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DEVELOPMENT OF AN IMPROVED 3D MOLODENSKY BADEKAS GEODETIC TRANSFORMATION MODEL

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Abstract

Mapping is a fundamental pillar of resource management and regional development, requiring high-precision spatial data to ensure the accurate extraction and governance of resources. Modern mapping has transitioned from classical local systems to global systems supported by Global Navigation Satellite System technologies, which provide precise 3D geodetic coordinates (latitude, longitude, and ellipsoidal height). This transition has created a legacy of maps with varying accuracies, necessitating a harmonized approach to map revision through rigorous coordinate transformation. The precision of such transformations is largely dictated by the quality of control point data and the mathematical suitability of the transformation model employed. While models range from simple 3-parameter congruent transformations to complex polynomials, each possesses inherent strengths and limitations depending on regional data characteristics. This study focuses on the Molodensky-Badekas model, aiming to enhance its performance for the Kenyan Geodetic System. The methodology involves acquisition and processing of geodetic control datasets with common points in two coordinate systems: UTM and Cassini Arc 1960 for 13 counties within central, and southern part of Kenya. An Improved 3D Geodetic Molodensky-Badekas Transformation Model was developed by transitioning the model to a multi-scale parameter framework. This improved model was rigorously tested against existing 3D Cartesian-to-Cartesian similarity models. The findings indicate that the improved Molodensky-Badekas model significantly outperformed traditional similarity models, specifically the 10-parameter Molodensky-Badekas model by 93.02% for the Kenyan Geodetic System. It is recommended as a robust alternative for high-accuracy map revision and datum transformation in the region.

Keywords: 3D Cartesian Transformation, Bursa wolf, Geodetic Transformation, Molodensky Badekas model, Kenya Geodetic Network.

Introduction

The calculation of physical size and related gravity field of the Earth is one of the main problems of geodesy. The figure of the Earth is a complex aspect to characterize by the virtue of its irregular geometry that cannot be represented by simple mathematical solids (Ziggah et al. 2013). Moreover, the geodetic changes in the gravity field over time require regular geodetic updating of the spatial frameworks and cartographic products. For this to be accomplished a coordinate system is defined (Hassan et al. 2020)

Horizontal and vertical positioning is based on reference surfaces. Horizontal coordinates (usually geodetic latitude and longitude) are in reference to a given reference ellipsoid and datum (Mina 2006; Ono 2009). To these ends, a mathematically amenable approximation to the surface of the Earth, flattened at the poles, is an ellipsoid of revolution. Conversely, vertical coordinates are traditionally measured with respect to the geoid, which is a surface of equal gravitational potential that is very similar to mean sea level as determined by tide gauges. Although in practice horizontal (2D) and vertical (1D) components were considered independently, the combination of these reference surfaces enables a three-dimensional (3D) representation of space, which is necessary in the context of the contemporary geodetic use (Hassan et al. 2020). This historical separation has created a global challenge: harmonizing legacy 1D and 2D classical networks with modern 3D global geocentric systems, such as WGS84 and ITRS, which are enabled by space-geodetic measurements like GNSS.

Kenya's geodetic infrastructure reflects this complexity through various horizontal framework consisting of the Cassini-Soldner on Clarke 1858 spheroid (used for cadastral mapping), Universal Transverse Mercator (UTM) system on modified Clarke 1880 spheroid ellipsoid, (used for topographical mapping), the East African War System on Clarke 1880 which was developed to unify the coordinate system for the British Commonwealth territories in the South, East and Central Africa, and UTM on WGS84 ellipsoid which characterizes the modern GPS surveys (Matara et al. 2022). These networks were established in phases starting in 1893, evolving from early boundary delimitations to the Directorate of Overseas Surveys (DOS) mapping (1950–1983). Primarily anchored to the Arc 1960 datum on the Clarke 1880 (Modified) ellipsoid, these legacy networks are impaired by considerable local distortions and scale biases. To address these inconsistencies and conform to modern geospatial technologies, the Survey of Kenya established the Kenya National Geodetic Reference Frame (KENREF), a multi-phase modernization effort designed to provide a high-accuracy, GNSS-compatible geodetic foundation (Murgnier 2013).

In addition, the vertical geodetic infrastructure in Kenya is primarily established upon the Mombasa Chart Datum, where orthometric heights are referenced to the Mean Sea Level (MSL) as determined by historical tide gauge observations (Matara et al. 2022). While this serves as the primary national vertical reference, localized surveys have occasionally utilized different realizations of high-water marks or reciprocal levelling from the Anglo-German Boundary Commission (AGBC) era (Siriba 2021).

The contemporary transition to the Kenya National Geodetic Reference Frame (KENREF) and the widespread adoption of GNSS technology have exposed the limitations of these historical frameworks, which were often treated independently of horizontal systems. Because GNSS provides ellipsoidal heights, there is an urgent functional requirement for a precise national geoid model. This model acts as the critical mathematical link necessary to transform satellite-derived ellipsoidal heights into the local orthometric systems used for engineering, hydrologic

modelling, and national mapping, thereby ensuring vertical consistency across the horizontal coordinate systems (Odera et al. 2014; Siriba 2021).

The core geodetic problem lies in the mathematical transformation required to bridge these diverse datums with global reference frames. While 3D similarity transformations, such as the 7-parameter Bursa-Wolf model, are widely utilized, they are frequently characterized by pronounced parameter correlation when implemented within localized networks (Abbey and Featherstone 2020).

On the other hand, Helmert (7-parameter) model assumes a uniform scale factor across the entire network, which prevents it from accounting for non-linear, localized distortions common in terrestrial datasets. Additionally, in small-extent networks, the estimated parameters, particularly rotation and translation, become highly correlated, leading to numerical instability hence reduced transformation accuracy (Watson 2006; Tran et al. 2023; Banković et al. 2024).

Recent studies have consequently explored more complex models, such as the 10-parameter Molodensky-Badedas model, which introduces a network centroid to reduce this correlation, or affine transformations that allow for changes in position, size, and shape through multiple scale factors (Annan et al. 2016a; Abbey and Featherstone 2020).

Technical studies suggest that transformation accuracy is significantly influenced by the inclusion of height components and the mitigation of scale distortions (Hassan et al. 2020; Kumi-Boateng and Yevenyo Ziggah 2020; Abbey and Featherstone 2020; Haddad and Gahlouz 2025). For instance, 8-parameter affine transformations use different scales to account for low accuracy in vertical data, while improved centroid techniques have further refined the precision of the Molodensky-Badedas model. Despite these advancements, there remains a research gap in further optimizing the Molodensky-Badekas model to handle the non-uniform distortions inherent in terrestrial networks through more flexible parameterization.

This research addresses these gaps by developing an improved 3D Geodetic Molodensky-Badekas transformation model. By increasing the degrees of freedom—specifically by expanding the model to a 11-parameter framework with multiple scale factors—this study aims to enhance transformation precision in the Southern Part of Kenya (UTM Zone 37S). This improved model is evaluated against traditional similarity models providing a mathematically transparent and highly accurate alternative for national coordinate transformation and map revision.

Literature Review

Geodetic Datums

A geodetic datum serves as the fundamental geometric model of the Earth, providing the essential coordinate reference system for spatial data. This framework encompasses an ellipsoid of rotation centered on a polar (Z) axis, with the X and Y axes situated within the equatorial plane such that the positive X axis establishes the zero-meridian longitude (Kumi-Boateng and Yevenyo Ziggah 2020). Once this reference frame is established, positions can be mathematically expressed through three distinct categories of coordinates: geographic, Cartesian, and grid-based as shown in figure 1, 2 and 3 respectively. Geographic coordinates utilize latitude (ϕ), longitude (λ) and ellipsoidal height ($\hbar$) to define a point's location via meridians and parallels relative to the ellipsoidal surface. Alternatively, Cartesian coordinates provide a three-dimensional representation using three perpendicular axes (X, Y, and Z) originating from the ellipsoid's center. For practical mapping and engineering applications, these three-dimensional positions are

transformed into grid coordinates also known as plane or map coordinates which represent 2D positions resulting from specific map projections (Elshambaky et al. 2018).

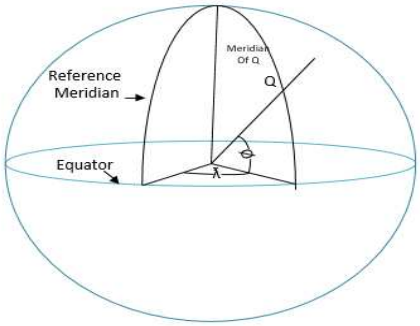


Fig. 1 *Geographic Coordinate System.*

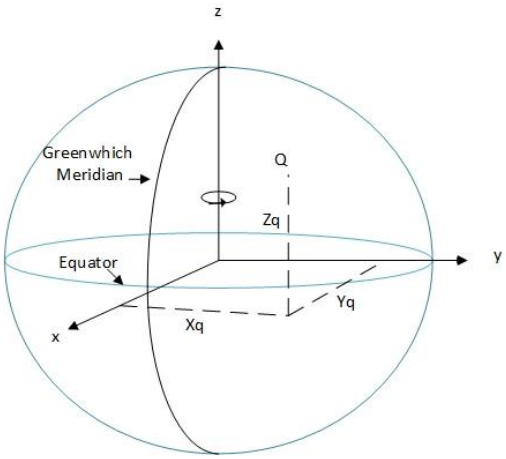


Fig. 2 *Cartesian Coordinate System*

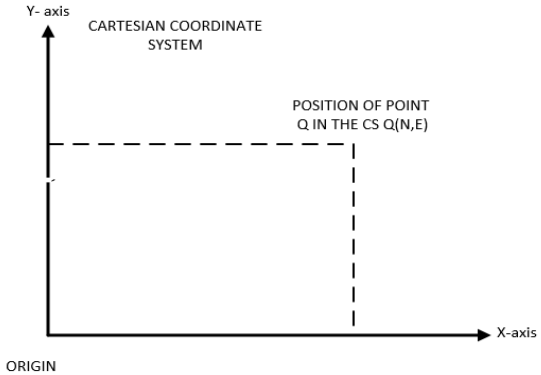


Fig. 3 *Planar/2 D/map Projection Coordinate System*

The geodetic datum definition is finalised by aligning the selected ellipsoid with the Earth, depending on whether the datum is designed for local or global application. Fitting the earth entails reducing the disparity between the ellipsoid

and the geoid, the equipotential surface that most accurately represents mean sea level. Local datums, often established before to the advent of satellites, utilise an ellipsoid that closely conforms to the geoid within a specific region, frequently including a single nation (Mina 2006).

This has led to a significant quantity of local datums. The advent of satellites has enabled the identification of ellipsoids that closely approximate the global geoid, to the extent feasible. This has resulted in global datums (Ruffhead 2023).

Geodetic Datums used in Kenya

In Kenya, the development and use of geodetic datums reflect the country's transition from classical terrestrial surveying to modern satellite-based positioning. Historically, Kenya adopted the Arc 1960 datum, which is based on the Clarke 1880 ellipsoid and forms part of the broader East African Arc datum network established during the colonial triangulation campaigns (Kinyua et al. 2025). In this datum, the reference ellipsoid was fitted to best approximate the geoid over the East African region rather than the entire Earth. As a result, while it provides good relative accuracy within Kenya and neighboring countries, it exhibits significant offsets when compared with global geocentric systems. Most cadastral surveys, topographic maps, and engineering datasets produced by the Survey of Kenya before the GNSS era are referenced to Arc 1960, commonly projected in the UTM coordinate system or the Cassini-Soldner projection for local mapping. Because Arc 1960 is not geocentric, direct use of modern GNSS coordinates (which are inherently global) requires datum transformation parameters to ensure compatibility (Odera et al. 2014; Thiong'o 2019; Matara et al. 2022).

With the advent of satellite geodesy and GNSS technology, Kenya increasingly adopted global reference systems, particularly the World Geodetic System 1984 (WGS 84). WGS 84 is a geocentric global datum, meaning its origin coincides with the Earth's center of mass, making it suitable for GPS positioning and international geospatial integration. However, discrepancies between Arc 1960 and WGS 84 can reach several hundred meters if transformation parameters are not applied. To address this limitation and modernize the national reference framework, Kenya has been implementing the Kenya Geodetic Reference Frame (KENREF), which is aligned with the International Terrestrial Reference Frame (ITRF) through the African Geodetic Reference Frame (AFREF) initiative. KENREF is realized through a network of Continuously Operating Reference Stations (CORS) distributed across the country, providing a unified, precise, and stable three-dimensional reference frame suitable for surveying, mapping, GIS, engineering, and scientific applications (Matara et al. 2022).

The coexistence of Arc 1960, WGS 84, and the emerging KENREF framework has created a practical need for accurate 2D and 3D coordinate transformation models within Kenya. Many legacy datasets remain on Arc 1960, while contemporary GNSS observations are referenced to WGS 84/ITRF (Matara et al. 2022). Consequently, reliable transformation parameters; whether based on three-parameter, seven-parameter Helmert, Molodensky, or grid-based models are essential to ensure spatial data consistency. The Kenyan case therefore illustrates the broader geodetic evolution from localized ellipsoid fitting toward globally integrated, geocentric reference systems, highlighting the importance of datum awareness in all geospatial applications (Kumi-Boateng and Yevenyo Ziggah 2020).

The quality of transformation is highly dependent on the model used, the quality of the geodetic data used, modelling of the network distortion and spatial distribution of the common points that cause parameter biases (Kinyua et al. 2025).

Geodetic Datum Transformation

A geodetic datum transformation refers to the process of converting location coordinates from one geodetic datum to another. Coordinates derived from measurements are often known in both datums at a series of common sites (Yilmaz 2023). A standard transformation technique is a mathematical model that aligns common points based on selected criteria, typically employing least-squares optimization. The data for which a transformation is intended are often denoted as "source" and "target". A transformation may be articulated in one of the following formats: coordinates in the destination datum as a function of coordinates in the source datum or variation in coordinates $\Delta\emptyset$ as a function of the coordinates in the original datum (Turgut 2010).

There are different types of transformation applied in geodesy today including geographic coordinates to Cartesian coordinate system and the reverse, the transformation between geographic datum to another geographic systems, the transformation from one Cartesian systems to another Cartesian systems that can be 2 dimensional or 3 dimension and lastly, the map projection systems. In 3D coordinate transformation, there are two categories of transformation namely the geocentric (Cartesian to Cartesian) transformations and geographic transformations (Lv and Zhang 2026). This can be simplified by use of the figure 4, where: A and C is Cartesian to Cartesian transformation, B is Geographic to geographic transformation, D is Cartesian to geographic transformation, E is Forward map projection, F is Reverse map projection, and G is Geographic to Cartesian transformation. The datum transformation models utilized in the current research are discussed in the following sections.

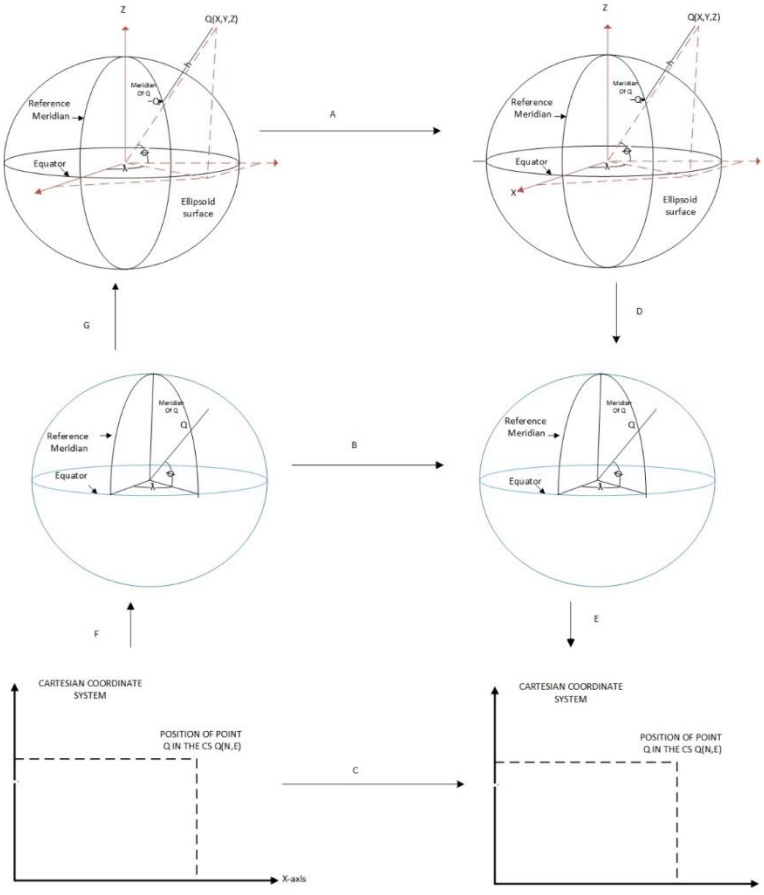


Fig. 4 *Illustrating various types of datum transformations*

Recent Studies

Kalu et al. (2022) evaluates the efficacy of the Bursa Wolf and Molodensky-Badedas transformation models in improving coordinate transformation between Nigeria's local Minna datum (Clarke 1880 ellipsoid) and the global WGS84 system. By analyzing 100 first-order control points in Southern Nigeria, Kalu et al. (2022) derived new sets of seven transformation parameters and validated them using an additional seven independent points. The results demonstrate a significant 68% improvement in coordinate correlation compared to previously used official parameters, with the Molodensky-Badekas model showing superior predictive accuracy and lower summation of residuals. Ultimately, the study recommends the adoption of these refined regional parameters to minimize data loss and distortion.

Ruffhead (2023) proposed a new method for deriving 9-parameter affine 3D datum transformations using ordinary least-squares to address distortions between traditional terrestrial and GPS-derived networks for Western Australia, Great Britain, and Sweden region. Ruffhead (2023) introduced an optimization process based on the equivalent-enlargement hypothesis to transition from a 7-parameter conformal model to a full 9-parameter affine model. The methodology involves four stages: 7-parameter Helmert optimization, followed by sequential 8-parameter and 9-parameter optimizations, and finally a scale-ratios correction. Findings indicate that while the 9-parameter model significantly improves the quality-of-fit for datasets in Western Australia and Great Britain, it actually increased the standard error in the Swedish case, suggesting that the model's necessity depends on the specific geodetic environment. Ruffhead (2023) recommends comparing the standard error of the affine model against the 7-parameter model to ensure statistical significance, identifying a research gap in the method's performance regarding very large rotations which requires further investigation.

Ruffhead (2019) developed a novel optimization method for deriving 7-parameter conformal (Helmert) transformations that overcomes the complexities of non-linearity and high parameter correlation. The study employs a diverse range of test areas, including actual datasets from Great Britain (OSGB 36 to WGS 84), Western Australia (AGD 84 to WGS 84), and Ghana (Accra datum to WGS 84), alongside 18 simulated datasets designed to test measurement errors. The findings demonstrate that the distance-analysis scale-change is optimal in almost all cases, requiring only a single iteration for the rotation parameters to converge with high precision. A key recommendation is the adoption of this method for national geodetic agencies, as it provides a straightforward, rigorously conformal alternative to near-conformal Bursa-Wolf models. A noted research gap or exception occurs in transformations covering very small areas (less than a degree square), where short inter-point distances maximize the impact of measurement errors, necessitating the quadratic verification stage to complete the optimization.

The geodetic research landscape in Kenya is undergoing a positive transformation, moving from a period of relatively scarce documentation to a vibrant era of inquiry. As Kenya modernizes its spatial data infrastructure, there has been a notable surge in local research focused on refining coordinate transformation models, representing a crucial step forward in the country’s technical self-reliance and geodetic precision. A foundational empirical study by, by Lwangasi (1993) computed 3- and 7-parameter transformation parameters relating the Clarke 1880 Arc Datum 1960 to the WGS-72 geocentric datum using a dataset of 25 geodetic control points distributed across Kenya. The methodology involved least-squares adjustment of transformation parameters, providing the first published set of

translation, rotation, and scale values that can be used to align historical geodetic surveys with global references. Key findings included quantified translation parameters (e.g., X = −179.1 m, Y = −39.7 m, Z = −304.1 m) and small rotation values, which allowed improved integration of legacy survey data within global systems. Recommendations emphasized the importance of regularly updating transformation parameters as new geodetic measurements become available, and the research gap remains in modernizing these parameters using current high-precision GNSS/CORS techniques.

Although primarily historical, Matara et al. (2022) Kenyan geodetic review has direct implications for coordinate transformation because it documents the heterogeneous legacy datums, coordinate systems, and control surveys used across the country and the challenges of datum unification. Covering networks from early triangulation chains to modern CORS, the paper analyses how different datums and projections have created inhomogeneous coordinate references, necessitating transformation operations whenever geospatial data are integrated. Methodologically, it synthesizes historical geodetic records, national survey campaigns, and recent efforts toward the Kenya Geodetic Reference Frame (KENREF), identifying inconsistent datum usage as a core problem. The authors recommend harmonization of geodetic networks and integration of national and private CORS, and highlight a research gap in establishing standardized national transformation models that can consistently convert coordinates across the multiple legacy systems.

Thiong'o (2019) investigated the 3D coordinate transformation necessary to relate the Kenya Power and Lighting Company (KPLC) CORS coordinate system realized on ITRF2008 to the Kenya national geodetic system (UTM Arc 1960 datum) used for official mapping using six common control points from both systems. Using the standard Bursa-Wolf (7-parameter Helmert) transformation, the research computed translation (X, Y, Z), rotation, and scale parameters, revealing distinct differences between the global CORS and national frames. Findings highlighted that establishing transformation parameters facilitates integrating CORS data with legacy geodetic datasets within GIS environments, though the validity of parameters needs testing with independent test data. The study recommended further research using additional common points over a broader geographic region to strengthen parameter reliability and address datum distortion across Kenya.

Materials and Methods

Study Area

Kenya comprises 47 counties and is geographically located between longitudes 35° East and 42° East, and approximately between latitudes 5° North and 4° South, straddling the Equator in East Africa. The present study focuses on parts of the central and southern regions of Kenya, situated within UTM Zone 37. Specifically, the study area covers 13 of the 47 counties and includes a significant portion of the Nairobi Metropolitan Region and its surrounding highlands and rift landscapes. The region was selected due to its substantial contribution to the national economy, the availability of high-quality spatial datasets, and the relatively dense and well-maintained geodetic control network.

Physically, the study area is characterized by diverse topography, including the central highlands, portions of the Great Rift Valley, and adjoining semi-arid lowlands. Elevations range from approximately 1,500 meters to over 2,500 meters

above sea level in the highland zones, while the rift floor and southern plains lie at comparatively lower altitudes. This variation in elevation significantly influences both climate and geodetic observation conditions. The climate of the central highlands is generally moderate, with temperatures typically ranging between 10°C and 26°C, and a bimodal rainfall regime consisting of long rains (March–May) and short rains (October–December). Annual rainfall in these areas ranges between 800 mm and 1,500 mm (Njoroge 2025). In contrast, the southern and rift valley areas experience warmer and drier conditions, with annual rainfall between 400 mm and 900 mm and greater variability in precipitation (Danube et al. 2026).

Socio-economically, the central and southern regions of Kenya represent the country's most productive and urbanized corridor. The Nairobi Metropolitan Region serves as the administrative, financial, technological, and infrastructural hub of the country. Rapid urbanization, expansion of transportation networks, energy infrastructure development, and large-scale housing projects have increased the demand for precise geospatial referencing. Additionally, the surrounding central highlands and rift valley areas are key agricultural zones, producing tea, coffee, horticultural crops, and dairy products that contribute significantly to Kenya's GDP and export earnings. Accurate land parcel delineation and cadastral management are therefore critical in these areas. The strong economic base has also contributed to better maintenance of geodetic control networks, more frequent re-observation campaigns, and improved data quality compared to less developed regions (World Bank 2025).

The southern Kenyan region offers relatively favorable conditions for transformation studies because it has a dense and economically supported geodetic infrastructure. The availability of well-distributed common control points enhances parameter stability and reduces extrapolation errors.

Data Sources

The study area has 1250 control points that is in both Cassini and UTM coordinate systems.

Table 1 *Data Sources*

Data	**Data source**	**Data type**	**Data period**
UTM data (N, E, H)	Survey of Kenya	Excel/coordinate point data	existing
Cassini data (X, Y, Z)	Survey of Kenya	Excel/coordinate point data	existing

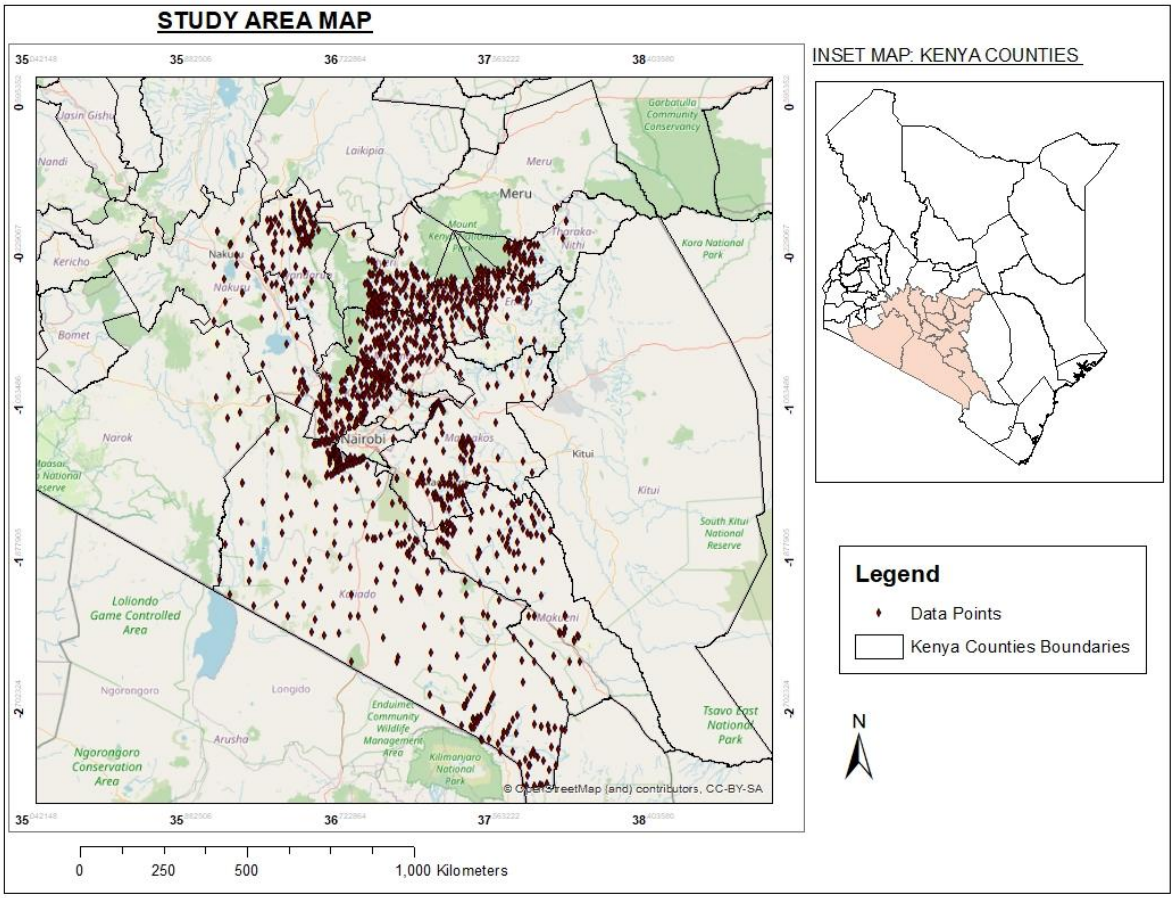


Fig. 5 *Study area Map*

Molodensky and the Abridged Molodensky

The Abridged Molodensky transformation model is used to directly convert coordinates from one geodetic datum to another by relating the ellipsoidal coordinates of the source datum to those of the target datum. The model assumes that the two reference ellipsoids differ only by three translation parameters, meaning their relative positions vary solely through shifts along the X, Y, and Z axes (Deakin 2004)**.** It requires the three-dimensional geocentric datum shift parameters *(*ΔX*,* ΔY*,* ΔZ), as well as the differences in the semi-major axes *(*Δa*)* and flattening *(*Δf*)* between the two reference ellipsoids. This approach enables transformation between three-dimensional Cartesian coordinate systems without the need for full rotational and scale parameters (Boon and Setan 2007).

Three Molodensky models appear in literature. Standard Molodensky, abridged Molodensky and Molodensky Badekas model. Abridged Molodensky Badekas is used for direct transformation of geographic data to geographic data. This model does not contain the ellipsoidal height h of the point to be transformed (Deakin 2006)**.** Standard molodensky badekas is a five-parameter transformation. The translation parameters between the axes are linked to the ellipsoidal p

$$\Delta\emptyset = \frac{1}{\rho\, sin\, 1''}(-\Delta X sin\emptyset cos\lambda - \Delta Y sin\emptyset sin\lambda + \Delta Z cos\emptyset + (a.\Delta f.\Delta a) sin2\emptyset \quad (1)$$

$$\Delta\lambda = \frac{1}{N cos\emptyset sin1''}(-\Delta X sin\lambda + \Delta Y cos\emptyset) \quad (2)$$

$$\Delta h = \Delta X cos\emptyset cos\lambda + \Delta Y cos\emptyset sin\lambda + \Delta Z sin\emptyset + (a\Delta f + f\Delta a) sin^2\emptyset - \Delta a \quad (3)$$

With $\rho = \frac{a(1-e^2)}{\sqrt{1-e^2\, sin^2\emptyset}}$ (4)

Where, ρ is the radius of curvature in the meridian, *(Δ∅, Δλ,Δh)* is the set of corrections to transform *(∅,λ,h)* ellipsoid 1 to *(∅,λ,h)* ellipsoid 2 and *(ΔX, ΔY, ΔZ)* is the set of corrections to transform *(X,Y,Z)* ellipsoid 1 to *(X, Y, Z)* ellipsoid 2. The estimated *Δh* values were then used to compute h for the War Office 1926 ellipsoid through given by:

H ellipsoid 2 = h ellipsoid 1 - Δh. (5)

Affine 9 parameter

9-parameter affine transformation include three translations, three rotations and three scales. Affine 9 parameters can be expressed in the equation:

$$X_2 + V_X = T_X + R(\propto_1, \propto_{2,}, \propto_3).S(\mu_1, \mu_2, \mu_3).X_1 \quad (6)$$

Where X_2 is the three dimension coordinates of system 2, V_X is the re residual factor , T_X the translations , X_1 is the coordinates in datum 1, $R(\propto_1, \propto_{2,}, \propto_3)$are the rotations in three axis , and $S(\mu_1, \mu_2, \mu_3).X_1$different scale factors in each axis given by (Andrei 2006).

$$S(\mu_1, \mu_2, \mu_3) = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} vx \\ vy \\ vz \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} \\ 0 & 1 & 0 & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} & a_{29} \\ 0 & 0 & 1 & a_{34} & a_{35} & a_{36} & a_{37} & a_{38} & a_{39} \end{bmatrix} \begin{bmatrix} T_X \\ T_Y \\ T_Z \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \end{bmatrix} + \begin{bmatrix} X_1 - X_2 \\ Y_1 - Y_2 \\ Z_1 - Z_2 \end{bmatrix} \quad (10)$$

Bursa wolf transformations

Bursa wolf also known as similarity transformation, conformal or Helmert 7 parameter transformation preserves shape and angles but size and positions of points may change. The model is popular because of the number of parameters

involved are minimal.it is a simple model that is adequate for relating two homogenous coordinate systems. The Bursa-wolf model consists The Helmert 7 parameters (also known as similarity transformation or Bursa wolf accounts for the rotation and scale change between the source and target stations (Andrei 2006):

$$\begin{pmatrix} \boldsymbol{X_2} \\ \boldsymbol{Y_2} \\ \boldsymbol{Z_2} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu_1} & \boldsymbol{\propto_3} & \boldsymbol{-\propto_2} \\ \boldsymbol{-\propto_3} & \boldsymbol{\mu_1} & \boldsymbol{\propto_1} \\ \boldsymbol{\propto_2} & \boldsymbol{-\propto_1} & \boldsymbol{\mu_1} \end{pmatrix} \begin{pmatrix} \boldsymbol{X_1} \\ \boldsymbol{Y_1} \\ \boldsymbol{Z_1} \end{pmatrix} + \begin{pmatrix} \boldsymbol{T_x} \\ \boldsymbol{T_y} \\ \boldsymbol{T_z} \end{pmatrix} \quad (11)$$

And can be written in the form:

$$\begin{bmatrix} \boldsymbol{vx} \\ \boldsymbol{vy} \\ \boldsymbol{vz} \end{bmatrix} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{a_{14}} & \boldsymbol{a_{15}} & \boldsymbol{a_{16}} & \boldsymbol{a_{17}} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{a_{24}} & \boldsymbol{a_{25}} & \boldsymbol{a_{26}} & \boldsymbol{a_{27}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{a_{34}} & \boldsymbol{a_{35}} & \boldsymbol{a_{36}} & \boldsymbol{a_{37}} \end{bmatrix} \begin{bmatrix} \boldsymbol{T_X} \\ \boldsymbol{T_Y} \\ \boldsymbol{T_Z} \\ \boldsymbol{\mu_1} \\ \boldsymbol{\alpha_1} \\ \boldsymbol{\alpha_2} \\ \boldsymbol{\alpha_3} \end{bmatrix} + \begin{bmatrix} \boldsymbol{X_1 - X_2} \\ \boldsymbol{Y_1 - Y_2} \\ \boldsymbol{Z_1 - Z_2} \end{bmatrix} \quad (12)$$

Where $\begin{pmatrix} \mathrm{X}_2 \\ \mathrm{Y}_2 \\ \mathrm{Z}_2 \end{pmatrix}$ is the target coordinates, $\begin{pmatrix} \mathrm{X}_1 \\ \mathrm{Y}_1 \\ \mathrm{Z}_1 \end{pmatrix}$ is the source coordinates, $\begin{pmatrix} \mathrm{T}_x \\ \mathrm{T}_y \\ \mathrm{T}_z \end{pmatrix}$ are the translations and $(\propto_1, \propto_{2,}, \propto_3)$ are the rotations which need to be applied. μ_1 is the scale corrections in parts per million given by $\mu_1 = (1 + \Delta\mathrm{s} \times 10^{-6})$ and ΔS is a scale correction.

The Helmert 7 parameters can be modified to account for the tectonic plate motions.(Snay 2003). This results to a 15 parameters transformation where the additional parameters are the rates of the original 7 parameter of 7 parameters of transformation that include 3 translations, 3 rotations and a uniform scale factor.

Molodensky-Badekas

Molodensky-Badekas model derives the rotation at a location within points used in the determination of the parameters, rather than the geocentric origin. (Sahl 2014). It introduces a centroid to cater for the high correlations between the parameters. The conventional centroid in the Molodensky-Badekas has conventionally been determined using Arithmetic mean. There have however been current developments where the centroid has been determined using other methods like Geometric mean, harmonic mean , quadratic mean and median (Ziggah and Yakubu 2017). Other models of determining centroids are Arithmetic Quadratic mean centroid and Harmonic Quadratic mean centroid (Annan et al. 2016b). Molodensky Badekas model has been referred to as 10 parameters Model in some literature and 7 parameter model in others (Andrei 2006; Sahl 2014; Annan et al. 2016b; Deakin 2017)

$$\begin{pmatrix} \boldsymbol{X_2} \\ \boldsymbol{Y_2} \\ \boldsymbol{Z_2} \end{pmatrix} = \begin{pmatrix} \boldsymbol{X_c} \\ \boldsymbol{Y_c} \\ \boldsymbol{Z_c} \end{pmatrix} + \begin{pmatrix} \boldsymbol{T_x} \\ \boldsymbol{T_y} \\ \boldsymbol{T_z} \end{pmatrix} + \boldsymbol{\eta R} \begin{bmatrix} \boldsymbol{X_1 - X_c} \\ \boldsymbol{Y_1 - Y_c} \\ \boldsymbol{Z_1 - Z_c} \end{bmatrix} \quad (13)$$

Where $\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}$ is the target coordinates, $\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$ is the source coordinates, $\begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$ are the translations, η is the scale factor, R is the rotation angle. $\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}$ is the centroid of the first datum.

This can be re written as general least square as

$$\begin{bmatrix} vx \\ vy \\ vz \end{bmatrix} = \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}_1 - \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} + \eta R \begin{bmatrix} X_1 - X_c \\ Y_1 - Y_c \\ Z_1 - Z_c \end{bmatrix} \tag{14}$$

Which can be linearized and represented in matrix form as

$$\begin{bmatrix} \mathrm{vx} \\ \mathrm{vy} \\ \mathrm{vz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \mathrm{a}_{14} & \mathrm{a}_{15} & \mathrm{a}_{16} & \mathrm{a}_{17} \\ 0 & 1 & 0 & \mathrm{a}_{24} & \mathrm{a}_{25} & \mathrm{a}_{26} & \mathrm{a}_{27} \\ 0 & 0 & 1 & \mathrm{a}_{34} & \mathrm{a}_{35} & \mathrm{a}_{36} & \mathrm{a}_{37} \end{bmatrix} \begin{bmatrix} \mathrm{T_X} \\ \mathrm{T_Y} \\ \mathrm{T_Z} \\ \mu_1 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} \mathrm{X_c} - \mathrm{X_2} \\ \mathrm{Y_c} - \mathrm{Y_2} \\ \mathrm{Z_c} - \mathrm{Z_2} \end{bmatrix} \tag{15}$$

Improved Molodensky 11-parameters model

The hybrid transformation models developed combines parts of Molodensky-Badekas centroid and the affine model. These include an improved model with 11 parameters. This involves 3 rotations, 3 translations, 2 scale factors and 3 centroids as the model parameters. The following equations were used to determine the transformation parameters.

$$\begin{bmatrix} dvx \\ dvy \\ dvz \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \boldsymbol{a_{14}} & \boldsymbol{a_{15}} & \boldsymbol{a_{16}} & \boldsymbol{a_{17}} & \boldsymbol{a_{18}} & \boldsymbol{a_{19}} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \boldsymbol{a_{24}} & \boldsymbol{a_{25}} & \boldsymbol{a_{26}} & \boldsymbol{a_{27}} & \boldsymbol{a_{28}} & \boldsymbol{a_{29}} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \boldsymbol{a_{34}} & \boldsymbol{a_{35}} & \boldsymbol{a_{36}} & \boldsymbol{a_{37}} & \boldsymbol{a_{38}} & \boldsymbol{a_{39}} \end{bmatrix} \begin{bmatrix} \boldsymbol{T_X} \\ \boldsymbol{T_Y} \\ \boldsymbol{T_Z} \\ \boldsymbol{\alpha_1} \\ \boldsymbol{\alpha_2} \\ \boldsymbol{\alpha_3} \\ \boldsymbol{\mu_1} \\ \boldsymbol{\mu_2} \end{bmatrix} + \begin{bmatrix} \mathbf{X_G} - \boldsymbol{X_2} \\ \boldsymbol{Y_G} - \boldsymbol{Y_2} \\ \boldsymbol{Z_G} - \boldsymbol{Z_2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{X_1} - \mathbf{X_c} \\ \boldsymbol{Y_1} - \mathbf{Y_c} \\ \boldsymbol{Z_1} - \mathbf{Z_c} \end{bmatrix} \tag{16}$$

Where $\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}$ is the target coordinates, $\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$ is the source coordinates, $\begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$ are the translations, $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ is the scale factor, $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$ is the rotation angle. $\begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}$ is the centroid of the first datum, $\begin{pmatrix} X_G \\ Y_G \\ Z_G \end{pmatrix}$ is the centroid of the second datum (target).

Methodology

Data Preprocessing

The data collected was in two systems that include the Clarke 1880 datum and WGS84 Datum. This data was modelled to eliminate distortion errors using grid distortion models. The data was then converted to the geographic coordinates with latitude and Longitude. It is then converted from 3D geodetic to the 3D Cartesian coordinates. The data is represented in the map of the area of study in order to evaluate the point's distribution.

Data Partitioning

To evaluate the model performances, transformation models are trained and tested using parts of the datasets. It is therefore necessary to carefully partition the dataset to decide which data is used to train and test the models effectively. The data used as training set is 75% and 25% for testing, which serves to provide enough data points for model training as well as significant test data points for performance evaluation.

Model comparative evaluation

This procedure was performed to compare the performance of the proposed method with the existing models. The models were compared based on their respective residuals generated using the various statistical methods such as Horizontal position residual, Maximum and Minimum errors, Standard Deviation of the parameters, Reference Standard Deviation of the computed points, Standard Deviation of position residual, and Root Mean Square error. These metrics were used to quantify the degree of dispersion of model predictions to observed data in a system. The formula applied are shown in the equations below.

The Root Mean Square Error is used to quantify the discrepancies between data observed and those projected by a model or estimate. The formula is provided by:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Oi - Pi)^2}, \tag{17}$$

where, O and P are the actual and predicted coordinates from the various procedures respectively.

The positional accuracy is assessed using the horizontal position error index. It is determined by contrasting the horizontal coordinates between the transformed coordinates produced by the suggested models and those produced by the actual coordinates. Its formulae is given by:

$$\text{HPE} = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2}, \tag{18}$$

Where E_1, N_1 show the predicted horizontal coordinates while E_2, N_2 the actual horizontal coordinates.

The Standard Deviation measures the amount of variation or dispersion of a set of values. Its formula is given by:

$$SD=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(e-\bar{e})^2}, \quad (19)$$

Where e represents the deviations between the actual and predicted coordinates; and $\bar{e}$ is the mean of the deviations.

Numerical Investigation

Model Evaluation

8-parameters Affine Model Parameters Evaluation

The table 2 below shows the 8 parameters for the 8-parameter model determined using the training data. Their standard deviations are also included.

Table 2 *8-parameter affine model parameters metrics*

Parameter	Value	SD
Tx	-6.40	0.07
Ty	-5.85	0.07
Tz	8.6E-02	0.10
Rx	-1.8E-05	1.2E-08
Ry	2.3E-05	1.3E-08
Rz	-1.5E-05	1.2E-08
Sx	-1.5E-05	1.1E-08
Sy	-1.5E-05	1.1E-08
Sz	-3.5E-05	1.4E-08

The translation parameters show shifts of approximately -6.40 m in the X-direction, -5.85 m in the Y-direction, and 0.086 m in the Z-direction, with relatively low standard deviations ranging from 0.07 m to 0.10 m, indicating good precision in the estimation of positional offsets. The standard error in the three translations is 0.07 in both X and Y and 0.10 Z translations. the rotation in Z has a higher standard error compared to the other rotations while the scale factor in Z has lower Standard deviation in comparison to Sx and Sy.

8-parameters Affine Model Performance Evaluation

The table 3 below shows error metrics obtained after comparing the values of the transformed coordinates using 8-parameter models and the actual coordinates.

Table 3 *8 parameter model error metrics*

ERROR METRICS	Error value (m)
REF.STD	0.09
STD VX	0.06
STD VY	0.05
STD VZ	0.05
MEAN(HE)	0.08
MAX(HE)	0.32
MIN(HE)	0.01
RMSE(HE)	0.09
STD (HE)	0.05

The error metrics for the 8-parameter affine model indicate that the 3D geodetic transformation achieved a high level of accuracy and consistency. The reference standard deviation of 0.09 m and RMSE of 0.09 m show that the transformed coordinates closely matched the reference coordinates with average discrepancies below 10 cm. The coordinate component standard deviations of 0.06 m in X and 0.05 m in both Y and Z further demonstrate stable residual distribution across all axes. The mean horizontal error of 0.08 m confirms good horizontal positional agreement, while the low standard deviation of horizontal error (0.05 m) indicates minimal dispersion and consistent transformation performance. Although the maximum horizontal error reached 0.32 m, suggesting the presence of a few localized deviations, the minimum error of 0.01 m shows that some points were transformed with high precision.

Affine 9-parameter model Parameters Evaluation

The parameter estimates for the 9-parameter affine model indicate that the transformation was able to capture significant positional differences between the coordinate systems while maintaining relatively stable rotational and scale components.

Table 4 *8-parameter affine model parameters metrics*

parameters	Value (m)	SD
Tx	-88.92	2.04
Ty	61.15	1.66
Tz	-1.00	0.09
Rx	-1.7E-05	1.3E-08
Ry	2.3E-05	1.6E-08
Rz	-6.5E-06	2.0E-07
Sx	-4.5E-06	2.5E-07
Sy	-2.1E-05	1.7E-07

Sz	-3.5E-05	1.2E-08

The translation parameters show large shifts of approximately (-88.92) m in the X-direction, (61.15) m in the Y-direction, and (-1.00) m in the Z-direction, suggesting substantial coordinate offsets between the two systems. However, the comparatively larger standard deviations of 2.04 m and 1.66 m for (Tx) and (Ty) indicate lower precision in the estimation of the horizontal translations, possibly due to spatial distortions, point distribution effects, or parameter correlations within the affine formulation. In contrast, the vertical translation parameter (Tz) exhibited a much smaller standard deviation of 0.09 m, indicating better stability in the vertical component. The low standard deviations associated with the scale parameters confirm stable parameter estimation despite the larger uncertainty observed in the translation components.

Affine 9-parameter model performance evaluation

The performance evaluation of the 9-parameter affine model indicates that the transformation achieved a generally high level of accuracy and consistency in transforming the common points between the coordinate systems.

Table 5 *9-parameter model error metrics*

ERROR METRICS	Error value (m)
REF.STD	0.09
STD VX	0.05
STD VY	0.05
STD VZ	0.05
MEAN(HE)	0.07
MAX(HE)	0.35
MIN(HE)	0.01

RMSE(HE)	0.09
STD (HE)	0.05

The reference standard deviation of 0.09 m shows that the overall transformation precision was maintained within approximately 9 cm, demonstrating a reliable fit of the model to the observed data. The residual standard deviations in the X, Y, and Z components were all approximately 0.05 m, indicating a balanced and stable distribution of errors across the three coordinate axes and suggesting that the model effectively minimized residual inconsistencies in both horizontal and vertical directions.

The horizontal error statistics further confirm the good performance of the model. The mean horizontal error of 0.07 m indicates that, on average, the transformed coordinates differed from the reference coordinates by only 7 cm in the horizontal plane, reflecting strong positional agreement. Similarly, the RMSE of 0.09 m demonstrates that the combined effect of random and systematic errors remained low and within acceptable engineering and geodetic tolerances. The standard deviation of the horizontal errors, was 0.05 m, indicating low variability among the transformed points and confirming the consistency of the model performance across the dataset.

Although the minimum horizontal error was as low as 0.01 m, indicating excellent agreement for some points, the maximum horizontal error reached 0.35 m, suggesting that a few points experienced comparatively larger residual discrepancies. These larger residuals may be attributed to localized distortions, uneven point distribution, or limitations of the affine model in fully accounting for all spatial deformations. Nevertheless, since the maximum error is significantly higher than the mean and RMSE values, it is likely that only a small number of outlier points contributed to this deviation.

10-parameter Molodensky Badekas model Evaluation

The parameter estimates for the 10-parameter Molodensky Badekas model indicate a highly stable and well-conditioned transformation with strong precision across all estimated parameters.

Table 6 *10-parameter molodensky badekas model parameters metrics*

Parameters	Values	SD
Tx	0.00	0.04
Ty	0.00	0.04

Tz	0.00	0.04
Rx	-2.1E-05	2.2E-07
Ry	2.0E-05	2.4E-07
Rz	-1.4E-05	2.6E-07
Sx	-2.4E-05	1.7E-07
Sy	-2.4E-05	1.7E-07
Sz	-2.4E-05	1.7E-07

The translation parameters (Tx, Ty, Tz) are effectively zero, each with a low standard deviation of 0.04 m, suggesting that the model successfully minimized translational biases through centroid-based transformation and achieved a strong alignment between the coordinate systems. The rotation parameters are very small, on the order of 10^5, indicating only negligible rotational differences between the systems, while their low standard deviations on the order of 10^5 demonstrate high reliability and estimation precision. Similarly, the scale parameters are uniformly small negative values, showing minimal and consistent scale distortion across all axes, with very low standard deviations confirming stable parameter estimation.

10-parameter Molodensky Badedas performance evaluation

Table 7 10-parameter molodensky badekas model error metrics

ERROR METRICS	**Error value (m)**
REF.STD	0.86
STD VX	0.45
STD VY	0.53

STD VZ	0.51
MEAN(HE)	0.71
MAX(HE)	3.79
MIN(HE)	0.08
RMSE(HE)	0.86
STD (HE)	0.49

As shown in table 7, the performance evaluation of the 10-parameter Molodensky Badekas model indicates moderate transformation accuracy with relatively larger residual errors compared to the other models. The reference standard deviation of 0.86 m shows that the overall transformation precision was below the sub-decimeter level achieved by other models. The residual standard deviations in the X, Y, and Z directions, ranging from 0.45 m to 0.53 m, indicate moderate dispersion of coordinate residuals and suggest that the transformation errors were relatively consistent across all coordinate components, although still considerably larger than desirable for high-precision geodetic applications.

The horizontal error (0.71) m further reflect this moderate performance. Similarly, the of 0.86 m confirms the presence of substantial combined systematic and random errors within the transformation process. The standard deviation of horizontal errors, of 0.49 m demonstrates considerable variability among the transformed points, indicating inconsistent transformation accuracy across the dataset. In addition, the maximum horizontal error of 3.79 m reveals the existence of some points with significantly large residual discrepancies, which may be attributed to localized distortions, outlier observations, or limitations of the model in fully representing the spatial relationship between the coordinate systems.

11-parameter Molodensky Badekas Parameters Evaluation

This model involves 3 rotations, 3 translations, 2 scale factors and 3 centroids as the model parameters.

Table 8 *11-parameter molodensky badekas model parameters metrics*

Parameters	**Values**	**SD**

Tx	0.00	0.00
Ty	0.00	0.00
Tz	0.00	0.00
Rx	-1.8E-05	1.6E-08
Ry	2.3E-05	1.7E-08
Rz	-1.5E-05	2.4E-08
Sx	-1.5E-05	2.1E-08
Sy	-1.5E-05	2.1E-08
Sz	-3.4E-05	1.3E-08

The parameter estimates for the 11-parameter Molodensky Badekas model indicate an exceptionally stable and highly optimized transformation solution, supporting the hypothesis that this model performs better than the alternative transformation models. The translation parameters were effectively zero with zero standard deviations, suggesting that the transformation successfully minimized global translational biases through the use of the centroid-based Molodensky–Badekas formulation. This demonstrates that the model was able to localize the transformation around the centroid of the common points, thereby reducing systematic distortions. The rotation parameters remained extremely small, on the order of 10^5, indicating only negligible rotational discrepancies between the coordinate systems, while their very small standard deviations on the order of 10^8 reflect very high estimation precision and parameter stability. Similarly, the scale parameters were also extremely small negative values with correspondingly low standard deviations, confirming minimal scale distortion and a highly consistent geometric fit. Compared to conventional transformation models, the near-zero translations and extremely precise parameter estimates suggest that the Molodensky–Badekas model better absorbed local distortions and provided a more rigorous alignment of the common points.

Table 9 11-parameter molodensky badekas model error metrics

ERROR METRICS	Error value (m)
REF.STD	0.06
STD VX	0.05
STD VY	0.03
STD VZ	0.03
MEAN(HE)	0.05
MAX(HE)	0.31
MIN(HE)	0.00
RMSE(HE)	0.06
STD (HE)	0.04

The performance evaluation of this transformation model indicates a very high level of accuracy and consistency, with all error metrics remaining at sub-decimetre levels. The reference standard deviation of 0.06 m reflects a strong fit between the transformed coordinates and the reference dataset. The standard deviations of the residuals in the X, Y, and Z components (0.05 m, 0.03 m, and 0.03 m respectively) indicate a well-balanced error distribution across all coordinate axes, with slightly better stability in the Y and Z directions. This suggests that the model effectively minimized directional bias and produced uniform accuracy in all spatial dimensions.

The horizontal error statistics further confirm the strong performance of the model. The mean horizontal error of 0.05 m and the RMSE of 0.06 m reinforces the conclusion that both systematic and random errors were well controlled, resulting in highly accurate coordinate transformation precision. The horizontal error standard deviation of 0.04 m also shows low dispersion of residuals, meaning that most transformed points closely cluster around the reference positions.

In addition, the minimum horizontal error of 0.00 m demonstrates that some points were transformed with near-perfect alignment, while the maximum horizontal error of 0.31 m indicates the presence of a few localized deviations, though still within acceptable limits for many engineering applications. Overall, the model exhibits excellent performance characterized by high precision, strong internal consistency, and low residual variability, making it a highly reliable transformation solution for the dataset under consideration.

Comparative analysis

The comparative evaluation of the four transformation models shows a clear hierarchy in performance, with the 11-parameter Molodensky–Badekas model emerging as the best-performing solution, followed by the 9-parameter affine model, while the 10-parameter Molodensky–Badekas variants show comparatively weaker performance.

The 11-parameter Molodensky–Badekas model demonstrates the highest accuracy and most stable error distribution, with a reference standard deviation of 0.06 m and an RMSE of 0.06 m, indicating very high transformation precision. The mean horizontal error of 0.05 m and a low horizontal error standard deviation of 0.04 m further confirm excellent consistency across the dataset. The relatively small maximum error of 0.31 m suggests that even the worst-performing points remain within acceptable limits. A key distinguishing feature of this model is its enhanced parameter structure, particularly the inclusion of two distinct scale representations, as reflected in the SX, SY, SZ compared to the other models. This allows the model to better absorb subtle anisotropic distortions in the dataset, improving its ability to represent real-world deformation more realistically and leading to superior overall performance.

The 9-parameter affine model performs well but slightly below the 11-parameter model, with a reference standard deviation and RMSE of 0.09 m and a mean horizontal error of 0.07 m. While still accurate and consistent, its higher maximum error of 0.35 m and slightly larger dispersion indicate that it is less capable of capturing localized distortions compared to the more flexible Molodensky–Badekas formulation.

In contrast, the 10-parameter Molodensky–Badekas models perform significantly worse, with RMSE values of 0.86 m and 1.78 m respectively, and mean horizontal errors of 0.71 m and 1.57 m. The presence of large maximum errors (3.79 m and 5.37 m) indicates poor fit and instability in the transformation, likely due to parameter sensitivity, data distribution effects, or over-parameterization without sufficient constraint. Although theoretically more sophisticated, these variants did not perform well for this dataset.

The 8-parameter affine model results are competitive and place this model as the third-ranked solution in the comparative evaluation, marginally below the 9-parameter affine model whose mean horizontal error was 0.07 m. The close agreement between the 8- and 9-parameter affine results is noteworthy: adding the third independent scale factor (Sz) in the 9-parameter formulation yielded only a marginal improvement in mean horizontal error (0.01 m), but introduced considerably larger standard deviations in the horizontal translation parameters (2.04 m and 1.66 m for Tx and Ty respectively).

Limitations of the study and further research directions

Limitations

Despite the strong performance of the improved 11-parameter Molodensky–Badekas transformation model, several limitations must be acknowledged. First, the spatial extent is limited to 13 counties within the central and southern regions of Kenya, falling within UTM Zone 37S. The derived transformation parameters may therefore not be directly

applicable to other geodetic zones across Kenya, particularly the northern arid and semi-arid counties where geodetic control network density is considerably lower and terrain conditions are markedly different.

Second, the study relies exclusively on existing control point datasets sourced from the Survey of Kenya, comprising 1,250 common points in both Cassini and UTM coordinate systems. While this dataset is substantial, it represents historical observations that may carry inherent positional errors accumulated over decades of separate and sometimes inconsistent survey campaigns.

Third, the vertical component of the transformation is constrained by the existing orthometric height framework referenced to the Mombasa Chart Datum. The absence of a precise national geoid model limits the accuracy of height transformation, introducing uncertainties in the ellipsoidal-to-orthometric height conversion that cannot be resolved by the coordinate transformation model alone. Finally, the study does not account for temporal variations in the geodetic reference frame arising from tectonic plate motion, crustal deformation, or co-seismic displacement events along the East African Rift System, which could introduce time-dependent errors in long-term applications of the derived parameters.

Future works

Future research should prioritize the extension of the improved 11-parameter Molodensky–Badekas model to cover all 47 counties of Kenya, particularly the northern regions that were excluded from the current study due to limited control point availability. Establishing a comprehensive national transformation framework requires a denser and more uniformly distributed geodetic control network, which could be achieved through collaboration with the Survey of Kenya and leveraging the expanding KENREF Continuously Operating Reference Station (CORS) infrastructure.

Future studies should integrate the improved transformation model with gravimetric geoid solutions to enable consistent ellipsoidal-to-orthometric height conversion across all geodetic zones. Integration with machine learning and artificial neural network (ANN) approaches presents another promising avenue for future work. As demonstrated by recent studies such as Kinyua et al. (2025) on ANN-optimized 2D affine models, hybrid computational methods could be adapted to further refine the 11-parameter model's performance, particularly in areas with complex spatial distortion patterns that purely parametric models cannot fully capture.

Additionally, future work should investigate the application of time-dependent transformation models incorporating velocity field parameters to account for crustal motion along the East African Rift System.

Discussion

The results of this study demonstrate that the choice of transformation model has a substantial influence on the accuracy of geodetic coordinate transformation within the Kenyan geodetic system. The improved 11-parameter Molodensky–Badekas model achieved the highest overall accuracy among the four models evaluated, with an RMSE of 0.06 m and a mean horizontal error of 0.05 m. This outcome is consistent with findings reported by Kalu et al. (2022) for Nigeria and Annan et al. (2016) for Ghana, where variants of the Molodensky–Badekas model consistently outperformed standard Helmert-type similarity transformations, reinforcing the broader geodetic principle that centroid-based formulations mitigate parameter correlation and improve transformation stability in localized networks.

The comparatively weak performance of the 10-parameter Molodensky–Badekas variant, with an RMSE of 0.86 m, underscores an important modelling insight: the conventional arithmetic-mean centroid formulation, while theoretically sound, may not optimally represent the geometric center of a network exhibiting non-uniform spatial distortions. The introduction of dual-scale factors in the 11-parameter model effectively decouples horizontal and vertical scale distortions, allowing a more faithful geometric fit to the empirical data. This finding is aligned with the theoretical rationale advanced by Ruffhead (2023), who demonstrated that flexible scale parameterization is critical in networks affected by anisotropic distortions, even at the cost of marginal increases in model complexity.

The 9-parameter affine model performed comparably to the 11-parameter model in terms of overall error statistics, with an RMSE of 0.09 m, but exhibited a slightly larger maximum horizontal error and greater parameter uncertainty in the horizontal translation components. This suggests that affine transformations, while capable of accommodating anisotropic scaling, are more sensitive to spatial data distribution and can produce less stable parameters when horizontal translations correlate with scale factors in localized networks. The results therefore highlight that model performance is not solely a function of parameter count, but is strongly conditioned by the geometric design of the transformation model and the spatial distribution of common control points.

From a practical geodetic management perspective, the sub-decimeter accuracy of the 11-parameter model positions it as a viable candidate for integration into Kenya's national coordinate transformation workflow, particularly for map revision, cadastral surveys, and engineering projects that require compatibility between the legacy Arc 1960 / Cassini datum and the modern WGS84-based KENREF system. The study further affirms that the quality and density of common control points remain the most critical determinant of transformation reliability, consistent with the conclusions of Lwangasi (1993) and Matara et al. (2022), and underscores the importance of continued investment in Kenya's geodetic control network.

Conclusion

This study successfully developed and evaluated an improved 11-parameter Molodensky–Badekas transformation model for the Kenyan geodetic system, addressing the long-standing challenge of accurately converting coordinates between the local Arc 1960 / Cassini datum and the modern WGS84-based reference framework within the central and southern regions of Kenya. By extending the conventional Molodensky–Badekas formulation to a multi-scale parameter framework that incorporates dual-scale factors alongside three translations, three rotations, and dual centroids, the improved model demonstrated a significantly enhanced ability to absorb the anisotropic and localized distortions inherent in Kenya's terrestrial geodetic network.

The comparative evaluation against the 8-parameter affine, 9-parameter affine, and conventional 10-parameter Molodensky–Badekas models confirmed that the improved 11-parameter model achieved the highest transformation accuracy, with a reference standard deviation and RMSE of 0.06 m and a mean horizontal positional error of 0.05 m.

The improved 11-parameter Molodensky–Badekas model is recommended as a robust and mathematically transparent alternative to conventional similarity transformations for high-accuracy map revision, datum transformation, and geospatial data integration within the study region. Its adoption into Kenya's national geodetic transformation workflow would significantly improve the consistency and accuracy of spatial data products derived from both legacy and modern GNSS-based surveys.

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Declarations I solemnly declare, to the best of my knowledge; no part of this report has been submitted in any institution for a degree or for consideration of any certification. All sources of knowledge used have been acknowledged.

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References

Abbey DA, Featherstone WE (2020) Comparative Review of Molodensky–Badekas and Burša–Wolf Methods for Coordinate Transformation. J Surv Eng 146:04020010. https://doi.org/10.1061/(ASCE)SU.1943-5428.0000319

Andrei C (2006) 3D affine coordinate transformations. Geometria. https://doi.org/10.1016/j.vaccine.2014.02.089

Annan RF, Ziggah YY, Ayer J, Odutola CA (2016a) A Hybridized Centroid Technique for 3D Molodensky-Badekas Coordinate Transformation in the Ghana Geodetic Reference Network using Total Least Squares Approach. South African Journal of Geomatics 5:269–284

Annan RF, Ziggah YY, Ayer J, Odutola CA (2016b) A Hybridized Centroid Technique for 3D Molodensky-Badekas Coordinate Transformation in the Ghana Geodetic Reference Network using Total Least Squares Approach. South African Journal of Geomatics 5:269–284. https://doi.org/http://dx.doi.org/10.4314/sajg.v5i3.1

Banković T, Dubrović M, Banko A, Pavasović M (2024) Detail Explanation of Coordinate Transformation Procedures Used in Geodesy and Geoinformatics on the Example of the Territory of the Republic of Croatia. Applied Sciences 14:1067. https://doi.org/10.3390/app14031067

Boon PS, Setan H (2007) 3D coordinate transformation using molodensky badekas transformation model: MBT07. In: Joint International Symposium and Exhibition on Geoinformation and International Symposium and Exhibition on Gps/Gnss, Persada Johor International Convention Centre, Johor Bahru, Johor

Danube NW, Kehbila A, Sitati C, Osano P (2026) State of the Climate in Kenya 2025

Deakin R (2017) A NOTE ON THE BURSA-WOLF AND MOLODENSKY-BADEKAS TRANSFORMATIONS BURSA-WOLF AND MOLODENSKY-BADEKAS

Deakin RE (2004) The standard and abridged molodensky coordinate transformation formulae. Department of Mathematical and Geospatial Sciences, RMIT University 1–21

Deakin RE (2006) A note on the Bursa-Wolf and Molodensky-Badekas transformations. School of Mathematical and Geospatial Sciences, RMIT University 1:21

Elshambaky HT, Kaloop MR, Hu JW (2018) A novel three-direction datum transformation of geodetic coordinates for Egypt using artificial neural network approach. Arab J Geosci 11:110. https://doi.org/10.1007/s12517-018-3441-6

Haddad M, Gahlouz M (2025) A comparison of transformation models between geodetic reference frames: case study in Illizi region (Algeria). Geodesy and Cartography 51:234–242. https://doi.org/10.3846/gac.2025.21550

Hassan A, Mustafa EK, Mahama Y, et al (2020) Analytical Study of 3D Transformation Parameters Between WGS84 and Adindan Datum Systems in Sudan. Arab J Sci Eng 45:351–365. https://doi.org/10.1007/s13369-019-04206-w

Kalu I, Ndehedehe CE, Okwuashi O, Eyoh AE (2022) A comparison of existing transformation models to improve coordinate conversion between geodetic reference frames in Nigeria. Model Earth Syst Environ 8:611–624. https://doi.org/10.1007/s40808-021-01090-y

Kinyua MN, Sichangi AW, Gachari MK (2025) Development of ANN optimized affine-6 2D coordinate transformation model. Appl Geomat 17:63–81. https://doi.org/10.1007/s12518-024-00600-8

Kumi-Boateng B, Yevenyo Ziggah Y (2020) A 3D Procrustean Approach to Transform WGS84 Coordinates to Ghana War Office 1926 Reference Datum. Ghana Mining Journal 20:1–10. https://doi.org/10.4314/gm.v20i1.1

Lv Z, Zhang M (2026) A novel total least absolute deviation solution for the 3D symmetric datum transformation. Survey Review 58:146–155. https://doi.org/10.1080/00396265.2025.2488665

Lwangasi AS (1993) DATUM TRANSFORMATION PARAMETERS FOR THE KENYA GEODETIC SYSTEM. Survey Review 32:39–46. https://doi.org/10.1179/sre.1993.32.247.39

Matara SM, Siriba DN, Kiema JBK, Musyoka SM (2022) A Review of the History and Development of the Kenyan Geodetic Reference Frame

Mina E (2006) A Unified System of Transformation Parameters for Combining Different Geodetic Networks in Egypt

Murgnier C (2013) Geodetic Report of Kenya: Historical Analysis & Current Insights. In: Studocu. https://www.studocu.com/row/document/jomo-kenyatta-university-of-agriculture-and-technology/bsccivil-engineering/geodetic-reportof-kenya/40423709. Accessed 19 Apr 2026

Njoroge EK (2025) An Analysis of Climate Change and Its Effect on Economic Growth in Kenya. American Journal of Economics 9:88–104. https://doi.org/10.47672/aje.2743

Odera PA, Musyoka SM, Gachari MK (2014) PRACTICAL APPLICATION OF THE GEOMETRIC GEOID FOR HEIGHTING OVER NAIROBI COUNTY AND ITS ENVIRONS. JOURNAL OF AGRICULTURE, SCIENCE AND TECHNOLOGY 16:175–185

Ono MN (2009) On Problems of Coordinates, Coordinate Systems and Transformation Parameters in Local Map Production, Updates and Revisions in Nigeria

Ruffhead AC (2023) Derivation of 9-parameter affine 3D geodetic datum transformations. Survey Review 55:441–450. https://doi.org/10.1080/00396265.2022.2137094

Ruffhead AC (2019) Derivation of rigorously-conformal 7-parameter 3D geodetic datum transformations. Survey Review 53:8. https://doi.org/10.1080/00396265.2019.1665614

Sahl TA (2014) Advanced Positioning for Offshore Norway

Siriba D (2021) Renewing Kenya's Vertical Datum as an Integral Part of KENREF through the Definition of a Local Geoid Model. 2020:36–38

Snay R (2003) Horizontal Time-Dependent Positioning. 1984:

Thiong'o PN (2019) Continuously Operating Reference Stations (CORS): Transformation between KPLC CORS System and the Kenya National Geodetic System

Tran DT, Nocquet J-M, Luong ND, Nguyen DH (2023) Determination of Helmert transformation parameters for continuous GNSS networks: a case study of the Géoazur GNSS network. Geo-spatial Information Science 26:125–138. https://doi.org/10.1080/10095020.2022.2138569

Turgut B (2010) A back-propagation artificial neural network approach for three-dimensional coordinate transformation. Scientific Research and Essays 5:3330–3335

Watson GA (2006) Computing Helmert transformations. Journal of Computational and Applied Mathematics 197:387–394. https://doi.org/10.1016/j.cam.2005.06.047

World Bank (2025) Kenya Economic Update, May 2025. https://openknowledge.worldbank.org/entities/publication/e3b6f83e-96b1-4ade-9c3a-8ca1ef63474d. Accessed 26 May 2026

Yilmaz N (2023) 2d Helmert and Affine Coordinate Transformation Using the Least Squares Method: Trabzon Province in Turkey

Ziggah YY, Yakubu I (2017) Alternative Methods of Determining Translation Parameters for Geocentric Translation Model Applications *. Ghana Journal of Technology Vol. 2:38–50

Ziggah YY, Youjian H, Amans C, Fan D (2013) Determination of GPS Coordinate Transformation Parameters of Geodetic Data between Reference Datums-A Case. International Journal of Engineering Sciences and Research Technology Vol. 2:956

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