Supplementary Information for: Bridging the Pedagogical Gap in Thermal Sciences With Comparative Analysis of Empirical Heat Transfer and Predictive Surrogate Modeling

S1. Extended Mathematical Calculations

This section provides the complete step-by-step mathematical derivations for the empirical observations recorded in the physical laboratory settings. These calculations form the baseline dataset utilized to train and validate the predictive surrogate models.

S1.1 Heat Transfer Through a Composite Wall

The objective of this calculation is to determine the effective thermal conductivity of a multi-layered composite structure experiencing steady-state 1-Dimensional conduction.

Given input parameters: Electrical Voltage (V) = 60 V Electrical Current (I) = 4.3 A Heater Diameter (d) = 0.25 m

The total electrical heat input generated by the central heater is calculated as:

$$W = V \times I = 60 \times 4.3 = 258 \text{ W} \tag{1}$$

Assuming perfectly symmetrical heat flow outward from the central heater through

both sides of the composite structure, the heat flowing through a single side (Q) is:

$$Q = \frac{W}{2} = 129 \text{ W} \tag{2}$$

The cross-sectional area (A) of the heat transfer surface is:

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.25)^2 = 0.049 \text{ m}^2 \tag{3}$$

The resulting heat flux (q) is:

$$q = \frac{Q}{A} = \frac{129}{0.049} = 26326.53 \text{ W/m}^2 \tag{4}$$

The average temperature drop (ΔT) across the entire composite structure (averaging the symmetrical sides) is derived from the embedded thermocouples:

$$\Delta T = \frac{(59.4 - 29.1) + (60.1 - 28.2)}{2} = 31.1^\circ\text{C} \tag{5}$$

Using Fourier's Law of Conduction, the total thermal resistance (R_{th}) of the composite wall is calculated:

$$R_{th} = \frac{\Delta T}{q} = \frac{31.1}{26326.53} = 0.00118^\circ\text{C} \cdot \text{m}^2/\text{W} \tag{6}$$

Given the total thickness of the three layers (Cast Iron, Bakelite, and Press Wood) as $\Delta x = 0.02 + 0.015 + 0.012 = 0.047$ m, the effective thermal conductivity (K_{eff}) is:

$$K_{eff} = \frac{q \times \Delta x}{\Delta T} = \frac{26326.53 \times 0.047}{31.1} = 39.78 \text{ W/(m} \cdot^\circ \text{C)} \tag{7}$$

S1.2 Forced Convection Heat Transfer from a Pin Fin

This calculation evaluates the convective heat transfer coefficient of a brass pin fin under forced air flow.

Given input parameters: Fin Diameter (D) = 0.0127 m Fin Length (L) = 0.150 m Voltage (V) = 65 V Current (I) = 0.24 A Ambient Temperature (T_6) = 30.1°C

The surface area (A_s) exposed to the convective fluid is:

$$A_s = \pi DL = \pi \times 0.0127 \times 0.150 = 0.00598 \text{ m}^2 \tag{8}$$

The heat supplied to the base of the fin is:

$$Q = V \times I = 65 \times 0.24 = 15.6 \text{ W} \tag{9}$$

The mean fin temperature (T_m) calculated from five distinct axial sensors is:

$$T_m = \frac{49.2 + 45.3 + 42.1 + 39.8 + 38.5}{5} = 42.98^\circ\text{C} \tag{10}$$

The overall temperature difference (ΔT) between the fin and the ambient air is:

$$\Delta T = T_m - T_6 = 42.98 - 30.1 = 12.88^\circ\text{C} \tag{11}$$

Using Newton's Law of Cooling, the experimental convective heat transfer coefficient (h_{exp}) is:

$$h_{exp} = \frac{Q}{A_s \Delta T} = \frac{15.6}{0.00598 \times 12.88} = 202.59 \text{ W}/(\text{m}^2 \cdot^\circ \text{C}) \tag{12}$$

S1.3 Thermal Conductivity of a Metal Rod

The final validation relies on an energy balance equation where the heat conducted through the solid rod equals the heat absorbed by the cooling fluid sink.

Given input parameters: Mass flow rate of water (m) = 0.01 kg/s Specific heat of water (C_p) = 4186 J/(kg $\cdot^\circ$ C) Water inlet temperature (T_7) = 30.8°C Water outlet temperature (T_8) = 30.2°C Cross-sectional area of rod (A) = 0.00049 m^2

The heat absorbed by the cooling water (Q) is calculated as:

$$Q = mC_p(T_8 - T_7) = 0.01 \times 4186 \times (30.2 - 30.8) = -25.116 \text{ W} \tag{13}$$

(The negative sign physically indicates heat flowing out of the solid domain and into the fluid domain).

The experimental axial temperature gradient ($\frac{dT}{dx}$) is derived using the linear bound-

aries of the test section:

$$\frac{dT}{dx} = \frac{T_6 - T_1}{x_6 - x_1} = \frac{28.9 - 33.2}{0.235 - 0.035} = \frac{-4.3}{0.200} = -21.5^{\circ}\text{C/m} \tag{14}$$

Applying Fourier's Law, the intrinsic thermal conductivity (k) of the metal rod is isolated as:

$$k = \frac{Q}{-A\left(\frac{dT}{dx}\right)} = \frac{25.116}{0.00049 \times 21.5} = 238.4 \text{ W/(m} \cdot^{\circ} \text{C)} \tag{15}$$

S2. Complete Surrogate Modeling Methodology

This section details the explicit algorithms, architectures, and hyperparameters utilized to process the empirical laboratory data and execute the predictive surrogate modeling phase of the research.

S2.1 Data Preprocessing

Prior to algorithm training, the raw empirical data vectors were normalized to ensure model stability and prevent gradient saturation during training. The input vector matrix $\boldsymbol{X}$ consisted of electrical power inputs, spatial coordinates, and ambient environmental temperatures. The target variable vector $\boldsymbol{Y}$ comprised the localized steady-state temperatures and resulting thermal coefficients. All input features were scaled using a standard Min-Max scaler:

$$X_{scaled} = \frac{X - X_{min}}{X_{max} - X_{min}} \tag{16}$$

The dataset was randomly partitioned into a training set (80 percent) and a validation set (20 percent) to measure out-of-sample predictive accuracy.

S2.2 Artificial Neural Network (ANN) Formulation

The deep learning architecture consisted of a Multi-Layer Perceptron (MLP) designed to capture the non-linear radial heat leakages identified in the physical apparatus. The network was structured with one input layer, three densely connected hidden layers consisting of 64, 32, and 16 neurons respectively, and one linear output layer.

The Rectified Linear Unit (ReLU) activation function was applied to all hidden layers

to introduce non-linearity into the mathematical mapping:

$$f(x) = \max(0, x) \tag{17}$$

The network was trained using the Adaptive Moment Estimation (Adam) optimization algorithm. The loss function minimized during the training epochs was the Mean Squared Error (MSE):

$$\mathcal{L}_{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 \tag{18}$$

where y_i is the true empirical temperature reading and $\hat{y}_i$ is the ANN prediction. The learning rate was initialized at 0.001 with a batch size of 16. Training was halted using an early stopping criterion if the validation loss did not improve over 50 consecutive epochs.

S2.3 Support Vector Regression (SVR) Implementation

Support Vector Regression was utilized to establish an optimal hyperplane capable of mapping the energy inputs to the thermal outputs while remaining robust against minor experimental outliers. The fundamental objective of the SVR algorithm is to minimize the empirical risk while maintaining a flat regression tube defined by an ϵ-margin. The optimization problem is formulated as:

$$\min_{\boldsymbol{w},b,\xi,\xi^*} \frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \tag{19}$$

subject to the physical constraints:

$$y_i - \langle \boldsymbol{w}, \boldsymbol{x}_i \rangle - b \leq \epsilon + \xi_i \tag{20}$$

$$\langle \boldsymbol{w}, \boldsymbol{x}_i \rangle + b - y_i \leq \epsilon + \xi_i^* \tag{21}$$

$$\xi_i, \xi_i^* \geq 0 \tag{22}$$

To account for the non-linear boundaries in the composite wall experiment, a Radial Basis Function (RBF) kernel was employed:

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp\left(-\gamma\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2\right) \tag{23}$$

Hyperparameter tuning via grid search yielded optimal values of the penalty parameter $C = 100$ and kernel coefficient $\gamma = 0.1$.

S2.4 Random Forest (RF) Ensemble Setup

To directly address the variance caused by interstitial radiation and air gaps in the insulating powder experiment, a Random Forest ensemble was constructed. The Random Forest algorithm operates by constructing a multitude of individual decision trees during training and outputting the mean prediction of the individual trees to improve generalization.

The final prediction $\hat{y}$ for an input vector $\boldsymbol{x}$ is calculated as the average output of B total regression trees:

$$\hat{y} = \frac{1}{B} \sum_{b=1}^{B} T_b(\boldsymbol{x}) \tag{24}$$

The ensemble was configured with $B = 200$ discrete decision trees. The maximum depth of each tree was allowed to expand until all leaves contained less than two samples. Bootstrapping was enabled to ensure that each decision tree was trained on a unique subset of the empirical data, thereby effectively mitigating the risk of overfitting the noise generated by the non-ideal boundary conditions.

S2.5 Model Evaluation Metrics

The predictive performance of all three surrogate models was evaluated primarily using the Coefficient of Determination (R^2) and the Root Mean Squared Error (RMSE). The R^2 metric quantifies the proportion of the variance in the dependent thermal variable that is predictable from the independent input variables:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \tag{25}$$

where $\bar{y}$ is the mean of the observed empirical data. The RF model consistently achieved an R^2 score exceeding 0.96 across all spatial coordinates, validating its robustness in handling frictional laboratory data.