

Supplementary Information for "Quantized heat flow in moiré chern bands of bilayer graphene"

Santanu Samai¹, Debangana Sarkar¹, Abhijit Halder¹, T. Taniguchi², K. Watanabe², Subroto Mukerjee¹, Saurabh Kumar Srivastav³, Anindya Das^{1*}

¹*Department of Physics, Indian Institute of Science, Bangalore, India.*

²*National Institute of Material Science, Tsukuba, Japan*

³*Department of Physics, Indian Institute of Technology (BHU), Varanasi, India*

Supplementary Note 1. Device fabrication, characterization, and measurement setup

To investigate the Chern insulator (CI) and symmetry-broken Chern insulator (SBCI) states, we fabricated bottom-graphite-gated, hexagonal boron nitride (hBN)-encapsulated bilayer graphene (BLG) heterostructures using the standard dry-transfer technique¹. For Device 1, the top and bottom hBN flakes were obtained from a single hBN crystal that contained a natural crack, such that the two flakes shared a common crystallographic edge. During assembly, one of the sharp edges of the top hBN flake was aligned with the longest straight edge of the BLG^{2,3}. The bottom hBN flake was subsequently aligned so that the common edges of the top and bottom hBN flakes were oriented at an odd multiple of 30°. This alignment strategy ensures that the BLG is crystallographically aligned with only one of the hBN layers, thereby avoiding the formation of a double moiré superlattice. For Device 2, a different alignment strategy was employed. The BLG flake was first cut into two pieces. The first piece was aligned with the top hBN, while the second piece was picked up after a 30° rotation with respect to the first BLG piece. The bottom hBN was then picked up at an angle of approximately 15° relative to the top hBN to suppress the formation of a double moiré superlattice. Following assembly, atomic force microscopy (AFM) was performed to determine the thicknesses of the hBN flakes. Electron-beam lithography (EBL) was then used to define the contact patterns. After EBL, reactive ion etching (RIE) using a mixture of CHF₃ and O₂ gases (flow rates of 40 sccm and 4 sccm, respectively) was carried out at 25°C with an RF power of 60 W to expose the graphene edges for one-dimensional edge contacts. After the contact etching process, the remaining bottom hBN thickness was approximately 7 nm for both devices. Subsequently, Cr/Pd/Au (6/12/65 nm) metal contacts were deposited by thermal evaporation in a chamber with a base pressure of approximately 1×10^{-7} mbar, followed by a lift-off process in acetone and isopropyl alcohol (IPA). A second EBL step, followed by reactive ion etching, was performed to define the final device geometry. Electrical transport measurements were carried out in a dilution refrigerator using a standard low-frequency lock-in technique (~ 31 Hz) in constant-current mode with an excitation current of 2 nA. Thermal conductance measurements were performed using Johnson-noise thermometry based on

*anindya@iisc.ac.in

an LC resonant circuit with a resonance frequency of approximately 730 kHz. The thermal noise signal was amplified by a custom-built cryogenic preamplifier mounted at 4 K, followed by a room-temperature amplifier, and finally detected using a spectrum analyzer. Detailed measurement setup is shown in Supplementary Fig. 3.

Supplementary Note 2. Detailed gain calibration of the measurement chain and electron temperature determination

To calibrate the gain of the measurement chain, we performed temperature-dependent Johnson noise measurements in the absence of any injected current. Under these equilibrium conditions, the measured voltage noise spectral density is given by⁴⁻⁶

$$S_V = g^2 (4k_B T R + V_n^2 + I_n^2 R^2) \text{ BW}, \quad (1)$$

where g is the voltage gain of the measurement chain, k_B is the Boltzmann constant, T is the electron temperature of the sample, R is the sample resistance, V_n and I_n are the intrinsic voltage and current noise of the amplifiers present in the measurement line, respectively. BW is the resonance bandwidth of the LC tank circuit. For a well-developed quantum Hall plateau, the sample resistance remains essentially constant over the temperature range of the measurement. Consequently, only the thermal noise term, $4k_B T R$, varies with temperature. Therefore, a plot of (S_V/BW) as a function of T exhibits a linear dependence. The slope of the linear fit is related to the measurement-chain gain through

$$g = \sqrt{\frac{\text{slope}}{4k_B R}}. \quad (2)$$

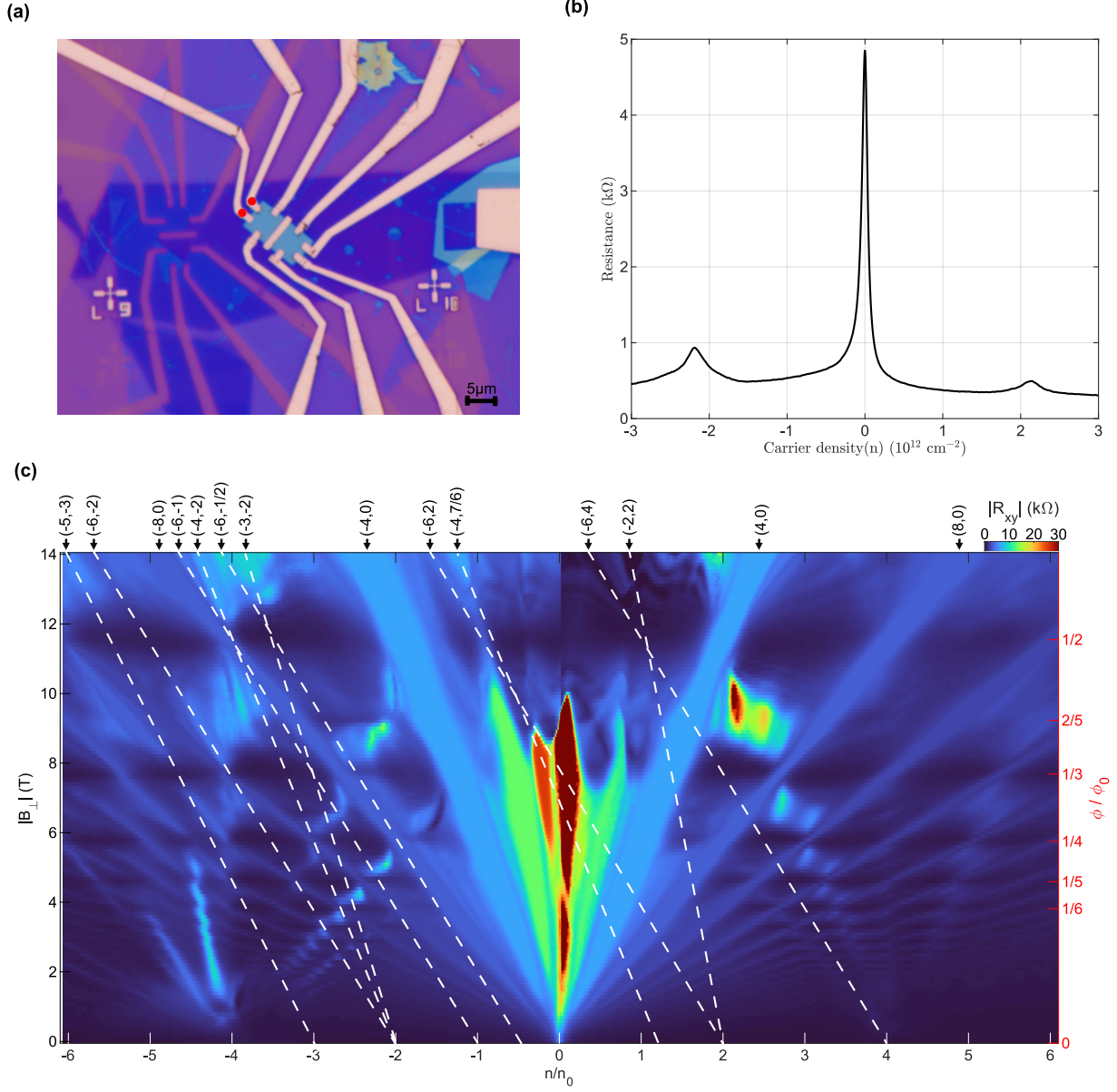
The intercept of the linear fit corresponds to the amplifier noise contribution and is given by

$$V_n^2 + I_n^2 R^2 = \frac{\text{intercept}}{g^2}. \quad (3)$$

Using the calibrated values of g and $V_n^2 + I_n^2 R^2$, the electron temperature can be determined from the measured voltage noise spectral density according to

$$T_0 = \frac{\frac{S_V}{g^2 \text{ BW}} - (V_n^2 + I_n^2 R^2)}{4k_B R}. \quad (4)$$

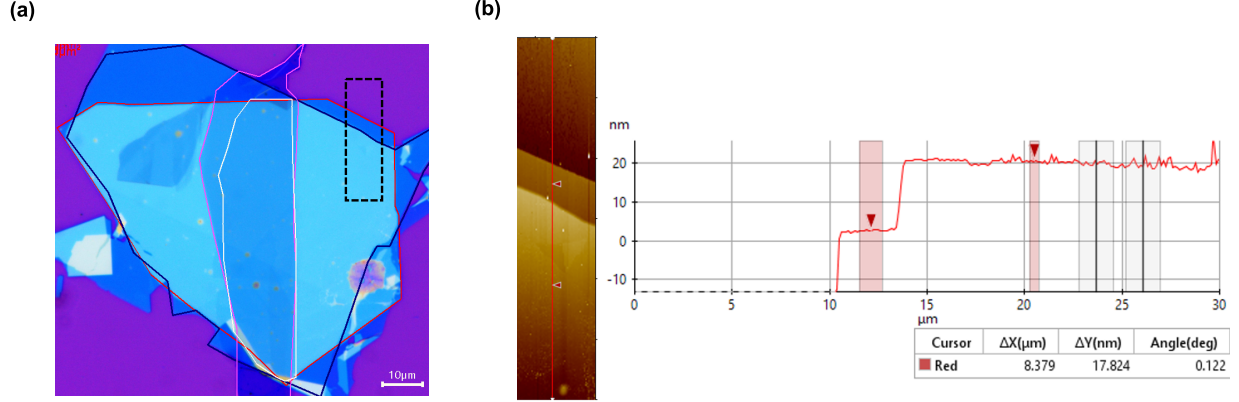
The calibrated electron temperatures for Device 1 obtained at different bath temperatures are summarized in Supplementary Table 1.



Supplementary Fig. 1: Optical image and characterization of Device 1. (a) Optical microscope image of the fabricated Device 1. The red solid circles indicate the electrical contacts used for the two-probe measurement. (b) Two-probe resistance as a function of carrier density (n) measured between the contacts marked by the red solid circles in panel (a) at $T = 6$ K and $B = 0$ T. In addition to the central charge neutrality peak, two satellite resistance peaks are observed, corresponding to complete filling of the moiré superlattice. The positions of the satellite peaks yield a moiré wavelength of approximately 14 nm. (c) 2-D color plot of $|R_{xy}| = |dV_1/dI_{SD}|$ (see Figure 1(a) in the main text) as a function of the carrier numbers per moiré unit cell, n/n_0 , and perpendicular magnetic field, $B_{\perp}$. The right axis shows the magnetic flux per moiré unit cell, ϕ/ϕ_0 . White dashed lines denote selected Diophantine trajectories corresponding to Chern insulating states, labelled by their Diophantine indices (t, s) .

Bath Temperature, T_{bath} (mK)	20	30	40	50	60
Electron Temperature, T_0 (mK)	37.6	41.4	47.8	56.1	61.5

Supplementary Table 1: Electron temperature measured at different bath temperatures for (-1,0) state in Device 1.



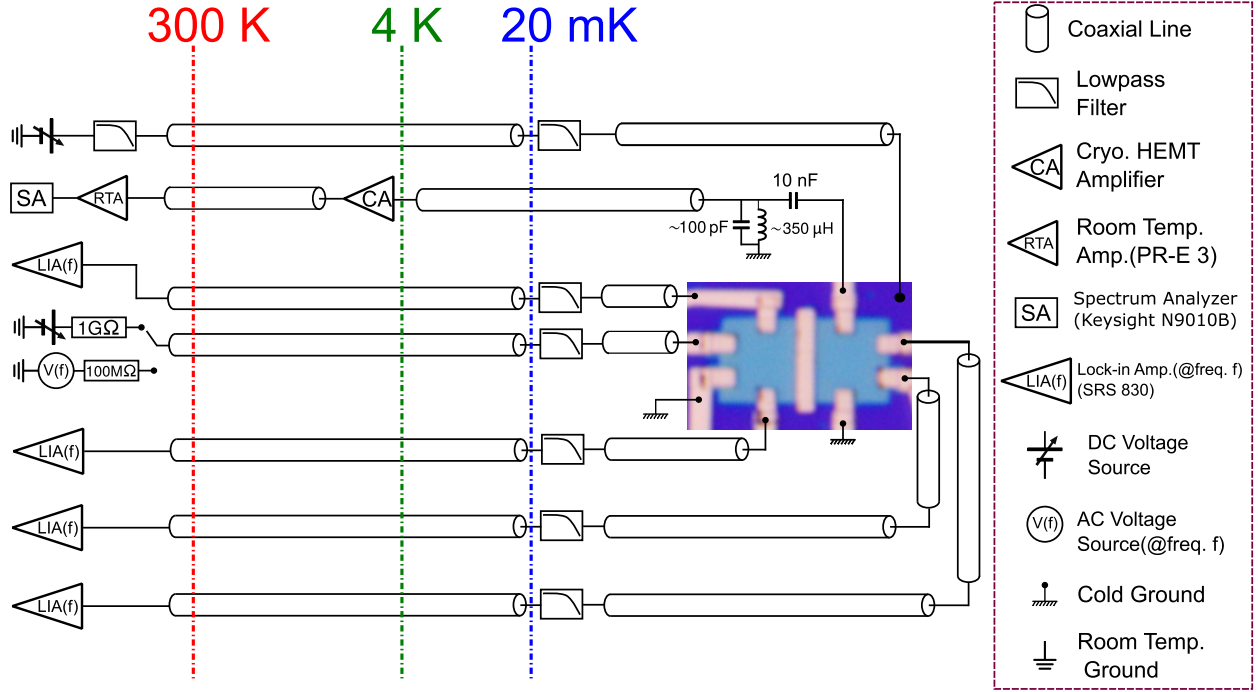
Supplementary Fig. 2: Optical image and selective AFM topography of the heterostructure for Device 1.

(a) Optical image of the fabricated heterostructure. The white, magenta, red, and black outlines indicate the bilayer graphene, graphite back gate, top hBN, and bottom hBN flakes, respectively. **(b)** Atomic force microscopy (AFM) was performed on the selected region marked by the black dashed rectangle in panel (a) to determine the flake thickness. The AFM height profile reveals that the thickness of the bottom hBN layer is 17.8 nm.

Supplementary Note 3. Discussion on Heat Coulomb Blockade (HCB)

Unlike electrical conductance, which is quantized in units of e^2/h (where e is the elementary charge and h is Planck's constant), thermal conductance is quantized in units of $\kappa_0 T = \frac{\pi^2 k_B^2}{3h} T$, where k_B is the Boltzmann constant and T is the temperature. In ballistic edge channels, each downstream chiral edge mode contributes one quantum of thermal conductance. However, when these edge channels are connected through a small floating metallic island, electron–electron interactions and charge conservation modify heat transport, giving rise to the phenomenon known as *Heat Coulomb Blockade* (HCB)⁷.

A floating metallic island possesses a finite charging energy, $E_C = \frac{e^2}{2C}$, where C is the capacitance of the island. Since the total charge on the floating island must be conserved, low-frequency charge fluctuations are suppressed. Heat transport is mediated by current fluctuations, which can be decomposed into one charge mode, corresponding to fluctuations of the total current, and $N - 1$ neutral modes, which transport energy without changing the total charge. Coulomb interactions suppress only the charge mode, while the neutral modes remain unaffected. Consequently, at sufficiently low temperatures ($k_B T \ll E_C$), exactly



Supplementary Fig. 3: **Experimental setup for electrical and thermal noise measurement.** Detailed schematic of the measurement setup for low-frequency electrical transport and high-frequency noise measurements. The device was mounted on a chip carrier connected to a custom-built cold finger thermally anchored to the mixing chamber plate of the dilution refrigerator. The ground contacts were directly connected to the cold finger to ensure both thermalization to the base temperature and a common ground potential. For low-frequency electrical measurements, an AC excitation current at 31 Hz was applied to the device, and the corresponding voltage signals were detected using lock-in amplifiers. For noise measurements, a DC current was passed through the device to heat the floating reservoir. The resulting Johnson noise voltage was measured to determine the electron temperature of the floating reservoir. The noise signal was first amplified by a homemade cryogenic voltage preamplifier mounted at the 4 K plate of the dilution refrigerator and subsequently amplified by a room-temperature voltage amplifier located at the top of the fridge. The amplified signal was then analyzed using a Keysight N9010B spectrum analyzer. All noise measurements were performed at the resonance frequency of an LC tank circuit with a bandwidth of 30 kHz. The resonant circuit consisted of a superconducting inductor with an inductance of approximately 365 μH , thermally anchored to the mixing chamber plate, and a parasitic capacitance of approximately 125 pF arising from the coaxial cables connecting the sample to the cryogenic preamplifier. A 10 nF ceramic capacitor was placed between the sample and the inductor to block the DC current in the measurement line while allowing the high-frequency noise signal to pass.

one quantum of thermal conductance is suppressed, leading to a total thermal conductance of $(N - 1)\kappa_0 T$ instead of $N\kappa_0 T$.

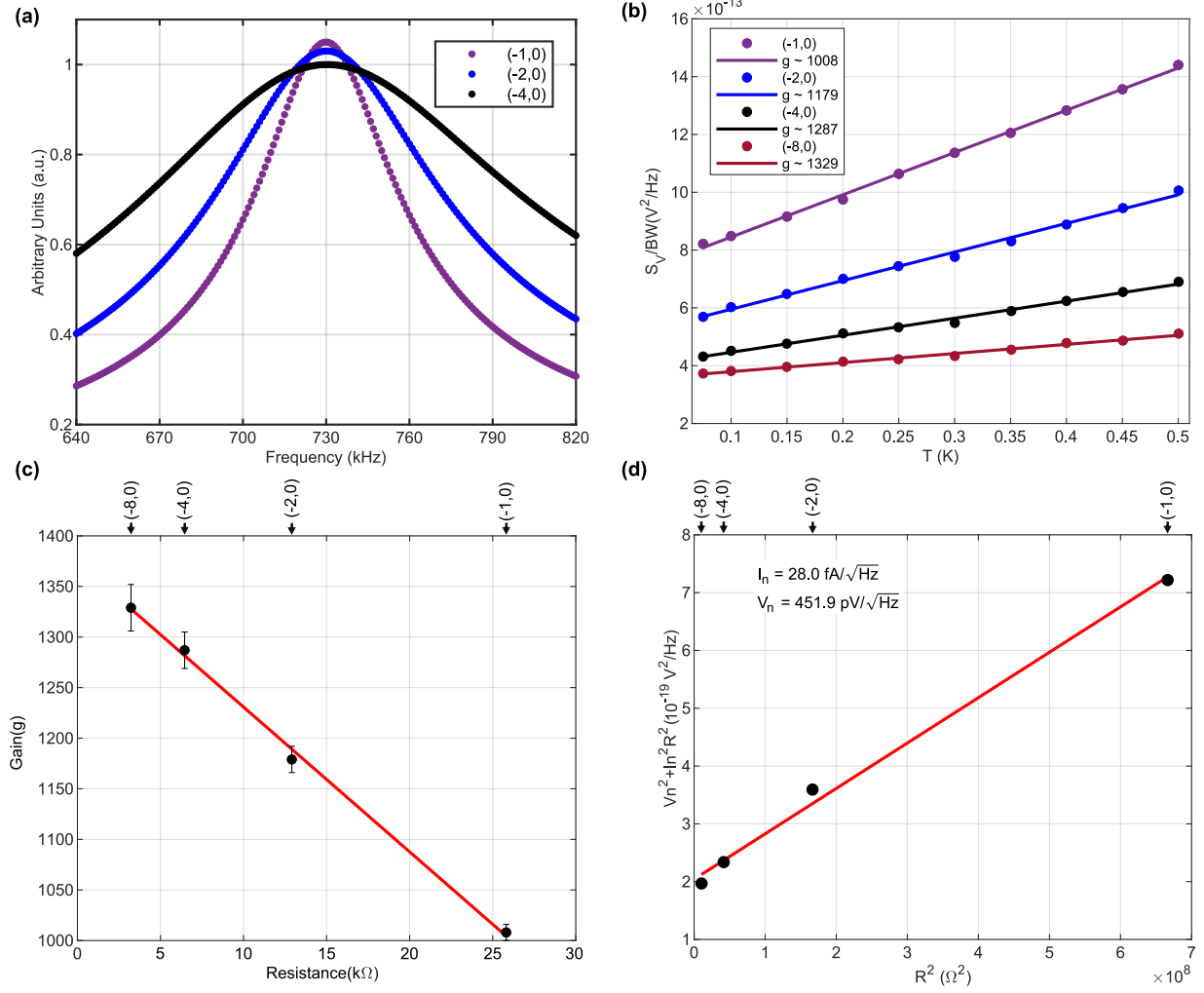
The characteristic crossover temperature for the onset of Heat Coulomb Blockade is given by $T_{CB} = \frac{Ne^2}{2\pi k_B C}$, where N is the total number of outgoing ballistic edge channels from the floating metallic island. When the electron temperature of the floating reservoir, T_M , is much lower than T_{CB} , the suppression of one quantum of thermal conductance becomes observable. At temperatures much higher than T_{CB} , thermal fluctuations overcome the charging constraint, and the full quantized thermal conductance is recovered.

For both of our devices, the initial bottom hBN thickness was approximately ~ 17 nm. During the reactive-ion etching (RIE-F) process used to define the contact geometry, the entire top hBN layer together with an additional ~ 10 nm of the bottom hBN was intentionally etched. As a result, the remaining bottom hBN thickness beneath the floating reservoir was approximately ~ 7 nm, which determines the separation between the floating metallic reservoir (FR) and the bottom graphite gate.

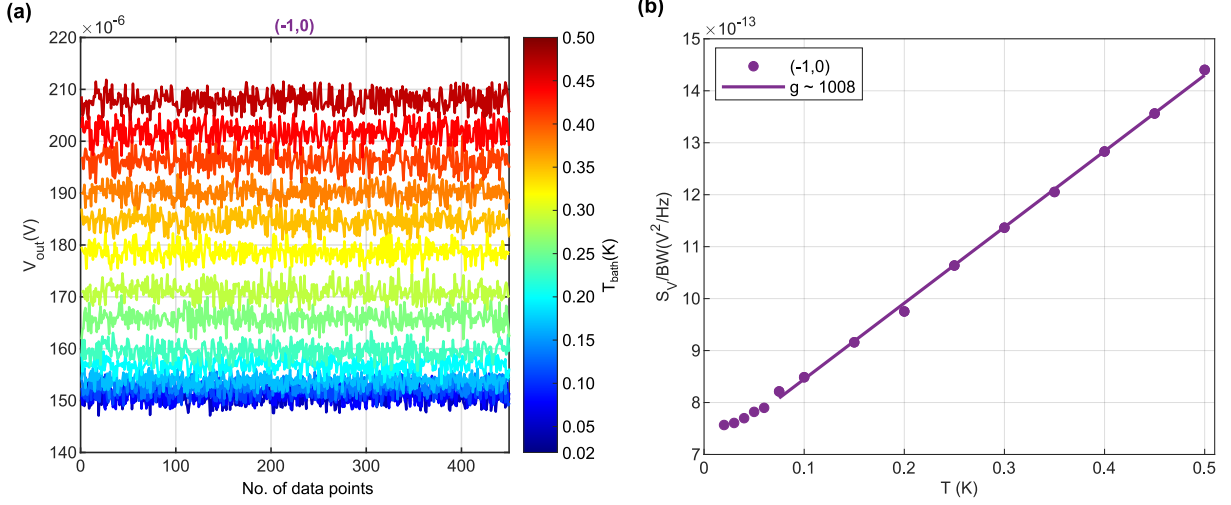
The area of the floating reservoir is approximately $9 \mu\text{m}^2$, corresponding to a geometrical capacitance of approximately 40 fF. Using this capacitance, the estimated crossover temperature is $T_{CB} \sim N \times 7$ mK, where N is the number of outgoing ballistic edge channels. The estimated values of T_{CB} for different values of N are summarized in Supplementary Table 2. From this estimation, for our devices, the effect of HCB is expected above $t > 3$ since our electron temperature was $\sim 37\text{mK}$ and $\sim 45\text{mK}$ for device 1 and device 2, respectively. However, in our measurement, we see the HCB started reducing the measured G_Q for $t \geq 5$. This can be seen further in Figure 2(e) of the main manuscript, where we have measured the G_Q for $(-4, 0)$ with the bath temperature variation from 20mK to 50mK , and we hardly see any change of its value from $\simeq 8\kappa_0 T$. This indicates that in our measurement, the HCB is not dominant for $t = 4$. This could be due to the fact that the actual capacitance of the FR could be higher than the estimation. This may arise from the uncertainty of separation between the FR and graphite gate, as well as the fringing effect around the FR, which may increase the effective capacitance.

t in (t,s) state	1	2	3	4	5	6	7	8
Number of outgoing channels, N	2	4	6	8	10	12	14	16
Crossover Temperature, T_{CB} (mK)	14	28	42	56	70	84	98	112

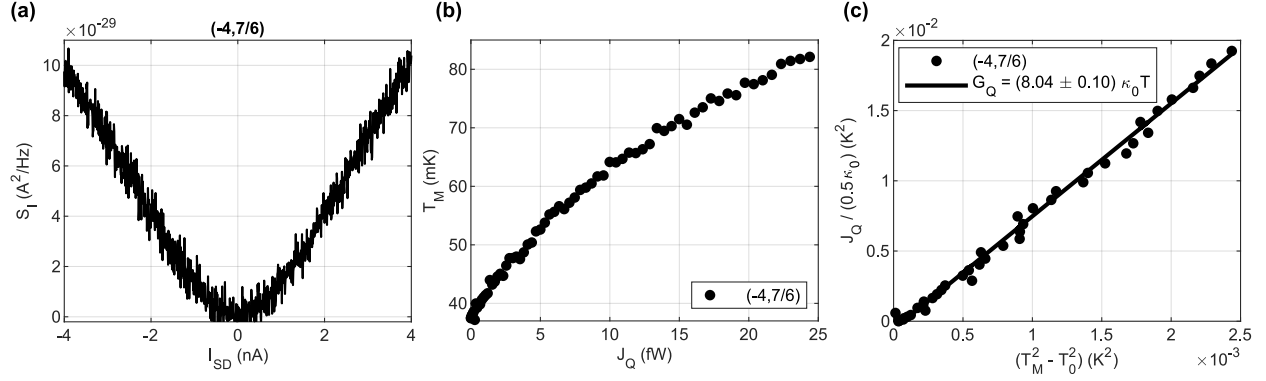
Supplementary Table 2: Estimated crossover temperature T_{CB} for different edge states. Here, $N = 2|t|$ is the total number of outgoing ballistic edge channels from the floating reservoir.



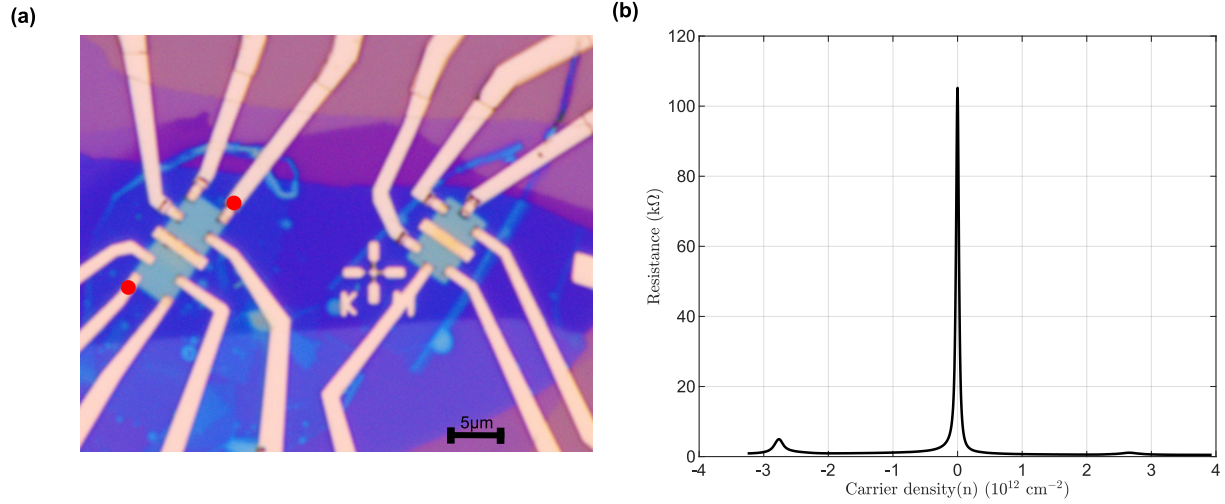
Supplementary Fig. 4: **Resonance frequency, gain, and intrinsic noise calibration of the measurement circuit for Device 1.** (a) Frequency response of the LC tank circuit measured by injecting a known AC signal using a lock-in amplifier (SR865A) and recording the voltage response with a spectrum analyzer. The normalized responses (arbitrary units, a.u.) for the (-1,0), (-2,0), and (-4,0) states are shown in purple, blue, and black, respectively. All three traces exhibit a resonance peak at ($f \sim 730$ kHz). (b) Measured noise spectral density (solid circles) as a function of temperature. Solid lines are linear fits used to extract the total voltage gain of the measurement chain, yielding gains of 1008, 1179, 1287, and 1329 for the (-1,0), (-2,0), (-4,0), and (-8,0) states, respectively. (c) Measured gain (solid circles) with error bars plotted as a function of device resistance. The red solid line is a linear fit, from which the gain for the (-3,0), (-5,0) and (-6,0) states are estimated to be ~ 1250 , 1300 and 1312 respectively. (d) Intrinsic amplifier noise, $(V_n^2 + I_n^2 R^2)$, obtained from the intercepts of the linear fits in panel (b) and plotted as a function of R^2 . The red solid line is a linear fit, yielding $V_n \sim 451.9 \text{ pV}/\sqrt{\text{Hz}}$ and $I_n \sim 28.0 \text{ fA}/\sqrt{\text{Hz}}$. For this cool-down the V_{cc} for the cold amplifier was 2.0 V.



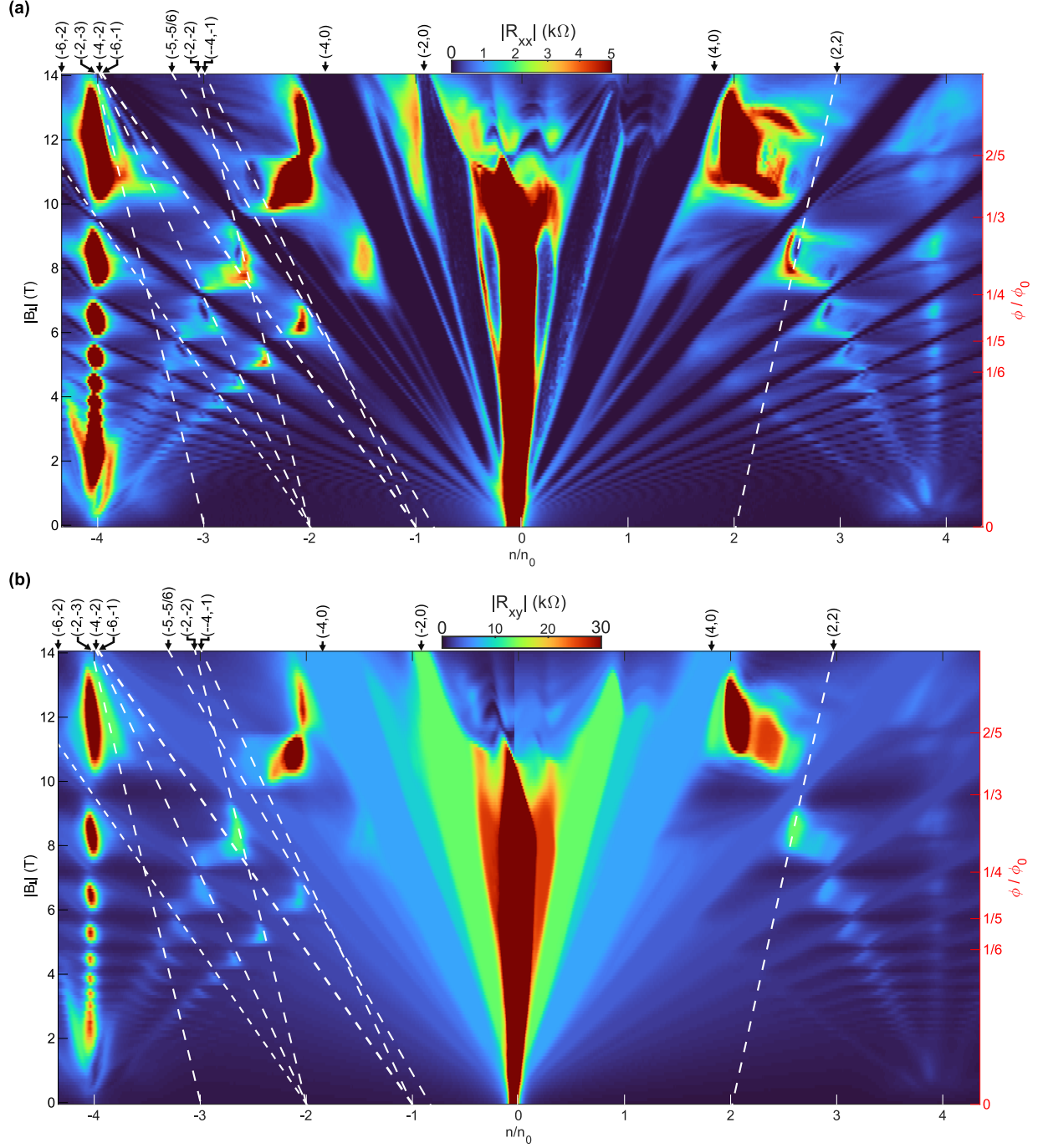
Supplementary Fig. 5: **Gain calibration of the measurement line and electron temperature estimation using the $(-1,0)$ state for Device 1.** (a) Output voltage, V_{out} , measured by the spectrum analyzer after the second stage of amplification for the $(-1,0)$ state at different bath temperatures, T_{bath} , ranging from 20 mK to 500 mK. The measured output voltage is converted to the noise spectral density, S_V , using $S_V = V_{\text{out}}^2$, which is subsequently used to determine both the total voltage gain of the measurement chain and the electron temperature, T_0 . (b) Noise spectral density, S_V , as a function of bath temperature. The solid circles represent the experimental data, while the solid line is a linear fit over the temperature range of 75–500 mK. The slope of the fit yields the total voltage gain of the measurement chain, $g \sim 1008$, for the $(-1,0)$ state. The same calibration is used to determine the electron temperature, T_0 , corresponding to each bath temperature, T_{bath} , as summarized in Supplementary Table 1.



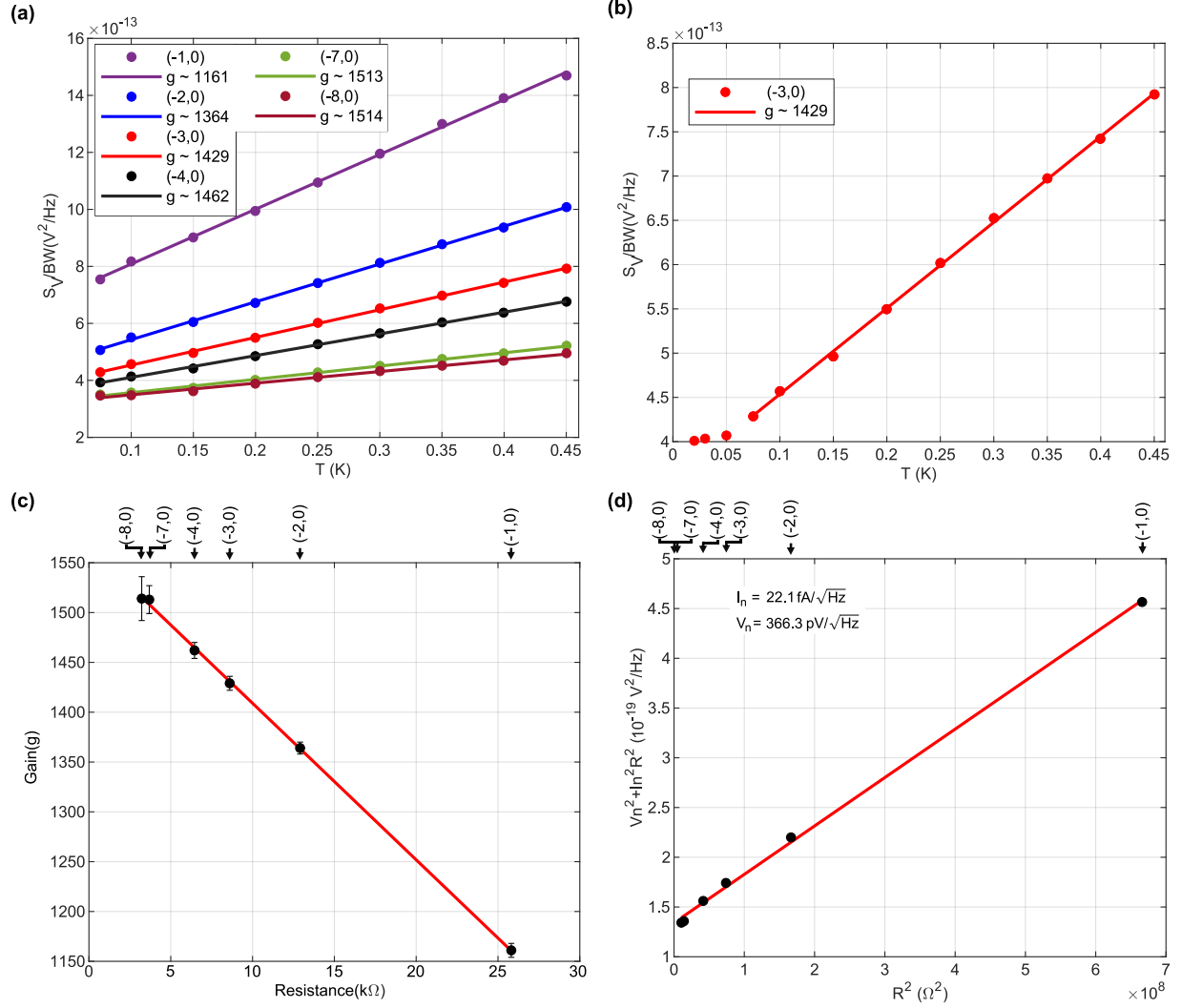
Supplementary Fig. 6: **Thermal conductance of symmetry-broken Chern insulator (SBCI) states in Device 1.** (a) Excess thermal noise S_I as a function of source current I_{SD} for $(t, s) = (-4, 7/6)$. (b) The temperature, T_M , of the floating-reservoir as a function of dissipated power, J_Q for $(-4, 7/6)$, ($N = 8$), where $N = 2|t|$ denotes the total number of outgoing edge modes from the floating reservoir. (c) Scaled dissipated power, $J_Q/0.5\kappa_0$, plotted as a function of $T_M^2 - T_0^2$ at electron temperature $T_0 \sim 37 \text{ mK}$. Linear fits (solid lines) yield the values of thermal conductances of, $G_Q = 8.04\kappa_0T$.



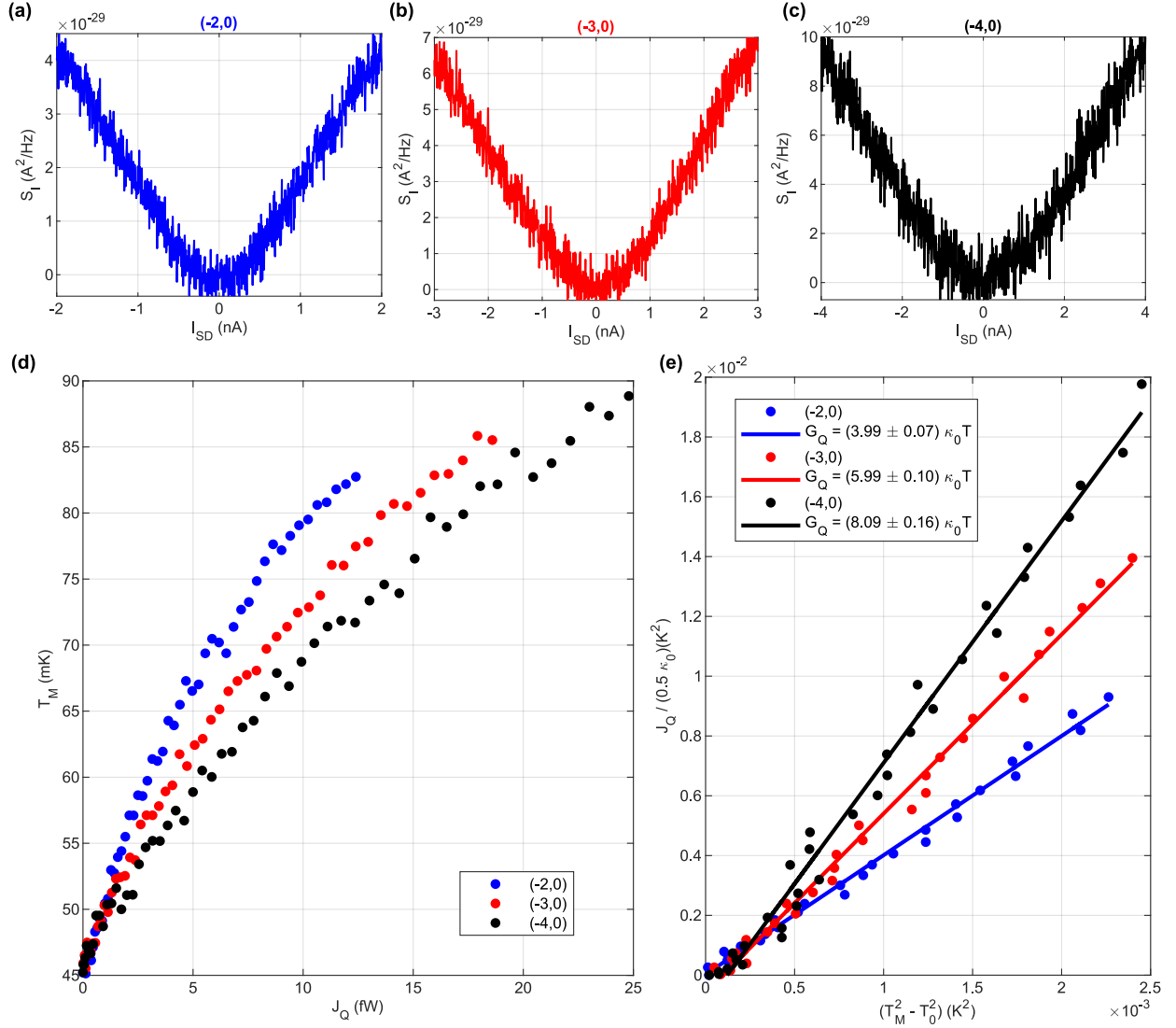
Supplementary Fig. 7: **Optical image and two probe gate response of Device 2.** (a) Optical microscope image of the fabricated Device 2 (left side Device). The red solid circles indicate the electrical contacts used for the two-probe measurement. (b) Two-probe resistance as a function of carrier density (n) measured between the contacts marked by the red solid circles in panel (a) at $T = 6 \text{ K}$ and $B = 0 \text{ T}$. In addition to the central charge neutrality peak, two satellite resistance peaks are observed, corresponding to complete filling of the moiré superlattice. The positions of the satellite peaks yield a moiré wavelength of $\sim 12.9 \text{ nm}$.



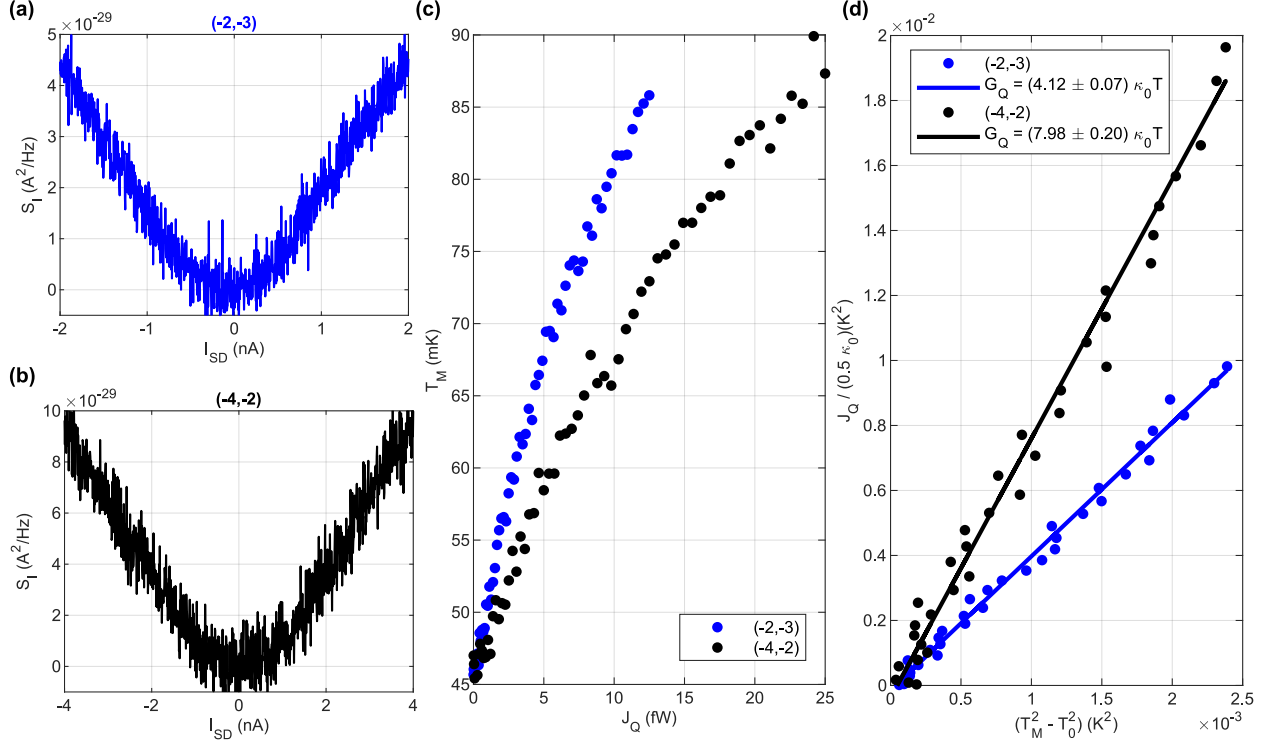
Supplementary Fig. 8: **Device characterization of Device 2.** (a) 2-D color map of $|R_{xx}| = |dV_{34}/dI_{SD}|$ (see Figure 1(a) in the main text), as a function of the carrier filling per moiré unit cell, n/n_0 , and perpendicular magnetic field, $B_{\perp}$. The right axis shows the magnetic flux per moiré unit cell, ϕ/ϕ_0 . White dashed lines indicate selected Diophantine trajectories corresponding to Chern insulating states, labelled by their Diophantine indices (t, s) . (b) 2-D color map of $|R_{xy}| = |dV_1/dI_{SD}|$ (see Figure 1(a) in the main text), plotted in the same way as in Supplementary Fig. 8(a).



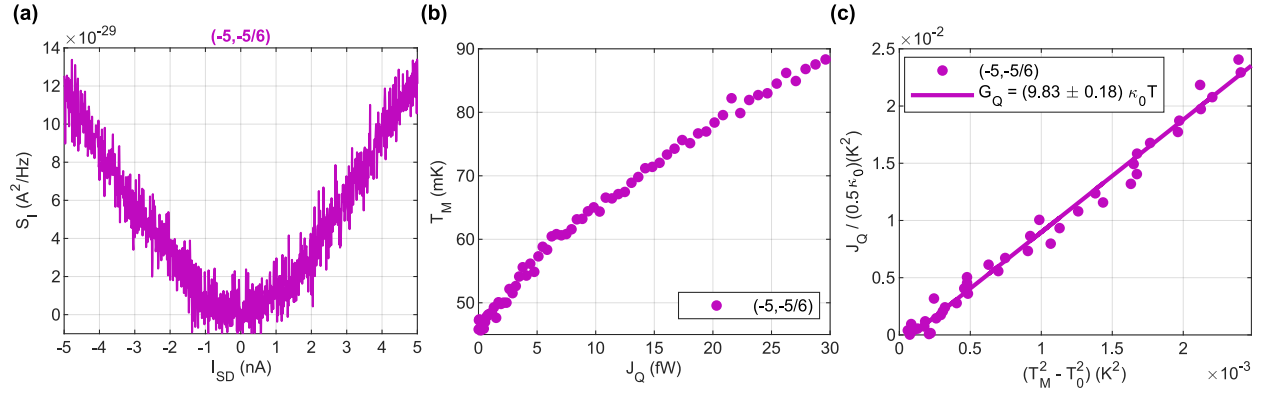
Supplementary Fig. 9: **Gain and intrinsic noise calibration of the measurement line for Device 2.** (a) Measured noise spectral density (solid circles) as a function of temperature. Solid lines are linear fits used to extract the total voltage gain of the measurement chain, yielding gains of 1161, 1364, 1429, 1462, 1513 and 1514 for the $(-1,0)$, $(-2,0)$, $(-3,0)$, $(-4,0)$, $(-7,0)$ and $(-8,0)$ states, respectively. (b) Noise spectral density, S_V , as a function of bath temperature for $(-3,0)$ state. The solid circles represent the experimental data, while the solid line is a linear fit over the temperature range of 75–450 mK. The slope of the fit yields the total voltage gain of the measurement chain, $G = 1429$. The same calibration is used to determine the electron temperature, $T_0 \sim 45 \text{ mK}$, corresponding to the base temperature, $T_{\text{bath}} = 20 \text{ mK}$. (c) Measured gain (solid circles) with error bars plotted as a function of device resistance. The red solid line is a linear fit to those data points. (d) Intrinsic amplifier noise, $V_n^2 + I_n^2 R^2$, obtained from the intercepts of the linear fits in panel (a) and plotted as a function of R^2 . The red solid line is a linear fit, yielding $V_n \sim 366.3 \text{ pV}/\sqrt{\text{Hz}}$ and $I_n \sim 22.1 \text{ fA}/\sqrt{\text{Hz}}$. For this cool-down the V_{cc} for the cold amplifier was 2.5 V.



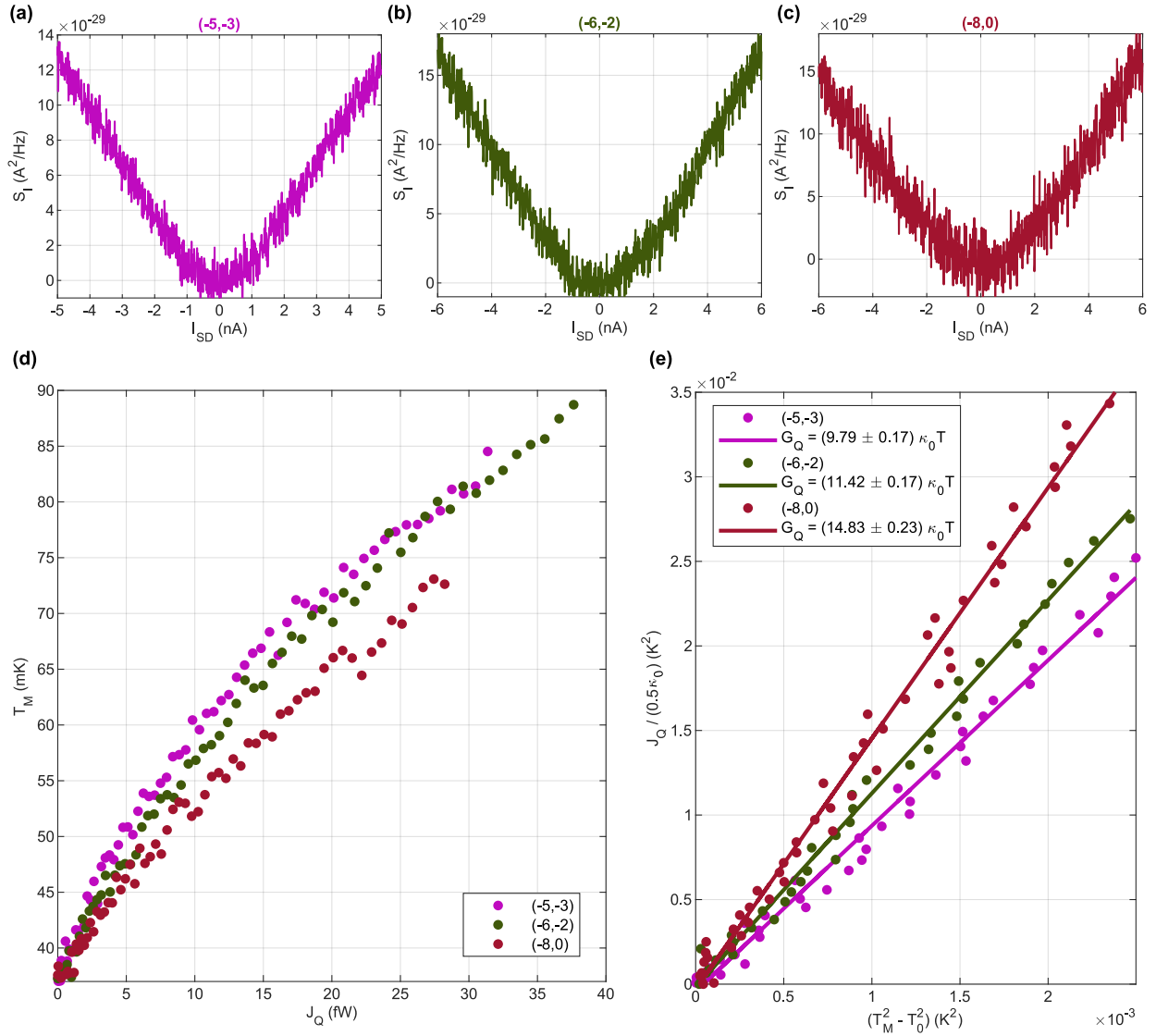
Supplementary Fig. 10: **Thermal conductance of conventional QH states in Device 2.** (a,b,c) Excess thermal noise S_I as a function of source current I_{SD} for $(t,s) = (-2,0)$ (blue) (a), $(t,s) = (-3,0)$ (red) (b), $(t,s) = (-4,0)$ (black) (c). (d) The temperature, T_M , of the floating reservoir as a function of dissipated power, J_Q for $(-2,0)$ ($N = 4$) (blue), $(-3,0)$ ($N = 6$) (red), $(-4,0)$ ($N = 8$) (black), where $N = 2|t|$ denotes the total number of outgoing edge modes from the floating reservoir. (e) Scaled dissipated power, $J_Q/0.5\kappa_0$, plotted as a function of $T_M^2 - T_0^2$ for the $(-2,0)$, $(-3,0)$ and $(-4,0)$ at electron temperature $T_0 \sim 45 \text{ mK}$. Linear fits (solid lines) yield the values of thermal conductances of $G_Q = 3.99\kappa_0 T$, $G_Q = 5.99\kappa_0 T$ and $G_Q = 8.09\kappa_0 T$ for $(-2,0)$, $(-3,0)$ and $(-4,0)$ states, respectively.



Supplementary Fig. 11: **Thermal conductance of Chern Insulator (CI) states in Device 2.** (a,b) Excess thermal noise S_I as a function of source current I_{SD} for $(t, s) = (-2, -3)$ (blue) (a), $(t, s) = (-4, -2)$ (black) (b). (c) T_M as a function of J_Q for $(-2, -3)$ ($N = 4$) (blue), $(-4, -2)$ ($N = 8$) (black) where $N = 2|t|$ denotes the total number of outgoing edge modes from the floating reservoir. (d) Scaled dissipated power, $J_Q/0.5\kappa_0$, plotted as a function of $T_M^2 - T_0^2$ at $T_0 \sim 45 \text{ mK}$. Linear fits (solid lines) yield values of thermal conductances of $(G_Q) = 4.12\kappa_0 T$, and $(G_Q) = 7.98\kappa_0 T$ for the $(-2, -3)$, and $(-4, -2)$ states, respectively.



Supplementary Fig. 12: **Thermal conductance of symmetry-broken Chern insulator (SBCI) states in Device 2.** (a) Excess thermal noise S_I as a function of source current I_{SD} for $(t, s) = (-5, -5/6)$. (b) The temperature, T_M , of the floating reservoir as a function of dissipated power, J_Q for $(-5, -5/6)$, ($N = 10$), where $N = 2|t|$ denotes the total number of outgoing edge modes from the floating reservoir. (c) Scaled dissipated power, $J_Q / (0.5\kappa_0)$, plotted as a function of $T_M^2 - T_0^2$ at an electron temperature of $T_0 \sim 45$ mK. The solid line is a linear fit, from which the thermal conductance is extracted as $G_Q = 9.83\kappa_0 T$. The slight reduction from the expected value of $10\kappa_0 T$ could be due to the heat Coulomb blockade (HCB) effect (see Supplementary Note 3).



Supplementary Fig. 13: **Thermal conductance of QH and CI states with $|t| > 4$ in Device 1.** (a,b,c) Excess thermal noise, S_I , as a function of source current, I_{SD} , for the $(t, s) = (-5, -3)$ (magenta) (a), $(-6, -2)$ (green) (b), and $(-8, 0)$ (maroon) (c) states. (d) Floating reservoir temperature, T_M , as a function of dissipated power, J_Q , for $(-5, -3)$ ($N = 10$), $(-6, -2)$ ($N = 12$), and $(-8, 0)$ ($N = 16$), where $N = 2|t|$ is the total number of outgoing edge modes. (e) Scaled dissipated power, $J_Q / 0.5 \kappa_0$, plotted against $T_M^2 - T_0^2$ at an electron temperature of $T_0 \sim 37$ mK. Linear fits (solid lines) yield thermal conductances of $9.79 \kappa_0 T$, $11.42 \kappa_0 T$, and $14.83 \kappa_0 T$ for the $(-5, -3)$, $(-6, -2)$, and $(-8, 0)$ states, respectively. The reduction from the expected values of $10 \kappa_0 T$, $12 \kappa_0 T$ and $16 \kappa_0 T$ for the $(-5, -3)$, $(-6, -2)$ and $(-8, 0)$ states could be due to the heat Coulomb blockade (HCB) effect (see Supplementary Note 3).

Supplementary references

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