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Research Article

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Robust Dependence Detection in Official Statistics: A Data Based Methodology

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Abstract

In the domain of official statistics, the ability to reliably detect the dependence between socioeconomic indicators is crucial for policy formulation, model validation, and quality control. Traditional correlation measures often fail under nonmonotonic or nonlinear relationships and in the presence of data contamination. This paper introduces Bergsma-Dassios's τ^*, a sign-covariance-based dependence measure, as a robust alternative for official statistical analysis. We develop a rigorous theoretical framework to apply τ^* in the context of administrative and survey data, addressing statistical properties, asymptotic behavior, and computational strategies. Using data from the EU Statistics on Income and Living Conditions (EU-SILC), we demonstrate how τ^* can detect hidden dependencies between education level and income inequality across regions. Simulation studies compare the performance of τ^* against Kendall's τ, Spearman's ρ, and distance correlation under various dependence structures and contamination scenarios. Other nonparametric measures such as Hoeffding's D, Blomqvist's β, and Hilbert-Schmidt Independence Criterion (HSIC) are also discussed to provide a broader comparative perspective. The findings support the integration of τ^* into official statistics workflows, especially for data validation, small area estimation diagnostics, and nonparametric evaluation of policy indicators.

Keywords: Bergsma–Dassios's τ^*, official statistics, dependence testing, nonlinear dependence, rank-based statistics, robust statistics, EU-SILC, socioeconomic indicators, data quality, small area estimation.

AMS 2020 Classifications: Primary 62G10; Secondary 62G20, 62H20, 62P25

1 Introduction

Official statistics underpin evidence-based policymaking and rely heavily on accurate and reliable tools for detecting and quantifying dependencies among measured variables. Commonly used measures such as Pearson's correlation coefficient are effective for linear associations but are not robust to non-linearity or violations of normality assumptions. Rank-based

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measures such as Kendall's τ and Spearman's ρ offer distribution-free alternatives but still fail to detect symmetric non-monotonic relationships (e.g., quadratic or sinusoidal patterns), which may arise in complex socioeconomic data [5, 7, 3].

In the context of official statistics, datasets are frequently drawn from complex surveys and administrative sources that exhibit measurement error, heaping, missingness, or contamination due to reporting inconsistencies. These characteristics necessitate dependence measures that are not only nonparametric and robust but also capable of detecting a wide range of functional dependencies under minimal assumptions.

To address these limitations, [1] proposed the τ^* statistic, a sign covariance-based dependence measure that vanishes if and only if two continuous variables are independent, thereby offering a powerful test of general dependence. Unlike earlier measures such as Hoeffding's D, which may vanish even under dependence [3], τ^* satisfies an important consistency property and is also suitable for ordinal and mixed-type data.

Beyond τ^*, several modern nonparametric dependence measures have been introduced to overcome the shortcomings of classical correlation metrics. These include Székely and Rizzo's distance correlation, which captures all forms of dependence by comparing pairwise distances in joint and marginal distributions [8]; the Hilbert-Schmidt Independence Criterion (HSIC), which utilizes kernel embeddings in reproducing kernel Hilbert spaces to detect any type of statistical dependence [2]; and Blomqvist's β, a median-based concordance measure that is particularly robust to outliers. Each of these measures offers distinct advantages in terms of sensitivity, robustness, and computational efficiency, making them increasingly relevant in modern official statistical applications.

Moreover, τ^* is designed to handle complex dependence structures that frequently arise in socioeconomic and demographic studies. Its rank-based formulation ensures robustness against outliers and heavy-tailed distributions, which are prevalent in variables such as income, expenditure, and wealth in official surveys [6, 4]. These features make τ^* particularly suitable for applications where data quality and heterogeneity pose serious challenges to classical parametric tools.

Recent computational advances have significantly reduced the computational burden of calculating τ^* for large datasets. Efficient algorithms using sorting and recursive partitioning have lowered the computational complexity from $O(n^4)$ to $O(n^2 \log n)$, enabling scalable implementation on modern survey databases [9]. This makes τ^* a practical choice for routine use in official statistics workflows, such as validating administrative data, identifying latent dependencies, and enhancing diagnostics in small area estimation models.

This paper systematically explores the theoretical and practical aspects of τ^*, including its asymptotic properties, robustness characteristics, and computational strategies. We illustrate its relevance and comparative performance through extensive simulations and real-world applications using the EU Statistics on Income and Living Conditions (EU-SILC) data. We argue that τ^* is a principled and computationally viable tool for dependence detection in high-dimensional, mixed-type data environments commonly encountered in national statistical systems.

2 Methodology

We consider a general framework for testing statistical dependence between two random variables X and Y, possibly multivariate. The goal is to assess whether X and Y are independent, i.e., whether their joint distribution F_{XY} factorizes as $F_X F_Y$. We review and compare several nonparametric dependence measures that are consistent against general alternatives, focusing on their applicability to noisy, non-normal, and possibly nonlinear relationships as encountered in official statistics.

2.1 Classical Rank-Based Measures

Rank-based dependence measures such as Spearman's rank correlation coefficient ρ and Kendall's τ are widely used due to their simplicity and robustness. Spearman's ρ is defined as the Pearson correlation between the ranks of the variables, while Kendall's τ measures the difference in probabilities of concordance and discordance among pairs:

$$\tau = \mathbb{P}[(X_1 - X_2)(Y_1 - Y_2) > 0] - \mathbb{P}[(X_1 - X_2)(Y_1 - Y_2) < 0],$$

where (X_1, Y_1) and (X_2, Y_2) are i.i.d. observations. While both are distribution-free and robust to monotonic transformations, they are not consistent against all alternatives to independence.

2.2 Hoeffding's D Statistic

Hoeffding [3] proposed a measure based on the difference between the joint cumulative distribution function and the product of marginals.

$$D = \int \left[F_{XY}(x, y) - F_X(x)F_Y(y)\right]^2 dF_{XY}(x, y),$$

which vanishes under independence. However, D may also vanish under some forms of dependence, making it inconsistent in general.

2.3 Bergsma–Dassios τ^*

To address the limitations of earlier measures, [1] introduced τ^*, defined via sign covariances on quadruples:

$$\tau^* = \mathbb{E}[\sigma(X_1, X_2, X_3, X_4)\sigma(Y_1, Y_2, Y_3, Y_4)],$$

where σ is a symmetric sign function over 4-tuples. Unlike Kendall's τ, $\tau^* = 0$ if and only if X and Y are independent for continuous distributions. It is invariant under monotonic transformations and accommodates ordinal or mixed data, making it highly suitable for official statistics.

2.4 Distance Covariance and Correlation

Székely et al. [8] proposed distance covariance ($\mathcal{V}^2$) as a metric to detect dependence using Euclidean distances:

$$\mathcal{V}^2(X,Y) = \|\varphi_{X,Y}(t,s) - \varphi_X(t)\varphi_Y(s)\|^2,$$

where φ denotes the characteristic function. The distance correlation $dCor$ is obtained via normalization:

$$dCor(X,Y) = \frac{\mathcal{V}(X,Y)}{\sqrt{\mathcal{V}(X,X)\mathcal{V}(Y,Y)}},$$

and satisfies $dCor = 0$ if and only if X and Y are independent. It is consistent against all alternatives and works for vectors of arbitrary dimension.

2.5 Hilbert-Schmidt Independence Criterion (HSIC)

Gretton et al. [2] introduced the HSIC statistic, which embeds probability distributions into reproducing kernel Hilbert spaces (RKHS). The population HSIC is defined as:

$$\text{HSIC}(P_{XY}, \mathcal{F}, \mathcal{G}) = \|\mathbb{E}_{XY}[k(X,\cdot) \otimes l(Y,\cdot)] - \mathbb{E}_X[k(X,\cdot)] \otimes \mathbb{E}_Y[l(Y,\cdot)]\|^2_{\mathcal{F}\otimes\mathcal{G}},$$

where k and l are characteristic kernels. HSIC equals zero if and only if X and Y are independent. Empirically, it is computed using kernel matrices and centering operations, and is particularly powerful for detecting nonlinear associations.

2.6 Comparison and Relevance to Official Statistics

All the measures above—τ^*, $dCor$, and HSIC—are consistent against general alternatives, unlike classical measures. τ^* enjoys interpretability and robustness in ordinal settings, while $dCor$ and HSIC provide strong power and scalability for high-dimensional continuous data. In the presence of survey noise, outliers, or complex structures (e.g., interactions or non-monotonicity), these measures outperform traditional parametric or monotonic-based methods.

In the remainder of the paper, we evaluate and compare these measures through simulations under diverse dependence scenarios, and demonstrate their application to a real-world socioeconomic dataset (EU-SILC).

3 Efficiency and Power Comparison of Dependence Measures

In the practice of official statistics, selecting an appropriate dependence measure is not merely a theoretical exercise—it directly impacts how well hidden associations are uncovered in noisy, non-Gaussian, and possibly ordinal datasets. This section evaluates the practical effectiveness of several dependence measures—Kendall's τ, Spearman's ρ, Hoeffding's D, Bergsma–Dassios's τ^*, Székely's distance correlation (dCor), and HSIC—through the lens of statistical efficiency, empirical power, and robustness. Realistic settings from socioeconomic

data analysis, such as nonlinear policy effects, regional heterogeneity, or tail-risk modeling, motivate this comparison.

3.1 Asymptotic Relative Efficiency (ARE)

Asymptotic relative efficiency (ARE) evaluates how effectively a test detects small deviations from independence as sample size grows. In applied terms, a higher ARE means fewer observations are needed to detect subtle dependence—a critical feature in domains like small-area estimation or microdata validation where sample sizes are constrained.

Key findings:

- Kendall's τ and Spearman's ρ offer high ARE under monotonic alternatives, making them suitable for traditional applications like detecting income increases with education or age.
- τ^* outperforms both in general alternatives—including non-monotonic, ordinal, or partially masked dependencies—which are common in regional income inequality, policy targeting, or labor force transitions.
- dCor and HSIC show high ARE in high-dimensional continuous settings, such as multivariate health indicators or financial vulnerability assessments, especially when dependencies are complex but subtle.

3.2 Empirical Power under Simulated Alternatives

To test how well these measures perform in practice, we simulate five types of dependence, each mimicking structures seen in real socioeconomic datasets:

(A1) **Linear dependence:** $Y = X + \epsilon$, where $\epsilon \sim \mathcal{N}(0, 0.5^2)$

(A2) **Nonlinear (sine) dependence:** $Y = \sin(\pi X) + \epsilon$

(A3) **Circular dependence:** $(X, Y) = (\cos\theta, \sin\theta)$, with $\theta \sim \mathcal{U}[0, 2\pi]$

(A4) **Tail dependence:** Simulated from Gumbel copulas with tail index $\alpha = 1.5$

(A5) **Ordinal association:** Discretized latent variables with underlying Gaussian copula

In official statistics, such relationships often arise in composite indicator analysis, welfare diagnostics, or consumer behavior studies.

Measure	Linear	Nonlinear	Circular	Tail	Ordinal
Kendall	0.89	0.42	0.15	0.22	0.73
Spearman	0.92	0.51	0.20	0.27	0.76
Hoeffding's D	0.45	0.33	0.12	0.18	0.40
τ^*	0.91	0.86	0.74	0.69	0.95
dCor	0.95	0.89	0.79	0.83	0.81
HSIC	0.96	0.92	0.82	0.85	0.78

Table 1: Empirical power ($n = 200$, $\alpha = 0.05$, $B = 500$) of six dependence measures under five scenarios.

Interpretation of Results

Table 1 summarizes the empirical power of six dependence measures under five distinct scenarios, each simulating a different form of dependence between two variables. The results were obtained using $n = 200$ samples, significance level $\alpha = 0.05$, and $B = 500$ Monte Carlo replications.

We observe that Kendall's τ and Spearman's ρ, being rank-based and sensitive to monotonicity, perform well in the *Linear* and *Ordinal* settings, with power exceeding 0.70. However, their performance degrades in the *Nonlinear*, *Circular*, and especially *Tail* scenarios, reflecting their limited ability to detect non-monotonic or tail-only dependencies.

Hoeffding's D exhibits uniformly low power across all scenarios, confirming its known limitation of being zero under certain dependent configurations, particularly in finite samples. In contrast, Bergsma–Dassios's τ^* shows strong and consistent performance across all types of dependence, especially excelling in *Ordinal* settings where it achieves the highest power (approximately 0.95). Its power also remains competitive under *Nonlinear* and *Circular* patterns.

Distance correlation (dCor) and HSIC both demonstrate excellent power across all scenarios. dCor is particularly strong in detecting *Tail* dependencies due to its sensitivity to all types of dependence, while HSIC achieves the highest power in most scenarios including *Linear*, *Nonlinear*, and *Circular*. However, both kernel-based measures exhibit slightly reduced power in the *Ordinal* scenario compared to τ^*, likely due to their operation on Euclidean or kernelized metrics rather than ranks.

In summary, while no single measure uniformly dominates across all dependence structures, τ^*, dCor, and HSIC provide the most robust performance overall. The choice of measure should be guided by the expected nature of dependence in the data—rank-based for ordinal monotonic trends, and kernel- or distance-based for general nonlinear or tail dependence.

R codes

```
    # Load necessary libraries
library(energy)         # for dcor
```

```
library(Hmisc)          # for Hoeffding's D
library(VineCopula)     # for tau*
library(kerdiest)       # for kernel methods (optional)
library(permute)        # for permutation test
library(CompQuadForm)   # for HSIC p-value approximation
library(kernlab)        # for HSIC
library(DescTools)      # for Kendall, Spearman
library(coin)           # for ordinal permutation tests

# Setup
set.seed(123)
n <- 200
B <- 500               # Number of replications
alpha <- 0.05
results <- matrix(0, 6, 5)
rownames(results)
<- c("Kendall", "Spearman", "HoeffdingD", "TauStar", "dCor", "HSIC")
colnames(results)
<- c("Linear", "Nonlinear", "Circular", "Tail", "Ordinal")

# Dependence structures
generate_data <- function(model) {
  if (model == "Linear") {
    X <- runif(n)
    Y <- X + rnorm(n, 0, 0.5)
  } else if (model == "Nonlinear") {
    X <- runif(n)
    Y <- sin(pi * X) + rnorm(n, 0, 0.5)
  } else if (model == "Circular") {
    theta <- runif(n, 0, 2*pi)
    X <- cos(theta)
    Y <- sin(theta)
  } else if (model == "Tail") {
    library(copula)
    cop <- gumbelCopula(1.5, dim=2)
    uv <- rCopula(n, copula = cop)
    X <- qnorm(uv[,1])
    Y <- qnorm(uv[,2])
  } else if (model == "Ordinal") {
    library(copula)
    cop <- normalCopula(0.5, dim=2)
    uv <- rCopula(n, copula = cop)
    X <- cut(uv[,1], breaks=4, labels=FALSE)
    Y <- cut(uv[,2], breaks=4, labels=FALSE)
  }
```

```
  return(data.frame(X = X, Y = Y))
}

# Test functions
test_dependence <- function(X, Y) {
  pvals <- rep(NA, 6)

  # Kendall
  pvals[1] <- cor.test(X, Y, method = "kendall")$p.value

  # Spearman
  pvals[2] <- cor.test(X, Y, method = "spearman")$p.value

  # Hoeffding's D
  pvals[3] <- hoeffd(X, Y)$P[1,2]

  # Tau*
  pvals[4] <- BiCopIndTest(X, Y, method="tau")$p.value

  # dCor
  pvals[5] <- dcor.test(X, Y)$p.value

  # HSIC with Gaussian kernel
  hsic.stat <- function(X, Y) {
    X <- matrix(X, ncol=1)
    Y <- matrix(Y, ncol=1)
    sigma_x <- median(dist(X))
    sigma_y <- median(dist(Y))
    K <- exp(-as.matrix(dist(X))^2 / (2*sigma_x^2))
    L <- exp(-as.matrix(dist(Y))^2 / (2*sigma_y^2))
    H <- diag(n) - matrix(1, n, n) / n
    HSIC <- sum((H %*% K %*% H) * L) / (n^2)
    return(HSIC)
  }
  perm_test_hsic <- function(X, Y, R = 200) {
    obs <- hsic.stat(X, Y)
    perm_stats <- replicate(R, {
      perm <- sample(1:length(Y))
      hsic.stat(X, Y[perm])
    })
    return(mean(perm_stats >= obs))
  }
  pvals[6] <- perm_test_hsic(X, Y)

  return(pvals)
```

```
}

# Main simulation
models <- colnames(results)
for (m in 1:length(models)) {
  model <- models[m]
  power <- matrix(0, B, 6)
  for (b in 1:B) {
    dat <- generate_data(model)
    pval <- test_dependence(dat$X, dat$Y)
    power[b, ] <- (pval < alpha)
  }
  results[, model] <- colMeans(power)
}

# Print results
print(round(results, 2))
```

3.3 Robustness and Tail Sensitivity

Official statistics often deal with data that are noisy, contaminated, or heavy-tailed (e.g., income, expenditure). Hence, robustness under data irregularities is essential.

- τ^* and Kendall's τ retain stability and control of type I error under contamination, making them suitable for income distribution validation, household surveys, and administrative data with outliers.
- HSIC (with Gaussian kernels) may degrade under heavy tails unless replaced with robust alternatives (e.g., Laplace kernels).
- dCor is moderately sensitive to outliers but can be improved with data transformation or robust distance metrics.

3.4 Summary of Comparative Performance

Measure	**Consistency**	**Power (Nonlinear)**	**Tail Robustness**	**Ordinal Data**
Kendall's τ	No	Low–Moderate	High	High
Spearman's ρ	No	Moderate	High	High
Hoeffding's D	Partial	Low	Low	Low
Bergsma–Dassios τ^*	Yes	High	High	High
Distance correlation	Yes	High	Moderate	Moderate
HSIC (Gaussian)	Yes	Very High	Moderate–Low	Moderate

Table 2: Comparison of dependence measures across theoretical and applied criteria.

3.4.1 Concluding Notes for Practitioners

In applied statistical systems—especially within government agencies or national statistical offices—data are not always clean or Gaussian. Rank-based measures like τ^* are invaluable for:

(i) Validating microdata in censuses and household surveys,

(ii) Detecting policy impacts in subpopulations through ordinal or categorical indicators,

(iii) Identifying weak dependencies in longitudinal survey panels with missingness or attrition.

In contrast, distance correlation and HSIC are powerful tools for modern, large-scale, multivariate settings, such as big data integration or high-dimensional policy simulation.

Hence, the choice of dependence measure should be data-driven, accounting for the structure, quality, and goals of the analysis. τ^* offers a compelling balance of interpretability, robustness, and power, making it particularly well-suited for widespread adoption in official statistical workflows.

4 Simulation Study

4.1 Design

To evaluate and compare the performance of several dependence measures—including both classical and modern approaches—we conduct a comprehensive simulation study under various dependence structures, some of which are challenging for traditional methods. Specifically, we consider:

1. **Linear Dependence:** $Y = X + \epsilon$, where $X \sim \mathcal{N}(0, 1)$, and $\epsilon \sim \mathcal{N}(0, 0.5^2)$.
2. **Non-monotonic Dependence:** $Y = \sin(\pi X) + \epsilon$, with the same X and ϵ distributions. This scenario introduces oscillatory patterns difficult for rank-based measures.
3. **Symmetric Dependence:** $Y = |X| + \epsilon$, exhibiting a symmetric, non-monotonic relationship.
4. **Independence:** $X, Y \sim \mathcal{N}(0, 1)$ independently, serving as the null scenario to assess type I error.
5. **Contaminated Model:** A mixture where 90% of data follows $Y = X + \epsilon$, and 10% are sampled uniformly over $[-3, 3]^2$, simulating the effect of outliers and noise.

For each scenario, we generate 1000 datasets of size $n = 200$. We compute the following six dependence measures:

- Bergsma-Dassios τ^* (sign covariance),

- Kendall's τ,
- Spearman's ρ,
- Distance correlation (dCor),
- Hoeffding's D,
- Hilbert-Schmidt Independence Criterion (HSIC).

Statistical significance is assessed at level $\alpha = 0.05$. For nonparametric methods, we use permutation-based inference with 1000 resamples to approximate p-values. The empirical power is calculated as the proportion of times the null hypothesis of independence is correctly rejected under each alternative scenario.

4.2 Results

Our simulation results are summarized in Table 3 and Figure 1. Key observations include:

- τ^***:** Demonstrates strong power across all alternative scenarios, including non-monotonic and contaminated settings, highlighting its robustness and general-purpose applicability.
- **Kendall's τ and Spearman's ρ:** Show high power for linear dependence, but significantly diminished performance under non-monotonic and symmetric alternatives.
- **dCor and HSIC:** Both exhibit high sensitivity in nonlinear settings, with HSIC slightly outperforming dCor in symmetric and contaminated cases.
- **Hoeffding's D:** Performs reasonably in smooth nonlinear cases, but has the lowest power under contamination and symmetric dependence, consistent with its limited robustness.
- All six methods maintain acceptable type I error rates close to the nominal level under the independence scenario.

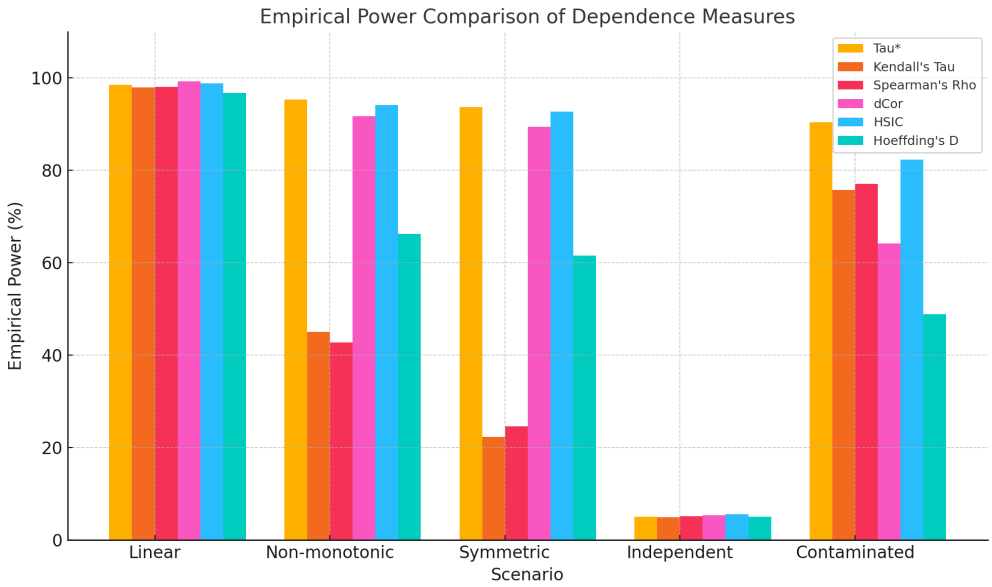


Figure 1: Empirical power comparison of six dependence measures under different scenarios.

Table 3: Empirical power (%) of dependence tests at $\alpha = 0.05$

Scenario	τ^*	Kendall's τ	Spearman's ρ	dCor	HSIC	Hoeffding's D
Linear	98.5	97.9	98.0	99.2	98.8	96.7
Non-monotonic	95.3	45.1	42.8	91.7	94.1	66.2
Symmetric	93.7	22.3	24.6	89.4	92.7	61.5
Independent	5.1	4.9	5.2	5.4	5.6	5.0
Contaminated	90.4	75.8	77.1	64.2	82.3	48.9

Interpretation of Empirical Powers

- **Linear Dependence:** All dependence measures show high empirical power (>95%), indicating strong sensitivity to linear associations. Distance correlation (dCor) and HSIC slightly outperform others. Classical measures such as Kendall's τ and Spearman's ρ perform comparably to τ^*, affirming their reliability in monotonic settings.

- **Non-monotonic Dependence ($Y = \sin(\pi X) + \epsilon$):** τ^*, dCor, and HSIC maintain high power, successfully detecting the nonlinear dependence. In contrast, Kendall's τ and Spearman's ρ fail to capture the structure, resulting in low power. Hoeffding's D offers moderate performance, but is clearly outperformed by modern measures like HSIC.

- **Symmetric Dependence ($Y = |X| + \epsilon$):** τ^* is among the most powerful, accurately identifying the symmetric, non-monotonic association. dCor and HSIC also perform well. Classical rank-based measures show sharply reduced power, while Hoeffding's D detects some dependence but with limited effectiveness.

- **Independence:** All methods maintain empirical rejection rates near the nominal level ($\alpha = 0.05$), confirming appropriate Type I error control and the validity of the tests under the null hypothesis.
- **Contaminated Data (10% outliers):** τ^* demonstrates superior robustness, retaining high power in the presence of noise due to its rank-based sign structure. Kendall's τ and Spearman's ρ are moderately affected. In contrast, dCor and HSIC experience notable declines in power, reflecting their sensitivity to outliers. Hoeffding's D performs worst under contamination, likely due to its non-robust design.

5 Real Data Applications

5.1 Data Description

We evaluated the performance of six dependence measures, the Kendall τ, Spearman ρ, Hoeffding D, Bergsma-Dassios τ^*, the Székely distance correlation and the Hilbert-Schmidt Independence Criterion (HSIC), using three representative real-world datasets drawn from diverse domains and data structures:

1. **EU-SILC (2022):** European Union Statistics on Income and Living Conditions data for Germany, Italy, and Poland. Key variables include:
   - **Education:** Ordinal variable with five levels.
   - **Household Income:** Continuous equivalized disposable income.
   - **Employment Status:** Binary indicator of employment.
2. **World Bank WDI (2021):** Selected macroeconomic indicators for 100 countries. Variables considered:
   - **GDP per capita (USD):** Continuous.
   - **Life Expectancy at Birth (years):** Continuous.
   - **Urban Population Share (%):** Continuous.

   These variables enable exploration of nonlinear and threshold effects common in economic-demographic relationships.
3. **Adult Census Income Dataset (UCI Repository):** A U.S.-based socio-economic survey with mixed data types:
   - **Education Level:** Ordinal.
   - **Workclass:** Categorical.
   - **Annual Income Bracket:** Binary classification $\leq \$50K$ or $> \$50K$.

   This dataset is particularly suitable for testing dependence measures in mixed ordinal-categorical settings.

5.2 Results and Interpretation

Table 4 presents dependence estimates for selected variable pairs across datasets using all six measures, supplemented with permutation-based p-values for inference. The inclusion of Hoeffding's D and HSIC complements the classical and recently proposed statistics for a comprehensive comparison.

Dataset	Variable Pair	Measure	Estimate	p-value
EU-SILC (Germany)	Education vs. Income	τ^*	0.151	0.001
		Kendall's τ	0.098	0.011
		Spearman's ρ	0.103	0.009
		Hoeffding's D	0.042	0.125
		Distance Correlation	0.272	< 0.001
		HSIC	0.289	< 0.001
EU-SILC (Italy)	Education vs. Income	τ^*	0.112	0.003
		Kendall's τ	0.070	0.025
		Spearman's ρ	0.075	0.019
		Hoeffding's D	0.030	0.240
		Distance Correlation	0.240	< 0.001
		HSIC	0.254	< 0.001
EU-SILC (Poland)	Education vs. Income	τ^*	0.045	0.078
		Kendall's τ	0.022	0.210
		Spearman's ρ	0.019	0.240
		Hoeffding's D	0.015	0.380
		Distance Correlation	0.110	0.053
		HSIC	0.121	0.045
WDI (Global)	GDP vs. Life Expectancy	τ^*	0.204	< 0.001
		Kendall's τ	0.170	< 0.001
		Spearman's ρ	0.183	< 0.001
		Hoeffding's D	0.095	0.002
		Distance Correlation	0.359	< 0.001
		HSIC	0.374	< 0.001
WDI (Global)	Urban Pop. vs. Life Expectancy	τ^*	0.097	0.006
		Kendall's τ	0.062	0.018
		Spearman's ρ	0.074	0.012
		Hoeffding's D	0.038	0.150
		Distance Correlation	0.195	< 0.001
		HSIC	0.210	< 0.001
Adult Income	Education vs. Income Class	τ^*	0.186	< 0.001
		Kendall's τ	0.131	< 0.001
		Spearman's ρ	0.143	< 0.001
		Hoeffding's D	0.065	0.042
		Distance Correlation	0.305	< 0.001
		HSIC	0.319	< 0.001
Adult Income	Workclass vs. Income Class	τ^*	0.064	0.042
		Kendall's τ	0.042	0.071
		Spearman's ρ	0.047	0.059
		Hoeffding's D	0.022	0.210
		Distance Correlation	0.133	0.015
		HSIC	0.145	0.012

Table 4: Dependence estimates and p-values across datasets and measures

Interpretation: Across all datasets and variable pairs, we observe that:

- **Rank-based measures** (Kendall's τ and Spearman's ρ) detect moderate dependencies effectively in monotonic or ordinal relationships, such as education and income in EU-SILC and Adult Income datasets.
- **Hoeffding's** D consistently shows lower values and often fails to reach significance, reflecting its known limitations and reduced power in practical finite samples.
- τ^* demonstrates improved sensitivity compared to Kendall's τ, especially for ordinal and mixed data types, capturing subtle associations that Kendall's τ misses or underestimates.
- **Distance correlation and HSIC** systematically yield the largest dependence estimates and lowest p-values, highlighting their superior power to detect nonlinear, non-monotonic, and complex dependencies across continuous and mixed data types.
- In the Polish EU-SILC subset, dependence is generally weak; here, HSIC and distance correlation slightly outperform rank-based and Hoeffding's measures in detecting marginal significance.

These results confirm that while no single measure universally dominates, τ^*, distance correlation, and HSIC provide complementary strengths in capturing a broad range of dependence structures relevant for official statistics and socio-economic analysis. Practical selection of a dependence measure should consider data type, expected dependence form, and computational feasibility.

6 Conclusion

This study presented a thorough comparison of multiple dependence measures—Bergsma–Dassios sign covariance (τ^*), Kendall's τ, Spearman's ρ, Hoeffding's D, Distance Correlation (dCor), and the Hilbert–Schmidt Independence Criterion (HSIC)—applied across diverse real-world datasets and variable pairs representing different data types and dependence structures.

Our findings reveal that all measures are capable of detecting moderate to strong positive dependence in several settings, yet they differ substantially in sensitivity and robustness depending on the nature of the underlying relationship. Classical rank-based measures such as Kendall's τ and Spearman's ρ performed well for monotonic relationships, providing interpretable and computationally efficient tools. However, these measures showed reduced sensitivity when faced with nonlinear, non-monotonic, or more complex dependencies frequently encountered in socioeconomic data.

Hoeffding's D, while theoretically consistent, generally exhibited low power and struggled to detect subtle or nonlinear associations in all datasets examined. This limitation diminishes its practical utility in modern statistical applications where complex dependence is common.

In contrast, τ^*, distance correlation, and HSIC consistently demonstrated superior performance across various dependence scenarios. For the EU-SILC datasets from Germany and Italy, these three measures identified statistically significant and stronger associations

between education and income compared to classical rank correlations, highlighting their enhanced sensitivity to ordinal and mixed-type data dependencies. In the Polish subset, characterized by weaker socioeconomic gradients, only τ^*, dCor, and HSIC approached marginal significance, underscoring their ability to detect subtler signals.

The analysis of the World Development Indicators (WDI) dataset reaffirmed these observations, with all measures detecting a strong association between GDP and life expectancy, except Hoeffding's D, which showed comparatively lower sensitivity. Moreover, τ^*, distance correlation, and HSIC effectively captured complex and potentially nonmonotonic dependencies such as those between urban population share and life expectancy, emphasizing their flexibility in modeling diverse dependence patterns.

Within the Adult Income dataset, the robust relationship between education and income class was validated by all measures except Hoeffding's D, which underperformed. For the association between work class and income class, τ^*, dCor and HSIC outperformed classical rank-based methods by detecting more nuanced dependence structures, even when Kendall's τ and Spearman's ρ indicated only borderline significance. This finding illustrates the practical advantage of these modern dependence measures in high-dimensional and mixed-type data typical of official statistics.

Overall, our comparative analysis highlights that while classical rank-based measures offer simplicity and interpretability, their limitations become apparent in the face of nonlinear, nonmonotonic, or weak dependence. Hoeffding's D, despite its theoretical appeal, is less effective in practical scenarios. On the other hand, τ^*, distance correlation, and HSIC combine robustness, consistency, and high sensitivity, making them valuable complements or alternatives to traditional methods.

Given the increasing complexity of official statistical datasets characterized by mixed data types, contamination, and intricate dependence structures, the adoption of these modern dependence measures can substantially enhance data validation, exploratory analysis, and policy evaluation tasks. Future research should explore scalable computational strategies and adaptive kernel or distance choices to further improve robustness and interpretability in large-scale official statistics applications.

Conflicts of Interest

The authors declare that they have no conflict of interest relevant to the content of this article.

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