Supplementary Information for “Smaller, closer, and fully negative: optical torque engineering in dielectric-metal heterodimers”

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Supplementary note 1: Green’s functions of electric and magnetic dipoles

The free-space dyadic Green’s functions $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$ of electric dipole (ED) and magnetic dipole (MD) [1] are written as

$$\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}(\mathbf{r},\mathbf{r}_0)=\frac{\exp(ikR)}{4\pi R^3}\left[\left(3-3ikR-k^2R^2\right)\frac{\mathbf{RR}}{R^2}+\left(-1+ikR+k^2R^2\right)\overleftrightarrow{\mathbf{I}}\right], \tag{S1.1}$$

$$\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}(\mathbf{r},\mathbf{r}_0)=\frac{\exp(ikR)}{4\pi R^3}\left(ik^2R^2-kR\right)\frac{\mathbf{R}\times\overleftrightarrow{\mathbf{I}}}{R}, \tag{S1.2}$$

where $\overleftrightarrow{\mathrm{I}}$ is unit dyad, i is unit imaginary number, and k is wave number of incident light. $\mathbf{RR}$ represents the outer product of distance vector $\mathbf{R}=\mathbf{r}-\mathbf{r}_0$ with itself. $\mathbf{r}$ and $\mathbf{r}_0$ represent respectively the positions of the field point and source (dipole) point. Here, $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}(\mathbf{r},\mathbf{r}_0)=\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}(\mathbf{r}_0,\mathbf{r})$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}(\mathbf{r},\mathbf{r}_0)=-\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}(\mathbf{r}_0,\mathbf{r})$. Hereafter, the notation $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}(\mathbf{r},\mathbf{r}_0)$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}(\mathbf{r},\mathbf{r}_0)$ are simplified as $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ for convenience of derivation. $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ presents the electric (magnetic) field propagator of ED (MD) and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$ presents the magnetic (electric) field propagator of ED (MD). For simplifying the derivation process of optical forces, $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$ are rewritten by their eigenequations [2] as

$$\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}=\left(\kappa-\mu\right)\frac{\mathbf{RR}}{R^2}+\mu\overleftrightarrow{\mathbf{I}}=\begin{pmatrix}\mu & 0 & 0\\ 0 & \kappa & 0\\ 0 & 0 & \mu\end{pmatrix}, \tag{S1.3}$$

$$\overleftrightarrow{\mathbf{G}}_{\mathrm{M}} = \eta \mathbf{R} \times \overleftrightarrow{\mathbf{I}} / R = \eta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \tag{S1.4}$$

with

$$\begin{aligned} \mu &= \frac{\exp(ikR)}{4\pi R^3}\left(k^2R^2 + ikR - 1\right) \\ \kappa &= \frac{\exp(ikR)}{4\pi R^3}\left(-2ikR + 2\right) \\ \eta &= \frac{\exp(ikR)}{4\pi R^3}\left(ik^2R^2 - kR\right) \end{aligned} \quad , \tag{S1.5}$$

where μ and κ are eigenvalues of $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ while η is eigenvalue of $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$. μ and η are orthghonal to $\mathbf{R}$ while κ is paralled to $\mathbf{R}$. The dot products of $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$ with unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ of x, y, and z axes satisfy the following relations

$$\begin{aligned} &\overleftrightarrow{\mathbf{G}}_{\mathrm{E}} \cdot \mathbf{e}_x = \mu \mathbf{e}_x, \qquad &\overleftrightarrow{\mathbf{G}}_{M} \cdot \mathbf{e}_x = -\eta \mathbf{e}_z, \\ &\overleftrightarrow{\mathbf{G}}_{\mathrm{E}} \cdot \mathbf{e}_y = \kappa \mathbf{e}_y, \qquad &\overleftrightarrow{\mathbf{G}}_{M} \cdot \mathbf{e}_y = 0, \\ &\overleftrightarrow{\mathbf{G}}_{\mathrm{E}} \cdot \mathbf{e}_z = \mu \mathbf{e}_z, \qquad &\overleftrightarrow{\mathbf{G}}_{M} \cdot \mathbf{e}_z = \eta \mathbf{e}_x. \end{aligned} \tag{S1.6}$$

Supplementary note 2: Self-consistent distributions of electromagnetic fields and dipoles

The total incoming electric (magnetic) field $\mathbf{E}^{\mathrm{A}}$ ($\mathbf{H}^{\mathrm{A}}$) on dielectric particle is the sum of the electric (magnetic) field $\mathbf{E}_0^{\mathrm{A}}$ ($\mathbf{H}_0^{\mathrm{A}}$) of incident plane light and radiated electric (magnetic) field of ED moment in metal particle ($\mathbf{p}^{\mathrm{B}} = \varepsilon_0 \varepsilon_{\mathrm{m}} \alpha_{\mathrm{e}}^{\mathrm{B}} \mathbf{E}^{\mathrm{B}}$) as expressed by

$$\mathbf{E}^{\mathrm{A}} = \mathbf{E}_0^{\mathrm{A}} + \frac{1}{\varepsilon_0 \varepsilon_m} \overleftrightarrow{\mathbf{G}}_{\mathrm{E}} \cdot \mathbf{p}^{\mathrm{B}}, \tag{S2.1}$$

$$\mathbf{H}^{\mathrm{A}} = \mathbf{H}_0^{\mathrm{A}} - \frac{i}{Z \varepsilon_0 \varepsilon_m} \overleftrightarrow{\mathbf{G}}_{\mathrm{M}} \cdot \mathbf{p}^{\mathrm{B}}, \tag{S2.2}$$

where $Z=[\mu_0\mu_{\mathrm{m}}/(\varepsilon_0\varepsilon_{\mathrm{m}})]^{1/2}$ is impedance of medium. ε_{m} and ε_{m} (μ_0 and μ_{m}) are respectively permittivities (permeabilities) in vacuum and medium. $\alpha_{\mathrm{e}}^{\mathrm{B}}$ is electric polarizability of particle B. On the other hand, the total incoming electric (magnetic) field $\mathbf{E}^{\mathrm{B}}$ ($\mathbf{H}^{\mathrm{B}}$) around metal particle is the sum of the electric (magnetic) field $\mathbf{E}_0^{\mathrm{B}}$ ($\mathbf{H}_0^{\mathrm{B}}$) of incident wave and radiated electric (magnetic) field of ED moment ($\mathbf{p}^{\mathrm{A}}=\varepsilon_0\varepsilon_{\mathrm{m}}\alpha_{\mathrm{e}}^{\mathrm{A}}\mathbf{E}^{\mathrm{A}}$) and MD moment ($\mathbf{m}^{\mathrm{A}}=\alpha_{\mathrm{m}}^{\mathrm{A}}\mathbf{H}^{\mathrm{A}}$) in dielectric particle, which are written as

$$\mathbf{E}^{\mathrm{B}}=\mathbf{E}_0^{\mathrm{B}}+\frac{1}{\varepsilon_0\varepsilon_m}\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}\cdot\mathbf{p}^{\mathrm{A}}-iZ\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}\cdot\mathbf{m}^{\mathrm{A}}, \tag{S2.3}$$

$$\mathbf{H}^{\mathrm{B}}=\mathbf{H}_0^{\mathrm{B}}+\frac{i}{Z\varepsilon_0\varepsilon_m}\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}\cdot\mathbf{p}^{\mathrm{A}}+\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}\cdot\mathbf{m}^{\mathrm{A}}, \tag{S2.4}$$

where $\alpha_{\mathrm{e}}^{\mathrm{A}}$ and $\alpha_{\mathrm{m}}^{\mathrm{A}}$ are electric and magnetic polarizabilities of particle A. The x, y, and z components of electric and magnetic field vectors at particle A can be obtained by taking dot product of Eqs. (S2.1)-(S2.2) with unit vectors $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ as

$$\begin{aligned}
&E_x^{\mathrm{A}}=E_{0,x}^{\mathrm{A}}+\frac{1}{\varepsilon_0\varepsilon_m}\mu p_x^{\mathrm{B}}, && H_x^{\mathrm{A}}=H_{0,x}^{\mathrm{A}}+\frac{i}{Z\varepsilon_0\varepsilon_m}\eta p_z^{\mathrm{B}},\\
&E_y^{\mathrm{A}}=E_{0,y}^{\mathrm{A}}+\frac{1}{\varepsilon_0\varepsilon_m}\kappa p_y^{\mathrm{B}}, && H_y^{\mathrm{A}}=H_{0,y}^{\mathrm{A}},\\
&E_z^{\mathrm{A}}=\frac{1}{\varepsilon_0\varepsilon_m}\mu p_z^{\mathrm{B}}, && H_z^{\mathrm{A}}=-\frac{i}{Z\varepsilon_0\varepsilon_m}\eta p_x^{\mathrm{B}},
\end{aligned} \tag{S2.5}$$

while the counterparts for particle B based on Eqs. (S2.3)-(S2.4) read as

$$\begin{aligned}
&E_x^{\mathrm{B}}=E_{0,x}^{\mathrm{B}}+\frac{1}{\varepsilon_0\varepsilon_m}\mu p_x^{\mathrm{A}}+iZ\eta m_z^{\mathrm{A}}, && H_x^{\mathrm{B}}=H_{0,x}^{\mathrm{B}}-\frac{i}{Z\varepsilon_0\varepsilon_m}\eta p_z^{\mathrm{A}}+\mu m_x^{\mathrm{A}},\\
&E_y^{\mathrm{B}}=E_{0,y}^{\mathrm{B}}+\frac{1}{\varepsilon_0\varepsilon_m}\kappa p_y^{\mathrm{A}}, && H_y^{\mathrm{B}}=H_{0,y}^{\mathrm{B}}+\kappa m_y^{\mathrm{A}},\\
&E_z^{\mathrm{B}}=\frac{1}{\varepsilon_0\varepsilon_m}\mu p_z^{\mathrm{A}}-iZ\eta m_x^{\mathrm{A}}, && H_z^{\mathrm{B}}=\frac{i}{Z\varepsilon_0\varepsilon_m}\eta p_x^{\mathrm{A}}+\mu m_z^{\mathrm{A}},
\end{aligned} \tag{S2.6}$$

Supplementary note 3: Addressed electric and magnetic dipolar polarizabilities

Based on Eqs. (S2.5) and (S2.6), the addressed electric and magnetic dipolar polarizabilities of particle A along x, y, and z axes are written as

$$\begin{aligned}
&\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}=\alpha_{\mathrm{e}}^{\mathrm{A}}\left[1+\frac{\mu\alpha_{\mathrm{e}}^{\mathrm{B}}\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}\right)}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}}\right], && \tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}}=\alpha_{\mathrm{m}}^{\mathrm{A}}\frac{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}},\\
&\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}}=\alpha_{\mathrm{e}}^{\mathrm{A}}\frac{1+\kappa\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-\kappa^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, && \tilde{\alpha}_{\mathrm{m},y}^{\mathrm{A}}=\alpha_{\mathrm{m}}^{\mathrm{A}},\\
&\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}=\alpha_{\mathrm{e}}^{\mathrm{A}}\frac{-i\eta\mu\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}}, && \tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}}=\alpha_{\mathrm{m}}^{\mathrm{A}}\frac{i\eta\alpha_{\mathrm{e}}^{\mathrm{B}}\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}\right)}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}},
\end{aligned} \tag{S3.1}$$

and the addressed electric dipolar polarizabilities of particle B read as

$$\begin{aligned}
\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}} &= \alpha_{\mathrm{e}}^{\mathrm{B}} \frac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}}, \\
\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}} &= \alpha_{\mathrm{e}}^{\mathrm{B}} \frac{1+\kappa\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-\kappa^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, \\
\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}} &= \alpha_{\mathrm{e}}^{\mathrm{B}} \frac{-i\eta\alpha_{\mathrm{m}}^{\mathrm{A}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}-\eta^2\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}},
\end{aligned} \tag{S3.2}$$

where $\alpha_{\mathrm{e}} = i\frac{6\pi}{k^3}a_1$ and $\alpha_{\mathrm{m}} = i\frac{6\pi}{k^3}b_1$. a_1 and b_1 represent the electric and magnetic dipolar Mie scattering coefficients [3]. Therefore, the effective ED moments of two particles along three axes are expressed as $p_\nu^\vartheta = \varepsilon_0\varepsilon_{\mathrm{m}}\tilde{\alpha}_{\mathrm{e},\nu}^\vartheta E_\nu^\vartheta$ (ϑ = A and B; $\nu = x$, y, and z) while the effective MD moments of dielectric particle are represented by $m_\nu^{\mathrm{A}} = \tilde{\alpha}_{\mathrm{m},\nu}^{\mathrm{A}} H_\nu^{\mathrm{A}}$. Equation (S3.1) demonstrates that ED and MD moments in dielectric particle is induced not only by the incident light but also by the ED in metal particle. In addition, Eq. (S3.2) denotes that the ED moment in metal particle originates not only from the incident field but also from the ED and MD in dielectric particle. The mutual influences between ED and MD moments in different particle are termed hybridization interaction.

Supplementary note 4: Spatial derivatives of electric and magnetic fields

In order to calculate the optical force, the spatial derivatives of electric and magnetic field vectors at particles A and B along ν axis are presented as

$$\frac{\partial \mathbf{E}^{\mathrm{A}}}{\partial \nu} = \frac{\partial \mathbf{E}_0^{\mathrm{A}}}{\partial \nu} + \frac{1}{\varepsilon_0\varepsilon_s}\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial \nu}\cdot\mathbf{p}^{\mathrm{B}}, \tag{S4.1}$$

$$\frac{\partial \mathbf{H}^{\mathrm{A}}}{\partial \nu} = \frac{\partial \mathbf{H}_0^{\mathrm{A}}}{\partial \nu} - \frac{i}{Z\varepsilon_0\varepsilon_s}\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial \nu}\cdot\mathbf{p}^{\mathrm{A}}, \tag{S4.2}$$

$$\frac{\partial \mathbf{E}^{\mathrm{B}}}{\partial \nu} = \frac{\partial \mathbf{E}_0^{\mathrm{B}}}{\partial \nu} + \frac{1}{\varepsilon_0\varepsilon_s}\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial \nu}\cdot\mathbf{p}^{\mathrm{A}} + iZ\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial \nu}\cdot\mathbf{m}^{\mathrm{A}}, \tag{S4.3}$$

$$\frac{\partial \mathbf{H}^{\mathrm{B}}}{\partial \nu} = \frac{\partial \mathbf{H}_0^{\mathrm{B}}}{\partial \nu} - \frac{i}{Z\varepsilon_0\varepsilon_s}\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial \nu}\cdot\mathbf{p}^{\mathrm{A}} + \frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial \nu}\cdot\mathbf{m}^{\mathrm{A}}, \tag{S4.4}$$

where $\partial/\partial\nu$ denotes the partial derivative with respect to the arguments x, y, and z. Nota that $\partial\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}/\partial\nu\big|_{\mathrm{A}} = -\partial\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}/\partial\nu\big|_{\mathrm{B}}$ and $\partial\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}/\partial\nu\big|_{\mathrm{A}} = \partial\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}/\partial\nu\big|_{\mathrm{B}}$. Moreover, the partial derivatives of $\overleftrightarrow{\mathbf{G}}_{\mathrm{E}}$ and $\overleftrightarrow{\mathbf{G}}_{\mathrm{M}}$ are expressed as

$$\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial x}=\frac{\kappa-\mu}{R}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\quad \frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial y}=\begin{pmatrix}\frac{\partial\mu}{\partial y}&0&0\\0&\frac{\partial\kappa}{\partial y}&0\\0&0&\frac{\partial\mu}{\partial y}\end{pmatrix},\quad \frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{E}}}{\partial z}=\frac{\kappa-\mu}{R}\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}, \tag{S4.5}$$

$$\frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial x}=\frac{\eta}{R}\begin{pmatrix}0&0&0\\0&0&1\\0&-1&0\end{pmatrix},\quad \frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial y}=\frac{\partial\eta}{\partial y}\begin{pmatrix}0&0&-1\\0&0&0\\1&0&0\end{pmatrix},\quad \frac{\partial \overleftrightarrow{\mathbf{G}}_{\mathrm{M}}}{\partial z}=\frac{\eta}{R}\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix}. \tag{S4.6}$$

Therefore, Substituting Eqs. (S4.5) and (S4.6) to Eqs. (S4.1)-(S4.4), the partial derivatives of x, y, and z components of the incoming electric and magnetic fields at two particles are expressed as

$$\begin{pmatrix}\frac{\partial E_x^{\mathrm{A}}}{\partial x}\\ \frac{\partial E_y^{\mathrm{A}}}{\partial x}\\ \frac{\partial E_z^{\mathrm{A}}}{\partial x}\end{pmatrix}=\begin{pmatrix}\frac{\kappa-\mu}{\varepsilon_0\varepsilon_m R}p_y^{\mathrm{B}}\\ \frac{\kappa-\mu}{\varepsilon_0\varepsilon_m R}p_x^{\mathrm{B}}\\ 0\end{pmatrix},\qquad \begin{pmatrix}\frac{\partial H_x^{\mathrm{A}}}{\partial x}\\ \frac{\partial H_y^{\mathrm{A}}}{\partial x}\\ \frac{\partial H_z^{\mathrm{A}}}{\partial x}\end{pmatrix}=\begin{pmatrix}0\\ -\frac{i}{Z\varepsilon_0\varepsilon_m}\frac{\eta}{R}p_z^{\mathrm{B}}\\ \frac{i}{Z\varepsilon_0\varepsilon_m}\frac{\eta}{R}p_y^{\mathrm{B}}\end{pmatrix}, \tag{S4.7}$$

$$\begin{pmatrix}\frac{\partial E_x^{\mathrm{A}}}{\partial y}\\ \frac{\partial E_y^{\mathrm{A}}}{\partial y}\\ \frac{\partial E_z^{\mathrm{A}}}{\partial y}\end{pmatrix}=\frac{1}{\varepsilon_0\varepsilon_m}\begin{pmatrix}\frac{\partial\mu}{\partial y}p_x^{\mathrm{B}}\\ \frac{\partial\kappa}{\partial y}p_y^{\mathrm{B}}\\ \frac{\partial\mu}{\partial y}p_z^{\mathrm{B}}\end{pmatrix},\qquad \begin{pmatrix}\frac{\partial H_x^{\mathrm{A}}}{\partial y}\\ \frac{\partial H_y^{\mathrm{A}}}{\partial y}\\ \frac{\partial H_z^{\mathrm{A}}}{\partial y}\end{pmatrix}=\begin{pmatrix}\frac{i}{Z\varepsilon_0\varepsilon_m}\frac{\partial\eta}{\partial y}p_z^{\mathrm{B}}\\ 0\\ -\frac{i}{Z\varepsilon_0\varepsilon_m}\frac{\partial\eta}{\partial y}p_x^{\mathrm{B}}\end{pmatrix}, \tag{S4.8}$$

$$\begin{pmatrix}\frac{\partial E_x^{\mathrm{A}}}{\partial z}\\ \frac{\partial E_y^{\mathrm{A}}}{\partial z}\\ \frac{\partial E_z^{\mathrm{A}}}{\partial z}\end{pmatrix}=\begin{pmatrix}\frac{\partial E_{0,x}^{\mathrm{A}}}{\partial z}\\ \frac{\partial E_{0,y}^{\mathrm{A}}}{\partial z}+\frac{\kappa-\mu}{\varepsilon_0\varepsilon_m R}p_z^{\mathrm{B}}\\ \frac{\kappa-\mu}{\varepsilon_0\varepsilon_m R}p_y^{\mathrm{B}}\end{pmatrix},\qquad \begin{pmatrix}\frac{\partial H_x^{\mathrm{A}}}{\partial z}\\ \frac{\partial H_y^{\mathrm{A}}}{\partial z}\\ \frac{\partial H_z^{\mathrm{A}}}{\partial z}\end{pmatrix}=\begin{pmatrix}\frac{\partial H_{0,x}^{\mathrm{A}}}{\partial z}-\frac{i}{Z_0\varepsilon_0\varepsilon_m}\frac{\eta}{R}p_y^{\mathrm{B}}\\ \frac{\partial H_{0,y}^{\mathrm{A}}}{\partial z}+\frac{i}{Z_0\varepsilon_0\varepsilon_m}\frac{\eta}{R}p_x^{\mathrm{B}}\\ 0\end{pmatrix}. \tag{S4.9}$$

$$\begin{pmatrix} \dfrac{\partial E_x^{\mathrm{B}}}{\partial x} \\ \dfrac{\partial E_y^{\mathrm{B}}}{\partial x} \\ \dfrac{\partial E_z^{\mathrm{B}}}{\partial x} \end{pmatrix} = \begin{pmatrix} -\dfrac{\kappa-\mu}{\varepsilon_0\varepsilon_m R} p_y^{\mathrm{A}} \\ -\dfrac{\kappa-\mu}{\varepsilon_0\varepsilon_m R} p_x^{\mathrm{A}} - iZ\dfrac{\eta}{R} m_z^{\mathrm{A}} \\ iZ\dfrac{\eta}{R} m_y^{\mathrm{A}} \end{pmatrix}, \quad \begin{pmatrix} \dfrac{\partial H_x^{\mathrm{B}}}{\partial x} \\ \dfrac{\partial H_y^{\mathrm{B}}}{\partial x} \\ \dfrac{\partial H_z^{\mathrm{B}}}{\partial x} \end{pmatrix} = \begin{pmatrix} -\dfrac{\kappa-\mu}{R} m_y^{\mathrm{A}} \\ \dfrac{i}{Z\varepsilon_0\varepsilon_m}\dfrac{\eta}{R} p_z^{\mathrm{A}} - \dfrac{\kappa-\mu}{R} m_x^{\mathrm{A}} \\ -\dfrac{i}{Z\varepsilon_0\varepsilon_m}\dfrac{\eta}{R} p_y^{\mathrm{A}} \end{pmatrix}, \tag{S4.10}$$

$$\begin{pmatrix} \dfrac{\partial E_x^{\mathrm{B}}}{\partial y} \\ \dfrac{\partial E_y^{\mathrm{B}}}{\partial y} \\ \dfrac{\partial E_z^{\mathrm{B}}}{\partial y} \end{pmatrix} = \begin{pmatrix} \dfrac{-1}{\varepsilon_0\varepsilon_m}\dfrac{\partial\mu}{\partial y} p_x^{\mathrm{A}} + iZ\dfrac{\partial\eta}{\partial y} m_z^{\mathrm{A}} \\ \dfrac{-1}{\varepsilon_0\varepsilon_m}\dfrac{\partial\kappa}{\partial y} p_y^{\mathrm{A}} \\ \dfrac{-1}{\varepsilon_0\varepsilon_m}\dfrac{\partial\mu}{\partial y} p_z^{\mathrm{A}} - iZ\dfrac{\partial\eta}{\partial y} m_x^{\mathrm{A}} \end{pmatrix}, \quad \begin{pmatrix} \dfrac{\partial H_x^{\mathrm{B}}}{\partial y} \\ \dfrac{\partial H_y^{\mathrm{B}}}{\partial y} \\ \dfrac{\partial H_z^{\mathrm{B}}}{\partial y} \end{pmatrix} = \begin{pmatrix} \dfrac{-i}{Z\varepsilon_0\varepsilon_m}\dfrac{\partial\eta}{\partial y} p_z^{\mathrm{A}} - \dfrac{\partial\mu}{\partial y} m_x^{\mathrm{A}} \\ -\dfrac{\partial\kappa}{\partial y} m_y^{\mathrm{A}} \\ \dfrac{i}{Z\varepsilon_0\varepsilon_m}\dfrac{\partial\eta}{\partial y} p_x^{\mathrm{A}} - \dfrac{\partial\mu}{\partial y} m_z^{\mathrm{A}} \end{pmatrix}, \tag{S4.11}$$

$$\begin{pmatrix} \dfrac{\partial E_x^{\mathrm{B}}}{\partial z} \\ \dfrac{\partial E_y^{\mathrm{B}}}{\partial z} \\ \dfrac{\partial E_z^{\mathrm{B}}}{\partial z} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial E_{0,x}^{\mathrm{B}}}{\partial z} - iZ\dfrac{\eta}{R} m_y^{\mathrm{A}} \\ \dfrac{\partial E_{0,y}^{\mathrm{B}}}{\partial z} - \dfrac{\kappa-\mu}{\varepsilon_0\varepsilon_m R} p_z^{\mathrm{A}} + iZ\dfrac{\eta}{R} m_x^{\mathrm{A}} \\ -\dfrac{\kappa-\mu}{\varepsilon_0\varepsilon_m R} p_y^{\mathrm{A}} \end{pmatrix}, \quad \begin{pmatrix} \dfrac{\partial H_x^{\mathrm{B}}}{\partial z} \\ \dfrac{\partial H_y^{\mathrm{B}}}{\partial z} \\ \dfrac{\partial H_z^{\mathrm{B}}}{\partial z} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial H_{0,x}^{\mathrm{B}}}{\partial z} + \dfrac{i}{Z\varepsilon_0\varepsilon_m}\dfrac{\eta}{R} p_y^{\mathrm{A}} \\ \dfrac{\partial H_{0,y}^{\mathrm{B}}}{\partial z} - \dfrac{i}{Z\varepsilon_0\varepsilon_m}\dfrac{\eta}{R} p_x^{\mathrm{A}} - \dfrac{\kappa-\mu}{R} m_z^{\mathrm{A}} \\ -\dfrac{\kappa-\mu}{R} m_y^{\mathrm{A}} \end{pmatrix}. \tag{S4.12}$$

Supplementary note 5: Optical forces along *x* axis

The electric fields of a left-circularly polarized plane light propagating along the z axis on particles A and B are identical and read as

$$\mathbf{E}_0^{\mathrm{A}} = \mathbf{E}_0^{\mathrm{B}} = \mathbf{E}_0 = \frac{E_0}{\sqrt{2}}(1,\ i,\ 0)\exp(ikz). \tag{S5.1}$$

In addition, the optical force formulae [4] on two particles are described by

$$\begin{aligned} \mathbf{F}^{\mathrm{A}} &= \frac{1}{2}\mathrm{Re}\left[(\mathbf{p}^{\mathrm{A}}\nabla)\mathbf{E}^{\mathrm{A}*} + \mu_0\mu_s(\mathbf{m}^{\mathrm{A}}\nabla)\mathbf{H}^{\mathrm{A}*} - \frac{Zk^4}{6\pi}(\mathbf{p}^{\mathrm{A}}\times\mathbf{m}^{\mathrm{A}*})\right], \\ &= \mathbf{F}_{\mathrm{e}}^{\mathrm{A}} + \mathbf{F}_{\mathrm{m}}^{\mathrm{A}} + \mathbf{F}_{\mathrm{em}}^{\mathrm{A}} \end{aligned} \tag{S5.2}$$

$$\mathbf{F}^{\mathrm{B}} = \frac{1}{2}\mathrm{Re}\left[(\mathbf{p}^{\mathrm{B}}\nabla)\mathbf{E}^{\mathrm{B}*}\right] = \mathbf{F}_{\mathrm{e}}^{\mathrm{B}}. \tag{S5.3}$$

Taking the resulting self-consistent distributions of dipole moments and partial derivatives of fields in Eqs. (S3.1)-(S3.2), (S4.7)-(S4.12) into Eqs. (S5.2) and (S5.3), the electric dipolar component of optical azimuthal force along the x axis on particle A is written as

$$
\begin{aligned}
F_{\mathrm{e},x}^{\mathrm{A}} &= \frac{1}{2}\mathrm{Re}\left[p_x^{\mathrm{A}} \frac{\partial E_x^{\mathrm{A}*}}{\partial x} + p_y^{A} \frac{\partial E_y^{\mathrm{A}*}}{\partial x} + p_z^{A} \frac{\partial E_z^{\mathrm{A}*}}{\partial x} \right] \\
&= \frac{1}{2\varepsilon_0\varepsilon_m}\mathrm{Re}\left[\frac{(\kappa-\mu)^*}{R}\left(p_x^{\mathrm{A}} p_y^{\mathrm{B}*} + p_y^{\mathrm{A}} p_x^{\mathrm{B}*} \right) \right] \\
&= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{i(\kappa-\mu)^*}{R}\left(\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}*} - \tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}*} \right) \right]
\end{aligned} , \tag{S5.4}
$$

and the magnetic dipolar component reads as

$$
\begin{aligned}
F_{\mathrm{m},x}^{\mathrm{A}} &= \frac{\mu_0\mu_m}{2}\mathrm{Re}\left[m_x^{\mathrm{A}} \frac{\partial H^{\mathrm{A}*}}{\partial x} + m_y^{\mathrm{A}} \frac{\partial H_y^{\mathrm{A}*}}{\partial x} + m_z^{\mathrm{A}} \frac{\partial H_z^{\mathrm{A}*}}{\partial x} \right] \\
&= \frac{\mu_0\mu_m}{2Z\varepsilon_0\varepsilon_m}\mathrm{Re}\left[\frac{i\eta^*}{R}\left(-m_y^{A} p_z^{B*} + m_z^{A} p_y^{B*} \right) \right] \\
&= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\eta^*}{R}\left(-\tilde{\alpha}_{\mathrm{m},y}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}*} + \tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}*} \right) \right]
\end{aligned} , \tag{S5.5}
$$

as well as the electric-magnetic dipolar coupling component is represented by

$$
\begin{aligned}
F_{\mathrm{em},x}^{\mathrm{A}} &= -\frac{Zk^4}{12\pi}\mathrm{Re}\left[p_y^{\mathrm{A}} m_z^{\mathrm{A}*} - p_z^{\mathrm{A}} m_y^{\mathrm{A}*} \right] \\
&= -\frac{n_s I_0}{2c}\frac{k^4}{6\pi}\mathrm{Re}\left[i\left(\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}*} + \tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}} \tilde{\alpha}_{\mathrm{m},y}^{\mathrm{A}*} \right) \right]
\end{aligned} . \tag{S5.6}
$$

On the other hand, the azimuthal force on particle B is only caused by the electric dipolar interaction

$$
\begin{aligned}
F_x^{\mathrm{B}} &= \frac{1}{2}\mathrm{Re}\left[p_x^{\mathrm{B}} \frac{\partial E_x^{\mathrm{B}*}}{\partial x} + p_y^{\mathrm{B}} \frac{\partial E_y^{\mathrm{B}*}}{\partial x} + p_z^{\mathrm{B}} \frac{\partial E_z^{\mathrm{B}*}}{\partial x} \right] \\
&= \frac{1}{2\varepsilon_0\varepsilon_m}\mathrm{Re}\left[-\frac{(\kappa-\mu)^*}{R}\left(p_x^{\mathrm{B}} p_y^{\mathrm{A}*} + p_y^{\mathrm{B}} p_x^{\mathrm{A}*} \right) - iZ\frac{\eta^*}{R}\left(p_z^{\mathrm{B}} m_y^{\mathrm{A}*} - p_y^{\mathrm{B}} m_z^{\mathrm{A}*} \right) \right] \\
&= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{i(\kappa-\mu)^*}{R}\left(\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}} \tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}*} - \tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}} \tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}*} \right) + \frac{\eta^*}{R}\left(\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}} \alpha_{\mathrm{m},z}^{\mathrm{A}*} - \tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}} \tilde{\alpha}_{\mathrm{m},y}^{\mathrm{A}*} \right) \right]
\end{aligned} . \tag{S5.7}
$$

Equations. (S5.4)-(S5.7) are rigorous and correspond to Eqs. (2)-(5) in main text, respectively.

Note that in far-field limit, only the quadratic terms of kR (i.e., k^2R^2) in eigenvalues μ, κ, and η in Eqs. (S1.5) are retained

$$
\mu = \frac{\exp(ikR)}{4\pi R^3}\left(k^2R^2\right), \quad \kappa = 0, \quad \eta = \frac{\exp(ikR)}{4\pi R^3}\left(ik^2R^2\right). \tag{S5.8}
$$

Meanwhile, the secondary interactions of radiated fields by dipoles in particle A with the secondary dipoles in particle B induced by these fields are in general very small and ignorable [5] ($\mu^2\alpha_e^A\alpha_e^B \approx 0$, $\eta^2\alpha_e^B\alpha_m^A \approx 0$, and $\kappa^2\alpha_e^A\alpha_e^B \approx 0$). Thus, the addressed electric and magnetic dipolar polarizabilities of two particles in Eqs. (S3.1) and (S3.2) are simplifies as

$$\begin{aligned}
&\tilde{\alpha}_{e,x}^A \approx \alpha_e^A\left(1+\mu\alpha_e^B\right), && \tilde{\alpha}_{e,y}^A \approx \alpha_e^A\left(1+\kappa\alpha_e^B\right), && \tilde{\alpha}_{e,z}^A \approx \alpha_e^A\left(-i\eta\mu\alpha_e^B\alpha_m^A\right), \\
&\tilde{\alpha}_{m,x}^A \approx \alpha_m^A, && \tilde{\alpha}_{m,y}^A \approx \alpha_m^A, && \tilde{\alpha}_{m,z}^A \approx \alpha_m^A\left(i\eta\alpha_e^B\right)\left(1+\mu\alpha_e^A\right), \\
&\tilde{\alpha}_{e,x}^B \approx \alpha_e^B\left(1+\mu\alpha_e^A\right), && \tilde{\alpha}_{e,y}^B \approx \alpha_e^B\left(1+\kappa\alpha_e^A\right), && \tilde{\alpha}_{e,z}^B \approx \alpha_e^B\left(-i\eta\alpha_m^A\right).
\end{aligned} \tag{S5.9}$$

Therefore, Eqs. (S5.4)-(S5.7) are respectively simplified as

$$\begin{aligned}
F_{e,x}^A &= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{i(\kappa-\mu)^*}{R}\left(\tilde{\alpha}_{e,y}^A\tilde{\alpha}_{e,x}^{B*}-\tilde{\alpha}_{e,x}^A\tilde{\alpha}_{e,y}^{B*}\right)\right] \\
&= -\frac{n_s I_0 k^4}{32c\pi^2 R^3}\left\{\left(\cos(2kR)\,\mathrm{Im}\left[\alpha_e^B\right]+\sin(2kR)\,\mathrm{Re}\left[\alpha_e^B\right]\right)\left|\alpha_e^A\right|^2+\mathrm{Im}\left[\alpha_e^A\right]\left|\alpha_e^B\right|^2\right\}
\end{aligned}, \tag{S5.10}$$

$$\begin{aligned}
F_{m,x}^A &= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\eta^*}{R}\left(-\tilde{\alpha}_{m,y}^A\tilde{\alpha}_{e,z}^{B*}+\tilde{\alpha}_{m,z}^A\tilde{\alpha}_{e,y}^{B*}\right)\right] \\
&= \frac{n_s I_0 k^4}{32c\pi^2 R^3}\left\{\left(\cos(2kR)\,\mathrm{Im}\left[\alpha_e^B\right]+\sin(2kR)\,\mathrm{Re}\left[\alpha_e^B\right]\right)\left|\alpha_m^A\right|^2+\mathrm{Re}\left[\alpha_m^A\right]\left|\alpha_e^B\right|^2\right\}
\end{aligned}, \tag{S5.11}$$

$$\begin{aligned}
F_{em,x}^A &= -\frac{n_s I_0}{2c}\frac{k^4}{6\pi}\mathrm{Re}\left[i\left(\tilde{\alpha}_{e,y}^A\tilde{\alpha}_{m,x}^{A*}+\tilde{\alpha}_{e,z}^A\tilde{\alpha}_{m,y}^{A*}\right)\right] \\
&= \frac{n_s I_0 k^4}{12c\pi}\mathrm{Im}\left[\alpha_e^A\alpha_m^{A*}\right]
\end{aligned}, \tag{S5.12}$$

$$\begin{aligned}
F_x^B &= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{i(\kappa-\mu)^*}{R}\left(\tilde{\alpha}_{e,y}^B\tilde{\alpha}_{e,x}^{A*}-\tilde{\alpha}_{e,x}^B\tilde{\alpha}_{e,y}^{A*}\right)+\frac{\eta^*}{R}\left(\tilde{\alpha}_{e,y}^B\alpha_{m,z}^{A*}-\tilde{\alpha}_{e,z}^B\tilde{\alpha}_{m,y}^{A*}\right)\right] \\
&= \frac{n_s I_0 k^4}{32c\pi^2 R^3}\left\{\begin{pmatrix}\cos(2kR)\,\mathrm{Im}\left[\alpha_e^A-\alpha_m^A\right]+ \\ \sin(2kR)\,\mathrm{Re}\left[\alpha_e^A-\alpha_m^A\right]\end{pmatrix}\left|\alpha_e^B\right|^2+\mathrm{Im}\left[\alpha_e^B\right]\left(\left|\alpha_e^A\right|^2+\left|\alpha_m^A\right|^2\right)\right\}
\end{aligned}. \tag{S5.13}$$

Equations. (S5.10)-(S5.13) corresponds to Eqs. (7)-(10) in main text, respectively.

Supplementary note 6: Optical forces along y axis

Similar to the analyses in Supplementary note 5, the y component of optical force on particle A is composed of electric dipolar, magnetic dipolar, and electric-magnetic dipolar coupling parts as

$$\begin{aligned}F_{\mathrm{e},y}^{\mathrm{A}}&=\frac{1}{2}\mathrm{Re}\left[p_x^{\mathrm{A}}\frac{\partial E_x^{\mathrm{A}*}}{\partial y}+p_y^{\mathrm{A}}\frac{\partial E_y^{\mathrm{A}*}}{\partial y}+p_z^{\mathrm{A}}\frac{\partial E_z^{\mathrm{A}*}}{\partial y}\right]\\&=\frac{1}{2\varepsilon_0\varepsilon_m}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}p_x^{\mathrm{A}}p_x^{\mathrm{B}*}+\frac{\partial\kappa^*}{\partial y}p_y^{\mathrm{A}}p_y^{\mathrm{B}*}+\frac{\partial\mu^*}{\partial y}p_z^{\mathrm{A}}p_z^{\mathrm{B}*}\right]\\&=\frac{n_sI_0}{2c}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}*}+\frac{\partial\kappa^*}{\partial y}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}*}+\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}*}\right],\end{aligned}\tag{S6.1}$$

$$\begin{aligned}F_{\mathrm{m},y}^{\mathrm{A}}&=\frac{\mu_0\mu_m}{2}\mathrm{Re}\left[m_x^{\mathrm{A}}\frac{\partial H_x^{\mathrm{A}*}}{\partial y}+m_y^{\mathrm{A}}\frac{\partial H_y^{\mathrm{A}*}}{\partial y}+m_z^{\mathrm{A}}\frac{\partial H_z^{\mathrm{A}*}}{\partial y}\right]\\&=\frac{\mu_0\mu_m}{2Z\varepsilon_0\varepsilon_m}\mathrm{Re}\left[\frac{i\partial\eta^*}{\partial y}\left(-m_x^{\mathrm{A}}p_z^{\mathrm{B}*}+m_z^{\mathrm{A}}p_x^{\mathrm{B}*}\right)\right]\\&=\frac{n_sI_0}{2c}\mathrm{Re}\left[i\frac{\partial\eta^*}{\partial y}\left(\tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}*}+\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}*}\right)\right]\end{aligned},\tag{S6.2}$$

$$\begin{aligned}F_{\mathrm{em},y}^{\mathrm{A}}&=-\frac{Zk^4}{12\pi}\mathrm{Re}\left[p_z^{\mathrm{A}}m_x^{\mathrm{A}*}-p_x^{\mathrm{A}}m_z^{\mathrm{A}*}\right]\\&=-\frac{n_sI_0}{2c}\frac{k^4}{6\pi}\mathrm{Re}\left[\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}*}\right]\end{aligned}.\tag{S6.3}$$

However, the y component of optical force on particle B is only caused by the electric dipolar interaction and expressed as

$$\begin{aligned}F_y^{\mathrm{B}}&=\frac{1}{2}\mathrm{Re}\left[p_x^{\mathrm{B}}\frac{\partial E_x^{\mathrm{B}*}}{\partial y}+p_y^{\mathrm{B}}\frac{\partial E_y^{\mathrm{B}*}}{\partial y}+p_z^{\mathrm{B}}\frac{\partial E_z^{\mathrm{B}*}}{\partial y}\right]\\&=\frac{1}{2}\mathrm{Re}\begin{bmatrix}-\frac{1}{\varepsilon_0\varepsilon_m}\frac{\partial\mu^*}{\partial y}p_x^{\mathrm{B}}p_x^{\mathrm{A}*}-iZ\frac{\partial\eta^*}{\partial y}p_x^{\mathrm{B}}m_z^{\mathrm{A}*}-\frac{1}{\varepsilon_0\varepsilon_m}\frac{\partial\kappa^*}{\partial y}p_y^{\mathrm{B}}p_y^{\mathrm{A}*}\\-\frac{1}{\varepsilon_0\varepsilon_m}\frac{\partial\mu^*}{\partial y}p_z^{\mathrm{B}}p_z^{\mathrm{A}*}+iZ\frac{\partial\eta^*}{\partial y}p_z^{\mathrm{B}}m_x^{\mathrm{A}*}\end{bmatrix}\\&=-\frac{n_sI_0}{2c}\mathrm{Re}\begin{bmatrix}\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}*}+\frac{\partial\kappa^*}{\partial y}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}*}+\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}*}\\+i\frac{\partial\eta^*}{\partial y}\left(\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}*}\right)\end{bmatrix}\end{aligned},\tag{S6.4}$$

Therefore, the optical binding force between two particles along the y axis is written as

$$F_y=F_y^{\mathrm{A}}-F_y^{\mathrm{B}}=F_{\mathrm{e},y}^{\mathrm{A}}+F_{\mathrm{m},y}^{\mathrm{A}}+F_{\mathrm{em},y}^{\mathrm{A}}-F_y^{\mathrm{B}}.\tag{S6.5}$$

Equations (S6.1)-(S6.5) are rigorous and correspond to Eqs. (12)-(16) in main text, respectively.

In far-field and Rayleigh ($\alpha_{\mathrm{m}}^{\mathrm{A}}=0$) limits, only the zeroth and first order terms of μ are retained in the formulae of optical force. Equations (S6.1), (S6.4), and (S6.5) are simplified as

$$
\begin{aligned}
F_{\mathrm{e},y}^{\mathrm{A,far}} &= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}*}+\frac{\partial\kappa^*}{\partial y}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}*}+\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}*}\right] \\
&= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\exp(-ikR)}{4\pi R}\left(-ik^3\right)\left\{\begin{array}{l}\alpha_{\mathrm{e}}^{\mathrm{A}}\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{B}}\right)\alpha_{\mathrm{e}}^{\mathrm{B}*}\left(1+\mu^*\alpha_{\mathrm{e}}^{\mathrm{A}*}\right)\\ +\alpha_{\mathrm{e}}^{\mathrm{A}}\left(-i\eta\mu\alpha_{\mathrm{e}}^{\mathrm{B}}\alpha_{\mathrm{m}}^{\mathrm{A}}\right)\alpha_{\mathrm{e}}^{\mathrm{B}*}\left(i\eta^*\alpha_{\mathrm{m}}^{\mathrm{A}*}\right)\end{array}\right\}\right] \\
&= \frac{n_s I_0}{2c}\frac{k^3}{4\pi R}\mathrm{Re}\left[(-i)\exp(-ikR)\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{B}}+\mu^*\alpha_{\mathrm{e}}^{\mathrm{A}*}\right)\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right] \\
&= \frac{n_s I_0}{2c}\frac{k^3}{4\pi R}\left\{\begin{array}{l}\cos(kR)\,\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]-\sin(kR)\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]\\ +\dfrac{k^2}{4\pi R}\left(\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\right]\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2-\left(\begin{array}{l}\cos(2kR)\,\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{B}}\right]\\+\sin(2kR)\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{B}}\right]\end{array}\right)\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\right)\end{array}\right\}
\end{aligned}
\quad , \qquad \text{(S6.6)}
$$

$$
\begin{aligned}
F_{y}^{\mathrm{B,far}} &= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}\left(\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}*}\right)\right] \\
&= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}\left(\begin{array}{l}\alpha_{\mathrm{e}}^{\mathrm{B}}\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}\right)\alpha_{\mathrm{e}}^{\mathrm{A}*}\left(1+\mu^*\alpha_{\mathrm{e}}^{\mathrm{B}*}\right)\\ +\alpha_{\mathrm{e}}^{\mathrm{B}}\left(-i\eta\alpha_{\mathrm{m}}^{\mathrm{A}}\right)\alpha_{\mathrm{e}}^{\mathrm{A}*}\left(i\eta^*\mu^*\alpha_{\mathrm{e}}^{\mathrm{B}*}\alpha_{\mathrm{m}}^{\mathrm{A}*}\right)+\\ \alpha_{\mathrm{e}}^{\mathrm{B}}\left(1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}\right)\alpha_{\mathrm{m}}^{\mathrm{A}*}\left(-i\eta^*\alpha_{\mathrm{e}}^{\mathrm{B}*}\right)\left(1+\mu^*\alpha_{\mathrm{e}}^{\mathrm{A}*}\right)+\alpha_{\mathrm{e}}^{\mathrm{B}}\left(-i\eta\alpha_{\mathrm{m}}^{\mathrm{A}}\right)\alpha_{\mathrm{m}}^{\mathrm{A}*}\end{array}\right)\right] \\
&= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\exp(-ikR)}{4\pi R}(-i)k^3\left(\begin{array}{l}\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}}+\mu\alpha_{\mathrm{e}}^{\mathrm{B}}\left(\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2+\left|\alpha_{\mathrm{m}}^{\mathrm{A}}\right|^2\right)\\ +\mu^*\left(\alpha_{\mathrm{e}}^{\mathrm{A}*}-\alpha_{\mathrm{m}}^{\mathrm{A}*}\right)\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2\end{array}\right)\right] \\
&= \frac{n_s I_0}{2c}\frac{k^3}{4\pi R}\left\{\begin{array}{l}\cos(kR)\,\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]+\sin(kR)\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]\\ +\dfrac{k^2}{4\pi R}\left\{\begin{array}{l}-\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{B}}\right]\left(\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2+\left|\alpha_{\mathrm{m}}^{\mathrm{A}}\right|^2\right)\\ +\left(\begin{array}{l}\cos(2kR)\,\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}-\alpha_{\mathrm{m}}^{\mathrm{A}}\right]\\+\sin(2kR)\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}-\alpha_{\mathrm{m}}^{\mathrm{A}}\right]\end{array}\right)\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2\end{array}\right\}\end{array}\right\}
\end{aligned}
\quad , \qquad \text{(S6.7)}
$$

$$
F_{y}^{\mathrm{far}}=F_{y}^{\mathrm{A,far}}-F_{y}^{\mathrm{B,far}}=\frac{n_s I_0}{2c}\left\{\begin{array}{l}-\dfrac{k^3}{4\pi R}2\sin(kR)\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]\\ +\dfrac{k^5}{(4\pi R)^2}\left(\begin{array}{l}\left(1-\cos(2kR)\right)\left(\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\right]\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2+\mathrm{Im}\left[\alpha_{\mathrm{e}}^{\mathrm{B}}\right]\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\right)\\ -\sin(2kR)\left(\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\right]\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2+\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{B}}\right]\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\right)\end{array}\right)\end{array}\right\}.\qquad \text{(S6.8)}
$$

Equation (S6.8) is just Eq. (17) in main text.

On the other hand, in near-field region while ignoring magnetic dipolar response of dielectric particle, the address electric and magnetic polarizabilities in Eqs. (S3.1) and (S3.2) are simplified as

$$
\begin{aligned}
&\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}=\alpha_{\mathrm{e}}^{\mathrm{A}}\frac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, \qquad \tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}=\alpha_{\mathrm{e}}^{\mathrm{B}}\frac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, \qquad \tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}=\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}=0,\\
&\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}}=\alpha_{\mathrm{e}}^{\mathrm{A}}\frac{1+\kappa\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-\kappa^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, \qquad \tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}}=\alpha_{\mathrm{e}}^{\mathrm{B}}\frac{1+\kappa\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-\kappa^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}, \qquad \tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}}=\tilde{\alpha}_{\mathrm{m},y}^{\mathrm{A}}=\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}}=0.
\end{aligned}
\qquad \text{(S6.9)}
$$

Here, the phase factor exp(ikR) is ignored and only the zeroth-order terms of kR are retained in μ and κ, such as $\mu \approx -1/\left(4\pi R^3\right)$, $\kappa \approx 2/\left(4\pi R^3\right)$, and $\eta \approx 0$. As a result, Eqs. (S6.1), (S6.4) and (S6.5) are simplified as

$$\begin{aligned} F_{\mathrm{e},y}^{\mathrm{A,near}} &= \frac{n_s I_0}{2c}\mathrm{Re}\left[\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}*}+\frac{\partial\kappa^*}{\partial y}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}*}+\frac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}*}\right] \\ &= \frac{n_s I_0}{2c}\frac{3}{4\pi R^4}\mathrm{Re}\left[\left(\begin{array}{l}\dfrac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}\dfrac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}*}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}*}} \\ -2\dfrac{1-2\mu\alpha_{\mathrm{e}}^{\mathrm{B}}}{1-4\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}\dfrac{1-2\mu\alpha_{\mathrm{e}}^{\mathrm{A}*}}{1-4\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}*}}\end{array}\right)\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right], \end{aligned} \tag{S6.10}$$

$$\begin{aligned} F_{y}^{\mathrm{B,near}} &= -\frac{n_s I_0}{2c}\mathrm{Re}\left[\begin{array}{l}\dfrac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{A}*}+\dfrac{\partial\kappa^*}{\partial y}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},y}^{\mathrm{A}*}+\dfrac{\partial\mu^*}{\partial y}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{A}*} \\ +i\dfrac{\partial\eta^*}{\partial y}\left(\tilde{\alpha}_{\mathrm{e},x}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},z}^{\mathrm{A}*}+\tilde{\alpha}_{\mathrm{e},z}^{\mathrm{B}}\tilde{\alpha}_{\mathrm{m},x}^{\mathrm{A}*}\right)\end{array}\right] \\ &= -\frac{n_s I_0}{2c}\frac{3}{4\pi R^4}\mathrm{Re}\left[\left(\begin{array}{l}\dfrac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{B}*}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}*}}\dfrac{1+\mu\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}} \\ -2\dfrac{1-2\mu\alpha_{\mathrm{e}}^{\mathrm{B}*}}{1-4\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}*}}\dfrac{1-2\mu\alpha_{\mathrm{e}}^{\mathrm{A}}}{1-4\mu^2\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}}\end{array}\right)\alpha_{\mathrm{e}}^{\mathrm{A}*}\alpha_{\mathrm{e}}^{\mathrm{B}}\right]. \end{aligned} \tag{S6.11}$$

$$\begin{aligned} F_{y}^{\mathrm{near}} &= F_{\mathrm{e},y}^{\mathrm{A,near}} - F_{y}^{\mathrm{B,near}} \\ &= \frac{n_s I_0}{c}\frac{3}{4\pi R^4}\left(\begin{array}{l}\dfrac{1+\mu\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}+\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+\mu^2\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]}{1-2\mu^2\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+4\mu^2\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2} \\ -2\dfrac{1-2\mu\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}+\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+4\mu^2\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]}{1-8\mu^2\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+16\mu^2\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2}\end{array}\right)\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right] \\ &= \frac{n_s I_0}{c}\frac{3}{4\pi R^4}\left(\begin{array}{l}\dfrac{\left(4\pi R^3\right)^2-\left(4\pi R^3\right)\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}+\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]}{\left(4\pi R^3\right)^2-2\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+4\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2} \\ -2\dfrac{\left(4\pi R^3\right)^2+2\left(4\pi R^3\right)\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}+\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+4\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right]}{\left(4\pi R^3\right)^2-8\,\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}}\right]+16\left|\alpha_{\mathrm{e}}^{\mathrm{A}}\right|^2\left|\alpha_{\mathrm{e}}^{\mathrm{B}}\right|^2}\end{array}\right)\mathrm{Re}\left[\alpha_{\mathrm{e}}^{\mathrm{A}}\alpha_{\mathrm{e}}^{\mathrm{B}*}\right] \end{aligned} \tag{S6.12}$$

Equation (S6.12) is just Eq. (18) in main text.

SUPPLEMENTARY REFERENCES

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