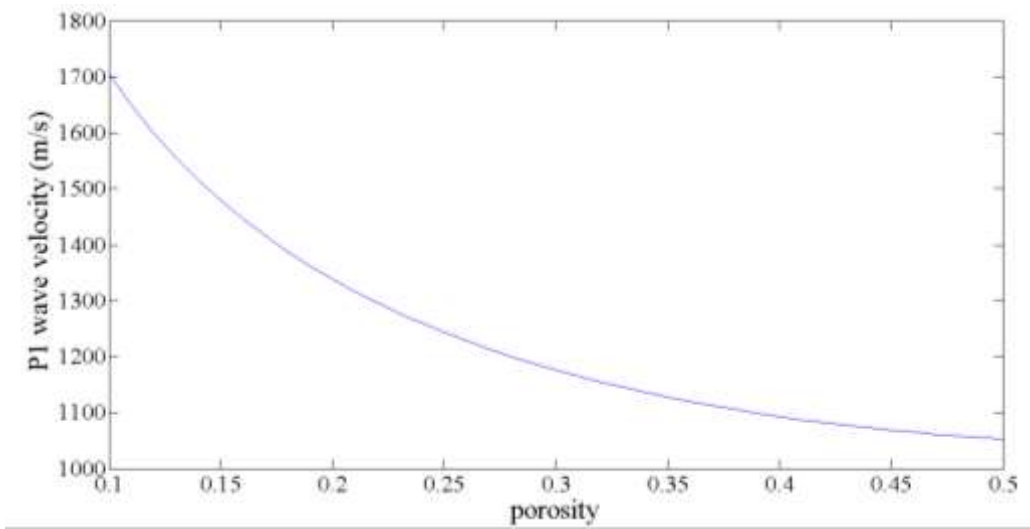


Figure 2 Relationships between wave velocity and porosity

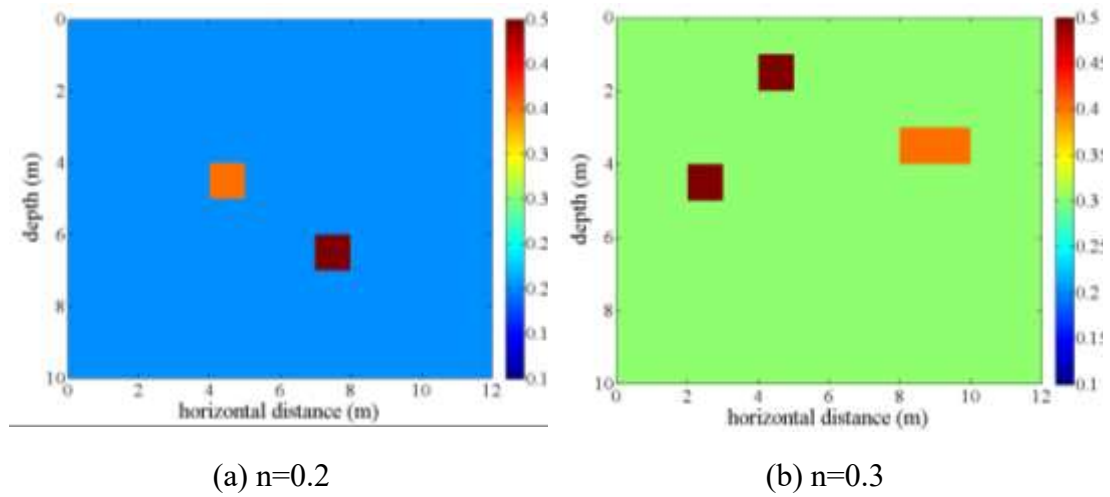


Figure 3 Theoretical model

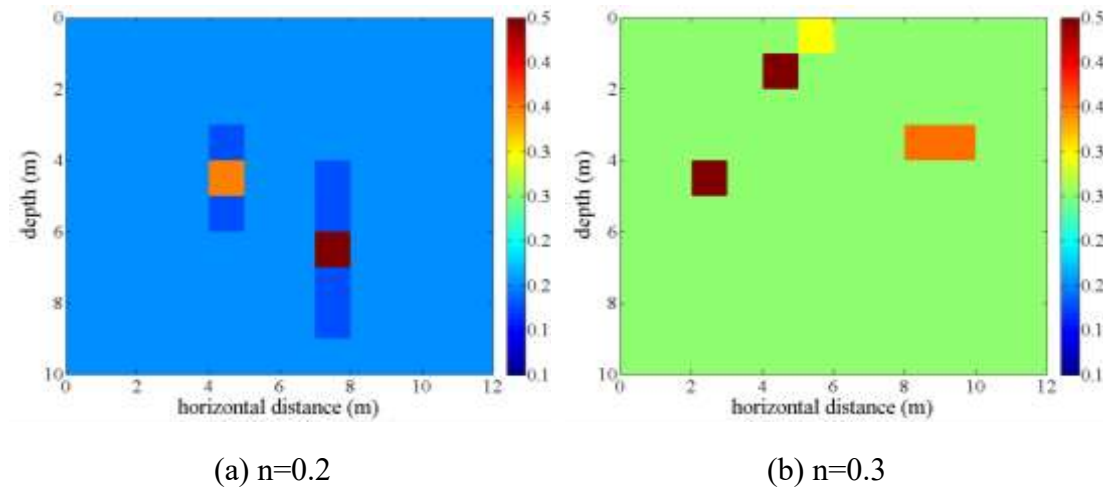


Figure 4 Inversion results