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When Crowd Size Stops Matter: Characterizing the Ceiling Effect in Crowd-Sourced Continuous Control — A Simulation Study

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Crowd-sourced continuous control systems enable multiple distributed participants to collectively generate control commands for a shared agent in real time, yet whether larger crowds control better under adversarial conditions remains insufficiently understood. Using a controlled simulation framework spanning four trajectory geometries (circle, square, lemniscate, and zigzag), three adversary models, adversarial ratios between 5% and 40%, and crowd sizes from 5 to 200, this study shows that crowd-size scaling follows a conditional structure rather than a monotonic improvement law. For smooth trajectories, a gradual adversarial transition was empirically observed around $tr \approx 10\text{--}20\%$ (centered near 15%), beyond which increasing crowd size produced a measurable RMSE reduction. The crowd size $\times$ adversarial ratio interaction was substantial ($\eta^2_{\text{interaction}} = 21.8\%$ for circle and 9.6% for square, two-way ANOVA on raw Monte Carlo realizations). In contrast, rapidly reversing zigzag trajectories exhibited practical crowd-size independence ($\Delta\text{RMSE} = 0.07\text{--}1.96\%$) across all tested conditions, indicating a geometry-imposed structural limit that additional participants do not overcome. As a simulation-based characterization without human-participant validation, these findings position crowd size as a secondary, conditional design factor jointly governed by adversarial participation, trajectory geometry, and adversary structure, with sizing implications for collective robotics, teleoperation, and large-scale interactive control platforms.

Keywords: adversarial robustness, collective control, crowd-size ceiling effect, crowd-sourced control, human-machine interaction, multi-operator single-robot systems.

Introduction

Collective control systems in which multiple distributed participants jointly generate commands for a shared agent have attracted increasing attention in robotics, human-machine systems, and interactive media platforms^{2–4}. Representative examples include robot swarm supervision, crowd-powered interface control, and real-time audience-driven control systems. These frameworks extend traditional single-user control into large-scale distributed coordination environments.

The classical theoretical basis for such systems is rooted in the wisdom-of-crowds principle, which suggests that aggregating multiple independent judgments can reduce individual noise and improve collective decision quality. From a statistical perspective,

ensemble averaging reduces variance as the number of contributors increases⁵⁻⁷, with peer-reviewed evidence that averaging and select-crowd strategies robustly outperform individuals²³. Under this view, increasing crowd size should generally improve system performance.

However, this expectation may not hold in real-time continuous control settings. Unlike static decision-making tasks, collective trajectory control requires temporally synchronized correction, robustness to disruptive inputs, and adaptation to rapidly changing geometric demands. Prior work has shown that social influence and adversarial disruption can substantially degrade collective performance⁸⁻⁹. In particular, distributed consensus and Byzantine fault literature suggest that malicious or structurally disruptive participants may fundamentally alter aggregation outcomes⁹⁻¹⁰.

A companion study (available as a preprint¹) reported that response speed accounts for a dominant share of performance variance in crowd-sourced control systems. The present study is self-contained and does not depend on that result; it is referenced only to motivate an unresolved question that it leaves open: whether the classical averaging benefit of larger groups persists after speed-related variance is controlled.

To address this question, the present study systematically investigates the joint effects of crowd size, adversarial participation ratio, and trajectory geometry using a controlled simulation framework. Our central hypothesis is that the effect of increasing crowd size is not monotonic but instead depends on adversarial pressure and task geometry.

The main contributions of this work are threefold:

- 1) Adversarial transition-region characterization. A gradual transition is empirically characterized around $tr \approx 10-20\%$ within the tested range $tr \in [5\%, 40\%]$, beyond which increasing crowd size is associated with measurable RMSE reduction over the tested grid.
- 2) Interaction quantification. The $N \times$ adversarial ratio interaction effect is quantified using raw Monte Carlo realizations ($\eta^2_{\text{interaction}} = 21.8\%$, circle; 9.6% , square at $MC = 50$; consistent with the initial $MC = 15$ estimate of $23.3\%/9.4\%$), indicating that crowd-size effects are conditioned by adversarial pressure.
- 3) Structural N-independence. Zigzag trajectory exhibits structural crowd-size independence ($\Delta RMSE = 0.07-1.96\%$) across all tested adversarial conditions, with low measurement variance (mean per-cell coefficient of variation $CV = 0.019$, the lowest among the four trajectories) indicating that noise is unlikely to be the explanation.

Together, these findings extend prior speed-dominance results¹ by positioning crowd size as a second-order design factor in collective continuous control systems.

Related work

Crowd-sourced control of shared agents originated in large-scale experiments such as Twitch Plays Pokémon and has since been formalized in real-time crowd-control platforms

²⁻⁴. The statistical premise — variance reduction through aggregation — descends from the wisdom-of-crowds literature ^{5, 7, 23}. That literature, however, concerns static judgment aggregation under predominantly cooperative conditions; social-influence effects ⁸ and collective behavior in games ¹¹ complicate but do not remove this assumption. A second line of work treats adversarial participants through the lens of Byzantine fault tolerance and robust distributed inference ^{9, 10, 12}, deriving resilience bounds for discrete consensus. A third line characterizes trajectory tracking under actuation constraints: path curvature and operator tracking error ¹³⁻¹⁵ and teleoperation under delay ¹⁶⁻¹⁸, extended to human-swarm interaction ¹⁹. The present study sits at the intersection: temporally synchronized continuous control, adversarial participation, and geometry-dependent tracking difficulty, a combination not systematically characterized by any of the three lines individually.

The relationship to the companion preprint ¹ is summarized in Supplementary Table S1.

Results

Threshold emergence in smooth trajectories

A gradual transition from crowd-size independence to crowd-size benefit was observed in the smooth trajectories within the tested range $tr \in [5\%, 40\%]$. Using the fitted N-scaling coefficient b of the ceiling model as a change-point target, a two-segment fit placed the transition in the region $tr \approx 10\text{--}20\%$ (circle change-point $tr \approx 17\%$, 95% CI $[17, 30]\%$; square $tr \approx 13\%$, CI $[13, 17]\%$; lemniscate $tr \approx 17\%$, with the bootstrap distribution concentrated on the 17% grid level; CIs from a parametric bootstrap that resamples the per-cell Monte Carlo means with their standard errors). Because the tr grid is coarse and, at small N , the realized adversarial count collapses (e.g., $N \leq 20$ yields identical integer adversary counts for $tr = 13/15/17\%$), a transition region is reported rather than a single threshold point. Ceiling-fit quality for the circle trajectory increased from $R^2 = 0.35$ at $tr = 5\%$ to $R^2 \geq 0.81$ at $tr \geq 15\%$ ($MC = 50$). This transition pattern is summarized in Table 1 and visualized in Fig. 1; the corresponding RMSE- N curves for the circle trajectory are shown in Fig. 2.

Interaction effect between crowd size and adversarial ratio

Two-way ANOVA conducted on raw $MC = 50$ realizations indicated that the effect of crowd size depends on adversarial conditions. For the circle trajectory, the interaction term explained $\eta^2_{\text{interaction}} = 21.8\%$ of total RMSE variance ($F = 80.8$, $p < 0.001$). For the square trajectory, the interaction accounted for $\eta^2_{\text{interaction}} = 9.6\%$ ($F = 19.4$, $p < 0.001$). For the square trajectory, 39.4% of RMSE variance remains unexplained by N , tr , and their interaction — nearly double the circle residual (21.7%). Corner-concentrated tracking error, which the smooth $N \times tr$ factorial cannot capture, is a likely contributor, so the square's lower interaction share should not be read as crowd size mattering less. These $MC = 50$ estimates agree with the initial $MC = 15$ values (23.3% and 9.4%) to within ~ 2 percentage points. A Monte Carlo stability check confirmed that mean RMSE changed by $< 1\%$ from $MC = 30$ to $MC = 50$ while the 95% confidence intervals narrowed by $\sim 49\%$.

Because large-sample normality and homoscedasticity were formally rejected, the interaction was re-confirmed with Welch-type heteroscedasticity-robust inference and distribution-free permutation tests. Both yielded the same qualitative conclusions (robust parametric tests remained highly significant; permutation tests rejected the null at the chosen permutation resolution). Detailed results are reported in Table 2 and visualized in Supplementary Fig. S1.

Geometry-dependent performance difference

A strong geometry-dependent pattern was observed across trajectory types (MC = 50). Smooth trajectories showed substantial scaling benefit above the transition region: ceiling R^2 reached 0.82–0.96 (circle) and 0.72–0.89 (lemniscate) at $tr \geq 20\%$, and 0.86–0.98 (square) over the same range. By contrast, the zigzag trajectory showed weak or absent ceiling behavior at all adversarial ratios ($R^2 \leq 0.83$, dropping to 0.15–0.23 at $tr \geq 25\%$). These differences indicate that trajectory curvature and directional continuity modulate collective correction efficiency, consistent with control-theoretic accounts of curvature-dependent tracking difficulty^{13–17}.

Structural n-independence in zigzag trajectory

The most important boundary-case result was observed in the zigzag trajectory. Across the full tested crowd-size range $N \in \{5 \dots 200\}$, the RMSE reduction remained limited to approximately 0.07–1.96% across all adversarial conditions ($tr \in [5\%, 40\%]$), indicating practical crowd-size independence. This near-flat response cannot be attributed to measurement noise: the mean per-cell coefficient of variation (CV = 0.019 for zigzag, the lowest among the four trajectories) confirms that the RMSE estimates were stable. Although some intermediate-ratio zigzag fits produced moderate ceiling R^2 values (Table 1), the absolute RMSE reduction from $N = 5$ to $N = 200$ remained within 0.07–1.96%. The zigzag is therefore characterized as practically N-independent on the basis of this absolute change rather than on fit quality. To examine the underlying mechanism, a time-resolved diagnostic analysis of vote consensus and tracking error was added (summarized in Discussion B). This analysis indicates that tracking error under rapid directional reversals is dominated by response lag rather than by consensus accuracy. This is presented as a mechanism consistent with the data rather than a directly proven causal claim. This finding is visualized in Fig. 3.

Speed robustness check across geometries

Additional robustness simulations indicated that, for the circle trajectory, the threshold and ceiling effects were stable after controlling for operating speed. This analysis was subsequently extended to the lemniscate and zigzag geometries (see below), where the qualitative N-scaling structure was likewise broadly preserved across the two tested speeds. Supplementary Table S2 compares the percentage RMSE reduction from $N = 5$ to $N = 200$ at $v = 2.0$ m/s (near-optimal speed per¹) and $v = 5.0$ m/s across geometries. The single positive Δ RMSE entry (lemniscate, $tr = 5\%$, $v = 5.0$ m/s; +1.1%, $p = 0.003$) is explained by realized-adversary discretization rather than a failure of ensemble averaging.

At $tr = 5\%$, integer rounding of adversary counts ($\text{round}(N \cdot tr)$) yields zero realized adversaries for $N \leq 10$ versus exactly 5.00% at $N = 200$. This means the comparison contrasts an adversary-free small crowd with a large crowd carrying its full nominal adversary load. Realized adversary fraction correlates with RMSE across N ($r = 0.80$, $p = 0.002$) and near-perfectly with consensus strength ($r = -0.997$). The two adversary-free cells ($N = 5, 10$), meanwhile, do not differ significantly from each other ($p = 0.08$). At $v = 2.0$ m/s, where vote noise rather than actuation lag limits performance, the large crowd overcomes the same handicap ($\Delta\text{RMSE} = -5.6\%$).

Extending the speed analysis to the lemniscate and zigzag geometries ($v = 2.0$ vs. 5.0 m/s; Supplementary Table S2) showed that the qualitative N -scaling structure is preserved under both speeds: the lemniscate retained tr -dependent ceiling behavior (fitted b grows with tr at both speeds). The zigzag retained near-zero N -scaling at both speeds. Importantly, operating speed strongly affects the absolute tracking error (the $v = 2.0$ m/s RMSE is roughly one-third of the $v = 5.0$ m/s value), consistent with the speed-dominance result of ¹. What is broadly preserved across the two tested speeds here is the qualitative scaling structure — whether additional crowd members help — not the absolute error level. Crowd size thus acts as a secondary, geometry-conditioned factor once the speed-related performance level is set.

F. ADVERSARIAL-MODEL LADDER: TARGETED AND PERSISTENT ATTACKS

Beyond the uniform baseline (T_0), two predeclared adversarial models were evaluated over the full grid ($MC = 50$). The committed anti-target adversary (T_1) votes, in an open-loop manner, for the direction opposite to the reference target, while the persistent stale anti-target adversary (T_2) holds an opposing direction for ten voting intervals before updating. Both produced a distinguishable degradation relative to T_0 at high adversarial ratios. At $tr = 40\%$ and $N = 200$, T_1 increased RMSE by approximately 7–13% across geometries. T_2 was strongly geometry-dependent: it raised circle RMSE by 82% (0.82 to 1.49) and square RMSE by 29%, but had little effect on the zigzag (about -2%), consistent with the lag-dominated regime of that geometry. Critically, the persistent adversary suppressed the crowd-size benefit itself. Under T_0 the circle RMSE fell by 29% from $N = 5$ to $N = 200$, but under T_2 this reduction collapsed to 2.5%, indicating that a coordinated persistent adversary can substantially suppress the averaging benefit on smooth trajectories. An adaptive consensus-opposite variant was also examined. Because it reduced rather than increased RMSE (acting as a negative-feedback stabilizer rather than an attack), it is reported only as a diagnostic observation and is not treated as a primary adversarial model.

Lag-axis sensitivity: direct test of the lag-domination mechanism

To test the lag-domination mechanism directly, the two parameters that jointly determine actuation lag — the response delay $\tau \in \{13, 26\}$ frames and the velocity-smoothing coefficient $\alpha \in \{0.10, 0.20\}$ — were varied for the lemniscate and zigzag trajectories at $tr = 20\%$ ($N \in \{5, 200\}$, 50 Monte Carlo runs per cell; Welch t-tests on the per-run RMSE samples). At the design point ($\tau = 26$ frames, $\alpha = 0.20$), the zigzag N -effect is small in

absolute terms ($\Delta\text{RMSE}(N = 5 \rightarrow 200) = 1.95\%$, $p < 0.001$) and vanishes entirely at the largest effective lag (-0.14% , $p = 0.75$ at $\alpha = 0.10$). Halving the delay roughly triples the zigzag benefit (5.33% , $p < 0.001$), and the lemniscate benefit grows from 4.39% to 27.3% . Across all eight conditions the N-benefit decreases monotonically in the total effective lag $\tau_{\text{eff}} = \tau - \Delta t / \ln(1 - \alpha)$ for both trajectories (perfect ordering; chance probability $1/24$ per trajectory), indicating that α and τ act through a single lag axis. These controlled manipulations upgrade the lag-domination account from a correlational observation to a directly supported mechanism within the tested $\text{tr} = 20\%$ condition.

* $p < 0.01$ (Welch); n.s. = not significant at 0.05.

Reconciling the ANOVA and ceiling-model frameworks

The two-way ANOVA treats N as a 12-level categorical factor, whereas the ceiling model treats it as a continuous variable with a prescribed $1/\sqrt{N}$ form; the two are reconciled here rather than left in parallel. Across all 40 T0 conditions a free-exponent model ($a + b \cdot N^{(-c)}$) attains lower AIC in 23 and the $1/\sqrt{N}$ model in 17, but the substantive conclusions are identical under either model. The ceiling coefficient b reverses sign in only one cell — lemniscate, $\text{tr} = 5\%$ — whose smallest crowd realizes zero adversaries ($\text{round}(N \cdot \text{tr}) = 0$ for $N \leq 10$). This is the discretization signature discussed for the Supplementary Table S2 anomaly, not genuine scaling behavior. Most importantly, the ANOVA interaction reappears in the ceiling frame as monotone growth of b with adversarial ratio: for the circle, b rises from 0.047 ($\text{tr} = 5\%$) to 0.356 ($\text{tr} = 20\%$) to 0.788 ($\text{tr} = 40\%$), a 17-fold increase. The zigzag b , by contrast, stays flat (0.043 – 0.077). A large categorical $N \times \text{tr}$ interaction ($\eta^2 = 21.8\%$, circle) and a strongly tr -dependent continuous coefficient $b(\text{tr})$ are therefore two projections of the same effect. The near-zero zigzag interaction corresponds to the flat zigzag $b(\text{tr})$.

Fitted ceiling-model parameters

At $\text{MC} = 50$ with twelve crowd-size levels, even sub-2% effects are statistically resolvable: the zigzag b -coefficients are small but their confidence intervals exclude zero (Table 4). The N -independence claim is therefore practical rather than statistical. The attainable RMSE reduction from $N = 5$ to the asymptote is at most 1.7% for zigzag versus 17 – 32% for the circle at matched adversarial ratios (a 17-fold difference in b at $\text{tr} = 40\%$). Consistent with our treatment of the ANOVA results, effect magnitude is prioritized over p -value resolution. The low weighted R^2 of near-flat cells reflects the smallness of the N -signal itself rather than model misfit.

The adversarial-model ladder comparison across the four geometries at $\text{tr} = 40\%$ is visualized in Fig. 4.

Discussion

The results are consistent with the conditional scaling interpretation. Within the tested adversarial range $tr \in [5\%, 40\%]$, gradual transitions were empirically observed around $tr \approx 10\text{--}20\%$ (centered near 15%). Below this level, the cooperative majority produces near-optimal directional consensus, and additional participants provide negligible signal improvement—consistent with ensemble averaging theory^{5–7} in low-noise regimes. Above this level, adversarial noise becomes the binding constraint on aggregation quality, and increasing N reduces it in a manner consistent with the $1/\sqrt{N}$ reference model. This threshold is empirical within the tested parameter range and may shift under different system configurations. The $N \times$ adversarial ratio interaction ($\eta^2_{\text{interaction}} = 21.8\%$, circle; $p < 0.001$ on raw $MC = 50$ data; 23.3% on the initial $MC = 15$ data) is consistent with this conditional structure.

A note on the square trajectory at $tr = 5\%$: Table 1 shows $R^2 = 0.533$, higher than circle (0.350) and lemniscate (0.356) at the same condition. However, the absolute $\Delta RMSE$ from $N = 5$ to 200 is only -2.1% , comparable to circle (-2.6%), indicating that the elevated R^2 reflects non-monotonic measurement variance rather than a structural ceiling tendency. The threshold narrative applies most cleanly to circle and lemniscate; square shows partial early ceiling tendency that should be acknowledged.

The geometry-dependent pattern is supported by formal trajectory descriptors. Curvature, reversal frequency, path discontinuity (corner count), turn rate, and the effective correction window were quantified for each geometry. Smooth trajectories have long correction windows relative to the response delay (circle $\approx 29\times$, square $\approx 7\times$), so vote noise — which decreases with N — is the limiting factor and larger crowds help. The zigzag, by contrast, has frequent reversals (7 per lap), the most corners (11 per lap), and the shortest correction window ($\approx 3.3\times$ the response delay), so the limiting factor shifts away from vote noise. Prior work on teleoperation delay and tracking control^{16–17} establishes that curvature fundamentally affects tracking difficulty; the present descriptors extend this by linking specific geometric quantities to whether crowd-size scaling is effective.

For zigzag trajectories, the structural N -independence reported above is unlikely to be explained by measurement noise. To examine the mechanism, a time-resolved analysis of vote-consensus magnitude ($\bar{R}$) and tracking error was added. The analysis indicates that $\bar{R}$ is nearly invariant to crowd size for both circle and zigzag, yet RMSE decreases with N only for the circle. For the zigzag, the per-interval turn demand is substantially larger than for the circle (mean turn-rate proxy $\approx 24^\circ$ vs. 10° per control interval), which makes the fixed response delay of ≈ 0.43 s more consequential. Tracking error also concentrates at corners (peak-to-mean ratio 2.6 vs. 2.0). These observations indicate that the tracking error under reversing geometry is dominated by response lag rather than by consensus accuracy, which would explain why increasing crowd size — which improves consensus but not lag — yields little benefit. We interpret this as a lag-domination mechanism consistent with the data rather than a fully proven causal claim.

To assess whether these conclusions depend on the fixed participant model, a behavioral-sensitivity panel (12,000 runs) was conducted. Honest participants were augmented with four behavioral variants: variable response delay, fatigue-like growth of angular noise, imitation in which a fraction of participants follow the public consensus, and learning-like reduction of noise over time. Across all variants, the qualitative regime structure was preserved: under the baseline and committed-adversary models the smooth trajectories retained their crowd-size benefit (circle RMSE reduction of about 5–8% from $N = 30$ to $N = 100$), under the persistent adversary this benefit remained suppressed on smooth trajectories, and the zigzag remained nearly N -independent throughout. Individual behavioral factors shifted absolute RMSE, with imitation producing the largest deviations in some high-adversarial conditions; nevertheless, the qualitative regime structure (N -benefit for smooth trajectories, N -independence for zigzag; Supplementary Table S3) was preserved across all variants. This indicates that the qualitative findings are not artifacts of a single fixed participant model, although full behavioral realism can only be established through human-participant studies (see Limitations).

The present results should be interpreted in direct relation to the companion preprint ¹, which reported that operating speed explains the dominant share of performance variance—a first-order relationship in which speed dominates baseline error. The present study extends this framework by showing that, once the speed-related performance floor is established, crowd-size scaling emerges as a second-order mechanism whose benefit is conditional on adversarial ratio and geometry. Together, these findings contribute toward a more complete system-level design picture for crowd-sourced continuous control.

From a system-design perspective, the present simulation findings suggest that simply increasing participation may not guarantee improved control quality. Within the tested range, the results indicate that system designers may need to consider: (1) expected adversarial participation level, (2) task geometry, and (3) response-time constraints ^{16, 18, 17}. For smooth, gradually varying tasks, increasing crowd size above $N \approx 20$ –30 may provide measurable benefit once the system enters the threshold regime ($tr \geq 15\%$). For rapidly reversing tasks, however, increasing participation alone may produce negligible gains regardless of adversarial conditions. These findings may inform the design of large-scale streaming-based interactive systems and collective teleoperation platforms ^{2–19}.

The transition near $tr \approx 10$ –20% admits an order-of-magnitude account linking adversarial vote dilution to the actuation-lag floor. Between consensus corrections the agent travels uncorrected for an effective dead time $T_c \approx \tau + T_v/2 \approx 0.58$ s (with the velocity time constant $t_s = -\Delta t/\ln(1 - \alpha) \approx 0.07$ s a minor addition), so tracking error composes approximately in quadrature: $e^2 \approx e_{lag}^2 + e_{adv}^2$, with an N -independent lag floor $e_{lag} \approx \kappa_g \cdot v \cdot \tau_{eff}$ and an adversarial term $e_{adv} \approx c_a \cdot tr \cdot v \cdot T_c / \sqrt{N}$. Jointly fitting κ_g and c_a by nonlinear least squares across the full circle grid (120 cells: ten adversarial ratios times twelve crowd sizes) gives $\kappa_g = 0.319$, $c_a = 1.67$, with $R^2 = 0.889$ for the fitted RMSE surface against the observed one — the two-term quadrature form captures the bulk of the crowd-size scaling across conditions. The fit is not uniform in quality across tr , however: the model's implied N -scaling coefficient $b(tr)$ tracks the empirical value closely at $tr = 40\%$ (predicted-to-observed ratio 1.13) but underestimates it at $tr = 5\%$ (ratio 0.40) and $tr =$

20% (ratio 0.71), indicating that the quadrature form omits a mechanism — plausibly direction-quantization noise, which is not modeled here — that strengthens with adversarial ratio.

Given this partial fit, the model is not treated as a precise predictor of the transition location; an earlier version of this account derived a point estimate for the transition threshold from an ad hoc noise-floor argument, but a more rigorous multi-cell calibration shows that estimate is not robust and is not reported here. Two qualitative conclusions remain supported by the fit. First, the transition is a continuous signal-to-noise crossover between dilution noise (proportional to $\text{tr}/\sqrt{N}$) and the lag floor (proportional to $v^*\tau_{\text{eff}}$), not a voting-threshold collapse. Discrete-consensus bounds such as the Byzantine $\sim 33\%$ or majority $\sim 50\%$ thresholds arise from voting semantics that do not apply to continuous vector averaging, consistent with the observed transition sitting well below them. Second, the same lag-floor/dilution-noise structure predicts that reducing effective lag should shift the visible transition to lower tr and strengthen N -dependence — the pattern independently confirmed by controlled manipulation in the lag-axis sensitivity experiment (Table 3). The precise location of the transition is established empirically by the change-point analysis (Table 1, Fig. 1) rather than predicted from first principles; closing that gap would require modeling the quantization mechanism explicitly, which is left to future work.

Several limitations should be acknowledged.

- 1) Simulation-only design: The present study is based entirely on a discrete-time simulation framework. Human behavioral dynamics—including social influence⁸, coordination patterns, and reaction time variability—are not fully captured by the stochastic participant model and its sensitivity variants. Human-participant user studies are essential for ecological validation, but controlled settings are also where the influence and weight of individual factors can be isolated while preserving internal validity^{20,21}. Uncontrolled variables—participant heterogeneity, task effects, and demand characteristics—can obscure or even dominate the isolated effect of target variables, and can affect objective performance measures, not only subjective ones²². The present simulation-based account is therefore positioned as an internal-validity-oriented first step for isolating parameter-level relationships, while subsequent human-participant studies remain necessary for ecological validation.
- 2) Parameter-dependent threshold: The $\text{tr} \approx 10\text{--}20\%$ transition region is an empirical observation within the tested range $\text{tr} \in [5\%, 40\%]$ and $N \in \{5, \dots, 200\}$. The threshold may shift under different system parameters, latency values, or trajectory scales.
- 3) Zigzag N -independence mechanism: the lag-domination mechanism is now supported by controlled manipulation of both lag-determining parameters (α, τ) for the two geometry extremes (Table 3); generalization of this manipulation across the full adversarial-ratio grid remains untested. Relatedly, the behavioral sensitivity panel covers honest-participant variants only. Adversarial behavioral variation — including the sensitivity of the persistent adversary’s holding duration — remains untested.

4) Adversarial model: The revised study extends the original uniform-random baseline (T0) with a committed anti-target adversary (T1) and a persistent stale anti-target adversary (T2). It additionally examined an adaptive consensus-opposite variant, which behaved as a stabilizer rather than an attack and is therefore reported only as a diagnostic observation. These remain simulated adversarial archetypes and do not exhaust the full space of strategic human behavior (e.g., fully coordinated Byzantine protocols^{9–10}).

5) **Crowd size range:** The tested range $N \in \{5, \dots, 200\}$ reflects practical streaming platform scales^{2,11}. Generalization to extreme scales ($N > 1,000$) requires additional investigation.

6) Speed scope: The speed analysis was extended from the circle to the lemniscate and zigzag geometries; the qualitative N-scaling structure was broadly preserved across the two tested speeds. A finer sweep of intermediate speeds remains future work.

In summary, the effect of crowd size in crowd-sourced continuous control is conditional rather than universally beneficial: on smooth trajectories it emerges only once adversarial pressure passes the $tr \approx 10\text{--}20\%$ transition, whereas on rapidly reversing trajectories it is structurally absent across all tested conditions.

Beyond the primary crowd-size finding, a predeclared adversarial-model ladder showed that a coordinated persistent adversary can substantially suppress the crowd-size benefit on smooth trajectories, while a behavioral-sensitivity panel indicated that the conditional structure is robust to variation in the participant model. As a simulation study, ecological validation through human-participant studies remains necessary future work; within that scope, the results offer design considerations for collective control architectures in robotics, teleoperation, and large-scale interactive platforms.

Methods

Problem statement

Objective. Determine whether, and under what conditions, increasing the number of participants N in a crowd-sourced continuous-control system reduces tracking error, when the crowd is subject to a controllable fraction tr of adversarial participants and the target trajectory geometry is varied.

Variables. Independent: crowd size N (twelve levels, 5–200); adversarial ratio tr (ten levels, 5–40%); trajectory geometry (circle, square, lemniscate, zigzag); adversary behavior model (T0 uniform random, T1 committed anti-target, T2 persistent stale anti-target). Dependent: root-mean-square tracking error (RMSE), consensus strength (γ), and the fitted ceiling-model coefficients (a , b).

Hypotheses. H1: the crowd-size benefit is not monotonic across conditions but is conditional on adversarial ratio and geometry. H2: geometries with short actuation-relevant correction windows relative to system delay (rapid-reversal trajectories) exhibit

attenuated or absent crowd-size benefit. H3: the adversarial-ratio threshold at which crowd-size benefit becomes visible admits an order-of-magnitude, lag-based account rather than a fixed voting-theoretic bound.

Evaluation outputs. RMSE as a function of N , tr , and geometry (Table 1, Fig. 1); the $N \times tr$ interaction via two-way ANOVA (Table 2, Supplementary Fig. S1); the adversarial-ratio transition region via change-point analysis on the fitted ceiling coefficient $b(tr)$; and controlled lag-manipulation results isolating the mechanism (Table 3).

System model

A crowd-sourced continuous control system is modeled in which participants collectively generate control commands for a shared agent. At each discrete timestep, the aggregated control input is defined as:

$$u(t) = \frac{1}{N} \sum_{i=1}^N a_i(t) \quad (1)$$

where N is the number of participants and $a_i(t)$ is the unit directional vote of participant i at time t . The agent state evolves as

$$x_{t+1} = f(x_t, u_t) \quad (2)$$

where x_t is the agent position and u_t the aggregated input from (1). The complete update law expands (2) as follows. Every voting interval ($V_I = 18$ frames = 0.3 s), participants observe the delayed agent state

$$\tilde{p}(t) = p(t - \tau), \quad \tau = 26 \text{ frames} \approx 433 \text{ ms} \quad (2a)$$

and the ideal heading is computed from the nearest-path projection with a 2.0 m look-ahead:

$$s^* = \operatorname{argmin}_s \| \tilde{p} - \Gamma(s) \|, \quad u = \frac{\Gamma(s^* + L) - \tilde{p}}{\| \Gamma(s^* + L) - \tilde{p} \|}, \quad L = 2.0 \text{ m} \quad (2b)$$

Honest participants split the non-adversarial fraction $(1 - tr)/0.95$ into accurate (70%; heading noise $U(-3^\circ, 3^\circ)$), slow (20%; partial rotation toward the ideal heading with retention factor $U(0.2, 0.5)$), and other (residual; noise $U(-30^\circ, 30^\circ)$). Adversaries follow the active adversary model (T0/T1/T2). All votes are quantized to the nearest of eight compass directions d_k (Eq. (2c)).

At ($N = 5$, $tr = 30\%$) the original rounding-based allocation produced six participants instead of five; the released engine guards this case: whenever the rounded group sizes would exceed N , the adversary count $nt = \operatorname{round}(N \cdot tr)$ is fixed first and the honest remainder $N - nt$ is split in the original accurate : slow : other proportions (70/95, 20/95, exact remainder), with any negative remainder absorbed into the accurate group; all other cells keep the original allocation bit-for-bit. The 12 affected cells (600 of 72,000 runs) were re-simulated with this corrected composition; all reported statistics use the corrected data (see Data availability).

$$v_i = \text{quant}_8(\theta_i) \quad (2c)$$

Votes are aggregated by the vector mean, whose magnitude defines consensus strength γ and direction the consensus heading $\hat{c}$:

$$\mathbf{b} = \frac{1}{N} \sum_{i=1}^N d_{v_i}, \quad \gamma = \|\mathbf{b}\|, \quad \hat{c} = \mathbf{b}/\gamma \quad (2d)$$

Every frame ($\Delta t = 1/60$ s), velocity follows the consensus target under exponential smoothing and position integrates velocity:

$$\mathbf{v}_{t+1} = \mathbf{v}_t + \alpha (v \hat{c} - \mathbf{v}_t), \quad \alpha = 0.2 \quad (2e)$$

$$p_{t+1} = p_t + \mathbf{v}_{t+1} \Delta t \quad (2f)$$

These equations are implemented verbatim in the released engine; the delay of $\tau = 26$ frames (≈ 433 ms) represents realistic end-to-end latency in streaming-based crowd-control platforms^{2,11}.

The complete engine, participant-model, and campaign parameters are listed in Supplementary Table S4.

Trajectory path equations (metres). Circle: $(x,y)=R(\cos t, \sin t)$, $R=10$. Square: half-side $h=10$, constant arc-rate. Lemniscate (Gerono): $(x,y)=(a \cos t/(1+\sin^2 t), a \sin t \cos t/(1+\sin^2 t))$, $a=7$, $n=800$. Zigzag: piecewise-linear, amplitude 5, 10 segments, step 5. Targets traversed at constant speed V ; agent tracks nearest path point.

Adversarial participant model

A proportion of participants are modeled as adversarial agents. In the initial campaign, adversarial agents vote for a direction drawn uniformly at random from 8 discrete cardinal directions (baseline model T0), following a simplified Byzantine-style disruption model^{9,10,12}. The revised campaign adds two predeclared adversarial models. T1, a committed anti-target adversary, votes in an open-loop manner for the quantized direction opposite to the instantaneous target-directed direction. T2, a persistent stale anti-target adversary, holds an opposing direction for ten consecutive voting intervals before updating, modeling a coordinated low-frequency disruptor. The adversarial ratio was systematically varied across a dense grid $tr \in \{5, 10, 13, 15, 17, 20, 25, 30, 35, 40\}\%$ to characterize the transition region. Non-adversarial participants are modeled as accurate (70%, $\pm 3^\circ$ angular noise), delayed (20%, lagged direction), and other (remaining), scaled as $sc = (1 - tr)/0.95$ to preserve the relative 70/20/5 proportions among non-adversarial participants as the adversarial ratio tr varies ($0.95 = 0.70 + 0.20 + 0.05$ is the baseline non-adversarial fraction at $tr = 5\%$).

Experimental design

A structured simulation campaign was employed across a dense adversarial-ratio grid (10 levels, $tr \in [5\%, 40\%]$). The primary independent variables were crowd size N , adversarial ratio tr , and trajectory geometry. Four trajectory types were examined:

- Circle ($R = 10$ m): constant curvature, stable optimal speed.
- Square ($h = 10$ m): four 90° corners.
- Lemniscate ($a = 7$ m): figure-eight with varying curvature.
- Zigzag (amp = 5 m): high-frequency directional reversals.

The study was conducted in two stages. An initial campaign used $MC = 15$ independent Monte Carlo realizations (4,230 runs). In response to reviewer feedback, the campaign was subsequently extended to $MC = 50$ over a denser adversarial-ratio grid and an explicit adversarial-model ladder (72,000 runs). The two are compared directly in the Monte Carlo stability analysis, where the $MC = 15$ and $MC = 50$ estimates agree to within $\sim 1\%$. Crowd size was varied across $N \in \{5, 10, 15, 20, 25, 30, 40, 50, 75, 100, 150, 200\}$. Seed handling uses independent NumPy SeedSequence streams per (trajectory, N , adversarial ratio, model, Monte Carlo index), ensuring observation independence. Supplementary Table S5 summarizes the experimental conditions.

The simulation pipeline from parameter grid to analysis outputs is summarized in Supplementary Fig. S2.

Performance metric

Performance was quantified using root mean square error (RMSE) between the target trajectory and the controlled agent path:

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^T \|x_t - x_t^{ref}\|^2} \quad (3)$$

where $T = 65$ s is the evaluation horizon, x_t the agent position and x_t^{ref} the target position at time t . To quantify the saturation (ceiling) strength, a model

$$RMSE(N) = a + \frac{b}{\sqrt{N}} \quad (4)$$

was fit to each (trajectory, adversarial ratio) condition, where a is the asymptotic error floor and b the crowd-size scaling coefficient; the coefficient of determination R^2 measures fit quality. The $1/\sqrt{N}$ form is a theoretically motivated baseline (the variance of an average of N approximately independent estimates scales as $1/N$, hence the error scales as $1/\sqrt{N}$) rather than an assumed universal law.

Statistical analysis

To evaluate whether the effect of crowd size depends on adversarial pressure, a two-way analysis of variance (ANOVA) was conducted, including the main effect of crowd size (N), the main effect of adversarial ratio (tr), and their interaction ($N \times tr$). A significant interaction indicates that the benefit of crowd size is conditional on the adversarial ratio. Effect sizes are reported using η^2 . The analysis was performed on raw $MC = 50$ realizations (1,800 observations per trajectory: 12 N -levels $\times$ 3 tr -levels $\times$ 50 runs) using the pingouin

library, and reproduced on the initial MC = 15 set with equivalent conclusions. No pseudo-observation reconstruction was employed; because large-sample normality and homoscedasticity tests are sensitive at this sample size, distribution-free permutation tests and Welch-type heteroscedasticity-robust inference were additionally applied and yielded the same qualitative conclusions.

Use of generative AI

During the preparation of this work, the author used Claude (Anthropic) as an AI writing assistant for language refinement and manuscript editing in the Introduction, Methods, Results, Discussion, and Conclusion sections. AI-generated text was not used for data analysis, statistical computation, or scientific interpretation. All scientific content, analysis, interpretation, and final revisions were independently reviewed and approved by the author, who takes full responsibility for the manuscript. The author reviewed and edited all AI-assisted text for factual accuracy and originality and takes full responsibility for the content of this work. No generative AI or AI-assisted image-generation or image-editing tools were used to create or alter any submitted figures; all data visualizations were generated from author-controlled scripts and archived datasets.

DATA AVAILABILITY

The simulation code, raw experimental data, and analysis scripts supporting the findings of this study are openly available in a dedicated repository for this paper at <https://github.com/betasbk-bkg/crowd-size-ceiling-control> (release tag v2.0.0), archived on Zenodo at doi:10.5281/zenodo.21337367. This archive is specific to the present paper and is distinct from the prior-work archive. The archive includes the corrected main-campaign dataset (N = 5, tr = 30% participant-composition fix; 12 cells re-simulated), the lag-axis experiment script and data, and the ceiling-fit, change-point, quadrature-calibration, and ANOVA assumption-diagnostic scripts (normality, homoscedasticity, and ceiling-model residual checks), so that every reported statistic maps to an executable script. The released version corresponds to release tag v2.0.0 of the repository; the README provides an explicit script-to-result mapping table linking each reported figure and table to its generating script.

AUTHOR CONTRIBUTIONS

B.S. conceived the study, designed and executed the simulation framework, performed all statistical analyses, and wrote the manuscript in its entirety.

COMPETING INTERESTS

The author declares no competing interests.

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REFERENCES

1. Song, B. Speed over strategy: why agent velocity dominates aggregation method in crowd-sourced continuous control. *Research Square* <https://doi.org/10.21203/rs.3.rs-9326125/v1> (2026). Preprint, not peer-reviewed.
2. Lasecki, W. S., Murray, K. I., White, S., Miller, R. C. & Bigham, J. P. Real-time crowd control of existing interfaces. In *Proc. 24th Annu. ACM Symp. User Interface Softw. Technol. (UIST)* 23–32 (ACM, 2011). <https://doi.org/10.1145/2047196.2047200>
3. Bigham, J. P., Lasecki, W. S. & Wobbrock, J. O. Target acquisition and the crowd actor. *Hum. Comput.* **2**, 135–154 (2015). <https://doi.org/10.15346/hc.v2i2.2>
4. Seering, J. et al. Audience participation games: blurring the line between player and spectator. In *Proc. 2017 Conf. Designing Interactive Syst. (DIS)* 429–440 (ACM, 2017). <https://doi.org/10.1145/3064663.3064732>
5. Surowiecki, J. *The Wisdom of Crowds* (Anchor Books, 2005).
6. Almaatouq, A. et al. Adaptive social networks promote the wisdom of crowds. *Proc. Natl Acad. Sci. USA* **117**, 11379–11386 (2020). <https://doi.org/10.1073/pnas.1917687117>
7. Bose, T., Reina, A. & Marshall, J. A. R. Collective decision-making. *Curr. Opin. Behav. Sci.* **16**, 30–34 (2017). <https://doi.org/10.1016/j.cobeha.2017.03.004>
8. Lorenz, J., Rauhut, H., Schweitzer, F. & Helbing, D. How social influence can undermine the wisdom of crowd effect. *Proc. Natl Acad. Sci. USA* **108**, 9020–9025 (2011). <https://doi.org/10.1073/pnas.1008636108>
9. Vempaty, A., Tong, L. & Varshney, P. K. Distributed inference with Byzantine data: state-of-the-art review on data falsification attacks. *IEEE Signal Process. Mag.* **30**, 65–75 (2013). <https://doi.org/10.1109/MSP.2013.2262116>
10. Blanchard, P., El Mhamdi, E. M., Guerraoui, R. & Stainer, J. Machine learning with adversaries: Byzantine tolerant gradient descent. In *Adv. Neural Inf. Process. Syst. (NeurIPS)* **30** (2017).
11. Aleta, A. & Moreno, Y. The dynamics of collective social behavior in a crowd controlled game. *EPJ Data Sci.* **8**, 22 (2019). <https://doi.org/10.1140/epjds/s13688-019-0200-1>
12. Lamport, L., Shostak, R. & Pease, M. The Byzantine generals problem. *ACM Trans. Program. Lang. Syst.* **4**, 382–401 (1982). <https://doi.org/10.1145/357172.357176>

13. Khalil, H. K. *Nonlinear Systems* 3rd edn (Prentice-Hall, 2002).
14. Samson, C. Control of chained systems: application to path following and time-varying point-stabilization of mobile robots. *IEEE Trans. Autom. Control* **40**, 64–77 (1995). <https://doi.org/10.1109/9.362899>
15. Morin, P. & Samson, C. Motion control of wheeled mobile robots. In *Springer Handbook of Robotics* (eds Siciliano, B. & Khatib, O.) 799–826 (Springer, 2008). https://doi.org/10.1007/978-3-540-30301-5_35
16. Slotine, J.-J. E. & Li, W. *Applied Nonlinear Control* (Prentice-Hall, 1991).
17. Niemeyer, G. & Slotine, J.-J. E. Stable adaptive teleoperation. *IEEE J. Ocean. Eng.* **16**, 152–162 (1991). <https://doi.org/10.1109/48.64895>
18. Anderson, R. J. & Spong, M. W. Bilateral control of teleoperators with time delay. *IEEE Trans. Autom. Control* **34**, 494–501 (1989). <https://doi.org/10.1109/9.24201>
19. Kolling, A., Walker, P., Chakraborty, N., Sycara, K. & Lewis, M. Human interaction with robot swarms: a survey. *IEEE Trans. Hum.-Mach. Syst.* **46**, 9–26 (2016). <https://doi.org/10.1109/THMS.2015.2480801>
20. Hornbæk K (2013), "Some Whys and Hows of Experiments in Human—Computer Interaction". *Foundations and Trends in Human-Computer Interaction*, Vol. 5 No. 4 pp. 299–373, <https://doi.org/10.1561/11000000043>
21. Subramanian, S., De Moor, K., Fiedler, M., Koniuch, K. & Janowski, L. Towards enhancing ecological validity in user studies: a systematic review of guidelines and implications for QoE research. *Qual. User Exp.* **8**, 6 (2023). <https://doi.org/10.1007/s41233-023-00059-2>
22. Ji, K., Spina, D., Hettiachchi, D., Salim, F. D. & Scholer, F. Examining the impact of uncontrolled variables on physiological signals in user studies for information processing activities. In *Proc. 46th Int. ACM SIGIR Conf. Res. Dev. Inf. Retr.* 1971–1975 (ACM, 2023). <https://doi.org/10.1145/3539618.3591981>
23. Mannes, A. E., Soll, J. B. & Larrick, R. P. The wisdom of select crowds. *J. Pers. Soc. Psychol.* **107**, 276–299 (2014). <https://doi.org/10.1037/a0036677>

Figure legends

Figure 1. Ceiling effect strength (R^2) across trajectory types and adversarial ratios. Green indicates strong ceiling effect ($R^2 > 0.8$); red indicates no ceiling pattern ($R^2 < 0.2$). All values from the extended MC = 50 campaign over the dense adversarial-ratio grid.

Figure 2. RMSE as a function of crowd size N for circle trajectory at $v = 5.0$ m/s across three adversarial ratios. Error bars represent 95% confidence intervals (MC = 50). Dashed lines

show fitted ceiling model $RMSE(N) = a + b/\sqrt{N}$. At $tr = 5\%$, no meaningful ceiling forms ($R^2 = 0.35$); at $tr \geq 15\%$, the ceiling model fits well ($R^2 \geq 0.87$).

Figure 3. Structural N-independence of zigzag trajectory at $tr = 40\%$. Left: RMSE vs. N for circle (blue) and zigzag (orange) with fitted ceiling models (R^2 shown). Right: normalized $\Delta RMSE$ from $N = 5$, confirming near-zero crowd-size benefit in zigzag across the full N range.

Figure 4. RMSE versus crowd size N for the three adversary models (T0/T1/T2) at $tr = 40\%$ across the four trajectory geometries (MC = 50, 95% CI). The persistent stale adversary (T2) suppresses the crowd-size benefit on smooth geometries while the zigzag remains lag-dominated.

Tables

Table 1. Ceiling Fit R^2 by Trajectory and Adversarial Ratio (MC = 50, baseline adversary model T0). A representative six-level subset of the full ten-level grid is shown; see Supplementary Table S6 for all ten levels. The $tr = 30\%$ column reflects a corrected $N = 5$ participant composition (Methods); low- tr cells ($tr \leq 10\%$) are affected by realized-adversary discretization at small N ($\text{round}(N \cdot tr) = 0$ for $N \leq 10$), which depresses fit quality without indicating absence of a ceiling pattern. Ceiling-model residuals are approximately normal and free of systematic structure across conditions (Durbin–Watson 1.5–2.5; Shapiro $p > 0.05$) except at the near-flat circle $tr = 5\%$ cell, where the small residuals show mild oscillation.

Trajectory	tr=5%	tr=10%	tr=15%	tr=20%	tr=30%	tr=40%
Circle	0.350	0.025	0.875	0.939	0.815	0.957
Square	0.533	0.868	0.942	0.967	0.858	0.944
Lemniscate	0.356	0.419	0.821	0.788	0.722	0.892
Zigzag	0.398	0.300	0.712	0.824	0.226	0.160

Table 2. Two-Way ANOVA Results ($N \times$ Adversarial Ratio) — Raw MC = 50 Realizations. Residual normality and homoscedasticity were formally rejected at this sample size (Shapiro–Wilk $p < 10^{-14}$; Levene $p < 10^{-59}$; per-cell variance ratio up to 34), although the residuals are symmetric ($|\text{skew}| < 0.06$) and the rejection is partly an artifact of the large n (1,800). Welch-type heteroscedasticity-robust inference and distribution-free permutation tests were therefore used as the primary tests and yielded the same qualitative conclusions.

Factor	Circle η^2	Circle F	Circle p	Square η^2	Square F	Square p
N (crowd size)	33.6%	249.3	$<10^{-300}$	26.8%	108.8	2.5×10^{-189}
tr (adversarial)	22.9%	933.7	2.7×10^{-277}	24.2%	541.1	5.5×10^{-184}

Factor	Circle η^2	Circle F	Circle p	Square η^2	Square F	Square p
N $\times$ tr	21.8%	80.8	8.9×10^{-248}	9.6%	19.4	5.7×10^{-68}
Residual	21.7%	—	—	39.4%	—	—

Table 3. Lag-axis sensitivity (tr = 20%, MC = 50 per cell): crowd-size benefit Δ RMSE(N = 5 $\rightarrow$ 200) under manipulated delay τ and smoothing α . Asterisks denote Welch $p < 0.05$ between the N = 5 and N = 200 per-run RMSE samples; n.s. = not significant.

α	τ (frames)	τ_{eff} (s)	Lemniscate	Zigzag
0.20	13	0.291	27.32%*	5.33%*
0.10	13	0.375	12.45%*	4.47%*
0.20	26	0.508	4.39%*	1.95%*
0.10	26	0.592	1.32%*	-0.14% (n.s.)

Table 4. Fitted ceiling parameters a, b with 95% CIs (weighted least squares on MC = 50 cell means), and the attainable RMSE reduction from N = 5 to the asymptote.

Traj	tr	a [95% CI]	b [95% CI]	Attainable
circle	5%	0.8032 [0.800, 0.807]	+0.047 [+0.029, +0.064]	2.5%
circle	20%	0.7736 [0.769, 0.779]	+0.356 [+0.328, +0.385]	17.1%
circle	40%	0.7488 [0.742, 0.755]	+0.787 [+0.747, +0.828]	32.0%
zigzag	5%	1.9598 [1.955, 1.964]	+0.043 [+0.021, +0.065]	0.97%
zigzag	20%	1.9651 [1.959, 1.971]	+0.077 [+0.046, +0.107]	1.72%
zigzag	40%	1.9724 [1.965, 1.980]	+0.046 [+0.000, +0.091]	1.03%

Figures

Figure 1

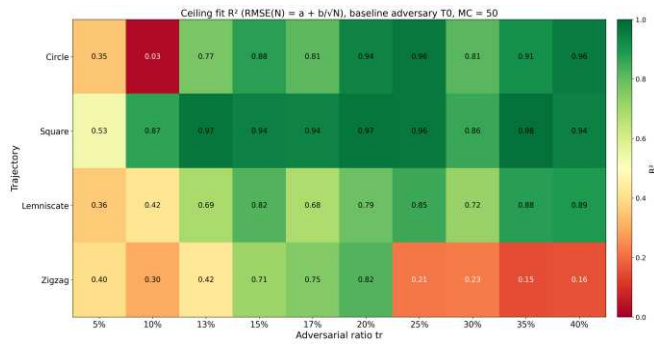


Figure 2

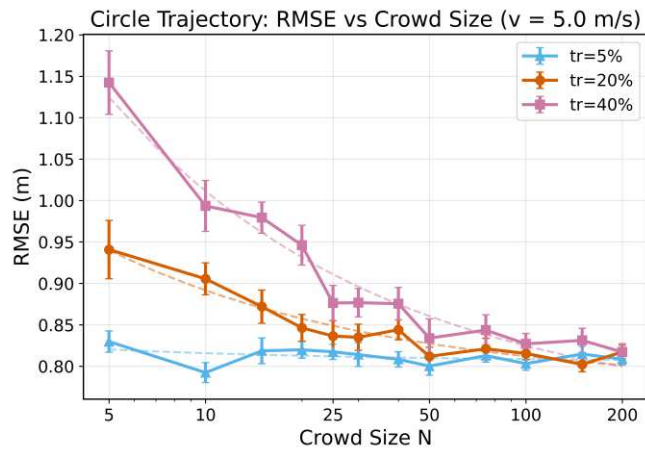


Figure 3

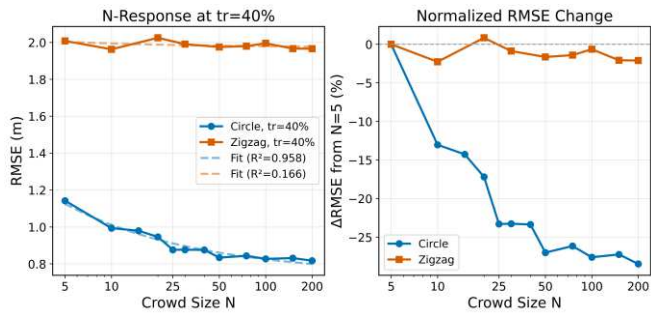


Figure 4

