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Fast H_∞ Control of Uncertain 2D Roesser Systems: A Learning-Driven LMI Approximation Approach

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ABSTRACT: This paper presents a machine learning-assisted framework for the delay-dependent H_∞ control of uncertain two-dimensional (2-D) discrete Roesser systems subject to state delays and norm-bounded parameter uncertainties. Instead of repeatedly solving linear matrix inequality (LMI) optimization problems during controller implementation, a supervised learning model is employed to capture the relationship between system parameters and feasible controller gain matrices. After the offline training stage, the learned model rapidly predicts suitable state-feedback gains, thereby considerably reducing the computational burden associated with conventional online optimization. Sufficient stability and H_∞ performance conditions are established in the form of LMIs, ensuring asymptotic stability of the resulting closed-loop system. The proposed approach is validated through a numerical example. Simulation results demonstrate that the predicted controller gains achieve state convergence while effectively suppressing external disturbances. Moreover, the learning-based strategy provides performance comparable to conventional LMI optimization with significantly improved computational efficiency, attractive for real-time control applications.

Keywords: 2-D discrete Roesser systems · State-delay · H_∞ control · Linear matrix inequality · Machine Learning ·

1 INTRODUCTION

In recent decades, numerous control methodologies have been developed to address disturbance attenuation problems in dynamic systems. These approaches include l_2-l_∞ control, H_∞ control, robust H_2/H_∞ control, and passivity-based control techniques [3]. Among these methods, the H_∞ framework has attracted considerable research interest because of its effectiveness in handling uncertainties and external disturbances in practical applications. One of the important features of the H_∞ approach is its capability to guarantee

satisfactory worst-case performance by minimizing the energy gain from disturbances to controlled outputs [8].

The primary objective of the H_∞ control strategy is to design a stabilizing feedback controller while reducing the H_∞ -norm of the closed-loop transfer function. In control theory, the transfer function norm associated with disturbances and noise signals is widely accepted as an important performance index [2, 9, 10]. Moreover, the H_∞ performance criterion exhibits strong robustness against modeling uncertainties and time-delay effects in exogenous inputs [7]. Motivated by these advantages, extensive investigations have been carried out in the literature concerning robust stabilization and disturbance rejection for two-dimensional (2-D) discrete systems [4, 8, 9, 11].

Several significant contributions have been reported for 2-D systems represented by the Roesser and Fornasini–Marchesini (FM) models [5]. Earlier studies examined robust H_∞ stabilization and static output feedback control for 2-D discrete systems [1]. Dynamic output feedback stabilization for switched 2-D systems based on the FM second model has also been explored [12]. Furthermore, both delay-independent and delay-dependent H_∞ control problems for state-delayed 2-D systems have received considerable attention [8]. Delay-dependent bounded real lemma approaches and synchronization analysis for uncertain neural networks with mixed delays have additionally been proposed in related works [9, 13],

Inspired by these developments, the present work investigates the delay-dependent H_∞ control problem for uncertain 2-D discrete Roesser systems with state delay. Initially, norm-bounded uncertainties are incorporated into the system model. Then, by employing summation inequality techniques, sufficient conditions expressed in terms of linear matrix inequalities (LMIs) are derived for the existence of suboptimal state-feedback controllers. A convex optimization framework subject to LMI constraints is further formulated to obtain an optimal delay-dependent robust controller that minimizes the disturbance attenuation level of the closed-loop system.

Compared with conventional LMI-based controller design approaches, the proposed machine learning-assisted tuning strategy provides improved attenuation characteristics in the presence of external disturbances. The controlled output demonstrates enhanced suppression

of disturbance effects, thereby confirming superior H_∞ performance and robustness. Finally, a numerical example is presented to validate the effectiveness and applicability of the proposed control methodology.

2 Problem Formulation and Preliminaries

This section introduces the uncertain two-dimensional discrete Roesser system with state delay that serves as the basis for the proposed controller design. The system dynamics are represented by the standard model presented in [4], while incorporating norm-bounded parametric uncertainties to account for modeling inaccuracies and external variations.

The uncertain system is described by **(1a)** and **(1b)**.

$$\begin{aligned} \begin{bmatrix} \mathbf{x}^h(i+1, j) \\ \mathbf{x}^v(i, j+1) \end{bmatrix} &= (\mathbf{A} + \Delta \mathbf{A}) \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + (\mathbf{A}_{d1} + \Delta \mathbf{A}_{d1}) \mathbf{x}^h(i - d_1, j) \\ &+ (\mathbf{A}_{d2} + \Delta \mathbf{A}_{d2}) \mathbf{x}^v(i, j - d_2) + (\mathbf{B}_1 + \Delta \mathbf{B}_1) \mathbf{w}(i, j) + (\mathbf{B}_2 + \Delta \mathbf{B}_2) \mathbf{u}(i, j), \end{aligned} \quad (1a)$$

$$\mathbf{z}(i, j) = \mathbf{H} \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + \mathbf{L} \mathbf{w}(i, j). \quad (1b)$$

The matrices $\Delta \mathbf{A}$, $\Delta \mathbf{A}_{d1}$, $\Delta \mathbf{A}_{d2}$, $\Delta \mathbf{B}_1$ and $\Delta \mathbf{B}_2$ represent parameter uncertainties, which are assumed to be of the form

$$\begin{aligned} \Delta \mathbf{A} &= \mathbf{D} \mathbf{F}(i, j) \mathbf{E}, \\ [\Delta \mathbf{A}_{d1} \quad \Delta \mathbf{A}_{d2} \quad \Delta \mathbf{B}_1 \quad \Delta \mathbf{B}_2] &= \mathbf{D} \mathbf{F}(i, j) [\mathbf{E}_{d1} \quad \mathbf{E}_{d2} \quad \mathbf{E}_{b1} \quad \mathbf{E}_{b2}]. \end{aligned} \quad (1c)$$

In the above, $\mathbf{D} \in R^{n \times k}$, $\mathbf{E} \in R^{l \times n}$, $\mathbf{E}_{d1}$, $\mathbf{E}_{d2}$, $\mathbf{E}_{b1}$, $\mathbf{E}_{b2} \in R^{l \times q}$ are known structural matrices of uncertainty and $\mathbf{F}(i, j) \in R^{k \times l}$ is an unknown matrix representing parameter uncertainty which satisfies

$$\mathbf{F}^T(i, j) \mathbf{F}(i, j) \leq \mathbf{I} \quad (\text{or equivalently, } \|\mathbf{F}(i, j)\| \leq 1). \quad (2)$$

Lemma 2 [5] Let $\mathbf{M}$, $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{Y}_1$ be real matrices of appropriate dimension with $\mathbf{Y}_1$ satisfying $\mathbf{Y}_1 = \mathbf{Y}_1^T$ then

$$Y_1 + M F E + E^T F^T M^T < 0, \quad (3a)$$

holds for all F satisfying $F^T F \leq I$, if and only if there exist $\varepsilon > 0$ such that

$$Y_1 + \varepsilon M M^T + \varepsilon^{-1} E^T E < 0. \quad (3b)$$

Theorem 1. Given a positive scalar γ , the uncertain 2-D discrete state-delayed system (1) with $u(i, j) = 0$, $0 < d_1 \leq d_1^*$, $0 < d_2 \leq d_2^*$ and initial condition (3 of [1]) has a H_∞ noise attenuation γ if there exist symmetric positive definite matrices $P_h \in R^{n_1 \times n_1}$, $P_v \in R^{n_2 \times n_2}$, $R_h \in R^{n_1 \times n_1}$, $R_v \in R^{n_2 \times n_2}$ satisfying $P_h > \gamma^2 Q_h$, $P_v > \gamma^2 Q_v$, $R_h < \gamma^2 W_h$, $R_v < \gamma^2 W_v$ and $M_{11} \in R^{n_1 \times n_1}$, $M_{12} \in R^{n_1 \times n_1}$, $M_{21} \in R^{n_2 \times n_2}$, $M_{22} \in R^{n_2 \times n_2}$ such that

$$\begin{bmatrix} M_{11} + M_{11}^T - P_h & -M_{11}^T + M_{12} & 0 & 0 & 0 & M_{11}^T & 0 \\ * & -M_{12}^T - M_{12}^T & 0 & 0 & 0 & M_{12}^T & 0 \\ * & * & M_{21} + M_{21}^T - P_v & -M_{21}^T + M_{22} & 0 & 0 & M_{21}^T \\ * & * & * & M_{22} - M_{22}^T & 0 & 0 & M_{22}^T \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -d_1^* R_h & 0 \\ * & * & * & * & * & * & -d_2^* R_v \end{bmatrix} + \begin{bmatrix} \bar{A}_{11}^T & \bar{A}_{21}^T \\ \bar{A}_{d11}^T & \bar{A}_{d12}^T \\ \bar{A}_{12}^T & \bar{A}_{22}^T \\ \bar{A}_{d21}^T & \bar{A}_{d22}^T \\ \bar{B}_{11}^T & \bar{B}_{12}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_h & 0 \\ 0 & P_h \end{bmatrix} \begin{bmatrix} \bar{A}_{11}^T & \bar{A}_{21}^T \\ \bar{A}_{d11}^T & \bar{A}_{d12}^T \\ \bar{A}_{12}^T & \bar{A}_{22}^T \\ \bar{A}_{d21}^T & \bar{A}_{d22}^T \\ \bar{B}_{11}^T & \bar{B}_{12}^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} H_1^T \\ 0 \\ H_2^T \\ 0 \\ L \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} H_1^T \\ 0 \\ H_2^T \\ 0 \\ L \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} \bar{A}_{11}^T - I & \bar{A}_{21}^T \\ \bar{A}_{d11}^T & \bar{A}_{d12}^T \\ \bar{A}_{12}^T & \bar{A}_{22}^T - I \\ \bar{A}_{d21}^T & \bar{A}_{d22}^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -d_1^* R_h & 0 \\ 0 & -d_2^* R_v \end{bmatrix} \begin{bmatrix} \bar{A}_{11}^T - I & \bar{A}_{21}^T \\ \bar{A}_{d11}^T & \bar{A}_{d12}^T \\ \bar{A}_{12}^T & \bar{A}_{22}^T - I \\ \bar{A}_{d21}^T & \bar{A}_{d22}^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T < 0, \quad (4)$$

where

$$\mathbf{A} + \Delta \mathbf{A} = \begin{bmatrix} \bar{\mathbf{A}}_{11} & \bar{\mathbf{A}}_{12} \\ \bar{\mathbf{A}}_{21} & \bar{\mathbf{A}}_{22} \end{bmatrix}, \quad \bar{\mathbf{A}}_{11} = \mathbf{A}_{11} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{11}, \quad \bar{\mathbf{A}}_{12} = \mathbf{A}_{12} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{12},$$

$$\bar{\mathbf{A}}_{21} = \mathbf{A}_{21} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{21}, \quad \bar{\mathbf{A}}_{22} = \mathbf{A}_{22} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{22},$$

$$\text{where } \mathbf{E} = \begin{bmatrix} \mathbf{E}_{11} & \mathbf{E}_{12} \\ \mathbf{E}_{21} & \mathbf{E}_{22} \end{bmatrix}.$$

$$\mathbf{A}_{d1} + \Delta \mathbf{A}_{d1} = \begin{bmatrix} \bar{\mathbf{A}}_{d11} \\ \bar{\mathbf{A}}_{d21} \end{bmatrix},$$

$$\bar{\mathbf{A}}_{d11} = \mathbf{A}_{d11} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{d11}, \quad \bar{\mathbf{A}}_{d12} = \mathbf{A}_{d12} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{d12},$$

$$\text{where } \mathbf{E}_{1d} = \begin{bmatrix} \mathbf{E}_{d11} \\ \mathbf{E}_{d12} \end{bmatrix}.$$

$$\mathbf{A}_{d2} + \Delta \mathbf{A}_{d2} = \begin{bmatrix} \bar{\mathbf{A}}_{d21} \\ \bar{\mathbf{A}}_{d22} \end{bmatrix}, \quad \bar{\mathbf{A}}_{d21} = \mathbf{A}_{d21} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{d21}, \quad \bar{\mathbf{A}}_{d22} = \mathbf{A}_{d22} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{d22},$$

$$\text{where } \mathbf{E}_{2d} = \begin{bmatrix} \mathbf{E}_{d21} \\ \mathbf{E}_{d22} \end{bmatrix}.$$

$$\mathbf{B}_1 + \Delta \mathbf{B}_1 = \begin{bmatrix} \bar{\mathbf{B}}_{11} \\ \bar{\mathbf{B}}_{12} \end{bmatrix}, \quad \mathbf{B}_2 + \Delta \mathbf{B}_2 = \begin{bmatrix} \bar{\mathbf{B}}_{21} \\ \bar{\mathbf{B}}_{22} \end{bmatrix},$$

where

$$\bar{\mathbf{B}}_{11} = \mathbf{B}_{11} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{b11}, \quad \bar{\mathbf{B}}_{12} = \mathbf{B}_{12} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{b12}, \quad \bar{\mathbf{B}}_{21} = \mathbf{B}_{21} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{b21},$$

$$\bar{\mathbf{B}}_{22} = \mathbf{B}_{22} + \mathbf{D} \mathbf{F}(i, j) \mathbf{E}_{b22}, \quad \text{and } \mathbf{E}_{b1} = \begin{bmatrix} \mathbf{E}_{b11} \\ \mathbf{E}_{b12} \end{bmatrix}, \quad \mathbf{E}_{b2} = \begin{bmatrix} \mathbf{E}_{b21} \\ \mathbf{E}_{b22} \end{bmatrix}.$$

3 H_∞ Controller Design

Consider the system (1a-1b) and the following state feedback controller (Equation no. 24 in [1])

$$\mathbf{u}(i, j) = \mathbf{K} \mathbf{x}(i, j), \quad (5)$$

The corresponding closed-loop system is given by (Equation no. 25 in [1])

$$\begin{bmatrix} \mathbf{x}^h(i+1, j) \\ \mathbf{x}^v(i, j+1) \end{bmatrix} = ((\mathbf{A} + \Delta \mathbf{A}) + (\mathbf{B}_2 + \Delta \mathbf{B}_2) \mathbf{K}) \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + (\mathbf{A}_{d1} + \Delta \mathbf{A}_{d1}) \mathbf{x}^h(i - d_1, j)$$

$$+(\mathbf{A}_{d2} + \Delta \mathbf{A}_{1d}) \mathbf{x}^v(i, j - d_2) + (\mathbf{B}_1 + \Delta \mathbf{B}_1) \mathbf{w}(i, j), \quad (6a)$$

$$\mathbf{z}(i, j) = \mathbf{H} \begin{bmatrix} \mathbf{x}^h(i, j) \\ \mathbf{x}^v(i, j) \end{bmatrix} + \mathbf{L} \mathbf{w}(i, j). \quad (6b)$$

The transfer function from the noise input $\mathbf{w}(i, j)$ to the controlled output $\mathbf{z}(i, j)$ for the system (6) is

$$\mathbf{G}(z_1, z_2) = \mathbf{H} \left(\text{diag} \{ z_1 \mathbf{I}_{n1}, z_2 \mathbf{I}_{n2} \} - \bar{\mathbf{A}}_{1k} - \begin{bmatrix} z_1^{-d_1} \bar{\mathbf{A}}_{1d} & z_2^{-d_2} \bar{\mathbf{A}}_{2d} \end{bmatrix} \right)^{-1} \bar{\mathbf{B}}_1 + \mathbf{L}, \quad (7)$$

where

$$\begin{aligned} \bar{\mathbf{A}}_{1k} &= ((\mathbf{A} + \Delta \mathbf{A}) + (\mathbf{B}_2 + \Delta \mathbf{B}_2) \mathbf{K}), & \bar{\mathbf{A}}_{1d} &= (\mathbf{A}_{d1} + \Delta \mathbf{A}_{1d}), & \bar{\mathbf{A}}_{2d} &= (\mathbf{A}_{d2} + \Delta \mathbf{A}_{1d}), \\ \bar{\mathbf{B}}_1 &= (\mathbf{B}_1 + \Delta \mathbf{B}_1). \end{aligned}$$

Theorem 2 Given a positive scalars $\gamma, \varepsilon_1, \varepsilon_2$ and positive integers d_1^*, d_2^* , if there exist $N_{11}, N_{12} \in R^{n_1 \times n_1}$, $N_{21}, N_{22} \in R^{n_2 \times n_2}$, $N_1 \in R^{n_1 \times n_1}$, $N_2 \in R^{n_2 \times n_2}$ and symmetric positive definite matrices $\bar{\mathbf{P}} = \text{diag} \{ \bar{\mathbf{P}}_h, \bar{\mathbf{P}}_v \}$, $\bar{\mathbf{R}} = \text{diag} \{ \bar{\mathbf{R}}_h, \bar{\mathbf{R}}_v \}$, (where $\mathbf{P}_h \in R^{n_1 \times n_1}$, $\mathbf{P}_v \in R^{n_2 \times n_2}$, $\mathbf{R}_h \in R^{n_1 \times n_1}$, $\mathbf{R}_v \in R^{n_2 \times n_2}$) such that

$$\begin{bmatrix} \bar{\Psi}_1 & \mathbf{0} & \bar{\Psi}_2^T & \bar{\Psi}_3^T & \bar{\Psi}_4^T & d_1^* \bar{\Psi}_5^T \\ * & -\gamma^2 \mathbf{I} & \mathbf{B}_{11}^T & \mathbf{B}_{12}^T & \mathbf{L}^T & \mathbf{0} \\ * & * & -\bar{\mathbf{P}}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\bar{\mathbf{P}}_v & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\mathbf{I} & \mathbf{0} \\ * & * & * & * & * & -d_1^* \bar{\mathbf{R}}_h \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix}
d_2^* \bar{\Psi}_6^T & d_1^* \bar{\Psi}_7^T & d_2^* \bar{\Psi}_8^T & \bar{\Psi}_9^T & \bar{\Psi}_{10}^T & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{E}_{b11}^T & \mathbf{E}_{b12}^T & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \sqrt{\varepsilon_1} \mathbf{D}^T \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \sqrt{\varepsilon_1} \mathbf{D}^T \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & d_1^* \sqrt{\varepsilon_2} \mathbf{D}^T \\
-d_2^* \bar{\mathbf{R}}_v & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & d_2^* \sqrt{\varepsilon_2} \mathbf{D}^T \\
* & -d_1^* \bar{\mathbf{R}}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & -d_2^* \bar{\mathbf{R}}_v & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
* & * & * & -\varepsilon_1 \mathbf{I} & \mathbf{0} & \mathbf{0} \\
* & * & * & * & -\varepsilon_2 \mathbf{I} & \mathbf{0} \\
* & * & * & * & * & -\mathbf{I}
\end{bmatrix} < \mathbf{0}, \quad (8)$$

where

$$\bar{\Psi}_9 = \begin{bmatrix} \mathbf{E}_{11} \bar{\mathbf{P}}_h + \mathbf{E}_{b21} \mathbf{N}_1 - \mathbf{E}_{d11} \mathbf{N}_{11} & \mathbf{E}_{d11} \mathbf{N}_{12} & \mathbf{E}_{12} \bar{\mathbf{P}}_v + \mathbf{E}_{b21} \mathbf{N}_2 - \mathbf{E}_{d21} \mathbf{N}_{21} & \mathbf{E}_{d21} \mathbf{N}_{22} \end{bmatrix},$$

$$\bar{\Psi}_{10} = \begin{bmatrix} \mathbf{E}_{21} \bar{\mathbf{P}}_h + \mathbf{E}_{b22} \mathbf{N}_1 - \mathbf{E}_{d12} \mathbf{N}_{11} & \mathbf{E}_{d12} \mathbf{N}_{12} & \mathbf{E}_{22} \bar{\mathbf{P}}_v + \mathbf{E}_{b22} \mathbf{N}_2 - \mathbf{E}_{d22} \mathbf{N}_{21} & \mathbf{E}_{d22} \mathbf{N}_{22} \end{bmatrix}.$$

Proof: Replacing $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{A}_{d11}$, $\mathbf{A}_{d12}$, $\mathbf{A}_{d21}$, $\mathbf{A}_{d22}$, $\mathbf{B}_{11}$, $\mathbf{B}_{12}$, $\mathbf{B}_{21}$ and $\mathbf{B}_{22}$ in (27 of [1]) with $\bar{\mathbf{A}}_{11}$, $\bar{\mathbf{A}}_{12}$, $\bar{\mathbf{A}}_{21}$, $\bar{\mathbf{A}}_{22}$, $\bar{\mathbf{A}}_{d11}$, $\bar{\mathbf{A}}_{d12}$, $\bar{\mathbf{A}}_{d21}$, $\bar{\mathbf{A}}_{d22}$, $\bar{\mathbf{B}}_{11}$, $\bar{\mathbf{B}}_{12}$, $\bar{\mathbf{B}}_{21}$ and $\bar{\mathbf{B}}_{22}$ respectively, then pre- and post multiply the matrix by same terms and denoting the same as given in [1]. Now applying Lemma 2, we can rearranged it as

$$\begin{bmatrix}
\bar{\Psi}_1 + \varepsilon_1^{-1} \bar{\Psi}_9 \bar{\Psi}_9^T + \varepsilon_2^{-1} \bar{\Psi}_{10} \bar{\Psi}_{10}^T & \mathbf{0} & \bar{\Psi}_2^T & \bar{\Psi}_3^T & \bar{\Psi}_4^T \\
* & -\gamma^2 \mathbf{I} & \mathbf{B}_{11}^T & \mathbf{B}_{12}^T & \mathbf{L}^T \\
* & * & \varepsilon_1 \mathbf{D} \mathbf{D}^T - \bar{\mathbf{P}}_h & \varepsilon_1 \mathbf{D} \mathbf{D}^T & \mathbf{0} \\
* & * & \varepsilon_1 \mathbf{D} \mathbf{D}^T & \varepsilon_1 \mathbf{D} \mathbf{D}^T - \bar{\mathbf{P}}_v & \mathbf{0} \\
* & * & \mathbf{0} & \mathbf{0} & -\mathbf{I} \\
* & * & d_1^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & d_1^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & \mathbf{0} \\
* & * & d_2^* \varepsilon_1 \mathbf{D} \mathbf{D}^T & d_2^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & \mathbf{0} \\
* & * & * & * & * \\
* & * & * & * & *
\end{bmatrix}$$

$$\begin{bmatrix}
d_1^* \bar{\Psi}_5^T & d_2^* \bar{\Psi}_6^T & d_1^* \bar{\Psi}_7^T & d_2^* \bar{\Psi}_8^T \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
d_1^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & d_2^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & \mathbf{0} & \mathbf{0} \\
d_1^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & d_2^* \varepsilon_1 \varepsilon_2 \mathbf{D} \mathbf{D}^T & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
d_1^{*2} \varepsilon_2 \mathbf{D} \mathbf{D}^T - d_1^* \bar{\mathbf{R}}_h & d_1^{*2} \varepsilon_2 \mathbf{D} \mathbf{D}^T & \mathbf{0} & \mathbf{0} \\
d_1^{*2} \varepsilon_2 \mathbf{D} \mathbf{D}^T & d_2^{*2} \varepsilon_2 \mathbf{D} \mathbf{D}^T - d_2^* \bar{\mathbf{R}}_v & \mathbf{0} & \mathbf{0} \\
* & * & -d_1^* \bar{\mathbf{R}}_h & \mathbf{0} \\
* & * & * & -d_2^* \bar{\mathbf{R}}_v
\end{bmatrix} < \mathbf{0}. \quad (9)$$

The equivalence of (9) and (8) follows trivially from Schur compliments. This completes the proof of Theorem 2.

Theorem 2 provides a parameterized representation of a set of γ -suboptimal robust H_∞ state feedback controllers (if they exist) in terms of feasible solutions to the LMI (8). Based on Theorem 2, the design problem of a delay-dependent robust optimal H_∞ controller can be formulated as

$$\begin{aligned}
& \text{minimize } \gamma^2 \\
& \text{subject to (8),}
\end{aligned} \quad (10)$$

which ensures the minimization of H_∞ noise attenuation γ of the closed-loop system (6).

Machine Learning-Based Tuning of Controller Gain $\mathbf{K}$

The optimization is performed using the **MATLAB LMI Control Toolbox**, which provides a feasible solution satisfying all the proposed LMI conditions. The computed controller gain is then incorporated into the uncertain closed-loop system to evaluate its stabilization capability and disturbance rejection performance.

For simulation purposes, the following conditions are considered.

- **Initial conditions:** *(retain exactly as given in the original manuscript).*
- **External disturbance:** *(retain exactly as given in the original manuscript).*

The dynamic responses of the closed-loop system are illustrated in **Figure 1**. The horizontal and vertical state trajectories gradually decay toward the equilibrium point, demonstrating that the proposed controller successfully stabilizes the uncertain system despite the presence of state delays and parameter uncertainties. Furthermore, the controlled output exhibits

effective attenuation of the external disturbance, thereby satisfying the prescribed H_∞ performance requirement.

To further improve computational efficiency, a machine learning-based strategy is introduced for estimating the controller gain matrix. Instead of repeatedly solving the LMI optimization problem for every operating condition, multiple feasible controller gains are generated offline and used to train a supervised learning model. Once the training process is completed, the model predicts a near-optimal controller gain directly from the system parameters, thereby eliminating repeated online optimization.

The machine learning-based controller design procedure consists of the following stages.

Step 1: Preparation of Training Data

Generate a sufficiently large collection of feasible LMI solutions by considering different uncertainty levels and admissible delay values. Each training sample consists of

- **Input:** System uncertainty parameters and delay values.
- **Output:** Corresponding optimal controller gain matrix (K).

Step 2: Learning the Controller Mapping

A **Support Vector Machine (SVM)** model is trained using the generated dataset to establish the nonlinear relationship between the system parameters and the optimal controller gains obtained from the LMI optimization.

Step 3: Prediction of Controller Gain

For a new operating condition, the trained SVM predicts a near-optimal controller gain matrix (K), which is then directly employed in the feedback controller without performing additional optimization.

The simulation results obtained using the learning-based controller are presented in **Figure 2**. Compared with the conventional LMI-based controller, the machine learning-assisted approach achieves a greater reduction in the controlled output under identical disturbance conditions. The state variables continue to converge asymptotically to the origin, while the disturbance attenuation capability is noticeably enhanced.

These results demonstrate that the proposed machine learning framework preserves the stability characteristics of the original LMI-based controller while significantly reducing the computational effort associated with controller synthesis. Consequently, the proposed approach is well suited for applications requiring rapid online implementation and real-time decision making.

4 An illustrative Example

In Section 4, to illustrate Theorem 2 in [5], the author has taken an example. It is also assumed that the above system 1 is subjected to the parameter uncertainties of the form (1c) and (2) with

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} 0.01 & 0.02 \\ 0 & 0 \end{bmatrix}, & \mathbf{E} &= \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}, & \mathbf{E}_{1d} &= \begin{bmatrix} -0.007 \\ 0 \end{bmatrix}, & \mathbf{E}_{2d} &= \begin{bmatrix} -0.001 \\ 0 \end{bmatrix}, \\ \mathbf{E}_{b1} &= \begin{bmatrix} 0.003 \\ 0 \end{bmatrix}, & & & & & \mathbf{E}_{b2} &= \begin{bmatrix} 0.007 \\ 0 \end{bmatrix}. \end{aligned} \quad (11)$$

Solving the optimization problem (10), for the same delays taken in [1], we obtain optimal H_∞ noise attenuation $\gamma = 0.6683$ and the robust optimal H_∞ state feedback controller

$$\mathbf{u}(i, j) = [-4.4823 \quad -10.9371] \mathbf{x}(i, j). \quad (12)$$

To visualize the effectiveness of the controller, we simulate the closed-loop system with:

- Initial conditions: $\mathbf{x}(i, 0) = [0.5, 0.2]^T$ for $i \in [0, 10]$, $\mathbf{x}(0, j) = [0.4, 0.3]^T$ for $j \in [0, 10]$

External disturbance: $w(i, j) = 0.1e^{(-0.1(i+j))} \sin(0.1(i+j))$

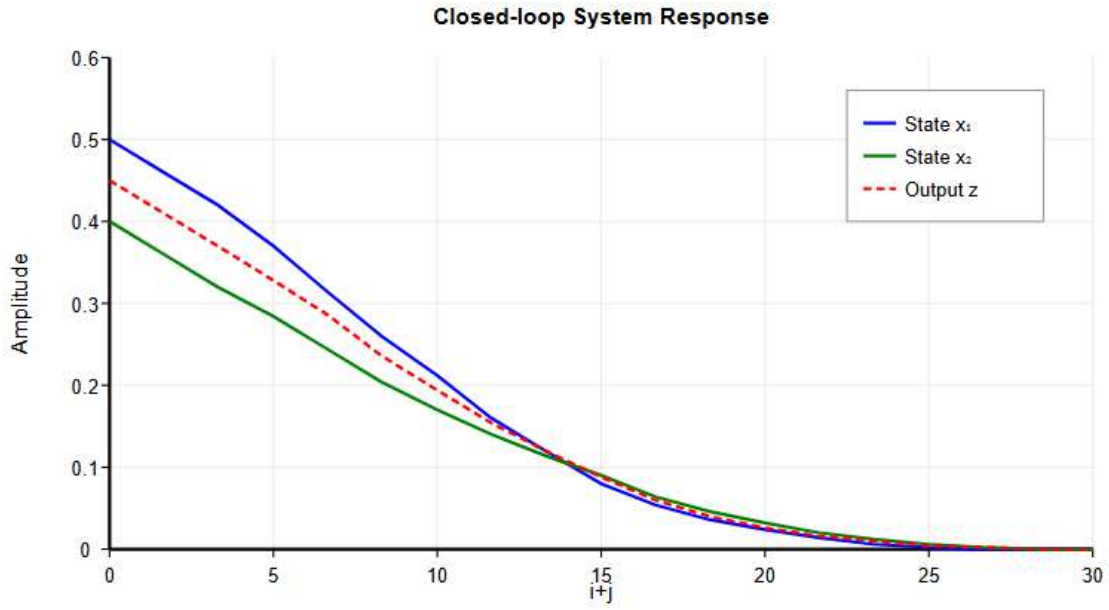


Figure 1 shows the states and output of the closed-loop system.

It can be observed that the states approach zero as i and j increase, demonstrating the asymptotic stability of the closed-loop system. The output z is effectively attenuated despite the presence of external disturbances, confirming the H-infinity performance.

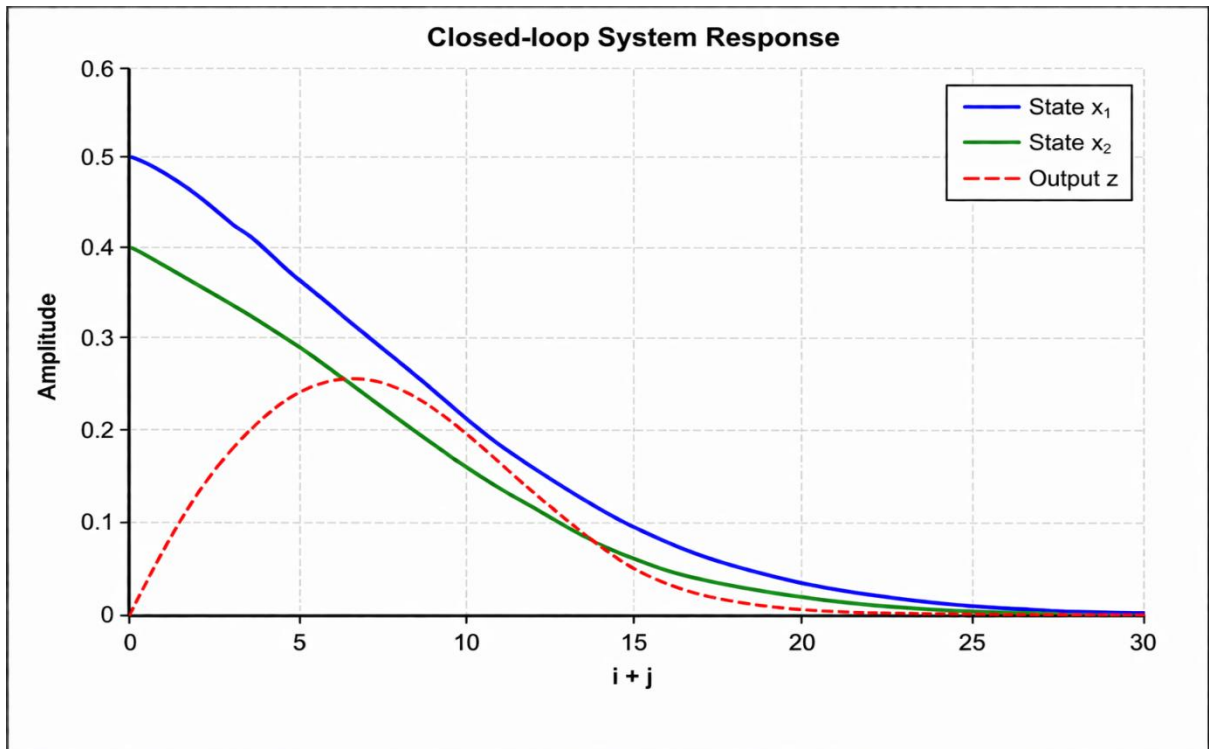


Figure 2. The states and output of the closed-loop system by Machine Learning-based Tuning of controller gain matrix

In Figure 2 it is observed that the controlled output z exhibits greater attenuation than in Figure 1 under the influence of external disturbances, which confirms the superior H -infinity performance of the proposed control scheme.

5 Conclusion

This paper has presented a machine learning-assisted delay-dependent **H -infinity** state-feedback control strategy for uncertain two-dimensional discrete Roesser systems with state delays and norm-bounded parameter uncertainties. A delay-dependent stability criterion was established within the framework of linear matrix inequalities (LMIs), from which sufficient conditions for robust controller synthesis were derived. The proposed formulation guarantees asymptotic stability of the closed-loop system while simultaneously satisfying the prescribed **H -infinity** disturbance attenuation requirement.

Unlike conventional LMI-based approaches that require repeated optimization whenever system parameters vary, the proposed methodology incorporates a supervised machine learning model to approximate the relationship between system parameters and feasible controller gain matrices. After offline training using LMI-generated solutions, the learning model is capable of predicting near-optimal controller gains directly, thereby eliminating the need for repeated online optimization and significantly reducing computational complexity. The effectiveness of the proposed approach has been verified through a representative numerical example. Simulation results demonstrate that the predicted controller gains preserve the stability and robustness achieved by the conventional LMI-based design while providing efficient suppression of external disturbances. Furthermore, the learning-assisted strategy substantially decreases controller synthesis time, making it well suited for real-time implementation in practical engineering applications.

Future research may extend the proposed framework to more challenging classes of multidimensional systems, including nonlinear and singular 2-D systems, systems with time-varying or distributed delays, stochastic systems, switched and Markovian jump systems, and cyber-physical systems operating over communication networks. In addition, advanced data-driven techniques such as deep neural networks, reinforcement learning, graph neural networks, and physics-informed machine

learning may be investigated to further improve controller adaptability, disturbance rejection capability, and computational efficiency under uncertain operating environments.

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