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Fifth-grade students' fairness judgments through probabilistic ideas: Maintaining and refining reasoning across two game contexts

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Research Article

Keywords: Fairness, informal probabilistic ideas, probability education, game-based learning, elementary students

Posted Date: September 25th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-10438502/v1

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Additional Declarations: No competing interests reported.

Abstract

Children frequently judge whether games are fair long before they receive formal instruction in probability, yet little is known about the role that probabilistic ideas play in these judgments. This study investigated the extent to which fifth-grade students drew on probabilistic ideas when evaluating the fairness of games and how their fairness judgments were maintained, revised, or reconstructed following gameplay. A qualitative exploratory design was employed with 68 fifth-grade students from a public elementary school. Students completed two game-based tasks, a rock-paper-scissors (RPS) game and a biased die game, and responded to written questions before and after gameplay. Data were analysed using thematic analysis.

The findings indicate that many students were able to identify the biased die game as unfair by intuitively comparing players' winning opportunities rather than relying on formal probability calculations. However, probabilistic ideas did not constitute students' only basis for judging fairness. Depending on the characteristics of the game, students also drew on procedural fairness, strategic competence, personal interpretations of chance, and gameplay experiences when justifying their decisions. Although gameplay rarely led students to abandon their initial judgments, it often supported the refinement or reconstruction of the reasoning underlying them.

The findings suggest that elementary school students possess informal probabilistic ideas that can support fairness judgments while also drawing on a range of non-probabilistic considerations. Fairness contexts could therefore provide a productive pedagogical entry point for eliciting and developing children's informal probabilistic ideas.

Introduction

Fairness is a fundamental concept through which children interpret social interactions, evaluate outcomes, and make judgments about justice. Although fairness is often associated with equal treatment or equal opportunities, many everyday situations require individuals to judge fairness under conditions of uncertainty. In games and other chance-based situations, fairness judgments depend on how individuals interpret chance, likelihood, and the distribution of possible outcomes.

Children's understanding of fairness has been widely examined within moral development, where conceptions of fairness are described as evolving from simple notions of equality toward more sophisticated judgments (Piaget, 1932; Kohlberg, 1969/1994). These perspectives, however, have focused primarily on distributive and social justice rather than fairness in situations governed by chance and uncertainty.

Evaluating fairness in chance-based situations requires probabilistic reasoning. Fairness cannot be inferred solely from observed outcomes because short-term results may appear unequal in fair games or balanced in unfair ones. Judging fairness requires reasoning about uncertainty, randomness, and long-run patterns rather than relying only on immediate outcomes.

Research in probability education has shown that children frequently experience difficulties when reasoning about uncertain events, including challenges in understanding randomness, coordinating theoretical and experimental probabilities, and recognizing variability (Fischbein, 1975; Jones et al., 1997; Pratt, 2000). Children's probabilistic reasoning also develops progressively from intuitive and outcome-based interpretations toward more sophisticated understandings of sample space, probability, and long-run frequencies (Langrall & Mooney, 2005). Consequently, evaluating whether a game is fair requires reasoning beyond observed outcomes toward the underlying probabilistic structure of the game.

Research focusing specifically on fairness in games suggests that students do not always evaluate fairness through formal probabilistic reasoning. Kazima (2006) found that students frequently interpreted fairness through everyday meanings of the term, drawing on game outcomes, participation opportunities, and personal experiences. Watson and Moritz (2003) reported that judgments ranged from intuitive beliefs to sophisticated understandings involving equal chances and long-run expectations. Similarly, Dayal and Sharma (2020) found that pre-service teachers' initial fairness judgments about a dice game were frequently affected by equiprobability bias, but many revised their judgments after engaging in gameplay and examining experimental data. Their findings indicate that conceptions develop over time and may be grounded in different forms of reasoning that can be reshaped through direct engagement with probabilistic evidence. Children's probabilistic judgments are likewise grounded in everyday experiences and intuitive reasoning (Nikiforidou, 2018; Kingston & Twohill, 2022; Tversky & Kahneman, 1974).

Research examining fairness in probabilistic contexts suggests that children's judgments are shaped through engagement with gameplay. Maher (1998) showed that children formulate and revise hypotheses about the fairness of games while playing. Similarly, Paparistodemou et al. (2008) found that children's ideas about fairness in random situations were dynamically negotiated through games and interactive environments, moving between deterministic interpretations (e.g., equal treatment or symmetry) and stochastic interpretations involving randomness and uncertainty. Therefore, these studies suggest that gameplay provides opportunities for children to reflect on and refine their reasoning about fairness.

Recent studies further emphasised that evaluating the fairness of games requires the coordination of multiple probabilistic ideas. Batista et al. (2022) found that judgments about game fairness depend on students' understanding of randomness, sample space, and the comparison of probabilities. Similarly, Gea et al. (2023) found that students' understanding of fair games was closely related to their proportional reasoning. Therefore, it could be claimed that judging the fairness of games involves coordinating multiple probabilistic ideas rather than relying solely on intuitive beliefs.

Despite the close relationship between fairness and probability, previous research has examined students' everyday conceptions of fairness (e.g., Kazima, 2006), the probabilistic components underlying fairness judgments (e.g., Batista et al., 2022; Gea et al., 2023), or how gameplay shapes the revision of

such judgments (e.g., Watson & Moritz, 2003; Dayal & Sharma, 2020). Consequently, less is known about the extent to which elementary school students spontaneously draw on probabilistic ideas when judging the fairness of games, how these ideas coexist with non-probabilistic forms of reasoning, and how gameplay shapes the reasoning underlying their judgments.

The present study makes three contributions to the literature. First, it examines the extent to which elementary school students' fairness judgments are informed by probabilistic and non-probabilistic ideas. Second, by following the same group of students across two gameplay experiences, it explores how the reasoning underlying these judgments is maintained, revised, or reconstructed through gameplay. Third, by identifying the probabilistic and non-probabilistic ideas underlying students' fairness judgments and how these evolve through gameplay, the study offers implications for how fairness can be used as a pedagogical context in probability teaching in the elementary grades.

In this study, fairness judgments are conceptualized as evaluations informed by probabilistic as well as non-probabilistic ideas. Because probability is not yet formally taught at this grade level, the study can capture students' spontaneous, informally developed probabilistic ideas. Accordingly, the aim of the study is to examine whether students use probabilistic ideas when judging the fairness of games and how these judgments occur through gameplay and change after gameplay. Based on this aim, the present study is guided by the following research questions: (1) To what extent are fifth-grade students' fairness judgments informed by probabilistic ideas? (2) What influences students to maintain, revise, or reconstruct their fairness judgments after gameplay?

Theoretical Framework

Fairness as a Probabilistic Concept

Fairness has traditionally been examined within the field of moral development, where researchers described children's understanding of fairness as developing from simple notions of equality toward increasingly sophisticated conceptions of justice. In probability, however, fairness has a more specific meaning. A game is considered probabilistically fair when all players have equal opportunities to win, regardless of short-term outcomes. This interpretation differs from everyday notions of fairness, which typically emphasise equal treatment, equal participation, or adherence to rules (Hourigan & Leavy, 2020). Distinguishing between everyday and probabilistic fairness is therefore essential when evaluating chance-based games, as games that appear fair in everyday terms may nevertheless provide unequal winning opportunities. Conversely, games with unequal short-term outcomes may still be probabilistically fair if each player has the same underlying chance of success. Accordingly, evaluating game fairness requires attention to the probabilistic structure of the game rather than to observed outcomes alone.

Probabilistic Reasoning and Fair Games

Judging whether a game is fair requires more than deciding whether its rules appear equitable; it also requires coordinating several probabilistic ideas. Bryant and Nunes (2012) identify four cognitive demands in probabilistic reasoning: understanding randomness, working out the sample space, quantifying probabilities, and reasoning about correlations. The first three of these, which they describe as the steps required in solving any probability problem, are directly relevant to judging the fairness of games: recognising that outcomes are uncertain, identifying the set of possible outcomes, and comparing players' probabilities of winning. The fourth demand, reasoning about correlations, is not typically implicated in fairness judgments and is therefore not addressed in this study. Together, these ideas enable individuals to evaluate whether players have equal opportunities to win and to distinguish the probabilistic structure of a game from the outcomes observed in a limited number of trials.

Recent research emphasised that children's probabilistic reasoning develops from informal ideas about chance and uncertainty that emerge through everyday experiences rather than formal instruction (Nikiforidou, 2018). As children's understanding develops, they progressively coordinate increasingly sophisticated ideas about sample space, theoretical and experimental probability, and long-run frequencies (Jones et al., 1997; Langrall & Mooney, 2005; Pratt, 2000). Consequently, children's judgments about game fairness may reflect different forms of probabilistic thinking, ranging from intuitive comparisons of winning opportunities to more formal probabilistic reasoning.

These results empirically support Bryant and Nunes's (2012) framework: Batista et al. (2022) demonstrated that these three components jointly shape students' fairness evaluations, while Gea et al. (2023) showed that proportional reasoning, which is closely tied to probability comparison, plays a similarly central role. Therefore, it could be claimed that fairness judgments provide a productive context for examining the probabilistic ideas students mobilise when reasoning about games of chance.

In the present study, the first three of these demands, which are understanding randomness, working out the sample space, and quantifying probabilities, serve as sensitizing concepts for the analysis. Although the analysis remained inductive, the framework provides the vocabulary for characterising the probabilistic quality of the ideas students mobilise when judging fairness.

Conceptual Framing of the Study

The present study conceptualizes fairness judgments as evaluations informed by both probabilistic and non-probabilistic ideas. When deciding whether a game is fair, children may draw on probabilistic ideas, such as attending to the composition of the sample space, comparing winning probabilities, or interpreting randomness, as well as on non-probabilistic ideas, including everyday notions of fairness such as equal rules, rights, and participation (Kazima, 2006), strategic competence, and observed gameplay outcomes. From this perspective, fairness judgments are not treated as fixed beliefs but as interpretations that reflect the probabilistic ideas children mobilise in response to a particular game situation.

Gameplay provides opportunities for these interpretations to be maintained, revised, or reconstructed as children encounter probabilistic evidence through repeated experiences. Rather than viewing fairness solely as a moral concept, the present study treats it as a pedagogical context that makes children's probabilistic ideas visible. Accordingly, the analysis focuses on whether children judge a game as fair or unfair, and on the probabilistic ideas underlying those judgments and how they evolve through gameplay.

Methodology

This study employed a qualitative exploratory design to investigate how fifth-grade students reason about the fairness of games and how they justify and revise their fairness judgments. The study was conducted in two fifth-grade classrooms in a public Imam Hatip elementary school in Türkiye, where girls and boys were taught in separate classrooms in this school. One classroom consisted of 36 boys and the other of 32 girls, with students ranging in age from 10 to 11 years. Students worked in pairs throughout the tasks, as collaborative discussion was considered essential for eliciting and documenting their fairness reasoning.

Participants

Fifth-grade students were selected because they are at a developmental stage in which both probabilistic reasoning and fairness-related judgments begin to become more sophisticated. Research suggests that children at this age increasingly attempt to coordinate intuitive beliefs, personal experiences, and emerging understandings of chance and uncertainty when making judgments about probabilistic situations (Nikiforidou, 2018; Jones et al., 1999). This makes them particularly suitable for investigating the relationship between probabilistic reasoning and fairness judgments.

Data Collection Procedure

Data were collected during two consecutive classroom sessions through two game-based tasks completed in pairs: a Rock–Paper–Scissors (RPS) task followed by a specially designed biased die task. In both tasks, students first evaluated whether they considered the game fair and justified their reasoning before gameplay. They then played ten rounds of the game, recorded the outcome of each round on a worksheet, and re-evaluated the fairness of the game after gameplay. In the RPS task, students were also asked to explain what they understood by a fair game in order to elicit their initial conceptions of fairness. Within each pair, the student seated on the left was designated as Player1 and the student seated on the right as Player2. The same roles were maintained across both tasks to ensure consistency. The RPS game was administered first and the biased die game second. This order allowed students to first articulate their reasoning about a familiar, structurally fair game before encountering a game whose fairness was grounded in a visible probabilistic structure.

The first task used the familiar game of RPS, which provided equal winning opportunities for both players. The second task used a custom-made six-sided die produced with a 3D printer, with a triangle

symbol on four faces and a square symbol on two faces. Player1 scored on triangles and Player2 on squares, creating unequal winning opportunities for the two players. (The full task worksheets are provided as appendix in Online Resource 1.)

The two games were selected to contrast the grounds on which fairness could be evaluated. In the biased die game, fairness judgments could be grounded in the game's probabilistic structure, which was perceptually available in the composition of the die. In the RPS game, fairness was grounded in the symmetric structure of the rules: both players had the same set of actions, moved simultaneously, and faced identical winning conditions, so that neither player was structurally advantaged, consistent with Batista et al.'s (2022) definition of a fair game as one in which players have the same chances of winning. At the same time, unlike the die game, outcomes in RPS are produced by players' choices rather than by a randomiser alone, which allowed us to examine whether students would mobilise probabilistic ideas even when no probabilistic structure was directly visible.

The primary data source consisted of students' written responses to the pre- and post-game questions. Whole-class discussions were audio-recorded but not systematically coded; they were used to contextualise pair-level interpretations and to check whether the ideas expressed in written responses were consistent with those voiced publicly.

Data Analysis

Data were analysed using thematic analysis following the procedures proposed by Braun and Clarke (2006). An interpretive-inductive approach guided the analysis, allowing patterns in students' fairness reasoning to emerge from their written responses. Rather than focusing solely on whether students judged a game as fair or unfair, the analysis examined the probabilistic and non-probabilistic ideas underlying those judgments across the two game contexts.

Each pair's initial and final fairness judgments and written explanations constituted the primary analytical material. Classroom discussions were consulted only where necessary to support the interpretation of students' written responses. The pair was treated as the primary unit of analysis, as students discussed and recorded their judgments collaboratively. Where members of a pair expressed divergent judgments, these were coded separately and retained as contrasting cases rather than being forced into consensus.

The analysis followed the six phases of thematic analysis outlined by Braun and Clarke (2006) and was conducted by the author using the analytic matrices described above. The credibility of the analysis was supported in three ways. First, coding was iterative: transcripts coded in the early stages were revisited as themes developed, and codes were refined accordingly. Second, negative cases, i.e. pairs whose judgments or reasoning contradicted the emerging themes, were actively sought and are reported explicitly in the Findings. Third, all coding decisions were documented in the analytic matrices, providing an audit trail from raw responses to themes. In addition, approximately 20% of the worksheets were

independently reviewed by a colleague familiar with the research context; interpretations were compared and discrepancies discussed until agreement was reached.

All written responses were produced in Turkish. Excerpts quoted in this article were translated into English by the author, keeping the wording as literal as possible while ensuring readability. Translations were cross-checked against the original responses during the review of themes.

Findings

Fairness judgments were analysed at the pair level (n = 34). Table 1 summarises the distribution of pairs' judgments before and after gameplay. In the biased die game, all 34 pairs initially judged the game as unfair. Following gameplay, 32 pairs maintained this judgment, one pair revised its judgment to fair (P15), and the members of one pair diverged (P8). In the RPS game, judgments were more heterogeneous from the outset: 19 pairs initially judged the game as fair, 11 as unfair, and four pairs expressed mixed or divergent views.

Table 1
Distribution of pairs' fairness judgments before and after gameplay

Game	Judgment	Pre-game	Post-game
Biased Die	Fair	0	1
	Unfair	34	32
	Divergent within pair	0	1
RPS	Fair	19	16
	Unfair	11	10
	Mixed/divergent	4	8

Following gameplay, 23 pairs maintained their initial judgment, whereas 11 pairs revised it, five of which reversing their judgment entirely and six shifting between a definite and a mixed or divergent position (Table 2). In the biased die game, gameplay outcomes were largely consistent with the game's structure: Player1 won in 23 pairs, six pairs tied, and the disadvantaged Player2 won in five pairs.

Table 2
Trajectories of pairs' fairness judgments across gameplay

Type of Change	*Description*	*Biased die*	*RPS*
Judgment maintained	Same judgment, same justification in essence	32	23
Judgment revised	Judgment changed after gameplay	2	11
Reversal (fair – unfair)	Fair ↔ unfair switch	1 (P15)	5 (P5, P29, P32; P15, P31)
Shift to/from mixed-divergent	Definite ↔ mixed/divergent	1 (P8)	6 (P1, P3, P6, P7, P8, P28)

Fairness reasoning in the biased die game

All 34 pairs initially judged the biased die game as unfair, and 32 maintained this judgment after gameplay (Table 1). However, the reasoning underlying these judgments was not uniform. Thematic analysis revealed four recurring patterns through which students justified their evaluations of fairness: (a) recognizing unequal game structure, (b) inferring unequal winning probabilities, (c) interpreting fairness through equal opportunity, and (d) coordinating randomness with structural unfairness. These themes illustrate how students progressed from noticing the structural characteristics of the game to constructing increasingly sophisticated probabilistic justifications for their fairness judgments. A small number of contrasting cases also emerged, in which students relied primarily on observed gameplay outcomes.

Read through the lens of Bryant and Nunes's (2012) framework, these themes reflect increasingly coordinated uses of the three components identified above: noticing the die's composition represents an intuitive engagement with the sample space; inferring unequal winning chances corresponds to comparing probabilities; and coordinating randomness with structural unfairness reflects an emerging understanding that short-term variability does not override a game's underlying probabilistic structure. The third theme, interpreting fairness through equal opportunity, marks the point at which probabilistic comparison is translated into the everyday moral language of fairness.

Recognizing unequal game structure

For many students, the first step in evaluating fairness was to examine the physical structure of the die. Rather than referring to probability or gameplay outcomes, the students focused on the unequal distribution of symbols on the die. Simply observing that four faces displayed triangles whereas only two displayed squares were considered sufficient evidence that the game was unfair. At this stage, students' reasoning remained close to the observable characteristics of the game and did not yet involve explicit probabilistic explanations.

For example, P1 justified their judgment by stating, “4 triangles, 2 squares.” Likewise, P26 argued, “Triangles are more.” and even described the die as “rigged”, indicating that the unequal arrangement of the symbols itself was perceived as the source of unfairness.

These responses suggest that students initially interpreted fairness through the visible properties of the game rather than through formal probabilistic reasoning.

Inferring unequal winning probabilities

For many students, identifying the unequal distribution of symbols was only the starting point of their reasoning. They went beyond describing the physical structure of the die and explicitly inferred that the unequal distribution resulted in unequal chances of winning for the two players. This marked a shift from describing the game to explaining why it was unfair.

Several pairs explicitly connected the number of symbols on the die to the players’ winning chances. For example, P2 stated, “Player1 is luckier.” and justified this by adding, “(Number of) squares and triangles are not the same.” Other pairs used explicit probabilistic language when justifying their judgments. For instance, P13 explained, “The probability of Player1’s winning is more.” while P24 stated, “Player1 is more likely to win because the die has more triangles.” Two pairs went further by framing winning chances in terms of the number of favourable outcomes, explaining that Player1 had more winning chances because “the number of moves that make [Player1] win is greater” (P2, P10), which is an intuitive approximation of sample-space reasoning. One pair (P17) provided the most sophisticated justification by expressing the players’ winning chances numerically as “One is 4/6, the other is 2/6.”

These responses indicate a progression from structural observation to probabilistic inference, as students increasingly interpreted fairness in terms of the probabilities generated by the game’s structure. Their arguments demonstrate an emerging understanding that fairness depends not only on the visible characteristics of a game but also on the probabilities that those characteristics generate.

Interpreting fairness through equal opportunity

As students’ reasoning became more elaborated, many moved beyond identifying unequal probabilities and began to interpret fairness in terms of equal opportunities. Rather than focusing solely on which player was more likely to win, these students evaluated whether both players had the same chance or conditions to succeed. Fairness was therefore understood as providing equal opportunities for all players rather than producing equal outcomes.

Several pairs explicitly framed their judgments using the language of equality, rights, and advantage. For example, P9 argued that the game was unfair because “Player2 is disadvantaged.” and explained that “winning conditions were not the same.” P11 expressed this idea even more explicitly by stating, “Our rights are fewer than Player1’s,” framing fairness in terms of unequal rights rather than probability alone. Likewise, P25 argued that “Player1 has more advantages,” emphasising that one player entered the game with greater opportunities to win.

Rather than treating unequal probabilities as purely mathematical differences, these students viewed them as unequal opportunities and unequal rights.

Coordinating randomness with structural unfairness

A smaller group of students demonstrated a more sophisticated form of reasoning by simultaneously acknowledging the role of randomness in individual game outcomes while maintaining that the game itself remained unfair. Rather than treating an unexpected outcome as evidence that the game was fair, these students distinguished between short-term variability and the long-term advantage created by the unequal distribution of symbols.

Several pairs explicitly coordinated these two ideas. For example, P6 acknowledged the possibility of random outcomes by stating, “we may be lucky, or we may not,” yet still concluded that the game was unfair because “triangles were more.” Similarly, P21 explained that “the lucky one wins,” while continuing to justify their fairness judgment by referring to the unequal structure of the die. One of the clearest examples was provided by P29 (Player2 won the game in this pair), who stated that although “Player2 was lucky, but squares were less than triangles.” the game nevertheless remained unfair because “squares were less than triangles.” Likewise, P18 maintained that the game was unfair despite a tied result, explaining that the unequal distribution of symbols, not the observed outcome, should determine judgments about fairness.

These responses indicate that some students distinguished random variation in individual trials from the structural probabilities embedded in the game, which is the most sophisticated pattern identified in the biased die game. A small number of pairs, however, interpreted gameplay outcomes differently.

In contrast, a small number of pairs interpreted unexpected gameplay outcomes as evidence that the game was fair. For example, P15 changed their judgment after observing that Player2 won, explaining, “Although Player2 had a lower chance of winning, they won.” The members of P8 diverged: one judged the game as fair, describing it as a matter of luck, whereas the other maintained that it was unfair, remarking that the unequal distribution of symbols “changes nothing.” Unlike the dominant pattern identified above, these students allowed isolated gameplay experiences to override the game’s underlying probability structure, relying more on observed outcomes when judging fairness.

Figure 2 summarises these patterns and their relation to the probabilistic demands identified by Bryant and Nunes (2012).

Maintained judgments, refined reasoning

Maintaining an initial judgment did not imply that gameplay left students’ reasoning unchanged. Among the 32 pairs who maintained their unfairness judgment in the biased die game, three patterns of refinement emerged (Table 3). First, four pairs shifted from intuitive references to luck or advantage towards more explicit probabilistic language after gameplay (P25, P27, P31, P33). Second, nine pairs encountered gameplay outcomes inconsistent with the game’s probabilistic structure, of which a tied

game (P2, P3, P11, P18, P19, P22) or a win by the disadvantaged player (P14, P21, P29), yet continued to justify their judgment through the die's unequal structure rather than the observed outcome. Only two of the 11 pairs who experienced such structure-inconsistent outcomes allowed the outcome to override the game's structure (P8, P15). Third, one pair (P32) reconstructed the basis of its judgment, shifting from a pre-game appeal to chance to a post-game appeal to the die's unequal composition.

Table 3
Patterns of reasoning refinement among pairs who maintained their unfairness judgment in the biased die game

Refinement pattern	***Pairs***	***n***
Shift towards explicit probabilistic language (e.g., luck → "probability")	P25, P27, P31, P33	4
Structural reasoning maintained under structure-inconsistent outcomes	P2, P3, P11, P14, P18, P19, P21, P22, P29	9
Reasoning reconstructed (chance-based → structure-based)	P32	1
Reasoning essentially unchanged	remaining pairs	18

Fairness reasoning in the Rock–Paper–Scissors (RPS) game

Unlike the biased die game, students' fairness judgments in the RPS game were not dominated by a single line of reasoning. Although 19 of the 34 pairs initially judged the game to be fair, pairs relied on different criteria when explaining their decisions. Thematic analysis identified four recurring patterns: (a) procedural fairness, (b) player competence, (c) chance as a fairness criterion, and (d) outcome-based fairness. Rather than representing a developmental progression, these themes illustrate complementary ways of interpreting fairness in a game that provided equal winning opportunities. Most students distinguished the fairness of the game from the causes of winning, attributing outcomes instead to strategy, prediction, experience, or chance.

Unlike the biased die, the probabilistic structure of the RPS game was not perceptually available to students: equal winning opportunities cannot be seen in the way that four triangles and two squares can. Accordingly, students' reasoning drew less directly on the components of probabilistic reasoning and more on the broader repertoire identified in the conceptual framing of this study: everyday procedural fairness, strategic competence, and intuitive interpretations of chance.

Procedural fairness

The most common basis for judging fairness in the RPS game was students' emphasis on procedural equality. Rather than focusing on who won the game, these students argued that fairness depended on whether both players followed the same rules, had equal rights, and participated under the same

conditions. Equal opportunities to choose rock, paper, or scissors and the simultaneous reveal of players' choices were frequently cited as evidence that the game was fair.

For example, P11 justified their judgment by stating, "both of us had the rights to rock, paper, and scissors." and "we played with the same rules." Similarly, P5 concluded after gameplay that "we played equally." Likewise, P22 argued that "a game is fair if everyone has equal rights." and further emphasised that "we both played rock, paper, scissors at the same time." Other students highlighted equal starting conditions more directly. For instance, P23 explained, "each player shows their hand at the same time."

These responses indicate that students primarily associated fairness with equal rules, equal rights, and equal conditions rather than with game outcomes.

Player competence

While many students agreed that the game itself was fair, they frequently attributed differences in game outcomes to the players rather than to the structure of the game. Success was commonly explained through strategy, prediction, planning, intelligence, experience, or the ability to anticipate an opponent's next move. In these responses, fairness and performance were treated as two distinct ideas: the game provided equal opportunities for both players, but the winner was determined by differences in player competence.

Several pairs explicitly associated successful performance with strategic thinking. For example, P3 argued that "the one who plays well and guesses well wins," emphasising both skill and prediction. Similarly, P12 stated that "the one who can read minds wins," suggesting that success depended on anticipating an opponent's decisions rather than on chance. Other students highlighted strategic adaptation during gameplay. For instance, P23 explained the outcome by referring to "because I'm smarter," whereas P20 argued that "Player2 got way ahead by using strategy." Likewise, P29 revised their initial judgment after gameplay, explaining that "I guessed what my opponent would do," and concluding that the game was fair because success could be achieved through strategic prediction rather than luck. These responses suggest that students viewed differences in performance as compatible with fairness because both players began the game with equal opportunities.

In contrast, a small number of students interpreted strategic behaviour itself as creating unfairness. For example, P16 argued that players who knew the game better or "she was making sneaky plans" possessed an unfair advantage. Similarly, P31 concluded that repeatedly making the same move ("it wasn't fair that he kept making the same moves.") reduced the fairness of the game. These contrastive cases indicate that not all students viewed strategic competence as compatible with fairness; for some, successful strategy itself became evidence that the game was unequal.

Chance as a fairness criterion

Another recurring pattern concerned the role that students attributed to chance when evaluating fairness. Unlike the biased die game, where randomness was generally coordinated with the game's

probability structure, students in the RPS game expressed contrasting interpretations of chance. Some regarded the presence of chance as evidence that the game was unfair, whereas others argued that chance actually supported fairness because it affected both players equally.

Several pairs consistently viewed chance as incompatible with fairness. For example, P9 argued that “luck (chance) is relative” and concluded that “luck (chance) cannot be equal.” Likewise, P21 maintained that “this game doesn’t depend on the ability” and repeatedly emphasised that “depending on luck (chance)” games could not be considered fair. Similarly, P27 justified their judgment by stating that “it’s a game of chance” and referred to “probabilities” as evidence that the game was unfair. These responses suggest that some students viewed randomness itself as conflicting with their intuitive understanding of fairness.

In contrast, other students interpreted chance differently. Rather than seeing randomness as a source of unfairness, they argued that chance contributed to fairness because both players were equally affected by it. For example, P30 explained that “the main factor in this game is chance” but immediately added that “the chances were equal.” Likewise, P34 argued that “everyone has a 50% chance” and emphasised that although “luck decided the winner,” the game remained fair because both players had equal chances. Similarly, P17 combined chance, strategy, and the tied outcome, concluding that “the chances were equal because we tied.” Students’ references to “50–50” chances treated winning and losing as the only relevant outcomes, setting aside ties; such statements represent informal equiprobability claims rather than computed probabilities.

Thus, students interpreted chance in two contrasting ways: for some it undermined fairness because outcomes were uncontrollable, whereas for others it supported fairness because both players were equally affected by it.

Revising fairness judgments through gameplay outcomes

A small number of pairs reconsidered their evaluations after observing gameplay outcomes, allowing their direct experiences during the game to reshape their understanding of fairness.

Several pairs revised their judgments after experiencing unexpected results. For example, P7 initially agreed that the game was fair because “we can’t break the rules,” but one student later concluded, “it’s not fair because I lost.” Likewise, P28 abandoned their initial belief that “everyone is in the same position” after gameplay, arguing instead that “Player1’s probability was very low” because one player had won. Similarly, P32 initially rejected the game because “it’s a game of chance,” yet after gameplay they reversed their position, concluding that “the chances were equal, the game was fair.” In another notable example, P29 shifted in the opposite direction. Initially, they regarded the game as unfair because it involved chance, but after recognising that they could anticipate an opponent’s behaviour (“I guessed what my opponent would do”), they concluded that the game was fair because success depended on strategic prediction rather than luck.

These responses suggest that some students reconstructed their fairness judgments in light of their gameplay experiences rather than maintaining a stable criterion throughout the task.

Summary of the Findings

Overall, students did not rely on a single criterion when evaluating fairness; the ideas they mobilised depended on the characteristics of the game. In the biased die game, whose probabilistic structure was perceptually salient, judgments centred on unequal winning opportunities, whereas the RPS game elicited a broader repertoire, which includes procedural fairness, strategic competence, and contrasting interpretations of chance. Gameplay rarely overturned judgments; its primary effect was to refine the reasoning underlying them.

Figure 3 illustrates how pairs moved between judgment categories from pre-game to post-game in each game context.

Discussion

The present study examined the extent to which fifth-grade students' fairness judgments were informed by probabilistic ideas and the factors that influenced the maintenance, revision, or reconstruction of these judgments following gameplay. Three findings are particularly noteworthy. First, the findings demonstrate that children draw on a range of probabilistic ideas when judging whether a game was fair, despite having received no formal probability instruction. Second, the probabilistic ideas students mobilised varied according to the characteristics of the game they were evaluating. Third, fairness provides a productive context for eliciting children's emerging probabilistic ideas before formal probability instruction.

Students' fairness judgments were informed by a range of probabilistic ideas

The findings suggest that children drew on a range of probabilistic ideas when evaluating whether a game was fair. Rather than relying on formal probability concepts or numerical calculations, they frequently reasoned through intuitive ideas such as unequal winning opportunities, chance, advantage, equal opportunities, and strategy. These findings suggest that meaningful probabilistic reasoning can emerge through everyday intuitions long before students acquire formal probability knowledge.

One particularly noteworthy finding was that many students identified the biased die game as unfair before receiving any formal instruction in probability. Rather than performing numerical probability calculations, they intuitively compared the players' winning opportunities and recognised that unequal chances of success made the game unfair. These intuitive comparisons are consistent with research showing that proportional reasoning provides a foundation for probabilistic reasoning before formal instruction (Begolli et al., 2021) and with studies demonstrating that even young children can distinguish fair from unfair probabilistic situations through intuitive reasoning (Mousoulides & English, 2009). The

present study extends this work by showing not only that elementary students can make such judgments, but also the diverse probabilistic and non-probabilistic ideas they mobilise when explaining why a game is fair or unfair.

In the RPS game, by contrast, many students reasoned through strategy, chance, procedural equality, and player behaviour, suggesting that the ideas children mobilise depend on the characteristics of the game they are evaluating. This interpretation is consistent with Pielsticker (2022), who similarly argued that students' understandings of fairness extend beyond probability calculations to include equal rules, equal opportunities, and equal treatment within game contexts.

One further distinction is noteworthy for probability education. In the RPS game, students mobilised the notion of chance in two contrasting ways: some interpreted chance as uncontrollability, arguing that a game whose outcomes cannot be controlled cannot be fair, whereas others interpreted chance as equal randomness, reasoning that fairness follows from both players being equally subject to randomness. These two readings correspond, respectively, to attending to the unpredictability of individual trials and to the symmetry of the underlying probability distribution, which is a distinction central to understanding randomness (Bryant & Nunes, 2012). That children spontaneously articulated both readings before instruction suggests that fairness discussions can make this foundational distinction explicit and open to examination in the classroom.

The findings also illustrate how the components of probabilistic reasoning identified by Bryant and Nunes (2012) can emerge in informal forms before instruction. Some students' explanations approximated an intuitive engagement with the sample space, as when two pairs justified the die game's unfairness by noting that "the number of moves that make [Player1] win is greater" (P2, P10). Others spontaneously compared winning probabilities, occasionally in numerical form (P17), or drew on proportional language such as unequal "ratios" (P34), which is a repertoire consistent with Gea et al.'s (2023) finding that understanding fair games is closely tied to proportional reasoning. Finally, the nine pairs who maintained structural justifications despite structure-inconsistent outcomes demonstrated an informal grasp of randomness as variability around a stable structure. That all three components identified above were observable in students' informal reasoning suggests that fairness contexts can make the building blocks of probabilistic reasoning visible before they are formally taught.

This context-dependence is consistent with research showing that probabilistic reasoning is sensitive to task characteristics (He & Chen, 2025), moderated by cognitive resources (Chiesi et al., 2011), and involves the coordination of multiple conceptual resources (Nilsson, 2009). The present findings extend these perspectives by showing that children's fairness judgments emerged through the flexible coordination of probabilistic, procedural, strategic, and experiential ideas. By following the same group of students across two game contexts, the study also extends research conducted within a single game context (e.g., Batista et al., 2022). Notably, whereas pre-service teachers often struggle with fair-game tasks (Hourigan & Leavy, 2020; Burgos et al., 2023), many students in the present study spontaneously attended to unequal winning opportunities.

Gameplay supported the refinement of children's fairness judgments

The second research question examined the factors that influenced students' maintenance, revision, or reconstruction of their fairness judgments following gameplay. The findings suggest that gameplay provided opportunities for students to test, justify, and sometimes reconsider the criteria they used when evaluating whether a game was fair.

Importantly, maintaining an initial judgment should not be interpreted as evidence that gameplay had little impact. Many students who retained their original evaluations nevertheless elaborated or refined their justifications after playing. In the biased die game, for example, students who initially recognised an unequal advantage frequently strengthened their explanations by referring more explicitly to unequal winning opportunities, probability, or fairness as equal opportunity. Gameplay often refined existing ideas rather than replacing them. This finding is consistent with Dayal and Sharma (2020), who likewise found that gameplay encouraged learners to reconsider and refine their initial judgments as they compared their predictions with experimental evidence; the present findings extend this work by showing that such refinement occurs even before formal probability instruction.

The contrasting cases identified in both games further illustrate the dynamic nature of students' fairness reasoning. Rather than simply reacting to winning or losing, these students reconsidered the criteria they regarded as relevant for evaluating fairness, including strategy, chance, gameplay experiences, and unequal winning opportunities. Similar patterns were reported by Maher (1998), who observed that gameplay encouraged students to revise initial hypotheses as new evidence emerged; the present study highlights how this reshaping involved the interaction of probabilistic and non-probabilistic ideas.

A small number of students evaluated fairness primarily through the immediate outcome of the game. In these cases, winning sometimes led students to perceive an objectively unfair game as fair, whereas losing occasionally prompted them to judge an otherwise fair game as unfair. Although relatively infrequent, these cases suggest that some children's fairness judgments were guided more by immediate outcomes than by the game's probabilistic structure.

Importantly, in many cases, it was not students' fairness judgments that changed, but the reasoning underlying those judgments. Its educational value therefore lies not only in exposing students to probabilistic outcomes but also in creating opportunities to articulate, examine, and refine their emerging probabilistic ideas about fairness.

The dual nature of the RPS game, which are symmetric in structure, yet contingent on players' choices in play, was directly reflected in students' reasoning: some construed the game as a game of chance in which luck was equal, others as a game of skill in which prediction and pattern recognition conferred an advantage, and still others as a rule-governed procedure. This heterogeneity does not indicate a flaw in the task; rather, it reveals that fairness judgments depend on how students construe the source of

outcomes, such as chance, strategy, or rules, when no probabilistic structure is visible. Notably, educational resources on RPS game similarly caution that its outcomes are not truly random, as players' choices are shaped by previous rounds and can be strategically exploited. The contrast with the die game, where the fairness-relevant structure was perceptually available, underscores the context-dependent nature of students' probabilistic reasoning about fairness.

Fairness as a productive context for probability learning

The findings have important implications for probability teaching in the elementary grades: participants who had not yet received formal probability instruction demonstrated a range of intuitive probabilistic ideas, suggesting that children's existing ideas about fairness, chance, and winning opportunities provide valuable starting points for developing probabilistic reasoning. Fairness discussions therefore offer a productive context in which children's informal probabilistic ideas can be elicited and examined before formal instruction.

Engaging students in discussions about fairness, equal opportunities, and chance before introducing formal probability concepts encourages children to articulate and justify their existing ideas, providing a foundation on which formal concepts can be built (Fischbein, 1975; Jones et al., 1997; Batanero et al., 2016). Addressing these ideas explicitly also matters for teacher education, as pre-service teachers themselves struggle to identify proportional reasoning in students' responses to fair-game tasks (Burgos et al., 2023).

Games are thus valuable not only for illustrating probabilistic concepts but also for revealing how children already reason about chance, advantage, and equal opportunities. Rather than viewing fairness merely as a context for applying probability, educators may treat it as a context for revealing and developing children's emerging probabilistic thinking (Nilsson, 2009).

Limitations and future research

Despite the contributions of this study, several limitations should be acknowledged. First, the study was conducted with fifth-grade students from a single public elementary school, limiting the transferability of the findings to other educational contexts and age groups. Second, the analysis relied on written explanations generated through two game-based tasks; individual clinical interviews could have provided a more detailed account of how students coordinated ideas such as fairness, chance, and strategy while constructing their explanations. Third, the school used single-sex classrooms; classroom and gender were therefore confounded. Gender was not treated as an analytic variable: the study asks how students reason about fairness, not how groups differ; I do not consider such a comparison necessary. Fourth, gameplay was limited to ten rounds per game; longer sessions could make long-run patterns more salient. This was a deliberate choice: it preserved sampling variability, allowing outcomes to deviate from the game's structure, such deviations that proved analytically productive (P15). Finally, the findings reflect students' intuitive reasoning prior to formal probability instruction and therefore should not be interpreted as evidence of formal probabilistic understanding.

Future research could examine how students' fairness judgments evolve across different grade levels, particularly after formal probability instruction. Comparative and longitudinal studies could clarify how intuitive probabilistic ideas develop into formal reasoning. In addition, future studies might investigate how different game contexts elicit different probabilistic ideas or explore whether engaging students in designing, modifying, or justifying games supports the development of probabilistic reasoning.

Conclusion

This study examined how fifth-grade students' fairness judgments drew on probabilistic ideas and how these judgments evolved through gameplay. Three findings stand out. First, children with no formal probability instruction spontaneously drew on a range of probabilistic ideas when judging game fairness. Second, the ideas they mobilised varied with the characteristics of the game: a visible probabilistic structure elicited structure-based reasoning, whereas a symmetric but strategy-laden game elicited procedural, strategic, and chance-based criteria. Third, gameplay rarely overturned judgments but refined the reasoning underlying them, suggesting that fairness contexts can make the building blocks of probabilistic reasoning visible before instruction. Fairness discussions therefore offer a productive pedagogical entry point for eliciting and developing children's informal probabilistic ideas in the elementary grades.

Declarations

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Ethics Approval and Informed Consent

The study adhered to the ethical guidelines set by the hosting institution. Ethical approval was obtained from the Mersin University Social and Human Sciences Ethics Committee on January 23, 2025 (Approval No. 25). The parents of all participants were informed about the aims of the study and voluntarily gave their written informed consent prior to participation.

Consent for Publication

Not applicable.

Competing Interests

The author declares no competing interests.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author used ChatGPT (OpenAI) to check language and grammar. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

Data availability statement

The data supporting the findings of this study are reported within the article and its supplementary material: the task worksheets are provided in Online Resource 1, and selected excerpts from students' written responses are presented throughout the manuscript. The completed worksheets containing students' full responses are not publicly available in order to protect the privacy and confidentiality of the participating children.

Acknowledgments

I am grateful to the administrators, mathematics teachers, and students of the participating elementary school for their willingness to engage with this research. Their contributions made this work possible.

Author Contributions

The author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

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Figures



Figure 1

Conceptual framing of the study: fairness judgments as evaluations informed by probabilistic and non-probabilistic ideas, developed through gameplay.

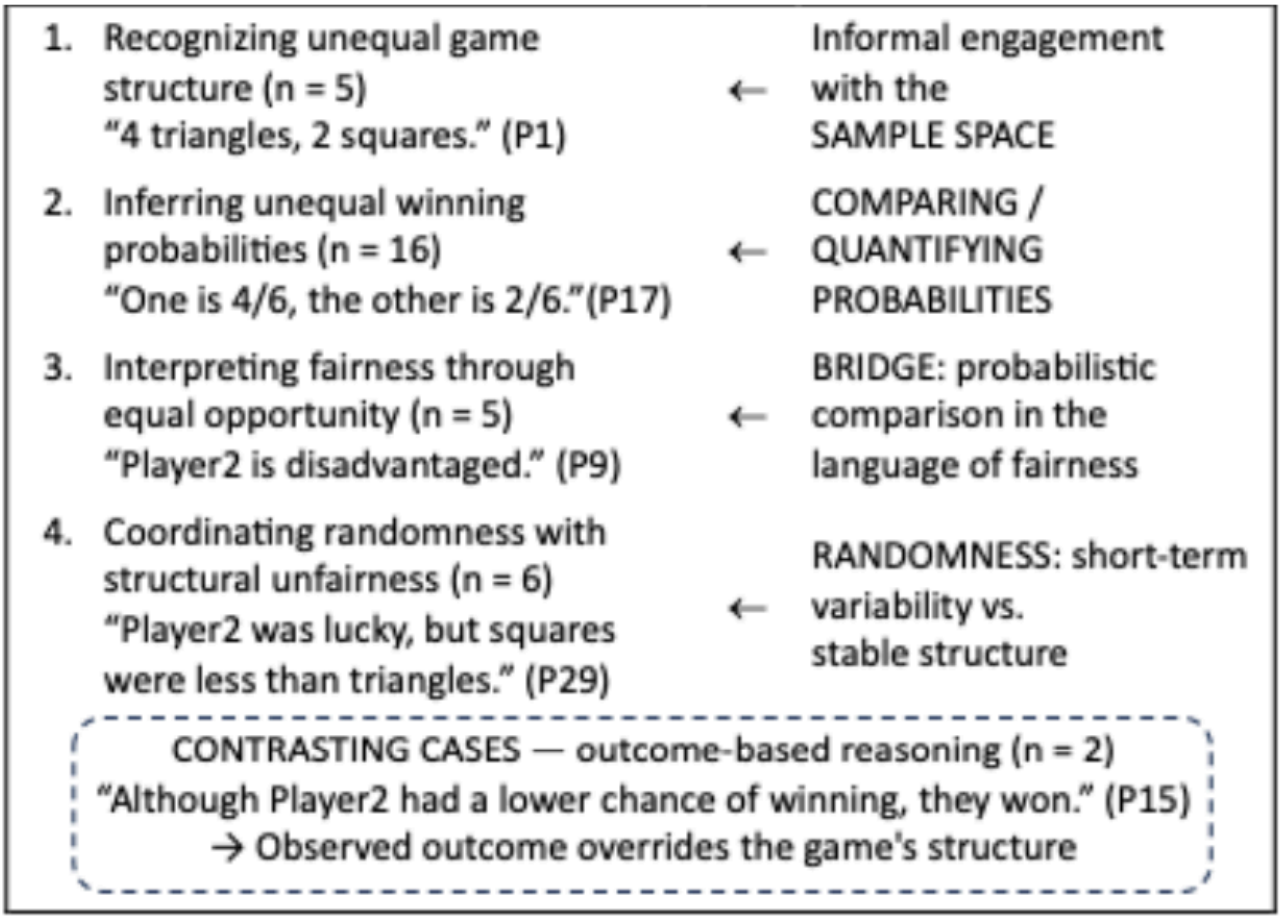


Figure 2

Patterns of fairness reasoning in the biased die game,
ordered by coordination of probabilistic demands (Bryant & Nunes, 2012).

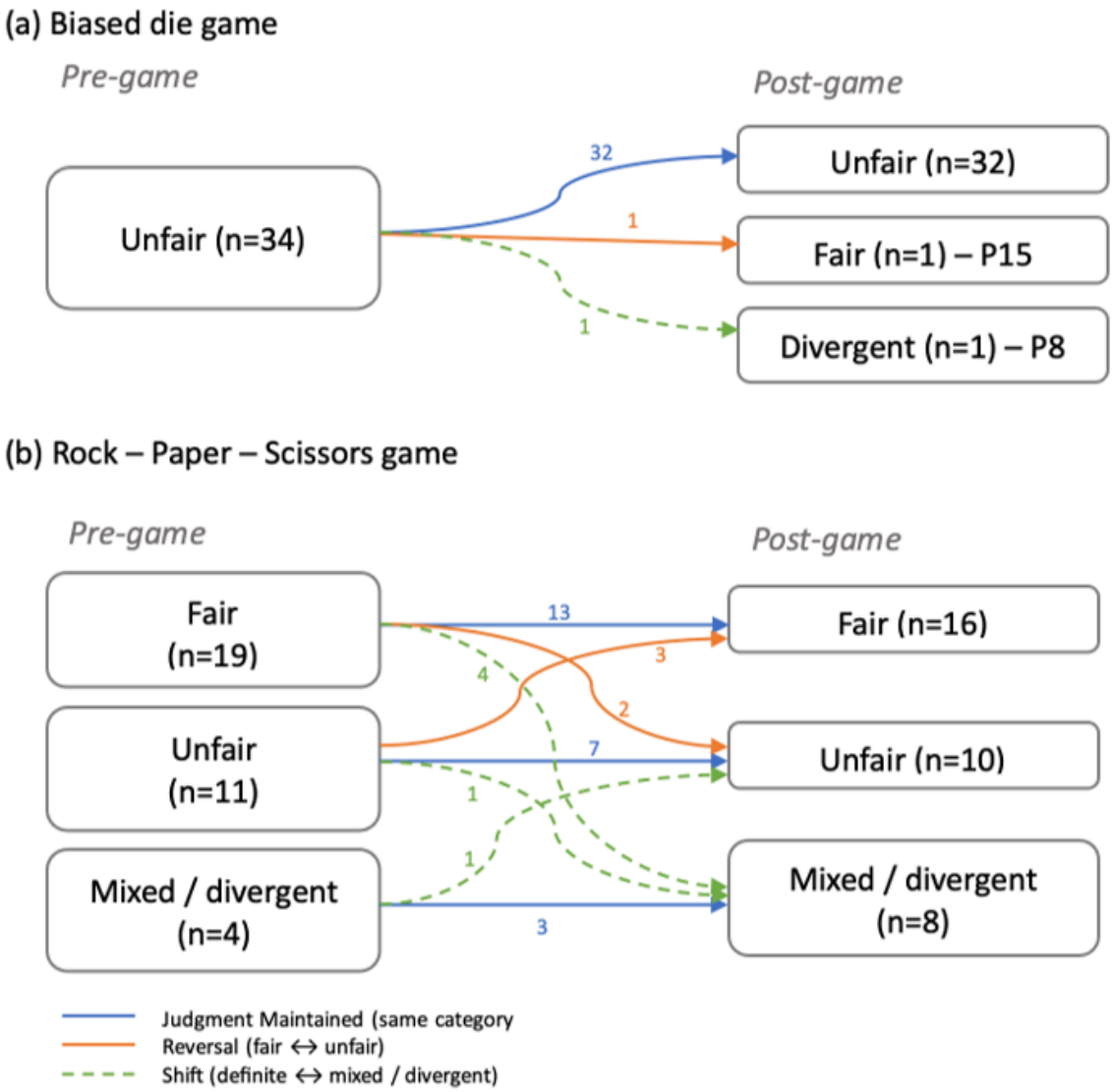


Figure 3

Movement of pairs' fairness judgments from pre-game to post-game in (a) the biased die game and (b) the rock–paper–scissors game. Labels on arrows indicate the number of pairs following each path. The classification of paths (maintained, reversal, shift) follows Table 2.