

Fractal Quantization Model (FQM)

Discrete Scale Invariance, Effective Relaxation and a Falsifiable Astrophysical Extension

Version 2.0 - Major update of the accepted preprint

Gilmar Bezerra da Cruz

GEC NeuroSpace Project - Independent Researcher

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Abstract

This Version 2.0 preserves the exact Fibonacci-relaxation sector and the dimensionally consistent Keplerian astrophysical extension of the previous manuscript, while adding a vector computational formulation, confidence-aware regularization and a complete falsifiable validation protocol. The exact sequence result remains $\gamma_n = \Phi^{-2}/R_n^2 \rightarrow \Phi^{-4}$ for a constant stiffness in the quadratic functional $E_n = \kappa \epsilon_n^2/2$. In the orbital extension, the physical radial curvature is $k_r = GMm/r^3$, whereas the energy-scale stiffness associated with dimensionless radial strain is $\kappa \epsilon_n = r^2 k_r = GMm/r = m v_c^2$. The revision adds radial and tangential kinematics, angular momentum, torque, soft-capped vector weights, effective sample size, normalized entropy and dominance. Standalone simulations test rotation and unit invariance, bootstrap stability, noise, missing frames, occlusion confidence and recovery of deliberately injected logarithmic periodicity. These tests support the numerical consistency and reproducibility of the operator. They do not demonstrate that Φ is present in natural astrophysical data. The planetary-node hypothesis remains falsifiable through locked index rules, simpler baselines, out-of-sample systems, permutation tests and dynamical constraints.

Keywords: Fibonacci; golden ratio; discrete scale invariance; effective potential; orbital dynamics; vector diagnostics; bootstrap; null models; falsifiability; reproducible computation.

1. Scope, continuity and epistemic separation

The present document is a major update of the same astrophysical-extension preprint. It does not replace the research question with a different theory. It strengthens the mathematical definitions, computational diagnostics and conditions under which the proposal can fail.

The framework contains four distinct layers: (i) exact identities of the Fibonacci sequence; (ii) an effective quadratic energy functional; (iii) a phenomenological astrophysical node hypothesis; and (iv) a vector-weighted computational diagnostic. Only the first layer is an exact theorem. The other layers must be calibrated, compared with alternatives and tested on independent data.

2. Exact Fibonacci relaxation sector

Let F_n be the Fibonacci sequence, $R_n = F_{n+1}/F_n$ and $\Phi = (1+\sqrt{5})/2$. Define $\epsilon_n = R_n - \Phi$. Binet's formula yields an alternating finite-generation error and the exact adjacent-energy ratio, for constant κ , $\gamma_n = E_{n+1}/E_n = \Phi^{-2}/R_n^2$. Therefore γ_n tends to Φ^{-4} .

For $E_n = \kappa \epsilon_n^2/2$, the asymptotic decrease of this selected functional is $1 - \Phi^{-4} = 0.8541019662$. This is a property of the sequence and functional. It is not by itself a universal law of physical heat dissipation.

3. Dimensionally consistent astrophysical extension

For a body of mass m in a circular Kepler orbit around central mass M , the effective radial potential has curvature $k_r = GMm/r^3$. If the displacement is represented as the dimensionless strain $\epsilon = (r - r^*)/r^*$, then the quadratic energy scale becomes $\kappa_\epsilon = r^{*2} k_r = GMm/r^* = m v_c^2$.

This distinction prevents the quantity proportional to m/r from being mislabeled as a physical radial spring constant. The optional topological or node term must be written explicitly in the Hamiltonian and constrained by orbital data.

4. Planetary-node hypothesis and baselines

The phenomenological node law is $r_n = A F_n$, with A estimated by a locked logarithmic criterion. Residuals must be compared with simpler models, including a one-parameter geometric progression and resonance-based descriptions. A visually suggestive alignment or a high energy leverage is not sufficient evidence.

In the standalone reference calculation included with this revision, the same ordered set is fitted by both the Fibonacci-node rule and a geometric baseline. The output reports their logarithmic root-mean-square errors. The hypothesis is disfavored whenever it does not improve predictive performance under equal or stricter complexity control.

5. Vector computational formulation

For observations i with position r_i , velocity v_i , effective mass m_i , deviation ϵ_i and confidence q_i , the updated diagnostic decomposes velocity into radial and tangential components. It also permits angular momentum $L_i = m_i r_i \times v_i$ and torque $\tau_i = r_i \times F_i$.

A balanced unnormalized weight is proportional to $m_i r_i^{-1} |v_i|^{1/2} q_i$ times a bounded directional factor. A continuous soft cap limits extreme leverage. After normalization, the score is $S = \sum_i w_i \epsilon_i^2$. It must be reported with $N_{\text{eff}} = 1/\sum_i w_i$, normalized entropy and dominance $\max_i(w_i)$.

6. Temporal reconstruction and confidence

Two-frame finite differences are sensitive to jitter and missing frames. Version 2.0 therefore uses local weighted linear regression over a sliding temporal window. Visibility, detector confidence and geometric occlusion evidence may be multiplied into a composite confidence. Low-confidence observations are downweighted rather than silently removed.

7. Validation protocol

Implementation validity is assessed by invariance, limiting cases, uncertainty propagation and null models. The tests are intentionally separated from the physical claim: passing them shows that the code behaves as specified, not that the Universe follows the proposed node law.

The package contains seven independent scripts. Each includes its own data generator and mathematical functions and can run without importing any other project file.

8. Reproduced numerical results

Test	Primary quantity	Result	Interpretation
Fibonacci relaxation	gamma limit	0.145897261	Matches Φ^{-4} within numerical tolerance.
Astrophysical nodes	Delta log-RMSE	0.228233	Positive values favor the simpler geometric baseline.
Vector invariance	rotation delta S	0.000e+00	Numerically invariant.
Bootstrap	median N_{eff}	36.081	Weight support remains distributed.
Torque	max central torque	5.551e-17	Central-force limiting case recovered.
Injected log periodicity	recovered base	1.61934	Detector sensitivity only; not natural-data evidence.

The reference results are machine-generated by the scripts shipped in the package. Small numerical differences may occur across platforms because of linear-algebra libraries, but the stated qualitative acceptance criteria should remain unchanged.

9. Statistical interpretation

A score without weight diagnostics is insufficient. A high or low value can be driven by one observation when N_{eff} is close to one. Bootstrap intervals quantify finite-sample sensitivity; permutation tests ask whether an equally good fit can arise after destroying the hypothesized ordering. Model comparison must use locked protocols and account for discretionary index gaps.

10. Falsifiable tests

- Locked-index Solar-System and exoplanet fits compared against geometric, resonance and migration baselines.
- Training on a subset of systems and prediction of held-out systems without changing the node rule.
- Permutation and bootstrap tests that include index uncertainty and penalties for missing nodes.
- N-body integration of an explicit perturbing potential with its amplitude constrained by observations.
- Independent measurement of any stiffness or damping parameter before testing the predicted relaxation ratio.
- Failure criteria published before inspecting the validation set.

11. Limitations

- The structural generation n is not physical time unless a kinetic mapping is independently derived.
- The quadratic functional is an effective ansatz and is not uniquely selected by Fibonacci algebra.
- The vector weights are diagnostic choices and require domain-specific calibration.
- The Solar-System sample is small, ordered and vulnerable to a posteriori index selection.
- Recovery of Φ from an injected signal only verifies detector sensitivity.

- No result in this revision establishes universal golden-ratio quantization, historical planetary clearing or universal 85.41% dissipation.

12. Conclusion

Version 2.0 converts the astrophysical extension into a more auditable and reproducible research program. The exact sequence sector remains intact; the orbital stiffness is dimensionally distinguished from the dimensionless-strain energy scale; and the computational layer now reports invariance, robustness and weight concentration. The strongest contribution of the revision is methodological: it specifies what is calculated, what is assumed and what observations would falsify the hypothesis. Physical support for the proposed node law still requires preregistered, out-of-sample and dynamically constrained tests.

Data and code availability

All scripts required to reproduce the numerical tables and figures are included in the accompanying Version 2.0 package. Every test is self-contained, uses explicit random seeds and produces machine-readable JSON and CSV outputs.

References

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