

Supplementary Methods

Additional barnacle feedback loop



with net reaction:



where P_r is the available substrate space, L_U and L_S are alien and native barnacle larvae, U_w and S_w are adult alien and native barnacles and W_a are predators. As long as we consider that the larvae are found in abundance and hence treating them as catalysts, that $\gamma + \delta = e$ and $f = 2$, Eq. 5 is equivalent to Eq. 5 of the AI-B reaction mechanism (in the main text).

Steady state solutions

Notations:

$$u_w = u, s_w = s, w_a = w, p_r = p, h_p^{1/\alpha} = A, h_p^{1/\beta} = B, c_t = C. \quad (6)$$

$$u' = 0, u^* = \frac{A}{w^a + C} \quad (7)$$

$$s' = 0, s^* = \frac{B + uC}{w^b} \quad (8)$$

Substituting u^* and s^* from Eq. (7)-(8):

$$w' = 0, p^e = (A + B)^f, p^* = (A + B)^{f/e} \quad (9)$$

$$p' = 0, p^e = \left(A \frac{w^a}{w^a + C} \right)^{1/c} + \left(\frac{B(w^a + C) + AC}{w^a + C} \right)^{1/d} \quad (10)$$

Equivalating Eq. (9) and (10):

$$(A + B)^f = \left(A \frac{w^a}{w^a + C} \right)^{1/c} + \left(\frac{B(w^a + C) + AC}{w^a + C} \right)^{1/d} \quad (11)$$

Eq. (11) cannot be solved explicitly in closed form.

Symbolic stability analysis of the reduced systems

Due to the fact that a full four-dimensional system is analytically difficult to classify symbolically (as found in our previous work), we focus on reductions in which two variables are held fixed and the remaining two are treated as dynamical. This does not imply that the fixed variables are fully constant, but that the corresponding to variables change on a much slower time scale. The reduced systems allow for trace-determinant analysis of the Jacobian, yielding explicit conditions for stable steady states, saddle points, and Hopf bifurcation candidates (see Table 1).

No.	Fixed var.	Dynamical var.	tr(J)	det(J)	Node type
1	p, w	u, s	-	+	Stable node/focus
2	p, s	u, w	-	+	Stable node/focus
3	s, w	u, p	-	+	Stable node/focus
4	p, u	s, w	-	+	Stable node/focus
5	u, w	s, p	-	+	Stable node/focus
6	s, u	w, p	-	+/-	Stable node/saddle point

Table 1: Symbolic stability classification of the reduced systems. In order of the columns: the number of the case (No.), the fixed variables in each configuration (Fixed var.), the dynamical variables (Dynamical var.), the sign of the trace of the Jacobian ($\text{tr}(J)$), the sign of the determinant of the Jacobian ($\text{det}(J)$) and the Node type. We see that only for case No. 6 could the node type be a saddle point under certain circumstances.

We choose to focus on case nr.6, for which the sign of the determinant is dictated by the following condition:

$$\text{sign}(\text{det}(J)) = \text{sign} \left[cdf p^{e-e/f} (auw^a + bs w^b) - \left(ad u^{1/c} w^{a/c} + bc s^{1/d} w^{b/d} \right) \right]. \quad (12)$$

Moreover, since $\text{tr}(J) < 0$ for all positive parameter values, a Hopf bifurcation is not expected in this reduced (w, p) system under the positive-parameter assumptions.

Local stability analysis for fixed variables $(u'_w = s'_w = 0)$ and dynamic variables w_a, p_r

$$F(w, p) = w' = -uw^a - sw^b + p^{e/f} = p^{e/f} - (A + B) \quad (13)$$

$$G(w, p) = p' = u^{1/c} w^{a/c} + s^{1/d} w^{b/d} - p^e = \left(\frac{Aw^a}{w^a + C} \right)^{1/c} + \left(\frac{(w^a + C)B + AC}{w^a + C} \right)^{1/d} - p^e \quad (14)$$

$$J = \begin{pmatrix} \frac{\partial F(w, p)}{\partial w} & \frac{\partial F(w, p)}{\partial p} \\ \frac{\partial G(w, p)}{\partial w} & \frac{\partial G(w, p)}{\partial p} \end{pmatrix} = \begin{pmatrix} 0 & \frac{e}{f} p^{e/f-1} \\ G_w & -e p^{e-1} \end{pmatrix} \quad (15)$$

where

$$G_w = \frac{d}{dw} \left(\frac{Aw^a}{w^a + C} \right)^{1/c} + \frac{d}{dw} \left(\frac{(w^a + C)B + AC}{w^a + C} \right)^{1/d} = \frac{aACw^{a-1}}{(w^a + C)^2} \left[\frac{1}{c} \left(\frac{Aw^a}{w^a + C} \right)^{1/c-1} - \frac{1}{d} \left(\frac{(w^a + C)B + AC}{w^a + C} \right)^{1/d-1} \right] \quad (16)$$