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A Classification-Regression Cooperative Fuzzy Surrogate-Assisted Evolutionary Algorithm for Expensive High-Dimensional Multi-Objective Optimization

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Abstract

Expensive high-dimensional multi-objective optimization problems pose a model-state mismatch challenge to surrogate-assisted evolutionary algorithms. To address this issue, this paper proposes HDFC-ASS, a classification-regression cooperative fuzzy surrogate-assisted evolutionary algorithm. The proposed method first designs a variable-correlation-guided random-subspace incremental Kriging strategy to acquire informative samples in high-dimensional decision spaces. Then, a fuzzy classifier-assisted local exploitation mechanism with a balanced-accuracy-based one-way transition is developed to prevent premature use of unreliable classification boundaries. Finally, a convergence-diversity-uncertainty cooperative model-management criterion is proposed to allocate expensive evaluations among global exploration, boundary correction and high-membership exploitation. The proposed framework is designed for expensive high-dimensional multi-objective optimization under a strictly limited evaluation budget.

Keywords: expensive multi-objective optimization; high-dimensional optimization; surrogate-assisted evolutionary algorithm; random subspace; incremental Kriging; fuzzy classification

1. Introduction

Multi-objective optimization problems (MOPs) widely arise in engineering design, control parameter tuning, energy management and industrial process optimization, where several conflicting objectives must be optimized simultaneously. However, in many real-world applications, each objective evaluation may require a time-consuming numerical simulation, a high-fidelity engineering analysis or a physical experiment. Under such expensive evaluation conditions, conventional EMOAs are often impractical because their search process usually requires thousands or even tens of thousands of objective function^[1-2] evaluations. Surrogate-assisted evolutionary algorithms (SAEAs) have been developed to reduce the number of expensive evaluations by replacing part of the original objective evaluations with inexpensive approximation models^[3]. Among different surrogate models, Kriging and Gaussian-process models are particularly attractive because they can provide both predicted objective values and uncertainty estimates^[4]. This feature makes them suitable for infill sampling and efficient global optimization^[5-9]. Classical Bayesian optimization and Kriging-assisted methods, such as EGO, ParEGO, MOEA/D-EGO^[7], K-RVEA^[8] and KTA2^[9], have demonstrated the effectiveness of regression-based surrogate modeling in expensive single-objective, multi-objective and many-objective optimization. Meanwhile, classification-assisted SAEAs, such as CSEA^[10], CPS-MOEA^[11], and MCEA/D^[12], avoid directly approximating exact objective values and instead learn

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whether candidate solutions are promising, which can reduce the modeling burden in complex objective spaces.

Recently, increasing attention has been paid to the scalability of SAEAs in high-dimensional decision spaces. Since the number of evaluated samples is usually very small compared with the number of decision variables, directly constructing full-dimensional surrogate models may lead to unreliable prediction and high computational cost. To alleviate this difficulty, variable-selection, variable-correlation analysis, random projection and random-subspace modeling have been introduced into expensive optimization. For example, some medium-scale SAEAs estimate the correlation between decision variables and objectives to reduce the effective modeling dimension^[13], while random-projection-based hierarchical SAEAs train local surrogate models in low-dimensional projected spaces to improve scalability. In addition, neural-network-based and ensemble surrogate strategies, such as dropout neural networks and heterogeneous surrogate ensembles, have been explored to improve modeling efficiency in high-dimensional expensive optimization^[14-16]. More recent learning-based SAEAs further attempt to use transfer learning, meta-learning, reinforcement learning or large language models to select surrogate models and infill criteria adaptively^[17-18].

Although these studies have advanced expensive multi-objective optimization, expensive high-dimensional MOPs with strictly limited evaluations still face a model-state mismatch challenge. Here, model-state mismatch denotes the inconsistency between surrogate reliability and the search decisions expected from the surrogate. Specifically, sparse high-dimensional archives make full-dimensional Kriging unstable, requiring regression modeling in informative subspaces. Early-stage dominance-based labels are often imbalanced and unstable, making classification boundaries unreliable and premature exploitation risky. Meanwhile, expensive evaluations must be adaptively allocated among global exploration, boundary correction and high-confidence exploitation. However, existing SAEAs often treat these tasks separately, and a unified model-management mechanism is still needed to coordinate regression modeling, fuzzy classification and infill sampling.

Motivated by the above observations, this paper proposes HD-FC-ASS, a classification-regression cooperative fuzzy surrogate-assisted evolutionary algorithm for expensive high-dimensional multi-objective optimization. Instead of permanently fusing regression and classification scores, HD-FC-ASS activates different surrogate roles according to their trustworthy states. The main contributions of this paper are summarized as follows.

(1) A variable-correlation-guided random-subspace incremental Kriging strategy is proposed for high-dimensional expensive multi-objective optimization. It estimates variable relevance from the evaluated archive, constructs informative low-dimensional subspaces and incrementally updates Kriging models, thereby reducing the instability and computational cost of full-dimensional Kriging under small-sample conditions.

(2) A fuzzy classifier-assisted local exploitation mechanism with a reliability-based one-way transition is developed. Unlike hard classification strategies, the proposed fuzzy classifier outputs positive-class membership values for candidate solutions. The transition from regression-assisted global exploration to fuzzy-classification-assisted local exploitation is triggered only when balanced

accuracy and class-support conditions are satisfied, which prevents unreliable early classification boundaries from misleading the search.

(3) A convergence-diversity-uncertainty cooperative model-management criterion is designed for expensive evaluation allocation. According to the current model state, the proposed criterion adaptively selects candidates for global exploration, fuzzy-boundary correction or high-membership exploitation, improving the coordination between surrogate modeling and expensive infill sampling.

2. Background

2.1 Overall Framework

The proposed HDFC-ASS consists of two sequential surrogate stages connected by a shared evaluated archive (A). The archive stores all solutions evaluated by the true expensive functions and provides data for regression modeling, fuzzy classification, diversity estimation, and final solution selection. In the first stage, a variable-correlation-guided random-subspace incremental Kriging strategy is used for global exploration. Instead of constructing a full-dimensional Kriging model in the original (D)-dimensional decision space, the algorithm estimates the relevance of each variable to the objective space and builds several low-dimensional subspaces. Kriging models are then trained and incrementally updated in these subspaces. In the second stage, fuzzy classifier-assisted local exploitation is activated only when the classifier satisfies the reliability condition. The fuzzy classifier outputs the membership degree of a candidate solution belonging to the promising class, which enables the algorithm to distinguish high-quality candidates, uncertain boundary candidates, and unpromising candidates. The two surrogate stages are not fused simultaneously. In the early stage, regression-assisted global exploration dominates the search. When the classifier reaches a reliable state, the algorithm switches to fuzzy classification-assisted local exploitation. During the whole process, a convergence-diversity-uncertainty model management strategy is used to select candidates for true expensive evaluation.

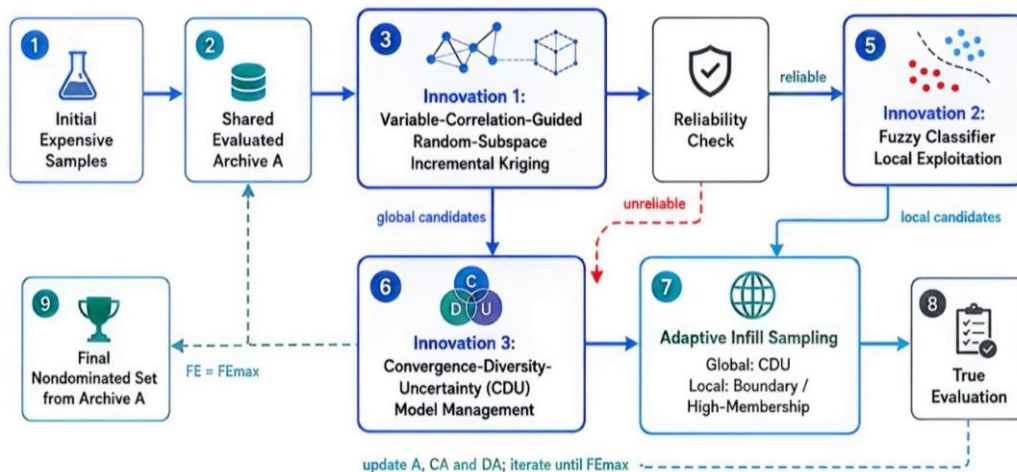


Fig. 1. Textual framework of the proposed HDFC-ASS.

2.2 Variable-Correlation-Guided Random-Subspace Incremental Kriging

The first innovation is designed for high-dimensional decision spaces. Instead of fitting a full- The first innovation aims to improve Kriging modeling in high-dimensional decision spaces. Given the evaluated archive

$$A = \{(x_i, F(x_i))\}_{i=1}^n \quad (1)$$

The relevance score of the k -th decision variable is calculated by combining Pearson correlation and Spearman rank correlation:

$$r_k = \frac{1}{M} \sum_{j=1}^M [\omega_p |\text{Pearson}(x_k, f_j)| + \omega_s |\text{Spearman}(x_k, f_j)|], \quad k = 1, \dots, D. \quad (2)$$

where x_k denotes the vector of the k -th decision variable values in the archive, f_j denotes the vector of the j -th objective values, and ω_p and ω_s are nonnegative weights. Variables with larger relevance scores form the core variable set S_c . For the b -th subspace, S_c is combined with randomly sampled complementary variables:

$$S_b = S_c \cup S_{r,b}, \quad |S_b| = d_s \leq D, \quad b = 1, 2, \dots, B. \quad (3)$$

where B is the number of subspaces and d_s is the subspace size. The core variables preserve important search directions, while the random complementary variables avoid search bias caused by a fixed reduced space. For each objective and each subspace, an individual Kriging model is trained. Let $m_{j,b}(x_{S_b})$ and $\sigma_{j,b}^2(x_{S_b})$ denote the predicted mean and variance of the b -th subspace model for the j -th objective. The ensemble prediction is

$$m_j(x) = \frac{1}{B} \sum_{b=1}^B m_{j,b}(x_{S_b}). \quad (4)$$

The uncertainty is defined as

$$U_j(x) = \frac{1}{B} \sum_{b=1}^B \sigma_{j,b}^2(x_{S_b}) + \frac{1}{B} \sum_{b=1}^B [m_{j,b}(x_{S_b}) - m_j(x)]^2. \quad (5)$$

The first term represents the prediction variance within each subspace Kriging model, while the second term measures the disagreement among different subspace models. When new expensive samples are added, the Kriging model is updated incrementally. Given the previous correlation matrix R , the correlation block C between old and new samples, and the correlation matrix D among new samples, the augmented correlation matrix is

$$R_{\text{new}} = \begin{bmatrix} R & C \\ C^T & D \end{bmatrix}. \quad (6)$$

Let

$$G = D - C^T R^{-1} C. \quad (7)$$

Then the inverse matrix can be updated as

$$R_{\text{new}}^{-1} = \begin{bmatrix} R^{-1} + R^{-1}CG^{-1}C^TR^{-1} & -R^{-1}CG^{-1} \\ -G^{-1}C^TR^{-1} & G^{-1} \end{bmatrix}. \quad (8)$$

This update avoids reconstructing the inverse correlation matrix from scratch and improves the efficiency of Kriging modeling under small-batch expensive evaluations.

2.3 Fuzzy Classifier-Assisted Local Exploitation and Reliability Transition

The second innovation introduces a fuzzy classifier-assisted local exploitation mechanism. Training labels are generated from the evaluated archive by nondominated ranking and density estimation. Solutions with good convergence and sparse distribution are labeled as positive samples, while dominated or crowded solutions are labeled as negative samples.

Let $y_i \in \{+1, -1\}$ be the label of x_i . For a candidate solution x , the MLPM-FKNN classifier estimates its positive-class membership as

$$\mu_+(x) = \frac{\sum_{i \in N_k(x)} w_i I(y_i = +1)}{\sum_{i \in N_k(x)} w_i}. \quad (9)$$

where $N_k(x)$ is the set of k -nearest labeled neighbors, $I(\cdot)$ is the indicator function, and the distance-based neighbor weight is

$$w_i = \frac{1}{\|x - x_i\|_2 + \varepsilon}. \quad (10)$$

Since positive and negative samples may be imbalanced in early iterations, balanced accuracy is used to measure classifier reliability:

$$\rho = \frac{1}{2} \left[\frac{TP}{TP + FN} + \frac{TN}{TN + FP} \right]. \quad (11)$$

where TP , TN , FP and FN are obtained by stratified cross-validation. The transition from the global Kriging stage to the local fuzzy-classification stage is triggered only when

$$\rho_{t-h} \geq \tau, \quad h = 0, 1, \dots, H-1, \quad \text{and} \quad \min(n_+, n_-) \geq n_{\min}. \quad (12)$$

where τ is the reliability threshold, H is the number of consecutive successful checks, and n_+ and n_- are the numbers of positive and negative samples. This one-way transition prevents unreliable early classifiers from misleading the search.

2.4 Convergence-Diversity-Uncertainty Model Management and Infill Sampling

The third innovation focuses on expensive evaluation allocation. A candidate should be selected not only because it is predicted to be good, but also because it may improve diversity, reduce surrogate uncertainty, or correct the fuzzy decision boundary. In the global stage, the convergence indicator is defined as

$$Cnv(x) = \|m(x) - z^*\|_2. \quad (13)$$

where $m(x) = (m_1(x), \dots, m_M(x))$ is the predicted objective vector and z^* is the estimated ideal point. The diversity indicator is

$$Div(x) = \min_{x_i \in A \cup C \cup DA} \|x - x_i\|_2. \quad (14)$$

The uncertainty indicator is

$$U(x) = \frac{1}{M} \sum_{j=1}^M U_j(x). \quad (15)$$

For a candidate x in the predicted elite set E , the global infill score is

$$Score_G(x) = -Cnv(x) + \lambda_D Div(x) + \lambda_U U(x) - \lambda_V CV(x). \quad (16)$$

where $CV(x)$ is the predicted constraint violation, and λ_D , λ_U and λ_V are nonnegative parameters. In the local stage, if the classifier is not sufficiently reliable, boundary sampling is adopted:

$$x_B = \arg \min_{x \in C} |\mu_+(x) - 0.5|. \quad (17)$$

If the classifier is reliable, high-membership positive candidates are preferred:

$$x_E = \arg \max_{x \in C_+} [\mu_+(x) - \kappa Cnv(x) + \lambda_D Div(x)]. \quad (18)$$

where C_+ is the set of candidates predicted as positive. Therefore, the proposed model management strategy adaptively allocates expensive evaluations among global exploration, boundary correction, and local exploitation.

Algorithm 1. Convergence-diversity-uncertainty model management and infill sampling

Input: Candidate set C , evaluated archive A , stage flag, Kriging predictions m and U , fuzzy membership μ_+ , classifier reliability ρ , batch size q .

Output: Infill solution set Q .

1. Remove duplicate candidates and candidates too close to the evaluated archive A .
2. Normalize $Cnv(x)$, $Div(x)$, $U(x)$, and $CV(x)$ for all $x \in C$.
3. Initialize $Q = \emptyset$.
4. If Stage = Global:
 1. Construct a predicted elite set E .
 2. Calculate $Score_G(x)$ for each $x \in E$ according to Eq. (13).
 3. Select the top q candidates into Q .
5. Else:
 1. If $\rho < \tau$, select candidates closest to the fuzzy boundary according to Eq. (14).
 2. Otherwise, select high-membership positive candidates according to Eq. (15).
 3. Use $Div(x)$ as the tie-breaking criterion.
6. If $|Q| < q$, fill Q with the most diverse unevaluated candidates.
7. Evaluate Q using true expensive functions.
8. Update A , CA , and DA .
9. Return Q .

4. Experimental Studies

4.1 Comparison of Algorithms

To comprehensively evaluate the performance of the proposed HDFC-ASS, several representative surrogate-assisted evolutionary algorithms are selected for comparison. These algorithms cover different types of surrogate modeling strategies, including classification-assisted methods, regression-assisted methods, decomposition-based surrogate-assisted methods, and ensemble surrogate-assisted methods. The comparison algorithms are briefly introduced as follows.

(1) **ABSAEA**^[19]: ABSAEA adopts an adaptive Bayesian strategy to dynamically adjust the hyperparameters in the acquisition function, so as to identify promising candidate solutions for true expensive function evaluations.

(2) **CSEA**^[10]: CSEA uses a single feedforward neural network model to preselect potentially promising offspring based on dominance relationships before expensive re-evaluation.

(3) **CPS-MOEA**^[11]: CPS-MOEA employs a K-nearest-neighbor model to filter candidate solutions and constructs nondominated clusters, from which promising solutions are selected for expensive evaluation.

(4) **MOEA/D-EGO**^[7]: MOEA/D-EGO adopts MOEA/D as the optimizer. It decomposes the multi-objective problem into a set of scalar subproblems and builds prediction models for the scalarized functions to select candidate solutions for true evaluation.

(5) **EIMEGO**^[20]: EIMEGO employs three expected improvement matrices as acquisition functions and embeds differential evolution operators as the optimizer to search for promising infill solutions.

(6) **K-RVEA**^[8]: K-RVEA uses reference vector-guided evolutionary algorithm as the optimizer and constructs Kriging models for each objective with a fixed amount of training data. When population diversity decreases, the solution with the maximum average standard deviation is selected as the infill sample; otherwise, the solution with the best APD value is selected for re-evaluation.

(7) **MCEA/D**^[12]: MCEA/D constructs multiple local SVM classifiers for the subproblems in MOEA/D-DE and selects solutions predicted to belong to the promising class. It is one of the representative classification-based SAEAs for high-dimensional expensive multi-objective optimization problems.

(8) **EDN-ARMOEA**^[21]: EDN-ARMOEA uses computationally efficient dropout neural networks instead of Gaussian process models to reduce the time consumption of model construction when the number of decision variables increases.

(9) **HeEMOEA**^[22]: HeEMOEA proposes a heterogeneous ensemble surrogate framework based on least-squares SVM and two RBF networks. A subset of decision variables and a group of transformed variables are selected as surrogate inputs to improve the diversity of the ensemble.

4.2 Experimental Environment and Test Problems

For each test problem, the number of decision variables ranges from 50 to 200, and the number of objectives is set to 3, 7, and 11. Testing surrogate-assisted evolutionary algorithms on computationally efficient benchmark problems is a common practice, rather than directly testing them on real-world expensive engineering problems. The key difference from traditional evolutionary algorithms is that only a very limited number of original expensive objective function evaluations is allowed, so as to simulate the assumption that objective evaluations are computationally expensive.

All experiments are conducted on the PlatEMO platform using a computer with 19 GB RAM and a 3.0 GHz CPU^[23]. The benchmark suites include DTLZ^[24], WFG^[25], and MaF^[29] test problems. These test suites are widely used to evaluate the convergence, diversity, and robustness of multi-objective optimization algorithms under different problem characteristics. The WFG test suite contains various complex properties, such as deception, multimodality, bias, nonseparability, and irregular Pareto fronts. The MaF test suite is further used to evaluate the scalability of the proposed algorithm on many-objective optimization problems.

4.3 Parameter Settings and Performance Metrics

The population size is set to ($N=100$), and the maximum number of expensive function evaluations is set to 500. The algorithm-specific parameters of HDfC-ASS are set as follows: ($k=3$), ($H=3$), ($n_{\min}=2$), and ($w_{\max}=30$). For all comparison algorithms, the parameter settings follow the recommendations in their original literature to ensure a fair comparison.

Each algorithm is independently executed 20 times on each test instance. The performance of the algorithms is evaluated using the inverted generational distance (IGD)^[27] and hypervolume (HV)^[28] indicators. A smaller IGD value indicates better convergence and diversity with respect to the reference Pareto front, while a larger HV value indicates better overall optimization performance.

To assess statistical significance, the Wilcoxon rank-sum test is performed at the 0.05 significance level^[29]. In the result tables, the best result on each test instance is highlighted in bold. The symbols “+”, “−”, and “ $\approx$ ” indicate that HDfC-ASS is significantly better than, significantly worse than, or statistically comparable to the corresponding comparison algorithm, respectively.

4.4 Effectiveness Analysis on DTLZ, WFG, and MaF Test Suites

First, the performance of HDfC-ASS is compared with representative classification-assisted and regression-assisted SAEAs on the DTLZ and WFG test problems. The statistical results are reported in Tables 1 and 2. From the results on all test instances, HDfC-ASS shows competitive or superior performance compared with the comparison algorithms according to the Wilcoxon rank-sum test.

For the DTLZ test suite, DTLZ1 and DTLZ3 are difficult to solve because of their multimodal landscapes, which may cause algorithms to converge to local Pareto fronts. Although some classification-assisted methods such as CSEA can achieve fast convergence on multimodal problems, HDfC-ASS obtains better average IGD values on these test instances. This indicates that the proposed

variable-correlation-guided random-subspace incremental Kriging strategy can effectively guide global exploration in high-dimensional decision spaces.

DTLZ2 is relatively easy to converge, but maintaining a well-distributed solution set remains important. HDFC-ASS achieves competitive results on DTLZ2, showing that the proposed convergence-diversity-uncertainty model management can balance convergence pressure and diversity preservation. DTLZ4 is a biased problem, where it is difficult to obtain uniformly distributed initial training samples. DTLZ5–DTLZ7 have irregular Pareto fronts, which pose greater challenges to diversity maintenance. The competitive performance of HDFC-ASS on these problems demonstrates the effectiveness of archive-assisted diversity preservation and fuzzy membership-guided local exploitation.

Table 1 Comparison of IGD values between HDFC-ASS and classification/regression-based SAEAs on the DTLZ benchmark problem

Problem	M	D	Classification-assisted SAEAs				Regression-assisted SAEAs			HDFC-ASS
			CPS-MOEA	CSEA	MCEA/D	ABSAEA	MOEA/D-EGO	EIMEGO	K-RVEA	
DTLZ1	3	50	9.7227e+2 (8.19e+1) -	1.1530e+3 (9.33e+1) -	7.3874e+2 (1.35e+2) -	1.1909e+3 (9.95e+1) -	1.2050e+3 (4.09e+1) -	1.1877e+3 (4.95e+1) -	1.1954e+3 (9.92e+1) -	1.0249e+2 (8.45e+1)
	7	100	1.7412e+3 (1.39e+2) -	1.9124e+3 (8.34e+1) -	1.5045e+3 (1.23e+2) -	1.9009e+3 (8.07e+1) -	1.9085e+3 (8.99e+1) -	1.9142e+3 (9.02e+1) -	1.9082e+3 (9.89e+1) -	2.7529e+2 (2.34e+2)
	11	200	3.5079e+3 (1.97e+2) -	3.7235e+3 (1.49e+2) -	3.0948e+3 (3.49e+2) -	3.7579e+3 (1.47e+2) -	3.9584e+3 (2.14e+2) -	3.7010e+3 (1.53e+2) -	3.9811e+3 (1.70e+2) -	6.7840e+2 (7.11e+2)
DTLZ2	3	50	2.1309e+0 (3.43e-1) -	2.9814e+0 (1.31e-1) -	7.4158e-1 (1.03e-1) -	2.7589e+0 (1.34e-1) -	2.7254e+0 (1.17e-1) -	2.7079e+0 (8.54e-2) -	2.7148e+0 (1.02e-1) -	4.3203e-1 (5.17e-2)
	7	100	5.9830e+0 (1.24e+0) -	9.1199e+0 (1.55e-1) -	1.8489e+0 (3.39e-1) -	9.1285e+0 (2.39e-1) -	9.1100e+0 (1.71e-1) -	9.0481e+0 (1.38e-1) -	9.0599e+0 (2.35e-1) -	1.3379e+0 (1.31e-1)
	11	200	1.2993e+1 (2.50e+0) -	1.3042e+1 (3.04e-1) -	3.9452e+0 (9.79e-1) -	1.3009e+1 (2.94e-1) -	1.2948e+1 (2.88e-1) -	1.3093e+1 (2.99e-1) -	1.3108e+1 (2.59e-1) -	2.1842e+0 (2.84e-1)
DTLZ3	3	50	3.0399e+3 (2.11e+2) -	3.7857e+3 (1.90e+2) -	1.9585e+3 (2.32e+2) -	3.9588e+3 (2.94e+2) -	3.7229e+3 (1.39e+2) -	3.7244e+3 (1.92e+2) -	3.9884e+3 (2.18e+2) -	2.3179e+2 (2.00e+2)
	7	100	9.8125e+3 (3.83e+2) -	7.9219e+3 (2.35e+2) -	3.9999e+3 (9.59e+2) -	7.9929e+3 (1.92e+2) -	7.9779e+3 (2.82e+2) -	8.0115e+3 (2.45e+2) -	7.9917e+3 (2.52e+2) -	3.7744e+2 (5.84e+2)
	11	200	1.4538e+4 (7.82e+2) -	1.7278e+4 (2.93e+2) -	8.4788e+3 (1.38e+3) -	1.7235e+4 (2.13e+2) -	1.7381e+4 (2.79e+2) -	1.7393e+4 (3.03e+2) -	1.7134e+4 (3.82e+2) -	5.1200e+2 (5.77e+2)
DTLZ4	3	50	2.3825e+0 (2.01e-1) -	3.0398e+0 (1.29e-1) -	1.0804e+0 (3.43e-2) -	3.0944e+0 (1.22e-1) -	2.9990e+0 (1.42e-1) -	3.0440e+0 (1.80e-1) -	3.1080e+0 (1.43e-1) -	6.9095e-1 (3.87e-3)
	7	100	5.0877e+0 (1.02e+0) -	9.3214e+0 (2.08e-1) -	1.5101e+0 (7.01e-2) -	9.3452e+0 (2.18e-1) -	5.7934e+0 (2.29e-1) -	9.3991e+0 (1.99e-1) -	9.3390e+0 (1.93e-1) -	1.2421e+0 (1.99e-2)
	11	200	9.8527e+0 (2.20e+0) -	1.3143e+1 (3.79e-1) -	1.7999e+0 (1.05e-1) -	1.3238e+1 (1.95e-1) -	1.2577e+1 (3.81e-1) -	1.3393e+1 (1.99e-1) -	1.3172e+1 (3.55e-1) -	1.4298e+0 (3.10e-2)
DTLZ5	3	50	2.0935e+0 (3.90e-1) -	2.9792e+0 (1.29e-1) -	5.2993e-1 (1.08e-1) -	2.9451e+0 (1.58e-1) -	4.1979e+1 (2.04e-1) -	2.9935e+0 (9.44e-2) -	2.9814e+0 (1.31e-1) -	2.2515e-1 (5.92e-2)
	7	100	5.2890e+0 (1.22e+0) -	5.7499e+0 (2.39e-1) -	1.2379e+0 (2.85e-1) -	5.7793e+0 (2.05e-1) -	8.2907e+1 (3.15e-1) -	5.7425e+0 (2.49e-1) -	5.7570e+0 (2.29e-1) -	7.9309e-1 (1.41e-1)
	11	200	1.0138e+1 (2.24e+0) -	1.2549e+1 (3.74e-1) -	3.1813e+0 (9.93e-1) -	1.2907e+1 (2.18e-1) -	1.9883e+2 (4.11e-1) -	1.2531e+1 (3.35e-1) -	1.2557e+1 (3.54e-1) -	1.5901e+0 (5.47e-1)
DTLZ9	3	50	3.2985e+1 (1.78e+0) -	4.1945e+1 (2.41e-1) -	2.4813e+1 (3.12e+0) -	4.1910e+1 (1.72e-1) -	9.3015e+0 (5.27e-1) +	4.1799e+1 (3.42e-1) -	4.1905e+1 (2.23e-1) -	1.7074e+1 (2.22e+0)
	7	100	7.0330e+1 (2.71e+0) -	8.2888e+1 (1.99e-1) -	5.3779e+1 (4.27e+0) -	8.2991e+1 (2.10e-1) -	2.4289e+1 (8.97e-1) +	8.2900e+1 (2.74e-1) -	8.2849e+1 (3.51e-1) -	3.9471e+1 (2.91e+0)
	11	200	1.4535e+2 (3.45e+0) -	1.9890e+2 (4.73e-1) -	1.1097e+2 (8.91e+0) -	1.9887e+2 (3.71e-1) -	4.0599e+1 (1.15e+0) +	1.9889e+2 (5.21e-1) -	1.9872e+2 (4.07e-1) -	8.7790e+1 (4.99e+0)
DTLZ7	3	50	9.3997e+0 (5.58e-1) -	9.9399e+0 (4.79e-1) -	8.9597e+0 (1.01e+0) -	9.1919e+0 (5.99e-1) -	1.2050e+3 (4.09e+1) -	9.2928e+0 (5.79e-1) -	9.3233e+0 (4.50e-1) -	6.4781e+0 (1.04e+0)
	7	100	2.5290e+1 (9.85e-1) -	2.4334e+1 (7.39e-1) -	2.4211e+1 (1.39e+0) -	2.4418e+1 (5.74e-1) -	1.9085e+3 (8.99e+1) -	2.4239e+1 (1.04e+0) -	2.4197e+1 (8.53e-1) -	1.9054e+1 (2.04e+0)
	11	200	4.2477e+1 (9.94e-1) -	4.0984e+1 (8.57e-1) -	4.1915e+1 (1.77e+0) -	4.1287e+1 (5.49e-1) -	3.9584e+3 (2.14e+2) -	4.0951e+1 (1.19e+0) -	4.0838e+1 (9.10e-1) -	3.9539e+1 (1.74e+0)
+/	-/≈		0/21/0	0/21/0	0/21/0	0/21/0	3/18/0	0/21/0	0/21/0	

Note: The symbols “+”, “-” and “≈” indicate that HDFC-ASS is significantly better than, significantly worse than and similar to the comparison

For the WFG test suite, HDFC-ASS also shows promising performance on problems with different complex characteristics. WFG1 has a biased transformation function, which makes it difficult for evolutionary algorithms to maintain a good distribution. WFG2 has a disconnected Pareto front,

while WFG3 replaces the disconnected front with a continuous one. WFG4 is a multimodal problem, WFG5 is a deceptive problem, and WFG9 and WFG9 are nonseparable reduction problems. WFG7 is a separable unimodal problem, whereas WFG8 is a nonseparable problem that makes diversity maintenance difficult. The results in Table 2 show that HDFC-ASS performs well on these WFG problems, confirming its robustness under different problem characteristics.

Table 2 Comparison of IGD values between HDFC-ASS and classification/regression-based SAEAs on the WFG benchmark problem

Problem	M	D	Classification-assisted SAEAs				Regression-assisted SAEAs			HDFC-ASS
			CPS-MOEA	CSEA	MCEA/D	ABSAEA	MOEA/D-EGO	EIMEGO	K-RVEA	
WFG1	3	50	2.3055e+0 (9.92e-2)-	2.2740e+0 (4.22e-2) -	2.2951e+0 (4.79e-2) -	2.2807e+0 (4.09e-2) -	2.2817e+0 (5.49e-2) -	2.2807e+0 (4.09e-2) -	2.2708e+0 (3.39e-2)-	1.7549e+0 (9.93e-2)
	7	100	2.9749e+0 (3.73e-2)+	2.8990e+0 (2.93e-2) +	2.9959e+0 (4.59e-2) ≈	2.8933e+0 (3.02e-2) +	2.8875e+0 (2.72e-2) +	2.8909e+0 (3.22e-2) +	2.8895e+0 (2.31e-2)+	3.0049e+0 (3.99e-2)
	11	200	3.9427e+0 (3.81e-2)+	3.5331e+0 (1.17e-2) +	3.9933e+0 (2.82e-2) ≈	3.5294e+0 (1.01e-2) +	3.5287e+0 (1.35e-2) +	3.5288e+0 (1.92e-2) +	3.5335e+0 (1.09e-2)+	3.0264e+0 (3.87e-2)
WFG2	3	50	8.7297e-1 (4.34e-2)-	8.3599e-1 (1.99e-2) -	7.5728e-1 (7.24e-2) -	8.3891e-1 (2.48e-2) -	8.4297e-1 (2.59e-2) -	8.4097e-1 (2.95e-2) -	8.3990e-1 (2.28e-2)-	5.1091e-1 (1.94e-2)
	7	100	2.0842e+0 (2.55e-1)-	1.9532e+0 (1.57e-1) ≈	2.0773e+0 (4.85e-1) -	1.9870e+0 (1.89e-1) -	1.8931e+0 (1.49e-1) ≈	1.9849e+0 (1.79e-1) -	1.9495e+0 (1.97e-1)≈	1.7981e+0 (3.48e-1)
	11	200	3.5995e+0 (3.57e-1)≈	3.1232e+0 (3.91e-1) ≈	3.8093e+0 (8.11e-1) ≈	3.0951e+0 (3.09e-1) ≈	3.1029e+0 (3.22e-1) ≈	3.1995e+0 (2.13e-1) ≈	3.3130e+0 (3.03e-1)≈	3.4972e+0 (7.41e-1)
WFG3	3	50	8.3139e-1 (3.04e-2)-	7.7521e-1 (7.73e-3) -	5.8754e-1 (3.41e-2) -	7.7845e-1 (1.05e-2) -	7.7504e-1 (1.23e-2) -	7.7475e-1 (1.22e-2) -	7.7439e-1 (1.13e-2)-	4.9548e-1 (3.18e-2)
	7	100	1.5975e+0 (9.50e-2)-	1.3395e+0 (7.91e-3) -	1.1158e+0 (5.94e-2) -	1.3424e+0 (1.95e-2) -	1.3394e+0 (1.18e-2) -	1.3383e+0 (1.75e-2) -	1.3384e+0 (1.55e-2)-	9.4329e-1 (3.80e-2)
	11	200	2.1242e+0 (1.07e-1)-	1.8125e+0 (1.39e-2) -	1.4279e+0 (9.50e-2) -	1.8122e+0 (1.09e-2) -	1.8128e+0 (1.39e-2) -	1.8103e+0 (1.07e-2) -	1.8103e+0 (1.31e-2)-	1.3289e+0 (5.11e-2)
WFG4	3	50	9.2212e-1 (1.08e-2)-	9.1084e-1 (2.50e-2) -	5.3234e-1 (2.73e-2) -	9.0995e-1 (1.40e-2) -	9.1271e-1 (2.51e-2) -	9.1215e-1 (2.32e-2) -	5.9794e-1 (1.98e-2)-	3.7479e-1 (2.82e-2)
	7	100	3.9038e+0 (2.02e-1)-	4.1090e+0 (1.37e-1) -	3.3439e+0 (2.75e-1) -	4.1559e+0 (1.93e-1) -	4.1377e+0 (1.99e-1) -	4.1792e+0 (1.51e-1) -	4.1582e+0 (1.80e-1)-	2.9545e+0 (1.52e-1)
	11	200	9.0053e+0 (4.98e-1)-	9.8512e+0 (3.51e-1) -	8.8182e+0 (1.05e+0) -	9.8292e+0 (2.78e-1) -	9.9219e+0 (2.04e-1) -	9.8732e+0 (3.40e-1) -	9.8552e+0 (2.09e-1)-	9.9339e+0 (9.43e-1)
WFG5	3	50	7.2740e-1 (2.10e-2)-	7.5533e-1 (9.09e-3) -	5.9419e-1 (4.52e-2) -	7.5492e-1 (7.15e-3) -	7.5255e-1 (7.48e-3) -	7.5557e-1 (9.22e-3) -	7.5500e-1 (1.14e-2)-	2.3991e-1 (2.77e-2)
	7	100	3.5743e+0 (1.78e-1)-	2.9970e+0 (9.01e-2) -	3.3804e+0 (1.04e-1) -	2.9992e+0 (9.02e-2) -	2.9819e+0 (9.93e-2) -	2.9774e+0 (7.31e-2) -	2.9990e+0 (7.81e-2)-	2.7742e+0 (1.39e-1)
	11	200	8.9339e+0 (3.87e-1) -	7.2508e+0 (1.97e-1) +	8.5128e+0 (3.75e-1) -	7.2723e+0 (1.88e-1) +	7.2135e+0 (1.82e-1) +	7.2887e+0 (1.92e-1) +	7.2984e+0 (2.17e-1)+	7.0032e+0 (3.28e-1)
WFG9	3	50	1.0094e+0 (1.95e-2)-	9.2952e-1 (8.99e-3) -	8.4491e-1 (3.29e-2) -	9.2912e-1 (9.99e-3) -	9.2873e-1 (9.79e-3) -	9.2950e-1 (5.99e-3) -	9.2914e-1 (9.22e-3)-	5.1942e-1 (1.40e-1)
	7	100	3.8390e+0 (2.49e-1)-	3.3009e+0 (9.19e-2) ≈	3.5951e+0 (2.59e-1) -	3.2922e+0 (7.72e-2) ≈	3.2938e+0 (8.99e-2) ≈	3.2792e+0 (8.39e-2) ≈	3.2511e+0 (7.10e-2)≈	3.2887e+0 (2.50e-1)
	11	200	9.0882e+0 (4.78e-1)≈	7.9949e+0 (1.83e-1) +	9.3813e+0 (7.88e-1) ≈	7.9129e+0 (1.77e-1) +	7.7209e+0 (2.10e-1) +	7.9879e+0 (1.78e-1) +	7.7793e+0 (1.91e-1)+	9.2915e+0 (8.85e-1)
WFG7	3	50	7.5895e-1 (1.48e-2)-	7.0432e-1 (8.09e-3) -	9.3120e-1 (3.52e-2) -	7.0333e-1 (9.93e-3) -	7.0985e-1 (9.19e-3) -	7.0758e-1 (1.09e-2) -	7.0144e-1 (5.78e-3)-	4.9300e-1 (1.99e-2)
	7	100	3.7997e+0 (2.83e-1) -	3.2434e+0 (1.18e-1) ≈	3.9444e+0 (3.59e-1) -	3.2920e+0 (1.08e-1) ≈	3.2959e+0 (1.07e-1) ≈	3.2295e+0 (1.03e-1) ≈	3.2377e+0 (9.13e-2)≈	3.2344e+0 (4.03e-1)
	11	200	9.2895e+0 (9.94e-1)≈	7.9185e+0 (2.99e-1) +	9.5117e+0 (8.85e-1) ≈	7.8098e+0 (2.02e-1) +	7.7888e+0 (2.40e-1) +	7.7532e+0 (2.27e-1) +	7.8240e+0 (2.07e-1)+	9.1993e+0 (9.53e-1)
WFG8	3	50	8.4771e-1 (2.18e-2)-	7.9979e-1 (1.33e-2) -	7.8028e-1 (3.81e-2) -	8.0317e-1 (8.72e-3) -	8.0095e-1 (1.20e-2) -	8.0471e-1 (1.39e-2) -	7.9937e-1 (1.39e-2)-	9.2902e-1 (3.58e-2)
	7	100	3.8248e+0 (2.74e-1)-	3.2290e+0 (9.32e-2) ≈	3.7515e+0 (3.29e-1) -	3.2570e+0 (9.79e-2) ≈	3.2472e+0 (5.30e-2) ≈	3.2079e+0 (9.19e-2) ≈	3.2819e+0 (9.08e-2)≈	3.2399e+0 (3.08e-1)
	11	200	9.0194e+0 (4.99e-1)≈	7.5784e+0 (1.85e-1) +	8.9118e+0 (8.43e-1) ≈	7.9111e+0 (1.80e-1) +	7.9194e+0 (2.31e-1) +	7.5992e+0 (1.91e-1) +	7.9304e+0 (2.09e-1)+	8.8294e+0 (8.01e-1)
WFG9	3	50	1.0054e+0 (2.91e-2)-	9.9912e-1 (1.20e-2) -	7.7843e-1 (9.38e-2) -	9.9493e-1 (2.24e-2) -	9.9399e-1 (1.89e-2) -	9.9898e-1 (1.92e-2) -	9.7049e-1 (1.94e-2)-	4.5485e-1 (4.13e-2)
	7	100	4.2485e+0 (2.31e-1)-	3.4473e+0 (1.02e-1) -	3.9943e+0 (2.74e-1) -	3.4835e+0 (7.90e-2) -	3.4757e+0 (1.09e-1) -	3.5040e+0 (1.38e-1) -	3.5035e+0 (1.35e-1)-	2.8309e+0 (1.95e-1)
	11	200	9.7907e+0 (4.13e-1)-	8.0020e+0 (1.55e-1) -	8.5239e+0 (5.00e-1) -	8.0433e+0 (1.72e-1) -	7.9339e+0 (1.99e-1) -	7.9435e+0 (2.54e-1) -	7.9212e+0 (2.29e-1)-	7.5179e+0 (4.33e-1)
+ / - / ≈			2/21/4	6/16/5	0/21/6	6/17/4	6/16/5	6/17/4	6/16/5	

Furthermore, to further analyze the algorithmic performance, we employed the MaF test function for comparative experiments. Table 3 presents the comparison results obtained using eight algorithms. The proposed HDFC-ASS demonstrates strong competitiveness in solving discontinuous problems

(e.g., MaF7) and transposed problems (e.g., MaF1 and MaF4). Its average IGD value significantly outperforms that of the other seven algorithms, particularly for discontinuous and transposed problems. These results highlight the critical importance of balancing exploration and utilization of MOPs within the search space to generate well-integrated and diverse solutions. Generally, for relatively lower-dimensional settings, regression-based SAEA achieves better performance than classification-based SAEA due to higher approximation accuracy. However, classification-based SAEA requires lower computational resources for high-dimensional problems. Overall, across most test cases, HDFC-ASS exhibits distinct advantages over the seven algorithms based on classification or regression.

Table 3 Comparison of IGD values between HDFC-ASS and classification/regression-based SAEAs on the MaF benchmark problem

Problem	M	D	Classification-assisted SAEAs				Regression-assisted SAEAs			HDFC-ASS
			CPS-MOEA	CSEA	MCEA/D	ABSAEA	MOEA/D-EGO	EIMEGO	K-RVEA	
MaF1	3	50	2.1388e+0 (2.90e-1)- 7.8823e+0	3.1999e+0 (1.97e-1) - 1.2955e+1	5.4559e-1 (9.55e-2) - 1.2348e+0	3.1502e+0 (1.92e-1) - 1.3229e+1	3.1502e+0 (1.92e-1) - 1.3159e+1	3.2199e+0 (1.49e-1) - 1.3179e+1	3.1173e+0 (1.84e-1) - 1.3299e+1	3.3872e-1 (4.97e-2) 7.1930e-1
	7	100	(8.97e-1)- 2.4940e+1	(4.78e-1) - 3.8340e+1	(1.98e-1) - 2.2215e+0	(3.70e-1) - 3.7958e+1	(4.91e-1) - 3.8097e+1	(9.01e-1) - 3.7779e+1	(5.39e-1) - 3.8022e+1	(5.47e-2) 1.1174e+0 (1.01e-1)
	11	200	(5.94e+0)- 2.4228e-1	(8.13e-1) - 2.4102e-1	(3.50e-1) - 1.2239e-1	(1.01e+0) - 2.4070e-1	(9.95e-1) - 2.4218e-1	(1.11e+0) - 2.3959e-1	(1.03e+0)- 2.3944e-1	(1.01e-1) 8.0755e-2 (7.49e-3)
MaF2	3	50	(2.42e-2)- 4.0389e-1	(9.30e-3) - 3.3389e-1	(9.37e-3) - 3.7519e-1	(5.95e-3) - 3.3492e-1	(9.00e-3) - 3.3197e-1	(9.72e-3) - 3.3273e-1	(5.07e-3) - 3.3219e-1	(7.49e-3) 3.2843e-1
	7	100	(1.45e-2)- 5.4810e-1	(9.71e-3) $\approx$ 4.9724e-1	(2.01e-2) - 4.9825e-1	(7.57e-3) $\approx$ 4.9711e-1	(9.87e-3) $\approx$ 4.9989e-1	(7.15e-3) $\approx$ 4.7000e-1	(7.12e-3) $\approx$ 4.9209e-1	(7.12e-3) $\approx$ 4.5747e-1 (2.41e-2)
	11	200	(3.00e-2)- 1.0990e+7	(7.79e-3) $\approx$ 2.2193e+7	(2.47e-2) - 2.7324e+9	(1.20e-2) $\approx$ 2.2790e+7	(9.13e-3) $\approx$ 2.1013e+7	(9.74e-3) - 2.1522e+7	(9.23e-3) $\approx$ 2.1522e+7	(2.77e-2) 8.5973e+5 (4.91e+5)
MaF3	3	50	(2.02e+9)- 9.3899e+7	(3.25e+9) - 8.3579e+7	(9.32e+5) - 1.7929e+7	(3.13e+9) - 8.9393e+7	(3.41e+9) - 8.9197e+7	(3.95e+9) - 8.9193e+7	(3.23e+9)- 8.9881e+7	(4.91e+5) 4.9989e+9 (1.59e+9)
	7	100	(9.29e+9)- 1.1091e+9	(7.85e+9) - 3.9292e+8	(9.75e+9) - 7.5592e+7	(1.11e+7) - 3.9808e+8	(7.81e+9) - 3.9118e+8	(9.31e+9) - 3.9739e+8	(7.47e+9) - 3.5953e+8	(7.47e+9) 1.7475e+7 (9.44e+9)
	11	200	(3.71e+9)- 9.5238e+3	(2.74e+7) - 1.1597e+4	(2.54e+7) - 9.0259e+3	(1.77e+7) - 1.1822e+4	(2.52e+7) - 1.1895e+4	(2.39e+7) - 1.2019e+4	(2.19e+7) - 1.2017e+4	(9.44e+9) 1.7920e+3 (1.29e+3)
MaF4	3	50	(1.09e+3)- 3.8748e+5	(8.99e+2) - 4.2349e+5	(1.24e+3) - 2.9382e+5	(5.90e+2) - 4.2791e+5	(8.10e+2) - 4.3320e+5	(3.79e+2) - 4.3235e+5	(5.90e+2)- 4.3274e+5	(1.29e+3) 2.8819e+4 (2.41e+4)
	7	100	(3.39e+4)- 1.2979e+7	(1.59e+4) - 1.4588e+7	(4.81e+4) - 9.7041e+9	(1.43e+4) - 1.4597e+7	(1.90e+4) - 1.4522e+7	(1.39e+4) - 1.4901e+7	(1.05e+4)- 1.4571e+7	(2.41e+4) 1.8888e+9 (1.72e+9)
	11	200	(1.29e+9)- 9.3039e+0	(3.04e+5) - 9.9954e+0	(1.80e+9) - 9.1109e+0	(3.00e+5) - 9.9434e+0	(3.33e+5) - 9.9792e+0	(3.98e+5) - 9.7918e+0	(4.24e+5)- 9.8497e+0	(1.72e+9) 4.4298e+0 (8.94e-1)
MaF5	3	50	(7.95e-1)- 4.9797e+1	(1.25e+0) - 5.5300e+1	(8.32e-1) - 4.8507e+1	(1.14e+0) - 5.7799e+1	(9.89e-1) - 5.3383e+1	(1.35e+0) - 5.9033e+1	(1.04e+0)- 5.5195e+1	(8.94e-1) 3.7932e+1 (5.90e+0)
	7	100	(5.75e+0)- 5.4252e+2	(1.38e+1) - 4.8348e+2	(2.79e+0) - 5.0321e+2	(1.92e+1) - 4.4911e+2	(1.37e+1) - 4.8019e+2	(1.41e+1) - 5.2225e+2	(1.54e+1)- 5.2982e+2	(5.90e+0) 4.3894e+2 (7.24e+1)
	11	200	(4.59e+1)- 1.5799e+2	(1.05e+2) $\approx$ 2.4735e+2	(8.24e+1) - 2.8999e+1	(1.09e+2) $\approx$ 2.5507e+2	(1.98e+2) $\approx$ 2.4817e+2	(2.09e+2) $\approx$ 2.5091e+2	(1.89e+2) $\approx$ 2.4981e+2	(7.24e+1) 9.8039e+0 (3.24e+0)
MaF9	3	50	(2.90e+1)- 5.9290e+2	(1.30e+1) - 5.5501e+2	(1.05e+1) - 8.0578e+1	(1.35e+1) - 5.5334e+2	(2.03e+1) - 5.5942e+2	(1.37e+1) - 5.9358e+2	(1.72e+1)- 5.5891e+2	(3.24e+0) 3.1981e+1 (1.33e+1)
	7	100	(1.09e+2)- 1.2051e+3	(1.75e+1) - 1.2293e+3	(2.80e+1) - 2.9357e+2	(2.21e+1) - 1.2340e+3	(1.84e+1) - 1.2339e+3	(1.35e+1) - 1.2499e+3	(2.29e+1)- 1.2339e+3	(1.33e+1) 1.1725e+2 (5.14e+1)
	11	200	(2.31e+2)- 8.9441e+0	(3.47e+1) - 9.3248e+0	(8.09e+1) - 9.2128e+0	(2.74e+1) - 9.2923e+0	(3.79e+1) - 9.4597e+0	(2.93e+1) - 9.3493e+0	(2.44e+1)- 9.3243e+0	(5.14e+1) 9.3011e+0 (1.03e+0)
MaF7	3	50	(9.90e-1)- 2.4845e+1	(5.37e-1) - 2.4295e+1	(8.07e-1) - 2.4199e+1	(5.01e-1) - 2.4229e+1	(5.52e-1) - 2.4292e+1	(4.08e-1) - 2.4249e+1	(5.31e-1) - 2.4347e+1	(1.03e+0) 1.8441e+1 (1.83e+0)
	7	100	(8.92e-1)- 4.2175e+1	(8.54e-1) - 4.0397e+1	(1.01e+0) - 4.1809e+1	(7.93e-1) - 4.0558e+1	(1.09e+0) - 4.0933e+1	(8.34e-1) - 4.0580e+1	(7.71e-1) - 4.0713e+1	(1.83e+0) 3.9858e+1 (2.08e+0)
	11	200	(1.29e+0)- + / - / $\approx$	(9.53e-1) - 0/21/0	(1.37e+0) - 0/18/3	(1.02e+0) - 0/21/0	(1.04e+0) - 0/18/3	(7.57e-1) - 0/19/2	(7.98e-1) - 0/18/4	(2.08e+0)

Figs.2 and 3 respectively illustrate the non-dominated solution sets of all comparison algorithms on the test functions with 3 and 11 objectives, along with corresponding parallel coordinate plots representing median IGD values. The figures demonstrate that HDFC-ASS yields a broader individual range than other comparison algorithms, indicating superior solution distribution properties. The practice of populating each agent update cycle with a solution has proven effective, as it fully leverages observed samples from original function evaluations to enhance both diversity preservation and convergence acceleration. Thus, our empirical study concludes that the population sampling strategy

combined with integration of classification and regression techniques significantly enhances evolutionary guidance efficiency while achieving balanced convergence and distribution across the population.

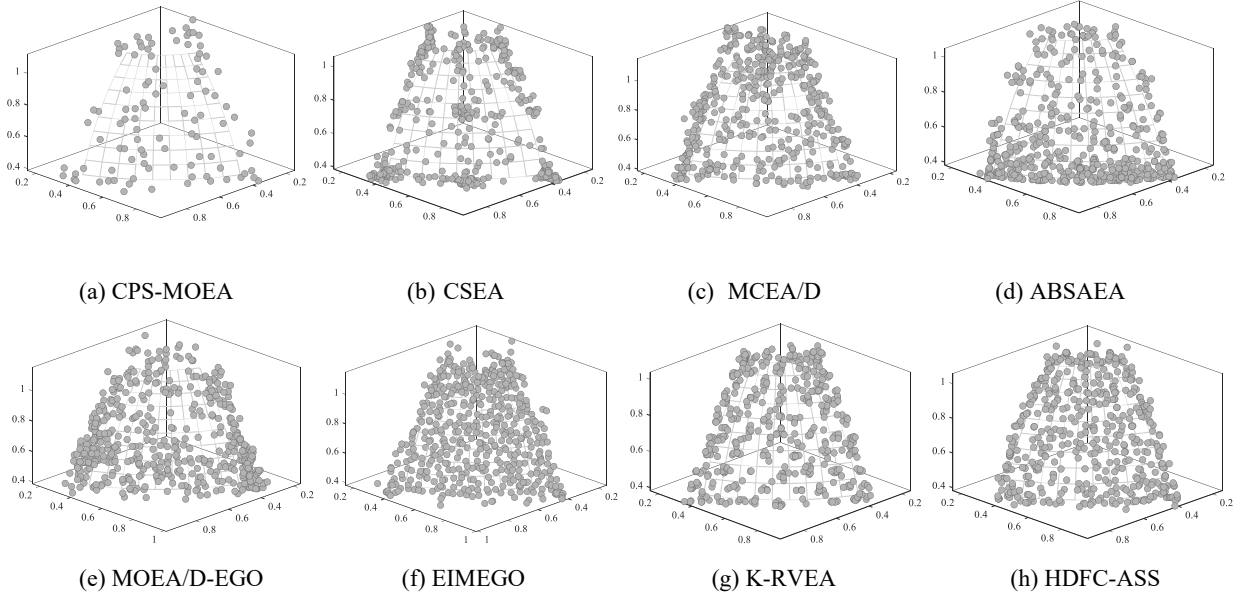


Fig. 2 Distribution plots of solution sets for the eight algorithms on the three-objective test function MaF2

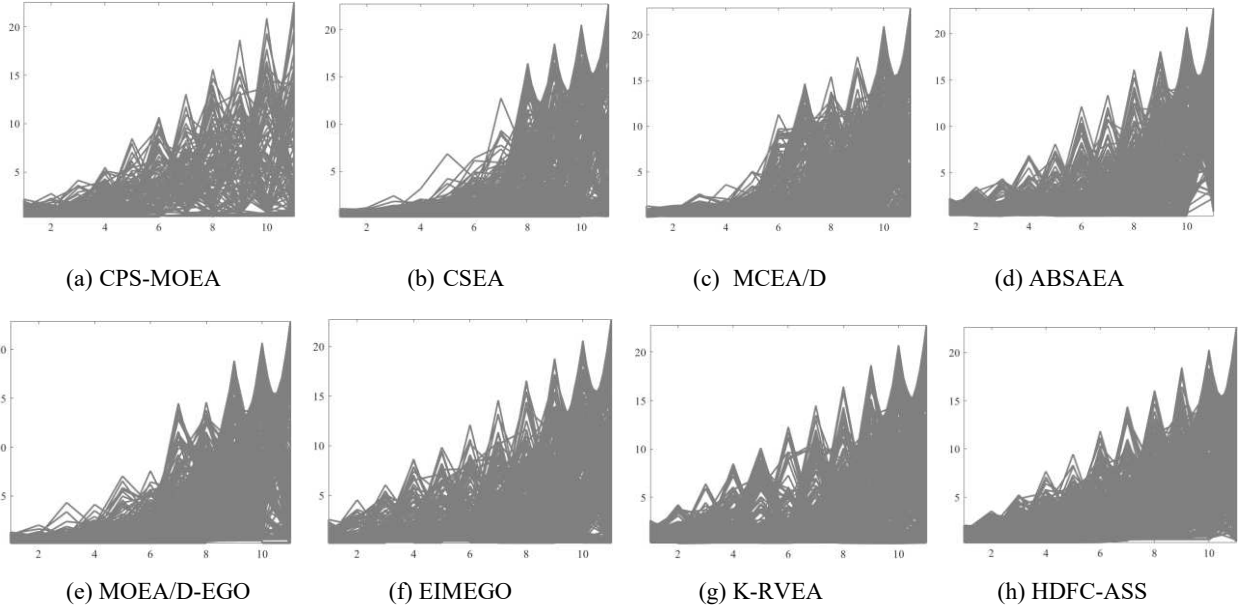


Fig. 3 Distribution plots of solution sets for the eight algorithms on the three-objective test function WFG9

The above results indicate that HDFC-ASS obtains competitive overall performance on DTLZ, WFG and MaF test suites under a limited evaluation budget. Its advantage is more evident on multimodal, biased, disconnected and irregular problems, where both global exploration and local exploitation must be carefully controlled. These results support the proposed model-state-aware cooperation mechanism: random-subspace incremental Kriging improves early-stage exploration in high-dimensional decision spaces, while the reliability-controlled fuzzy classifier strengthens local

screening after the classification boundary becomes trustworthy. Therefore, the performance improvement mainly comes from activating regression and classification surrogates according to their reliable states, rather than from simply combining two model outputs.

4.5 Comparative analysis of algorithmic time complexity

We conducted a comparative analysis of the average computational time (in seconds) required by four multi-model integrated SAEA algorithms on the DTLZ and WFG test functions for a set of experiments with $M = \{3, 7, 11\}$ and $D = \{50, 100, 200\}$. The computational-time comparison in Table 4 shows that HDFC-ASS requires more computation time than MCEA/D because it maintains random-subspace Kriging models and performs fuzzy-classification-based model management. However, its computational cost is much lower than that of HeEMOEa in high-dimensional settings. Therefore, HDFC-ASS does not aim to be the fastest algorithm; instead, it seeks a trade-off between computational overhead and optimization performance under expensive high-dimensional evaluation conditions.

Table 4 Average computation time (20 rounds) in seconds for integrating SAEAs across all dimensions of the DTLZ and WFG problems

M	D	MCEA/D	EDN-ARMOEA	HeEMOEa	HDFC-ASS
3	50	3.1322e+00	2.0999e+01	4.0184e+02	5.9805e+01
7	100	4.4181e+00	3.1322e+00	1.1455e+03	8.7870e+01
11	200	8.2793e+00	1.1852e+02	4.7087e+03	5.4501e+02

5. Conclusion

This paper proposes HDFC-ASS, a classification-regression cooperative fuzzy surrogate-assisted evolutionary algorithm for expensive high-dimensional multi-objective optimization. Motivated by the model-state mismatch caused by high dimensionality, small evaluated archives and imbalanced labels, HDFC-ASS combines variable-correlation-guided random-subspace incremental Kriging, balanced-accuracy-based fuzzy classification, and convergence-diversity-uncertainty model management. Experimental results on DTLZ, WFG and MaF benchmark suites show that HDFC-ASS achieves competitive or superior IGD performance against representative SAEAs and maintains a good balance between convergence and diversity on multimodal, biased, disconnected and irregular problems. Although it is not the fastest method, HDFC-ASS offers an effective trade-off between model-management cost and optimization performance. Future work will investigate adaptive parameter control and real-world expensive engineering applications.

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