

Open-Source Hardening Soil Material-Point Integrator and Neural-Network Surrogate for Ho Chi Minh City Soils

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Abstract

The Hardening Soil (HS) model is widely used in deep-excavation and metro-tunnel analyses, but its reliance on closed-source software limits independent verification, reproducibility, and integration with machine-learning workflows. This study developed and validated an open-source HS-based material-point integrator and assessed its use as a physics-based data generator for a neural-network surrogate. Methods The Python implementation incorporated the stress-dependent stiffness law, Mohr–Coulomb failure criterion, and hyperbolic primary-loading relation. It was evaluated through internal consistency checks and comparison with consolidated-drained triaxial data from three representative Ho Chi Minh City soils comprising 274 measured points. The validated model generated 20,000 stress–strain curves for training a multilayer perceptron. The implementation was limited to drained primary loading and did not reproduce the complete multisurface HS formulation. Results Leave-one-out validation yielded a pooled R^2 of 0.998, Pearson’s r of 0.999, and a mean absolute percentage error of 1.9%; the maximum internal consistency error was below 0.1%. The fitted exponent m ranged from 0.61 to 0.94. The surrogate achieved $R^2 = 0.9999$ and an RMSE of 1.47 kPa. The proposed platform provides a reproducible basis for inverse analysis and large-scale geotechnical simulations.

Keywords

Hardening Soil model; open-source constitutive modelling; triaxial test; neural-network surrogate; stress-dependent stiffness; Ho Chi Minh City soils

1 Introduction

The accuracy of finite element analyses of underground structures, including deep excavations, diaphragm walls, and metro tunnels, depends strongly on the selected soil constitutive model and its parameters [1]. Appropriate model selection is therefore critical [2], particularly because soil stiffness is nonlinear over a wide range of strains [3]. The Hardening Soil model [4] is widely adopted because it represents nonlinear primary loading, unloading–reloading behaviour, and stress-dependent stiffness through the reference moduli E_{50}^{ref} , $E_{\text{oed}}^{\text{ref}}$, and $E_{\text{ur}}^{\text{ref}}$ and the exponent m . The formulation was subsequently extended to account for small-strain stiffness [5].

Reliable prediction of ground movements and structural forces consequently requires these parameters to be determined for the site-specific ground conditions.

Two limitations remain in current practice. First, the HS model is used predominantly through closed-source commercial software, such as PLAXIS [6]. Because the constitutive integration algorithm is not openly accessible, independent verification and modification are difficult, whereas open-source finite element platforms such as OpenSees [7] and Kratos [8] permit direct customisation. Second, high-quality geotechnical test data are scarce and costly to obtain, which constrains the direct training of data-intensive deep-learning models. This limitation has been noted in reviews of artificial-intelligence applications in geotechnical engineering [9, 10], although neural networks have long been used to represent complex nonlinear relationships in the field [11].

One approach to addressing both limitations is to develop a verified open-source constitutive model and use it as a physics-based data generator for machine learning. Such a workflow improves transparency and reproducibility while alleviating data scarcity by producing simulated responses over a prescribed parameter domain. This strategy is credible only when the constitutive implementation has first been verified numerically and validated against experimental observations.

Accordingly, this study has three objectives. First, an open-source HS-based material-point integrator is implemented for the drained triaxial primary-loading response, incorporating the stress-dependent stiffness law, the Mohr–Coulomb failure criterion, and the hyperbolic stress–strain relation. Second, the implementation is evaluated using two complementary lines of evidence: an internal consistency check and statistical validation against consolidated-drained triaxial data for three representative Ho Chi Minh City soils. Third, the validated model is used as a data generator to train a neural-network surrogate that reproduces complete stress–strain curves. The principal contribution is a reproducible material-point framework and differentiable emulator intended to support subsequent inverse-analysis and large-scale simulation studies.

2 Materials and Methods

2.1 Experimental data and research workflow

The experimental dataset was obtained from a consolidated-drained (CD) triaxial testing programme conducted on three representative Ho Chi Minh City soils: flow-state soft clay, medium clay, and silty sand. Each soil group comprised 16 specimens, and tests were conducted

at confining stresses of $\sigma_3 = 100, 200$, and 300 kPa. These tests provided the secant modulus E_{50} , the unloading–reloading modulus E_{ur} , and the effective shear-strength parameters c' and ϕ' . Following data cleaning and physical-consistency screening, 274 measured points were retained. The physical properties of the three soil layers are summarised in Table 1.

Table 1 Physical properties of the three soil layers derived from CD triaxial tests

Soil layer	Samples	c' (kPa)	ϕ' (deg)	e_0	E_{50}^{ref} (MPa)	E_{ur}^{ref} (MPa)
Soft clay	16	0.1-1.2	20.2-24.1	1.94-2.24	2.7-4.4	10.2-19.2
Silty sand	16	3.2-7.1	28.2-31.1	-	12.3-18.2	42.5-69.5
Medium clay	16	3.1-4.7	28.0-29.6	-	7.8-14.6	28.6-51.0

Table 2 summarises the descriptive statistics of the measured parameters. The friction angle was the least variable parameter, with a coefficient of variation below 6% for all three layers. In contrast, the effective cohesion of the soft clay exhibited substantial relative variability (CV = 42%), primarily because the measured values were small in absolute magnitude. The reference moduli showed moderate variability, with coefficients of variation of 13%–19%, reflecting the natural heterogeneity of the soils. The initial void ratio is reported only for the soft clay, for which 16 reliable measurements were available; the single values available for each of the other two layers were excluded from the descriptive analysis.

Table 2 Descriptive statistics of the triaxial parameters by soil layer

Soil layer	Parameter	n	Mean	Std	Min	Max	CV (%)
Soft clay	c'	16	0.76	0.32	0.14	1.24	42.0
Soft clay	ϕ'	16	22.34	1.26	20.19	24.12	5.7
Soft clay	e_0	16	2.05	0.10	1.94	2.24	4.9
Soft clay	E_{50}^{ref}	16	3.58	0.50	2.66	4.36	14.1
Soft clay	E_{ur}^{ref}	16	15.51	2.35	10.24	19.21	15.2
Silty sand	c'	16	5.43	1.00	3.16	7.06	18.4
Silty sand	ϕ'	16	29.80	0.69	29.06	31.07	2.3
Silty sand	E_{50}^{ref}	16	14.92	1.91	12.26	18.24	12.8
Silty sand	E_{ur}^{ref}	16	53.59	9.08	42.54	69.46	16.9
Medium clay	c'	16	3.89	0.47	3.12	4.66	12.2
Medium clay	ϕ'	16	28.89	0.50	28.03	29.62	1.7
Medium clay	E_{50}^{ref}	16	11.03	2.14	7.76	14.62	19.4
Medium clay	E_{ur}^{ref}	16	39.15	6.90	28.61	51.03	17.6

The research workflow comprised seven sequential stages: preparation of the experimental dataset, implementation of the constitutive relations, internal verification, external validation, material-point data generation, surrogate training, and surrogate evaluation (Fig. 1).

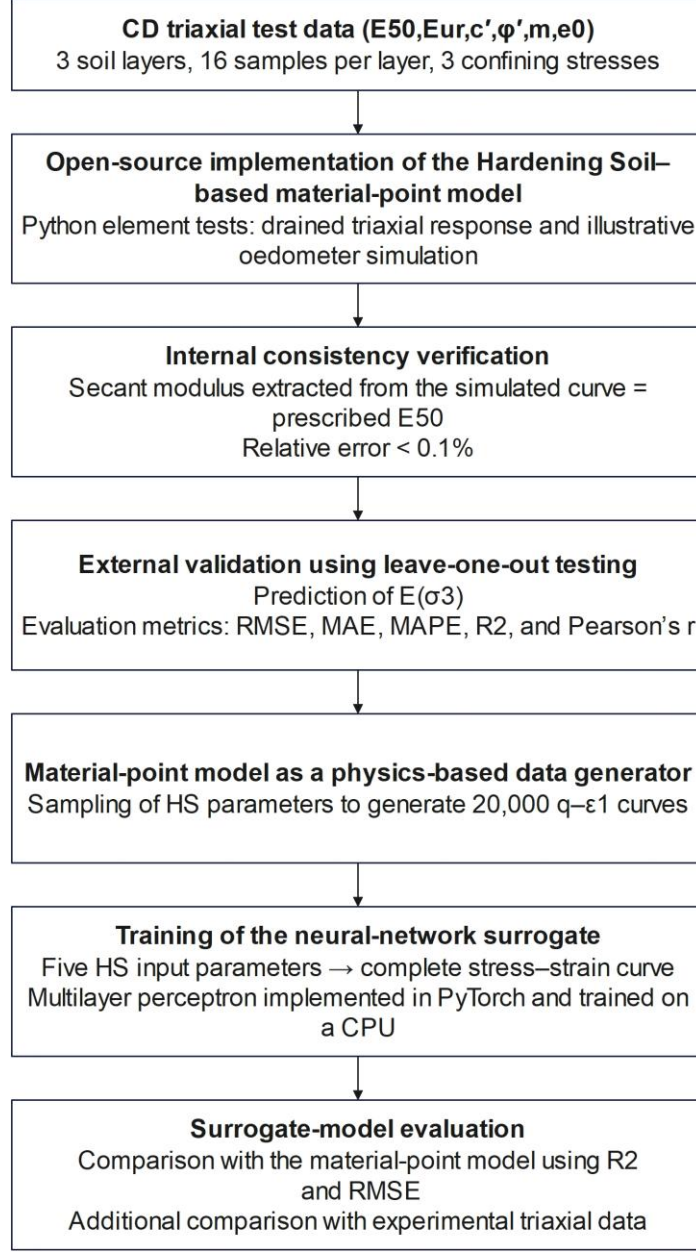


Fig. 1 Research workflow

2.2 HS formulation at the material-point level

A defining feature of the HS model is the dependence of stiffness on effective confining stress. This dependence is represented by the power law in Eq. (1), where E_{50}^{ref} denotes the reference secant modulus at the reference stress p_{ref} and m controls the degree of stress dependency.

$$E = E_{\text{ref}} \cdot \left(\frac{c \cos \varphi + \sigma_3 \sin \varphi}{c \cos \varphi + p_{\text{ref}} \sin \varphi} \right)^m \quad (1)$$

The failure condition is defined by the Mohr–Coulomb criterion, with the deviatoric stress at failure q_f calculated using Eq. (2). To ensure that the secant modulus obtained from the simulated curve equals the prescribed E_{50} at $q = 0.5 q_f$, the initial modulus E_i is related to E_{50} through the failure ratio R_f , as expressed in Eq. (3).

$$q_f = (c \cot \varphi + \sigma_3) \cdot \frac{2 \sin \varphi}{1 - \sin \varphi} \quad (2)$$

$$E_i = \frac{2E_{50}}{2 - R_f} \quad (3)$$

Primary loading is represented by the hyperbolic stress–strain relation in Eq. (4), which originates from the formulation proposed by Kondner [12] and subsequently developed by Duncan and Chang [13]. In this relation, $q_a = q_f/R_f$ defines the asymptotic deviatoric stress. The equation is inverted to compute q directly as a function of axial strain ε_1 , thereby providing the response used to simulate CD triaxial loading.

$$\varepsilon_1 = \frac{\frac{q}{E_i}}{1 - \frac{q}{q_a}} \quad (4)$$

The implementation therefore includes the three constitutive components required to represent the drained triaxial primary-loading response: the stress-dependent stiffness law, the Mohr–Coulomb strength criterion, and the hyperbolic hardening relation. The shear- and volumetric-hardening yield surfaces, cap mechanism, and dilatancy rule of the complete HS formulation are not implemented at this stage. Moreover, E_{ur} is not used to simulate a separate unloading–reloading branch and enters only through the scaling adopted for the primary-loading response. The resulting scope is appropriate for the present objective of generating and validating drained triaxial stiffness responses.

Figure 2 illustrates the stress-dependent stiffness law in Eq. (1). For a fixed reference modulus, E_{50} increases with σ_3 , and the rate of increase is controlled by m . The corresponding simulated CD triaxial curves at three confining stresses are shown in Fig. 3; each curve approaches its associated failure stress q_f .

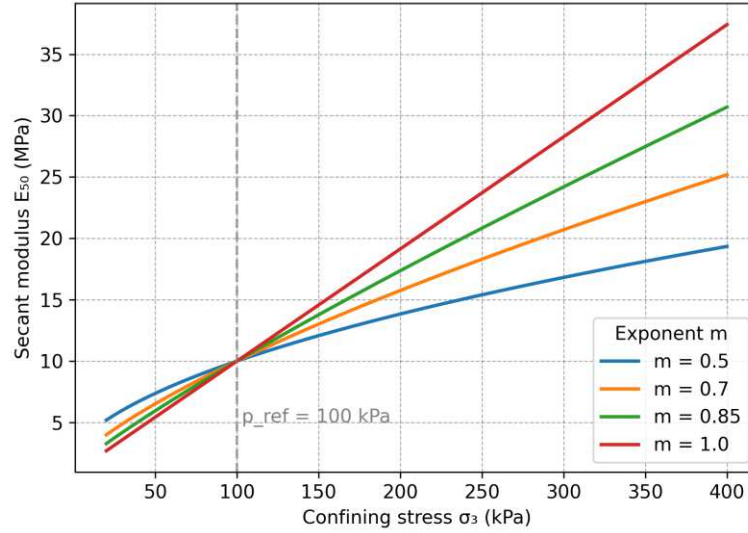


Fig. 2 Stress-dependent stiffness relation of the HS model

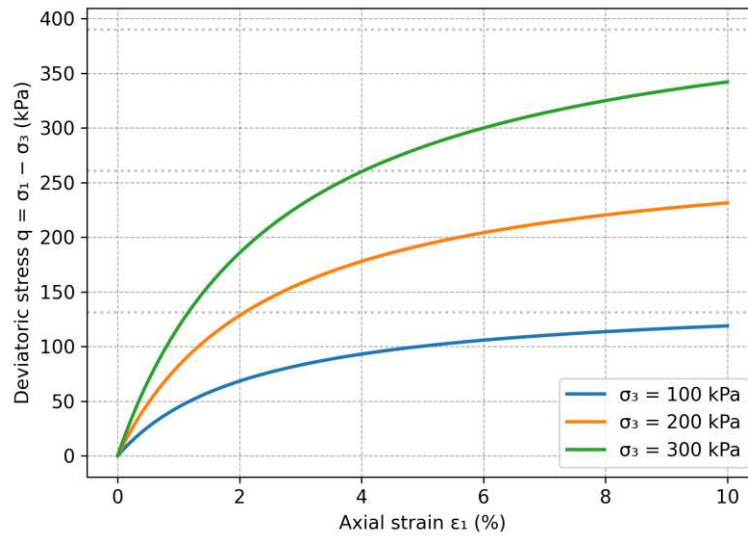


Fig. 3 Simulated CD triaxial responses at three confining stresses

Using the same stress-dependent stiffness concept, the model also represents one-dimensional consolidation through a tangent oedometer modulus E_{oed} that varies with vertical stress following Janbu's formulation [14]. Figure 4 presents (a) the simulated compression curves and (b) the corresponding stress-dependent tangent moduli. Because experimental oedometer data were not available, these results are illustrative rather than validation cases. The simulations used $E_{\text{oed}}^{\text{ref}}$ values of 8, 18, and 30 MPa for the soft clay, medium clay, and silty sand, respectively, with $m = 0.8$. Figure 5 further illustrates the combined effects of confining stress and m on normalised stiffness.

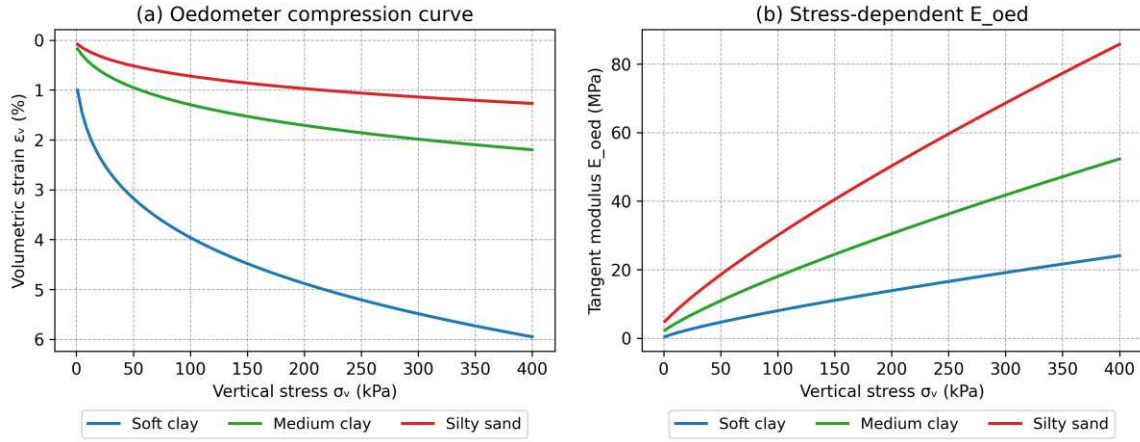


Fig. 4 Simulated one-dimensional consolidation: (a) compression curves and (b) tangent moduli

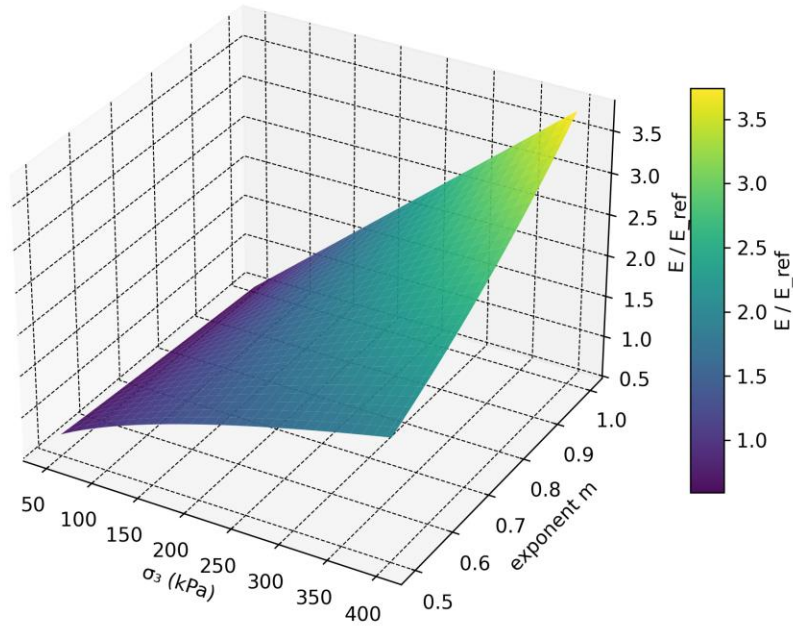


Fig. 5 Normalised stiffness surface as a function of confining stress and exponent m

The sensitivity analysis in Fig. 6 isolates the influence of each input parameter on the triaxial response. The reference modulus primarily controls the initial slope, the friction angle governs the failure stress, m controls the curvature associated with stress dependency, and the confining stress affects both stiffness and strength. Figure 7 presents the stress paths for conventional triaxial compression (CTC) and reduced-confinement extension (RTE), together with the Mohr–Coulomb failure envelope.

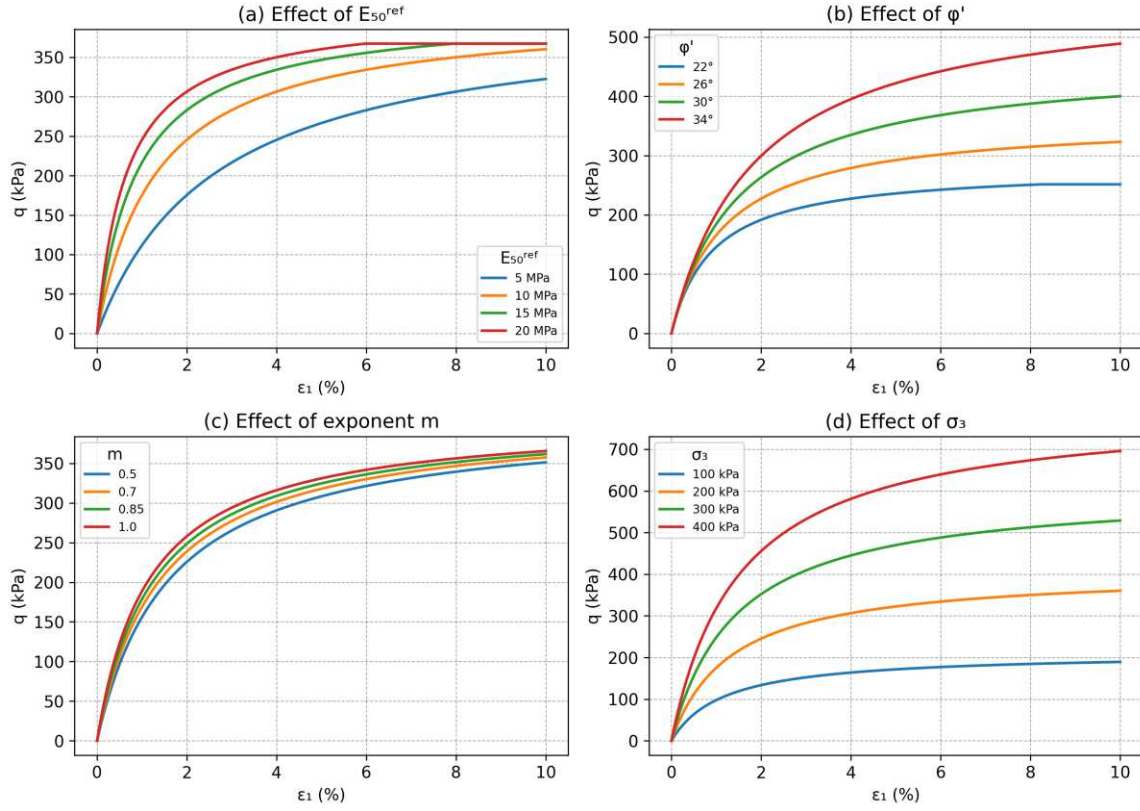


Fig. 6 Sensitivity of the triaxial response to HS parameters: (a) reference modulus, (b) friction angle, (c) exponent m , and (d) confining stress

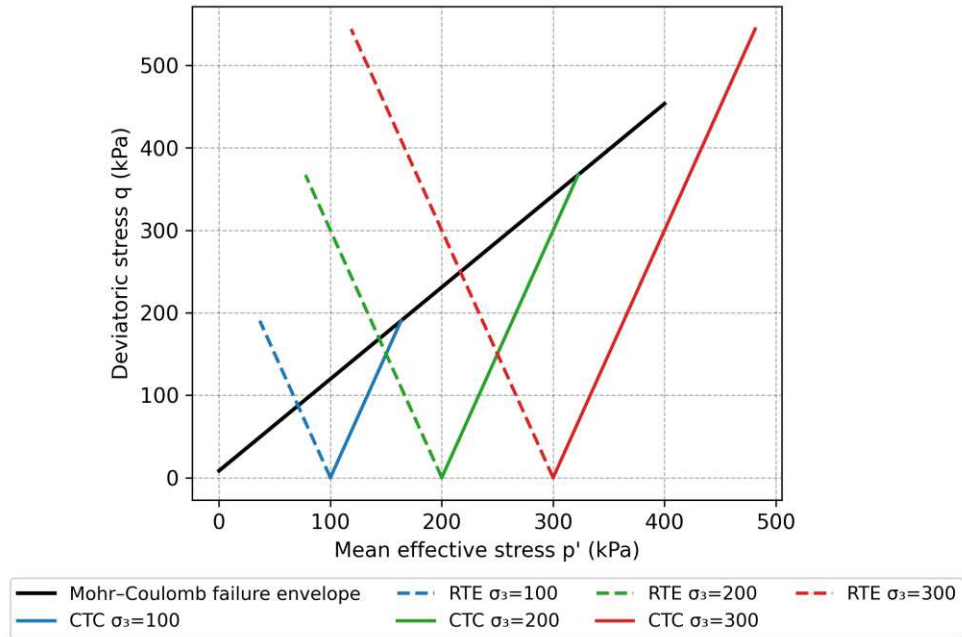


Fig. 7 CTC and RTE stress paths relative to the Mohr–Coulomb failure envelope

2.3 Calibration and leave-one-out validation of the stress-dependency exponent

For each specimen, the modulus measured at $\sigma_3 = 100$ kPa was adopted as the reference modulus. The exponent m was then estimated by linear regression after logarithmic transformation of Eq. (1). This all-point calibration, reported in the Results section as the fitted exponent, used all available confining stresses and quantified the goodness of fit of the power law using the regression coefficient of determination. Specimens without a reference value at $\sigma_3 = 100$ kPa or with fewer than two available confining stresses were excluded; consequently, 14–16 specimens were retained in each group.

Calibration and evaluation using the same observations would quantify only in-sample fit. Predictive performance was therefore assessed separately using a leave-one-out procedure. To predict the modulus at either 200 or 300 kPa, m was recalibrated using the 100 kPa reference point and the other available higher-confinement point; the omitted value was then predicted. Thus, every validation prediction was generated without using the corresponding observation during calibration. This procedure required measurements at all three confining stresses.

2.4 Material-point model as a data generator

After validation, the material-point model was used to generate the training dataset. The general strategy of replacing computationally intensive simulations with data-driven surrogates is established in surrogate-modelling theory [15] and has been applied to predictions of excavation response based on finite element datasets [16]. Five parameters controlling the drained triaxial primary-loading curve- E_{50}^{ref} , c' , ϕ' , σ_3 , and m -were sampled independently from uniform distributions over the extended ranges listed in Table 3.

Table 3 Parameter ranges used to generate 20,000 material-point samples

Parameter	Meaning	Range	Unit
E_{50}^{ref}	Reference secant modulus (sampled)	2 000 - 20 000	kPa
c'	Effective cohesion (sampled)	0 - 10	kPa
ϕ'	Effective friction angle (sampled)	20 - 35	deg
σ_3	Confining (cell) stress (sampled)	50 - 400	kPa
m	Stress-dependency exponent (sampled)	0.5 - 1.0	-
p_{ref}	Reference stress (fixed)	100	kPa
R_f	Failure ratio (fixed)	0.9	-

Because $E_{\text{ur}}^{\text{ref}}$ does not influence the primary-loading curve in the present implementation, it was excluded from the surrogate inputs. For each parameter set, the model generated a 40-point

stress–strain curve, yielding 20,000 samples. Figure 8 illustrates the resulting dataset and its coverage across the prescribed confining-stress range.

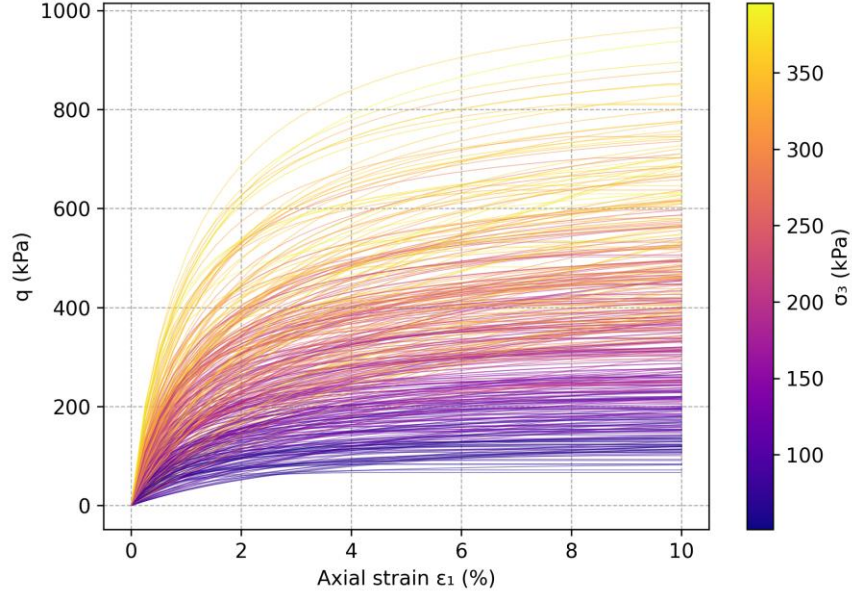


Fig. 8 Distribution of the 20,000 material-point curves used for surrogate training

2.5 Neural-network surrogate and evaluation metrics

The surrogate was formulated as a multilayer perceptron that maps the five HS input parameters to a 40-point stress–strain curve [17, 18]. The dataset was partitioned into training, validation, and test subsets in proportions of 70%, 15%, and 15%, respectively. The network was implemented in PyTorch 2.11 [20] with NumPy 2.4 and Python 3.14, trained using the Adam optimiser [19] with early stopping, and executed on a 20-core Intel CPU without GPU acceleration. A single random seed (20260605) was used for data generation, dataset partitioning, and network initialisation to ensure exact computational reproducibility.

Model performance was quantified using the root-mean-square error (Eq. (5)), coefficient of determination (Eq. (6)), mean absolute percentage error, and Pearson correlation coefficient. The same set of metrics was applied to the experimental validation of the stiffness law and to the evaluation of the neural-network surrogate against the material-point model.

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_{i,\text{pred}} - y_{i,\text{meas}})^2} \quad (5)$$

$$R^2 = 1 - \frac{\sum_i^n (y_{i,\text{meas}} - y_{i,\text{pred}})^2}{\sum_i^n (y_{i,\text{meas}} - \bar{y})^2} \quad (6)$$

3 Results

3.1 Stress-dependency exponent m

Table 4 summarises the fitted exponent m by soil layer and modulus type. For E_{50} , the mean values were 0.85 for soft clay, 0.83 for medium clay, and 0.76 for silty sand; the corresponding means for E_{ur} were 0.66, 0.80, and 0.73. Individual estimates ranged from 0.61 to 0.94, indicating that the group means describe the central tendencies rather than the full observed range. Within-group standard deviations were below 0.05, and the mean coefficient of determination for the power-law regressions was approximately 0.999. These results indicate an excellent fit of the stress-dependent stiffness relation to the measured moduli. As shown in Fig. 9, all fitted values fall within the commonly adopted range of 0.5–1.0, spanning typical sand-like to clay-like stress dependency.

Table 4 Fitted exponent m by soil layer and modulus type

Soil layer	Modulus	n	m mean	m std	m min	m max	R^2 fit mean
Soft clay	E_{50}	15	0.852	0.043	0.764	0.939	1.000
Soft clay	E_{ur}	16	0.660	0.032	0.606	0.732	1.000
Silty sand	E_{50}	16	0.765	0.033	0.711	0.847	0.999
Silty sand	E_{ur}	14	0.728	0.025	0.675	0.773	0.999
Medium clay	E_{50}	15	0.827	0.027	0.791	0.867	1.000
Medium clay	E_{ur}	14	0.801	0.025	0.760	0.839	0.999

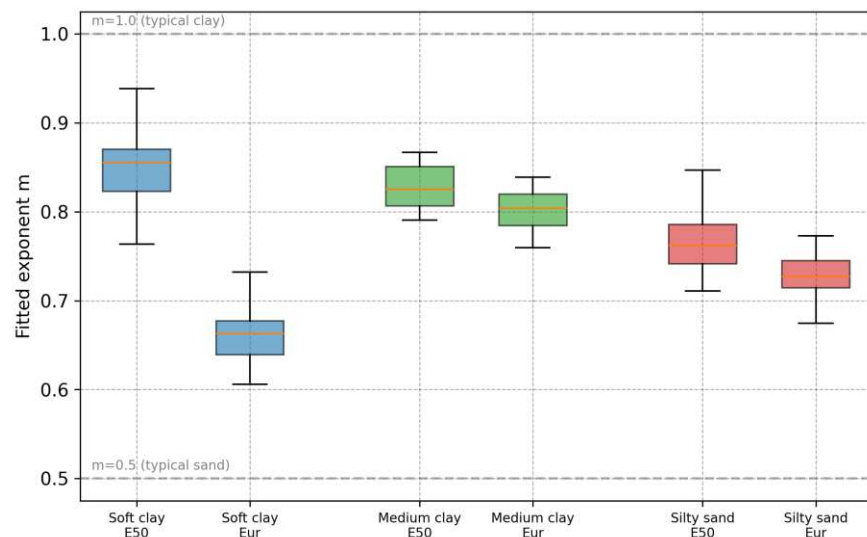


Fig. 9 Distribution of the fitted exponent m by soil layer and modulus type

The measured moduli in Table 5 directly demonstrate the stress dependency represented by the fitted exponents. Both E_{50} and E_{ur} increased substantially with confining stress. For example, the mean E_{50} of the soft clay increased from 3.58 MPa at $\sigma_3 = 100$ kPa to 8.90 MPa at 300 kPa, corresponding to an increase by a factor of approximately 2.5; comparable increases were observed for the medium clay and silty sand. At each confining stress, E_{ur} was approximately three to four times E_{50} , supporting the separate characterisation of the loading and unloading–reloading moduli.

Table 5 Measured secant (E_{50}) and unloading–reloading (E_{ur}) moduli by soil layer and confining stress (mean $\pm$ SD, MPa)

Soil layer	Modulus	100 kPa	200 kPa	300 kPa
Soft clay	E_{50}	3.58 ± 0.50	6.38 ± 0.81	8.90 ± 1.07
Soft clay	E_{ur}	15.51 ± 2.35	24.57 ± 3.45	31.64 ± 4.38
Silty sand	E_{50}	14.92 ± 1.91	24.84 ± 2.71	32.42 ± 2.85
Silty sand	E_{ur}	53.59 ± 9.08	87.52 ± 13.77	113.86 ± 16.72
Medium clay	E_{50}	11.03 ± 2.14	19.44 ± 3.52	26.26 ± 4.75
Medium clay	E_{ur}	39.15 ± 6.90	67.52 ± 11.73	90.88 ± 15.27

3.2 Validation of the stress-dependent stiffness law against experimental data

Under the leave-one-out protocol, the stiffness law predicted each modulus at $\sigma_3 = 200$ or 300 kPa using an exponent calibrated from the 100 kPa reference value and the remaining higher-confinement observation. After excluding the reference points and specimens without measurements at all three stresses, 176 out-of-sample predictions were obtained.

Table 6 Leave-one-out validation metrics for predicting the moduli at $\sigma_3 = 200$ and 300 kPa

Soil layer	Modulus	n	RMSE (kPa)	MAE (kPa)	MAPE (%)	R^2	Pearson r
Soft clay	E_{50}	30	102.5	88.2	1.118	0.9957	0.9992
Soft clay	E_{ur}	32	504.3	431.6	1.531	0.9906	0.9974
Silty sand	E_{50}	30	872.8	726.4	2.416	0.9658	0.9953
Silty sand	E_{ur}	28	2870.9	2566.9	2.437	0.9789	0.9960
Medium clay	E_{50}	28	539.7	468.6	1.997	0.9898	0.9976
Medium clay	E_{ur}	28	1989.5	1791.4	2.159	0.9872	0.9976
Overall	All	176	1471.5	985.3	1.930	0.9982	0.9993

Table 6 shows that agreement remained strong under this stricter evaluation. Group-specific R^2 values ranged from 0.966 for the silty-sand E_{50} predictions to 0.996 for the soft-clay E_{50} predictions, and MAPE ranged from 1.1% to 2.4%. The pooled results yielded $R^2 = 0.998$, Pearson $r = 0.999$, and MAPE = 1.9%. The pooled R^2 exceeded the group-specific values because the combined dataset spans a much wider modulus range and therefore has greater total variance; the

group-level metrics provide the more conservative assessment. In the parity plots in Fig. 10, predictions for both E_{50} and E_{ur} closely follow the 1:1 line, with no evident soil-specific bias.

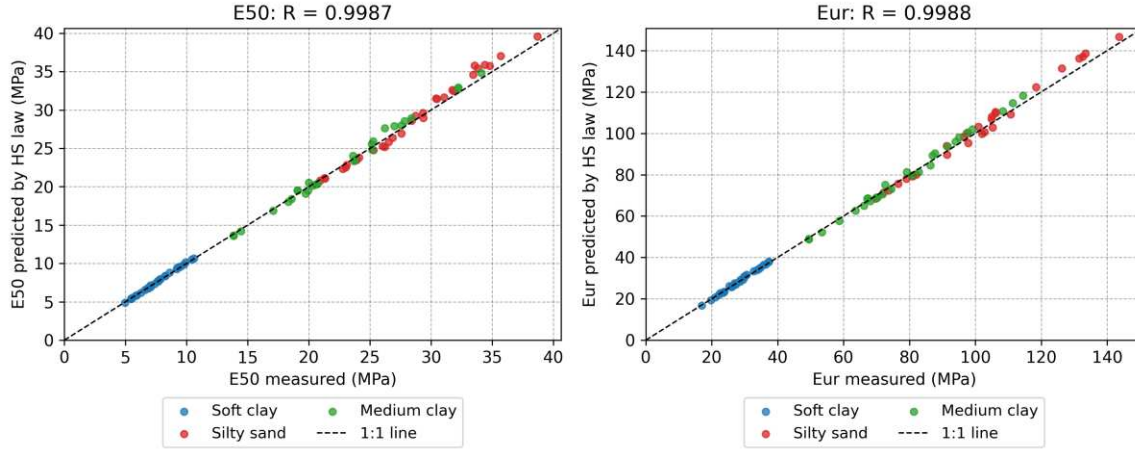


Fig. 10 Predicted versus measured moduli: (a) E_{50} and (b) E_{ur}

3.3 Internal consistency and relationship between E_{50} and E_{ur}

Internal numerical consistency was assessed by comparing the secant modulus extracted from each simulated curve at $q = 0.5 q_f$ with the E_{50} value prescribed by the stiffness law.

Table 7 Internal consistency of the material-point integrator

Item	Value
Number of check points	136
Mean relative error	0.0163 %
Maximum relative error	0.0711 %
Correlation (secant vs law)	> 0.9999

As shown in Table 7, the mean and maximum relative errors were 0.016% and 0.071%, respectively, confirming that the initial modulus defined by Eq. (3) reproduces the intended secant modulus on the adopted numerical grid. This calculation is a self-consistency check governed partly by grid resolution and is not an independent validation against measured behaviour; external validation is provided by the leave-one-out analysis in Section 3.2. Figure 11 and Table 8 describe the experimental relationship between E_{ur} and E_{50} . Across all layers, the mean E_{ur}/E_{50} ratio was 3.65, with individual ratios ranging from 2.87 to 4.80. Soft clay showed the highest mean ratio (3.89) and the greatest variability, whereas the medium clay and silty sand were more tightly clustered around 3.5. These values are within the range of 3–5 commonly adopted in HS calibration.

Table 8 Ratio of unloading–reloading modulus to secant modulus (E_{ur}/E_{50}) by soil layer

Soil layer	n	Mean	Std	Min	Max
Soft clay	46	3.89	0.45	2.87	4.80
Silty sand	43	3.52	0.24	3.11	4.11
Medium clay	43	3.51	0.10	3.36	3.75
All layers	132	3.65	0.35	2.87	4.80

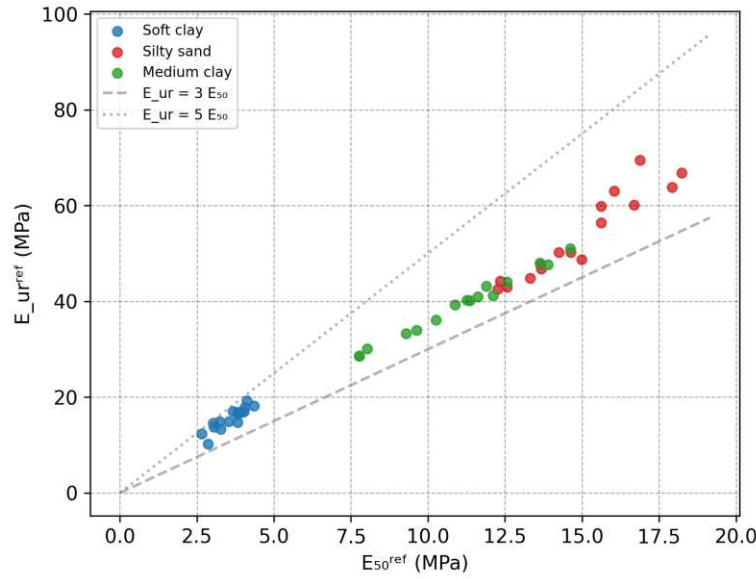


Fig. 11 Relationship between E_{ur} and E_{50} at the reference stress

3.4 Neural-network surrogate performance

The surrogate converged after 115 epochs and achieved high accuracy on the held-out test set of 3,000 samples drawn from the same parameter distribution. Across all test points, R^2 was 0.9999 and RMSE was 1.47 kPa (Table 9). The parity plot and representative curves in Fig. 12 show close agreement between the surrogate and material-point responses throughout the strain range. Figure 13 shows a sustained decrease in validation error without an evident divergence from the training trend. Because the analytical target is smooth and the test set was sampled from the same distribution as the training data, the near-perfect performance should be interpreted as accurate interpolation within the sampled domain rather than evidence of out-of-distribution generalisation. The principal value of the surrogate at this stage is its smooth and differentiable mapping, which is suitable for subsequent inverse analysis and optimisation. The reported accuracy does not constitute independent validation against additional laboratory or field observations.

Table 9 Neural-network architecture and performance on the held-out test set

Item	Value
Architecture	MLP 5-256-256-256-40, SiLU activation
Samples (train/val/test)	14 000 / 3 000 / 3 000
Optimizer	Adam, lr=1e-3, early stopping
Convergence epochs	115
Test RMSE (kPa)	1.465
Test MAE (kPa)	0.979
R^2 (all points)	0.99994
R^2 (per-curve mean)	0.99920

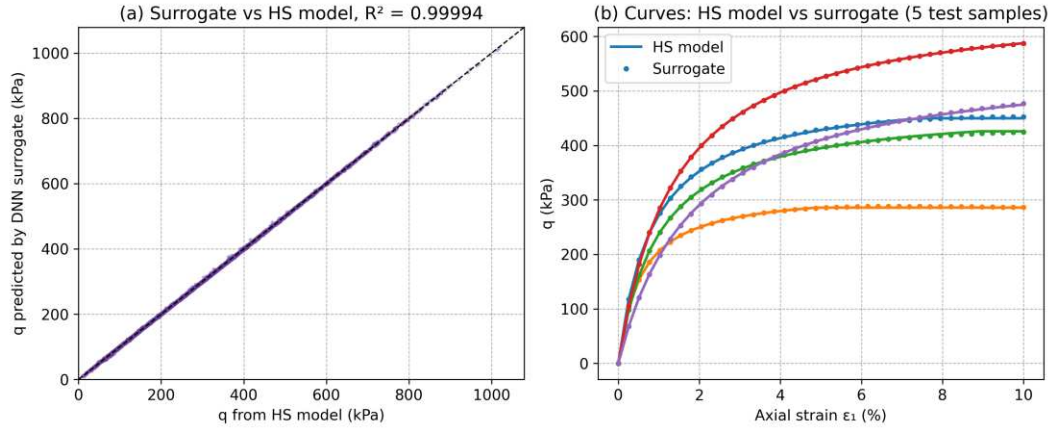


Fig. 12 Surrogate performance relative to the material-point model: (a) parity plot and (b) representative curves

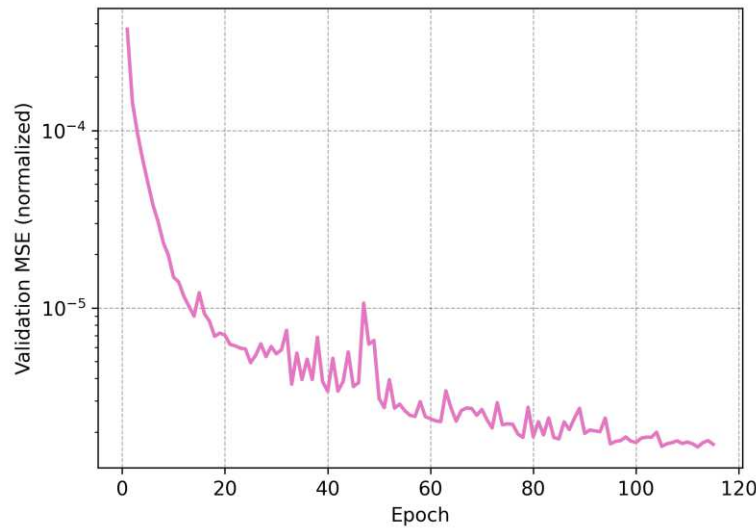


Fig. 13 Convergence history of the neural-network surrogate

The residual distribution in Table 10 confirms that the aggregate performance was not dominated by a small subset of easily predicted curves. The median absolute residual was 0.66 kPa, and the 99th percentile was 5.0 kPa; thus, fewer than 1% of the 120,000 test points exceeded

5.0 kPa. The maximum absolute residual was 18.4 kPa. At the curve level, 91.0% of the test curves had $R^2 > 0.999$ and 99.1% had $R^2 > 0.99$, while the minimum per-curve R^2 was 0.848. The relatively few larger errors occurred for curves with very low failure plateaux, for which small absolute deviations produce comparatively large reductions in R^2 .

Table 10 Error distribution of the neural-network surrogate on the held-out test set

Item	Value
Absolute residual, median (kPa)	0.659
Absolute residual, 90th percentile (kPa)	2.199
Absolute residual, 95th percentile (kPa)	2.998
Absolute residual, 99th percentile (kPa)	5.041
Absolute residual, maximum (kPa)	18.362
Per-curve R^2 , minimum	0.8479
Per-curve R^2 , 5th percentile	0.9976
Per-curve R^2 , median	0.9998
Curves with $R^2 > 0.999$ (%)	91.0
Curves with $R^2 > 0.99$ (%)	99.1

4 Discussion

The fitted exponent m had group means of 0.66–0.85 and individual values of 0.61–0.94. These estimates are consistent with the HS framework described by Schanz et al. [4], in which values near 0.5 and 1.0 are commonly associated with sand- and clay-like stress dependency, respectively. The observed range also agrees with values reported for soft and stiff Bangkok clays by Surarak et al. [21], who calibrated the same power-law relation using oedometer and triaxial data. The stiffness degradation documented for Bangkok clays over the small-strain range [22] further supports the use of a stress-dependent stiffness formulation. For the soft clay, the higher m values fitted to E_{50} than to E_{ur} indicate a stronger dependence of the primary-loading modulus on confinement than that of the unloading–reloading modulus. This difference is consistent with the distinct stiffness levels associated with primary loading and unloading–reloading [3].

Relative to conventional empirical correlations between stiffness and indices such as N_{60} or c_u , the present study explicitly implements the constitutive relations and evaluates them using two complementary forms of evidence. The hyperbolic response is based on the formulations of Kondner [12] and Duncan and Chang [13]. The complete HS framework extends the classical hyperbolic model by distinguishing primary-loading and unloading–reloading stiffness, an important feature for excavation analyses [2, 23]. In the present reduced-form implementation, however, the external validation addresses stress-dependent moduli and the primary-loading triaxial response rather than the complete unloading and volumetric behaviour of the HS model.

The group-specific leave-one-out results ($R^2 > 0.96$) indicate that the open-source integrator captures the measured stress dependency of the three soils. A direct numerical benchmark against a commercial implementation such as PLAXIS was beyond the scope of this study and remains necessary in future work.

The coupling of the material-point model with a neural-network surrogate is consistent with the growing use of deep learning in geotechnical engineering [9, 10]. Finite element datasets have previously been used to construct rapid predictive models for braced excavations [16]. In the present approach, the surrogate is trained on responses generated by the validated constitutive equations rather than directly replacing the mechanical formulation. It therefore retains the response structure imposed by the material-point model within the sampled parameter domain while enabling rapid and differentiable evaluation. Differentiability is advantageous for inverse analysis and gradient-based optimisation, in which response sensitivities with respect to soil parameters are required, and may also facilitate subsequent development of physics-informed learning frameworks [24].

Several limitations should be acknowledged. First, the experimental validation is restricted to conventional triaxial compression and does not include triaxial extension or oedometer measurements; consequently, the oedometer results are illustrative only. Second, reliable initial void-ratio measurements were available only for the soft clay, precluding a robust analysis of the relationship between m and porosity for the other two layers. Third, the implementation is currently limited to material-point simulations and does not reproduce the full boundary-value behaviour of excavation or tunnel problems. Extension to boundary-value analyses is therefore required before the computational benefit of the surrogate can be evaluated under realistic construction sequences and soil–structure interaction conditions.

5 Conclusions

This study implemented an open-source, reduced-form HS-based material-point integrator for the drained triaxial primary-loading response and evaluated it through internal consistency checks and leave-one-out validation. The maximum internal consistency error was below 0.1%. Across the three Ho Chi Minh City soils, the group-specific validation yielded R^2 values of at least 0.966, while the pooled R^2 and MAPE were 0.998 and 1.9%, respectively. The fitted stress-dependency

exponent had group means of 0.66–0.85 and individual values of 0.61–0.94, consistent with clayey and silty-sand behaviour and with published regional data.

The validated material-point model generated 20,000 stress–strain curves for training a neural-network surrogate. On the held-out test set, the surrogate achieved $R^2 = 0.9999$ and RMSE = 1.47 kPa. These results demonstrate the feasibility of using an open-source constitutive implementation as a reproducible data generator for machine learning in geotechnical engineering. The resulting platform and differentiable emulator provide a basis for future inverse analysis, reliability assessment, and large-scale simulations of excavation–metro-tunnel interaction. These conclusions are limited to the parameter ranges, loading paths, and model scope examined in this study.

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