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Low-Beta Plasma in Accretion Disks around Schwarzschild Black Holes: Magnetic Support, Stability, Angular-Momentum Transport, and Observational Constraints

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ABSTRACT

Magnetic fields are dynamically important in many black-hole accretion flows. They contribute to vertical support, transport angular momentum through Maxwell stress, heat the corona, and help determine whether an accretion flow is weakly magnetized, magnetically elevated, or magnetically arrested. This paper develops a self-consistent analytic treatment of low-beta plasma in a disk around a non-rotating Schwarzschild black hole, with plasma beta defined as the ratio of gas pressure to magnetic pressure. The Schwarzschild geometry is used as a control case because it has an event horizon and an innermost stable circular orbit but no frame dragging or Blandford-Znajek spin-energy extraction. Within a local, isothermal, constant-beta model, we derive the magnetic-pressure fraction, the effective hydrostatic scale height, the Alfvén-to-sound-speed ratio, and the exact fastest-growing wavelength of the ideal, axisymmetric, vertical-field magnetorotational instability (MRI) in a Newtonian Keplerian patch. The derivation distinguishes this exact wavelength from the frequently used estimate $2\pi v_A/\Omega$. As beta decreases, the disk becomes more strongly supported by magnetic pressure and the fastest-growing vertical MRI mode becomes too long to fit comfortably within one scale height. This does not imply universal stabilization: strong fields suppress parts of the local MRI spectrum while permitting magnetic buoyancy, reconnection, interchange motions, and global flux accumulation. Recent horizon-scale polarimetry of M87* and Sagittarius A* constrains ordered and time-dependent magnetic-field structure, but beta remains an inferred, model-dependent parameter. The results are analytic scalings, not a replacement for radiative general-relativistic magnetohydrodynamic simulations.

I. INTRODUCTION

Accretion converts gravitational binding energy into radiation, mechanical outflows, and magnetic activity around compact objects. Early quantitative treatments established the energetic importance of accretion [1], while thin-disk and relativistic disk models provided the standard descriptions of radiatively efficient flows [2, 3, 4, 5, 6, 7]. In a purely hydrodynamic Keplerian disk, outward angular-momentum transport is not supplied by molecular viscosity at an astrophysically useful rate. The alpha prescription therefore represents unresolved transport through an effective stress [2].

Magnetized disks provide a physical transport mechanism. The magnetorotational instability (MRI), anticipated in rotating magnetized Couette flow [8, 9, 10] and applied to

accretion disks by Balbus and Hawley [11, 12, 13, 14], destabilizes a weakly magnetized flow whenever angular velocity decreases outward under ideal-MHD conditions. Local simulations established that the nonlinear state contains correlated magnetic and velocity fluctuations and that Maxwell stress normally dominates the transport [15, 16, 17, 18]. Reviews of MRI theory and disk transport emphasize that field geometry, dissipation, stratification, and non-ideal terms can materially alter the outcome [13, 19, 20, 21, 22, 23]. Stratified calculations also show how magnetic energy can rise into and heat a corona [16, 24].

The plasma beta parameter, β, is a compact measure of the relative gas and magnetic pressures. High-beta plasma is gas-pressure dominated; low-beta plasma is magnetically dominated. Beta is useful but insufficient by itself. The topology and net flux of the field, the cooling rate, radiation pressure, optical depth, electron thermodynamics, and relativistic effects all influence the disk [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. In particular, a local low-beta region is not automatically a magnetically arrested disk (MAD). A MAD is a global state in which accumulated poloidal magnetic flux dynamically obstructs inflow near the black hole [25, 26, 27, 36, 37].

This paper restricts the central object to a Schwarzschild black hole. The choice removes frame dragging and makes the horizon angular velocity zero. Consequently, the Blandford-Znajek mechanism cannot extract rotational energy from the hole [38]. Disk-driven or coronal winds can still occur [30, 29, 39], but they must be powered by the accretion flow rather than black-hole spin. This separation provides a clean conceptual baseline before introducing the additional physics of Kerr spin, spin-dependent jet efficiency, and tilted flows [28, 32, 40, 41, 42].

The goal is deliberately limited. We construct a transparent local scaling model, correct several common algebraic and interpretive ambiguities, and compare the trends with the broader theoretical and observational literature. Section 2 defines the Schwarzschild length scales and pressure variables. Section 3 derives the local hydrostatic and MRI scalings. Section 4 presents the numerical trends. Sections 5 through 6 discuss stress, stability, winds, and the boundary with the Kerr problem. Section 7 summarizes current observational constraints. Section 8 states the limits of the calculation, and Section 9 gives the conclusions.

II. Theoretical Framework

A. Schwarzschild length scales and efficiency

For a black hole of mass M, define the gravitational radius $r_g = GM/c^2$, the Schwarzschild radius $r_s = 2r_g = 2GM/c^2$, and the innermost stable circular orbit (ISCO) $r_{ISCO} = 6r_g = 3r_s$. For a zero-torque, radiatively efficient thin disk terminating at the Schwarzschild ISCO, the specific orbital energy is $E_{ISCO} = \sqrt{(8/9)}$. The corresponding ideal efficiency is therefore [3, 4, 43, 44]

$$\epsilon_{NT} = 1 - E_{ISCO} = 1 - \sqrt{\frac{8}{9}} \simeq 0.05719 \quad (1)$$

Table 1 gives representative radii.

Table 1: Representative Schwarzschild length scales. Values use $r_g = GM/c^2$, $r_s = 2r_g$, and $r_{ISCO} = 6r_g$.

Object	r_g	r_s	r_{ISCO}
Stellar BH (10 $M_\odot$)	14.8 km	29.5 km	88.6 km
Sgr A* (4×10^6 $M_\odot$)	5.9×10^6 km	11.8×10^6 km	35.4×10^6 km
M87* (6.5×10^9 $M_\odot$)	9.6×10^9 km	19.2×10^9 km	57.6×10^9 km

For circular equatorial geodesics, the orbital angular frequency with respect to Schwarzschild coordinate time is

$$\Omega_K = \sqrt{\frac{GM}{r^3}} \tag{2}$$

The radial epicyclic frequency measured in the same coordinate time is

$$\kappa_t^2 = \Omega_K^2\left(1 - \frac{6r_g}{r}\right) \tag{3}$$

It vanishes at the ISCO. If the oscillation is instead measured per unit proper time of the circular orbit, the result contains the time-dilation factor

$$\kappa_\tau^2 = \Omega_K^2 \frac{1 - 6r_g/r}{1 - 3r_g/r} \tag{4}$$

These two expressions describe different clocks. Mixing them without specifying the observer leads to an apparent contradiction. The local MRI model below uses the Newtonian Keplerian limit and is not applied quantitatively at the ISCO.

B. Magnetic pressure and plasma beta

Magnetic pressure is $P_B = B^2/(8\pi)$ (Gaussian-cgs). In the local model we use Gaussian-cgs units and define

$$\beta \equiv \frac{P_g}{P_B} = \frac{8\pi P_g}{B^2} \tag{5}$$

If the model total pressure includes only gas and magnetic contributions, $P_{tot} = P_g + P_B$, then

$$\frac{P_B}{P_{tot}} = \frac{1}{1 + \beta} \tag{6}$$

In an isothermal gas, $P_g = \rho c_s^2$.

C. Alfvén speed

The non-relativistic Alfvén speed in Gaussian-cgs units is

$$v_A = \frac{B}{\sqrt{4\pi\rho}} \tag{7}$$

Combining this definition with $P_g = \rho c_s^2$ and $P_B = P_g/\beta$ gives

$$\frac{v_A}{c_s} = \sqrt{\frac{2}{\beta}} \tag{8}$$

This equality is exact within the stated non-relativistic, isothermal conventions. In relativistic MHD, using units with $c = 1$, the fluid-frame expression is $v_{A,rel}^2 = b^2/(w + b^2)$, where w is the relativistic enthalpy density.

III. Local Constant-Beta Scaling Model

A. Assumptions and vertical support

Consider a local disk patch that is isothermal, approximately Keplerian, threaded by a vertical magnetic field for the MRI calculation, and characterized by beta that is independent of height. For a gas-pressure-supported patch, the one-sided Gaussian scale height is

$$H_g = \frac{c_s}{\Omega_z} \tag{9}$$

where $\Omega_z = \Omega_K$ in the Newtonian Keplerian limit. At constant beta,

$$P_{tot} = \rho c_s^2 \left(1 + \frac{1}{\beta}\right) \tag{10}$$

so the effective squared support speed is

$$c_{eff}^2 = c_s^2 \left(1 + \frac{1}{\beta}\right) = c_s^2 + \frac{v_A^2}{2} \tag{11}$$

The resulting scale height is

$$H = \frac{c_{eff}}{\Omega_z} = H_g \sqrt{1 + \frac{1}{\beta}} \tag{12}$$

B. MRI dispersion relation and exact mode

For an incompressible, axisymmetric perturbation with vertical wavenumber k in a Newtonian disk, the ideal-MHD dispersion relation is [8, 11]

$$\omega^4 - \omega^2[\kappa^2 + 2(kv_A)^2] + (kv_A)^2 \left[(kv_A)^2 + \frac{d\Omega^2}{d\ln r}\right] = 0 \tag{13}$$

Let $q \equiv -d\ln\Omega/d\ln r$, and $\kappa^2 = 2(2 - q)\Omega^2$. For a Newtonian Keplerian disk, $q = 3/2$ and $\kappa = \Omega$. Therefore

$$\gamma_{max} = \frac{3}{4}\Omega, \qquad k_{max}v_A = \frac{\sqrt{15}}{4}\Omega \tag{14}$$

and the exact fastest-growing wavelength is

$$\lambda_{\max} = \frac{2\pi}{k_{\max}} = \frac{8\pi}{\sqrt{15}}\frac{v_A}{\Omega} \tag{15}$$

The often quoted $\lambda \sim 2\pi v_A/\Omega$ is a useful characteristic estimate, but it is about 3.3 percent shorter than the exact fastest-growing Keplerian value. Using $H = c_{\text{eff}}/\Omega$ gives

$$\frac{\lambda_{\max}}{H} = \frac{8\pi}{\sqrt{15}}\frac{v_A}{c_{\text{eff}}} = \frac{8\pi}{\sqrt{15}}\sqrt{\frac{2}{\beta + 1}} \tag{16}$$

IV. Numerical Scaling Results

All entries in Table 2 are dimensionless.

Table 2: Beta-dependent local scalings.

β	P_B/P_{tot}	H/H_g	v_A/c_s	λ_{max}/H
100	0.0099	1.005	0.141	0.91
30	0.0323	1.017	0.258	1.65
10	0.0909	1.049	0.447	2.77
3	0.2500	1.155	0.816	4.59
1	0.5000	1.414	1.414	6.49
0.3	0.7692	2.082	2.582	8.05
0.1	0.9091	3.317	4.472	8.75
0.03	0.9709	5.859	8.165	9.04
0.01	0.9901	10.050	14.142	9.13

At $\beta = 0.1$, magnetic pressure supplies 90.9 percent of $P_g + P_B$, the scale-height enhancement is $H/H_g = 3.317$, and $v_A/c_s = 4.472$. The exact result gives $\lambda_{max}/H = 1$ at $\beta = 128\pi^2/15 - 1 \simeq 83.22$. For smaller beta, the fastest-growing vertical wavelength exceeds one scale height. This is a finite-domain statement, not proof that all MRI activity vanishes.

V. Magnetic Stress and Stability

Define positive outward transport through the local r-ϕ stress

$$W_{r\phi} = \rho\langle \delta v_r \delta v_\phi \rangle - \frac{\langle B_r B_\phi \rangle}{4\pi} \tag{17}$$

The dimensionless alpha parameter may be normalized to gas pressure or to total pressure:

$$\alpha_g = \frac{W_{r\phi}}{P_g}, \qquad \alpha_{tot} = \frac{W_{r\phi}}{P_g + P_B} = \alpha_g \frac{\beta}{1 + \beta} \tag{18}$$

Thus identical physical stress can correspond to very different reported alpha values when beta is small.

It is inaccurate to say that a strong magnetic field simply stabilizes an accretion disk. Magnetic tension shifts the MRI spectrum to longer wavelength and can exclude local vertical

modes from a finite disk. At the same time, stratified magnetic fields are buoyant, current sheets can reconnect, and globally accumulated flux can drive interchange-like accretion [17, 25, 26, 33].

VI. Schwarzschild Winds and the Boundary with the Kerr Problem

A Schwarzschild disk may launch a magnetocentrifugal wind when an ordered large-scale field threads the disk at a suitable inclination [39]. Hot accretion flows may also drive thermally or magnetically assisted outflows [35].

The Blandford-Znajek mechanism is different. Its power depends on magnetic flux threading a rotating horizon and vanishes with the horizon angular velocity in the Schwarzschild limit [38]. For the Schwarzschild control problem, any persistent jet-like outflow must be attributed to the disk, corona, or infalling magnetic field rather than extraction of black-hole rotational energy.

VII. Observational Constraints

Horizon-scale observations now constrain the geometry and variability of magnetic fields close to black holes. The first EHT images of M87* and Sagittarius A* established a ring-scale morphology and ordered polarization patterns consistent with dynamically important magnetic fields [45, 46, 47, 48]. These measurements do not directly measure beta. Converting observed intensity and polarization into a pressure ratio requires a radiative model built on a dynamical model. A specific numerical beta is a model-dependent posterior quantity, not a direct observable.

VIII. Limitations and Scope

The hydrostatic model assumes isothermality, constant beta with height, and isotropic magnetic-pressure support. The MRI derivation is Newtonian, incompressible, axisymmetric, ideal MHD with a uniform vertical field. The model uses beta as an input and does not solve magnetic-flux transport. No energy equation or radiative transfer is solved. The Schwarzschild spacetime is used to define the horizon and ISCO, but the beta scalings are not a GRMHD solution.

IX. Conclusions

Within a local, isothermal, constant-beta closure, magnetic pressure increases the effective hydrostatic scale height by $\sqrt{(1 + 1/\beta)}$ and gives $v_A/c_s = \sqrt{(2/\beta)}$. For the exact fastest-growing axisymmetric, vertical-field MRI mode in a Newtonian Keplerian patch,

$$\frac{\lambda_{\max}}{H} = \frac{8\pi}{\sqrt{15}}\sqrt{\frac{2}{\beta + 1}} \tag{19}$$

Low-beta plasma shifts the disk toward magnetic support and global magnetic organization. It can suppress parts of the local MRI spectrum while enabling buoyancy, reconnection,

interchange motions, and intermittent accretion. The Schwarzschild restriction cleanly separates these disk processes from Blandford-Znajek extraction. Polarimetric observations demonstrate that magnetic fields near the emitting region are organized and dynamically important, but beta must still be inferred through radiative GRMHD modeling.

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