Mereological Syntax of Space (MSS): A Relational Framework for Translating Urban Spatial Structures into Architectural Configuration

Technical Appendix

1. Methodology

The MSS methodology is designed to provide a systematic process for analyzing urban spatial structures and translating their relational logic into architectural design. It consists of three sequential stages: Identification of Spatial Elements, Specification of Spatial Relations, and Construction of Relational Matrices, culminating in the derivation of spatial syntactic properties and their translation into architectural design.

1.1. Analytical Workflow

The methodology of Mereological Syntax of Space (MSS) is organized as a structured analytical workflow that translates urban relational patterns into architectural generative rules. In line with the formal definitions introduced in Section 3.3, the method proceeds from the identification of spatial elements to the derivation of relational matrices, syntactic measures, and a translation-based architectural configuration.

At the operational level, MSS is composed of four interdependent phases:

1. **Urban contextual reading:** the site is interpreted as a relational field rather than a neutral container.
2. **Spatial element extraction:** urban components are coded as discrete analytical units.
3. **Relational matrix construction:** mereological relations are encoded in matrix form.
4. **Generative translation:** urban relational logic is mapped into architectural elements and configurations.

This workflow is designed to ensure that architectural form is not produced as a detached formal gesture, but as a transformed expression of the relational syntax embedded in the urban context. Accordingly, MSS distinguishes between descriptive spatial data and operational relational structure: GIS layers, site observations, and plan-based readings are not directly converted into form; instead, they are first codified into analyzable relational structures.

Operational coding protocol
To support reproducibility, the following coding protocol is adopted:

- Source data: site plans, urban maps, aerial imagery, field observation, and contextual drawings.
- Unit of analysis: a spatial element representing a physically or functionally distinguishable component.
- Coding rule: each element is assigned an identifier and classified according to its spatial role.
- Relation coding: pairwise relations are recorded through binary or weighted matrix entries depending on relation type.
- Verification: ambiguous cases are resolved through repeated comparison of plan reading, field evidence, and contextual adjacency.

This protocol ensures that the MSS procedure is not purely interpretive, but sufficiently structured for replication and cross-case comparison.

1.2. Relational Matrices and Syntactic Measures

1.2.1. Spatial system and relational encoding

The urban spatial system is defined as:

$$S_U = \langle E_U, R_U \rangle$$

where $E_U = \{e_1, e_2, \dots, e_n\}$ denotes the set of urban spatial elements, and

R_U denotes the set of relations among them. The architectural system is similarly defined as:

$$S_A = \langle E_A, R_A \rangle$$

Each relation $r_k \backslash in\ R$ is represented through a relational matrix:

$$M^{(k)} = \left[w_{ij}^{(k)}\right] \in R^{n\times n}$$

where $w_{ij}^{(k)}$ expresses the presence, intensity, or directionality of relation r_k

between elements e_i and e_j.

1.2.2. Primary relational operators

MSS uses five primary relations as the base relational operators:

Adjacency $\boldsymbol{A}$: direct spatial contiguity or shared boundary.

Connectivity $\boldsymbol{C}$: functional or circulatory linkage between elements.

Containment $\boldsymbol{H}$: inclusion of one element within a larger spatial envelope.

Overlap $\boldsymbol{O}$: partial shared occupation of spatial domain.

Hierarchy $\boldsymbol{P}$: ordinal or dominance-based relational precedence among elements.

These five relations are treated as primary because they are sufficient to encode the minimum relational grammar required for MSS translation. Other notions, including liminality, are treated as derived indicators obtained from combinations of the primary matrices rather than as independent primitive relations.

1.2.3. Matrix properties

To reduce ambiguity, each matrix is defined according to its formal properties:

$\boldsymbol{A}$: binary and symmetric in most plan-based cases.

$\boldsymbol{C}$: binary or weighted, and potentially directed if circulation has orientation.

$\boldsymbol{H}$: binary and generally asymmetric, since containment has directionality.

$\boldsymbol{O}$: binary or weighted and typically symmetric.

$\boldsymbol{P}$: ordinal or weighted, representing relative spatial precedence or dominance.

Accordingly:

$$A = [a_{ij}] \in \{0,1\}^{n\times n}$$

$$C = [c_{ij}] \in [0,1]^{n\times n}$$

$$H = [h_{ij}] \in \{0,1\}^{n\times n}$$

$$O = [o_{ij}] \in [0,1]^{n\times n}$$

$$P = [p_{ij}] \in R^{n\times n}$$

where the exact range of values depends on the case-study coding scale.

1.2.4. Derived syntactic measures

Several syntactic indicators are derived from the relational matrices.

Connectivity index

For element e_i , connectivity is defined as:

$$\text{Conn}(e_i) = \sum_{j=1}^{n} c_{ij}$$

This measure captures the extent to which an element participates in spatial and functional linkage.

Adjacency degree

$$\text{Adj}(e_i) = \sum_{j=1}^{n} a_{ij}$$

This reflects the immediate relational exposure of an element within the spatial field.

Containment depth

A simple depth measure may be expressed as:

$$\text{Depth}(e_i) = \sum_{j=1}^{n} h_{ji}$$

where higher values indicate greater embeddedness within larger enclosing structures.

Hierarchy score

$$\text{Hier}(e_i) = \sum_{j=1}^{n} p_{ij}$$

This expresses the relative hierarchical position of an element in the spatial system.

Liminality index

Liminality is not treated as a primitive relation, but as a derived condition emerging from adjacency, overlap, and containment. A simplified liminality measure is defined as:

$$Lim(e_i) = \lambda_1 \sum_{j=1}^{n} a_{ij} + \lambda_2 \sum_{j=1}^{n} o_{ij} + \lambda_3 \sum_{j=1}^{n} h_{ij}$$

where $\lambda 1 , \lambda 2 , \lambda 3$ are non-negative weights satisfying:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

This index captures the degree to which a spatial element functions as a transitional or threshold condition.

1.2.5. Operational note on hierarchy

In MSS, hierarchy refers specifically to relational precedence and structural dominance, not merely to visibility or spatial centrality. This distinction is necessary because publicness gradients, centrality, and dominance are analytically related but not identical. Hierarchy is therefore encoded as an ordinal relational matrix rather than a generic descriptive quality.

1.3. Formal Mathematical Model

1.3.1. Urban and architectural genomes

The urban genome is defined as:

$$G_U = \langle E_U, R_U \rangle$$

and the architectural genome as:

$$G_A = \langle E_A, R_A \rangle$$

In MSS, the term "genome" does not function as a metaphorical label. It denotes the compact relational encoding of spatial organization, namely the minimum set of elements and relations required to reproduce the structural logic of a spatial system.

1.3.2. Translation operators

The generative transformation from urban to architectural space is expressed as a pair of mapping operators:

$$T_E: E_U \rightarrow E_A$$

$$T_M: M_U \rightarrow M_A$$

where T_E translates spatial elements and T_M translates relational matrices.

The overall translation is therefore:

$$T: G_U \rightarrow G_A$$

1.3.3. Objective function

The translation process is guided by an objective function that evaluates the degree of structural fidelity between urban and architectural genomes.

Let G_A^* denote a candidate architectural genome.

The optimization problem is formulated as:

$$G_A^* = \arg\min_{G_A} J\,(G_A)$$

where:

$$J(G_A) = \alpha D(E_U, E_A) + \beta \sum_k d!\left(M_U^{(k)}, M_A^{(k)}\right) + \gamma \Phi(G_A)$$

Here:

- $\boldsymbol{D(EU, EA)}$ measures the structural correspondence between the source and target element sets;
- $\boldsymbol{d\left(M_U^k, M_A^k\right)}$ measures the discrepancy between corresponding relational matrices;
- $\boldsymbol{\Phi(G_A)}$ is a penalty term for incompatibility with architectural constraints;
- $\boldsymbol{\alpha, \beta, \gamma}$ are non-negative weights.

This formulation makes the optimization direction explicit: MSS seeks to minimize discrepancy while preserving relational fidelity.

5.3.4. Reproducibility and validation

To improve reproducibility, the translation output should satisfy three criteria:

- Re**lational fidelity:** key urban relations should remain recognizable in the architectural genome.
- **Functional coherence:** the resulting configuration should satisfy programmatic and circulation requirements.
- **Structural consistency:** translated elements should preserve the relative organization of the source system.

A simple fidelity index may be defined as:

$$F = 1 - \frac{1}{K} \sum_{k=1}^{K} \frac{|M_U^{(k)} - M_A^{(k)}|}{|M_U^{(k)}| + \epsilon}$$

where K is the number of relational matrices and ϵ is a small constant to prevent division by zero.

1.4. MSS Generative Translation Algorithm

Algorithm 1. MSS Generative Translation Algorithm

Input K, J :

$$S_U = \langle E_U, R_U \rangle$$

Output:

$$S_A = \langle E_A, R_A \rangle$$

1: Read urban context and set analytical boundary

2: Extract urban spatial elements E_U

3: Code relations R_U using K

4: Build relational matrices A, C, H, O, P

5: Form urban genome $G_U = \langle E_U, R_U \rangle$

6: Define translation operator T_E and T_E

7: Generate candidate architectural elements E_A

8: Construct architectural matrices A_A, C_A, H_A, O_A, P_A

9: Compute objective $J(G_A)$ and fidelity F

10: While $F < F_t hresholdorJ(G_A) > J_t hreshold$ do

11: Modify E_A and relational matrices

12: Recompute $J(G_A)$ and F

13: end while

14: Return $S_A =< E_A, R_A >$

1.5. Methodological Notes on Case-Study Application

In the case-study application, MSS is implemented through plan reading, relational coding, and iterative translation. The case-specific data are first decomposed into analyzable spatial units, then encoded through the relational matrices introduced above. The derived matrices are subsequently used as the basis for generating the architectural configuration.

Because the method is case-sensitive, not every relation appears with equal intensity across all sites. Accordingly, the weighting of matrices and the selection of design parameters are adjusted according to site morphology, spatial program, and contextual constraints. This allows MSS to remain structurally consistent while remaining adaptable to the specific conditions of each project.

1.6. Summary

MSS provides a structured methodology for transforming urban spatial syntax into architectural form through relational coding, matrix-based analysis, and generative translation. Its main methodological contribution lies in the formalization of relational continuity between city and architecture, while preserving the interpretive dimension required in design research. By combining qualitative spatial reading with explicit mathematical representation, MSS offers a reproducible framework for case-based generative design.