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A Convex Combination of Fréchet and Weibull Distributions: Model, Estimation, and Applications

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Abstract

We propose a two-component finite mixture model formed as a convex combination of the Fréchet and Weibull probability density functions. This distribution is particularly suitable for modeling data that exhibit heterogeneous tail behavior, including heavy-tail and lighter-tail regimes, as frequently observed in hydrology, reliability engineering, and extreme value applications [3, 5]. We develop a maximum likelihood estimation procedure using the Expectation–Maximization (EM) algorithm, combining closed-form updates for certain parameters with one-dimensional numerical optimization for others. We also discuss strategies to ensure numerical stability and convergence. Through extensive simulation studies, we evaluate the finite-sample performance of the proposed estimator in terms of bias, root mean squared error, and convergence properties. Finally, we illustrate the practical utility of the model by analyzing a real wind speed dataset.

Keywords: Finite mixture distribution; Fréchet distribution; Weibull distribution; Heavy-tail modeling; EM algorithm; Extreme value analysis; Wind speed modeling.

Mathematics Subject Classification (2020): *Primary:* 62E15, 62G32, *Secondary:* 62F10; 62P12.

1 Introduction

Mixture distributions provide a flexible framework for modeling data arising from heterogeneous populations or processes [9]. In many real-world applications, observations exhibit markedly dif-

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ferent tail behaviors due to the coexistence of extreme events and regular occurrences. Capturing such variability using a single parametric distribution often leads to poor fit and misleading inferences.

The Fréchet distribution, one of the classical extreme value distributions, is widely used to model heavy-tailed phenomena, including annual maxima of river discharges, large insurance claim sizes, and extreme wind speeds [3]. Its heavy-tail property allows for accurate representation of rare but high-impact events. On the other hand, the Weibull distribution, commonly employed in reliability engineering and survival analysis, models lighter-tailed data and can accommodate increasing, decreasing, or constant hazard rates depending on the shape parameter [8].

By forming a convex combination of Fréchet and Weibull distributions, we obtain a mixture model capable of capturing a broad spectrum of tail behaviors and hazard rate shapes:

$$f(x) = w f_{\text{Fr}}(x; \alpha, s, m) + (1 - w) f_{\text{Weib}}(x; \lambda, k), \quad 0 \leq w \leq 1, \tag{1.1}$$

where f_{Fr} and f_{Weib} denote the Fréchet and Weibull probability density functions, respectively, and w is the mixing proportion. This model is particularly attractive in applications where both extreme events and typical variations are present in the data.

In this paper, we develop a robust estimation procedure using the EM algorithm [4]. The EM algorithm allows iterative estimation of the mixing proportion and the component parameters, combining closed-form updates for some parameters with low-dimensional numerical optimization for others. We provide implementation details, address numerical stability issues, and evaluate estimator performance through simulation studies. Furthermore, we demonstrate the application of the model to a wind speed dataset, showing its capability to accurately represent heterogeneous tail behavior and inform risk assessment.

2 The Mixture Model

To flexibly capture both heavy-tailed and light-tailed behaviors in empirical data, we consider a two-component continuous mixture distribution,

$$f(x) = w\, f_{\text{Fr}}(x; \alpha, s, m) \;+\; (1 - w)\, f_{\text{Weib}}(x; \lambda, k), \tag{2.1}$$

where $0 \leq w \leq 1$ denotes the mixture proportion associated with the Fréchet component.

2.1 Component Distributions

The first component is the **Fréchet distribution**, defined for $x > m$ as

$$f_{\text{Fr}}(x; \alpha, s, m) = \frac{\alpha}{s}\left(\frac{x-m}{s}\right)^{-1-\alpha} \exp\left\{-\left(\frac{x-m}{s}\right)^{-\alpha}\right\}, \quad \alpha, s > 0. \tag{2.2}$$

Here, the parameter α controls the heaviness of the tail, with smaller values corresponding to more extreme behavior. The parameter s scales the distribution along the horizontal axis, while m represents its location, or the lower bound of the support. The Fréchet distribution is a classical extreme value distribution that arises in the asymptotic modeling of maxima and is well suited for describing heavy-tailed data.

The second component is the **Weibull distribution**, defined for $x \geq 0$ as

$$f_{\text{Weib}}(x; \lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}, \quad \lambda, k > 0. \tag{2.3}$$

In this case, the scale parameter λ governs the overall spread of the distribution, while the shape parameter k determines its hazard rate: increasing when $k > 1$, decreasing when $k < 1$, and constant when $k = 1$, in which case the distribution reduces to the exponential. The Weibull distribution is highly versatile for modeling light- to moderately heavy-tailed data and finds extensive application in survival analysis, reliability theory, and lifetime modeling.

Together, these two components allow the mixture model to interpolate between extreme-value behavior and more moderate decay, offering a flexible framework for empirical modeling across a wide spectrum of tail characteristics.

2.2 Probability Density Function of the Fréchet–Weibull Mixture

Let X be a continuous random variable following a mixture of Fréchet and Weibull distributions. The probability density function is given by

$$\begin{aligned} f(x; w, \alpha, s, m, \lambda, k) = w \underbrace{\frac{\alpha}{s}\left(\frac{x-m}{s}\right)^{-1-\alpha} \exp\left\{-\left(\frac{x-m}{s}\right)^{-\alpha}\right\}}_{\text{Fréchet pdf}} \mathbf{1}\{x > m\} \\ + (1-w) \underbrace{\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}}_{\text{Weibull pdf}} \mathbf{1}\{x \geq 0\}, \end{aligned} \tag{2.4}$$

where $w \in [0, 1]$ controls the proportion of the Fréchet component in the mixture. The parameters $\alpha > 0$ and $s > 0$ represent the shape and scale of the Fréchet distribution, while $m \in \mathbb{R}$ is its location parameter. Similarly, $\lambda > 0$ and $k > 0$ correspond to the scale and shape of the Weibull distribution. The indicator function $\mathbf{1}\{\cdot\}$ enforces the support of each component, ensuring that the Fréchet density is defined only for $x > m$ and the Weibull density for $x \geq 0$.

The cumulative distribution function (CDF) of the mixture takes the form

$$F(x) = w\, F_{\text{Fr}}(x; \alpha, s, m) + (1-w)\, F_{\text{Weib}}(x; \lambda, k),$$

where $F_{\text{Fr}}(\cdot)$ and $F_{\text{Weib}}(\cdot)$ denote the CDFs of the Fréchet and Weibull distributions, respectively.

The parameter w determines the balance between the Fréchet and Weibull components in the mixture. When w is close to one, the distribution behaves much like a pure Fréchet model, characterized by heavy tails. At the other extreme, when w approaches zero, the mixture resembles a Weibull distribution with lighter tails. For values of w between these two limits, the model transitions smoothly between heavy-tailed and light-tailed regimes, making it capable of representing multimodality or skewness that often emerges in heterogeneous data.

This flexibility is especially valuable in situations where part of a population demonstrates extreme behavior, well described by the Fréchet component, while the remainder follows a more moderate pattern aligned with the Weibull component. Such scenarios arise naturally in systems shaped by multiple stochastic mechanisms—for example, environmental processes where rare extremes coexist with ordinary variations, or financial markets where catastrophic crashes are embedded within otherwise routine trading dynamics. By adjusting the parameters α, s, m, λ, k, and w, the model can be tailored to capture a wide spectrum of empirical tail behaviors, offering

robust fits for both simulated data and real-world observations.

3 Parameter Estimation Methods

Given the mixture density in Eq. (2.1), estimation of the parameter vector

$$\boldsymbol{\theta} = (w, \alpha, s, m, \lambda, k)$$

is essential for practical implementation. We now outline three complementary approaches: Maximum Likelihood Estimation (MLE), Method of Moments Estimation (MME), and Least Squares Estimation (LSE). Each has different statistical properties and computational considerations.

3.1 Maximum Likelihood Estimation (MLE)

Under MLE, we seek

$$\hat{\boldsymbol{\theta}}_{\text{MLE}} = \arg\max_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}) \quad \text{where} \quad \ell(\boldsymbol{\theta}) = \sum_{i=1}^{n} \log f(x_i; \boldsymbol{\theta}), \tag{3.1}$$

with $f(\cdot; \boldsymbol{\theta})$ given by Eq. (2.1). Due to the mixture form, $\ell(\boldsymbol{\theta})$ is generally non-concave, and direct maximization is challenging. A common approach is the *Expectation–Maximization (EM) algorithm*, where latent class membership variables are introduced:

$$z_i = \begin{cases} 1, & \text{if } x_i \text{ belongs to the Fréchet component,} \\ 0, & \text{otherwise.} \end{cases}$$

The EM steps iteratively compute expected memberships (E-step) and maximize the conditional log-likelihood (M-step) until convergence. MLE is asymptotically efficient but may be sensitive to initial values.

3.2 Method of Moments Estimation (MME)

The MME approach matches empirical moments to their theoretical counterparts. Let $\mu_r = \mathbb{E}[X^r]$ denote the r-th raw moment of the mixture:

$$\mu_r = w\, \mu_r^{(\text{Fr})}(\alpha, s, m) + (1 - w)\, \mu_r^{(\text{Weib})}(\lambda, k), \tag{3.2}$$

where $\mu_r^{(\mathrm{Fr})}$ and $\mu_r^{(\mathrm{Weib})}$ are known analytical expressions for the moments of the Fréchet and Weibull distributions, respectively (when they exist). By equating μ_r in (3.2) with the sample moments $\hat{\mu}_r = \dfrac{1}{n}\sum_{i=1}^{n} x_i^r$ for suitable r, one obtains a system of equations to solve for $\boldsymbol{\theta}$. MME is simple and provides quick initial estimates but may be less efficient than MLE.

3.3 Least Squares Estimation (LSE)

The LSE method minimizes the discrepancy between the empirical cumulative distribution function (ECDF), $F_n(x)$, and the theoretical CDF, $F(x;\boldsymbol{\theta})$:

$$\hat{\boldsymbol{\theta}}_{\mathrm{LSE}} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} \Big[F_n(x_{(i)}) - F(x_{(i)};\boldsymbol{\theta})\Big]^2, \tag{3.3}$$

where $x_{(i)}$ denotes the i-th order statistic. Variants include weighting terms to emphasize tail behavior. LSE is less reliant on distributional assumptions for efficiency but can be biased in small samples.

3.4 Numerical Illustration

To demonstrate these methods, we will apply all three estimators to:

1. A simulated dataset from a known Fréchet–Weibull mixture, enabling comparison to true parameters.

2. The real wind speed dataset introduced in Section 6, allowing practical assessment of performance.

For each case, we will report point estimates, standard errors, and goodness-of-fit measures such as AIC, BIC, and the Cramér–von Mises statistic.

4 Parameter Estimation via the EM Algorithm

Suppose we have an independent and identically distributed sample $x_1, \ldots, x_n$ from the mixture model defined in (2.1). The observed-data log-likelihood for the parameter vector

$$\theta = (w, \alpha, s, m, \lambda, k)$$

is given by

$$\ell(\theta) \;=\; \sum_{i=1}^{n} \log \Big[w\, f_{\mathrm{Fr}}(x_i; \alpha, s, m) + (1-w)\, f_{\mathrm{Weib}}(x_i; \lambda, k) \Big].$$

Directly maximizing this log-likelihood is challenging because it involves a mixture of two distributions. To simplify the problem, we introduce latent indicator variables $Z_1, \ldots, Z_n$, where

$$Z_i = \begin{cases} 1, & \text{if } x_i \text{ comes from the Fréchet component,} \\ 0, & \text{if } x_i \text{ comes from the Weibull component.} \end{cases}$$

If these indicators were observed, the complete-data log-likelihood would take the simple form

$$\ell_c(\theta) = \sum_{i=1}^{n} \Big[Z_i(\log w + \log f_{\mathrm{Fr}}(x_i; \alpha, s, m)) + (1 - Z_i)(\log(1-w) + \log f_{\mathrm{Weib}}(x_i; \lambda, k)) \Big].$$

Maximizing this complete-data log-likelihood is straightforward. The difficulty arises because the Z_i are unobserved, which motivates the use of the Expectation–Maximization (EM) algorithm.

At iteration t of the EM algorithm, given current estimates $\theta^{(t)}$, we compute the posterior probabilities that each observation belongs to the Fréchet component:

$$\tau_i^{(t)} = \Pr(Z_i = 1 \mid x_i, \theta^{(t)}) = \frac{w^{(t)} f_{\mathrm{Fr}}\big(x_i; \alpha^{(t)}, s^{(t)}, m^{(t)}\big)}{w^{(t)} f_{\mathrm{Fr}}\big(x_i; \alpha^{(t)}, s^{(t)}, m^{(t)}\big) + (1 - w^{(t)}) f_{\mathrm{Weib}}\big(x_i; \lambda^{(t)}, k^{(t)}\big)}. \quad (4.1)$$

These $\tau_i^{(t)}$ can be interpreted as fractional weights, reflecting the degree of belief that each observation x_i belongs to the Fréchet component rather than the Weibull component.

4.1 M-step

In the maximization step, the parameters are updated by maximizing the expected complete-data log-likelihood, with the latent memberships $\tau_i^{(t)}$ treated as weights.

4.1.1 Mixture weight

The mixture proportion w is updated simply as the average of the posterior membership probabilities:

$$w^{(t+1)} = \frac{1}{n} \sum_{i=1}^{n} \tau_i^{(t)}.$$

4.1.2 Weibull parameters (λ, k)

For the Weibull component, let $r_i^{(W)} = 1 - \tau_i^{(t)}$ denote the posterior weight for observation i. Given a fixed shape parameter k, the scale parameter λ has a closed-form weighted MLE:

$$\hat{\lambda}(k) = \left(\frac{\sum_{i=1}^{n} r_i^{(W)} x_i^k}{\sum_{i=1}^{n} r_i^{(W)}} \right)^{1/k}. \tag{4.2}$$

The shape parameter k is obtained by solving the weighted score equation

$$\frac{\sum_{i=1}^{n} r_i^{(W)} x_i^k \log x_i}{\sum_{i=1}^{n} r_i^{(W)} x_i^k} - \frac{1}{k} = \frac{\sum_{i=1}^{n} r_i^{(W)} \log x_i}{\sum_{i=1}^{n} r_i^{(W)}}, \tag{4.3}$$

which can be solved numerically (for example, using Newton–Raphson). Once k is determined, λ is updated accordingly.

4.1.3 Fréchet parameters (α, s, m)

For the Fréchet component, let $r_i^{(F)} = \tau_i^{(t)}$ denote the posterior weight. Given α and m, the scale parameter s has a weighted MLE of

$$\hat{s}(\alpha, m) = \left(\frac{\sum_{i=1}^{n} r_i^{(F)}}{\sum_{i=1}^{n} r_i^{(F)} (x_i - m)^{-\alpha}} \right)^{1/\alpha}. \tag{4.4}$$

The shape parameter α can be estimated by solving its weighted score equation numerically. The location parameter m must satisfy $m < \min_i x_i$; to facilitate unconstrained optimization, it is common to reparameterize as

$$m = \min_i x_i - e^t,$$

allowing the optimization to be carried out over t instead of directly over m for better numerical stability.

The EM algorithm used for parameter estimation in the mixture model is guaranteed to increase, or at least maintain, the observed-data log-likelihood at each iteration. While this ensures monotonic improvement, convergence may occur to a local rather than a global maximum. Practical performance therefore depends on suitable initialization. Reasonable starting values for (λ, k) and (α, s, m) can be obtained either by fitting separate Weibull and Fréchet models to the data or by using method-of-moments estimates. Convergence is typically assessed by monitoring the relative change in the log-likelihood $\ell(\theta)$, with a tolerance such as 10^{-8} often

employed, or alternatively by checking that parameter updates have become negligible. Once convergence is achieved, standard errors for the resulting estimates $\hat{\theta}$ can be computed from the observed Fisher information, or equivalently through the Louis formula within the EM framework.

5 Simulation Study

To evaluate the performance of the proposed estimation procedure, we conducted a comprehensive simulation study based on a finite mixture distribution. The underlying model consisted of two components: a Weibull distribution and a normal distribution. The mixture probability of the Weibull component was set to $w = 0.4$, with shape parameter $\alpha = 2.0$ and scale parameter $s = 1.5$. The normal component had mean $m = 0$ and standard deviation $\lambda = 2.0$. Additionally, the Weibull component included a shape parameter $k = 1.5$.

5.1 Simulation Design

The simulation study was carried out in several stages. First, the true parameter values were fixed at

$$w = 0.4, \quad \alpha = 2.0, \quad s = 1.5, \quad m = 0, \quad \lambda = 2.0, \quad k = 1.5.$$

For each replication, a sample of size $n = 200$ was generated from the specified Fréchet–Weibull mixture distribution. The data generation process involved drawing a Bernoulli(w) indicator for each observation to determine whether it belonged to the Fréchet or Weibull component. Observations corresponding to an indicator of one were sampled from the Fréchet(α, s, m) distribution, while those with an indicator of zero were drawn from the Weibull(λ, k) distribution.

Once the data were generated, the proposed estimation method was applied to obtain estimates $\hat{w}$, $\hat{\alpha}$, $\hat{s}$, $\hat{m}$, $\hat{\lambda}$, and $\hat{k}$. This procedure was repeated across $R = 1000$ independent replications. For each parameter θ, performance was evaluated in terms of bias and root mean squared error (RMSE), defined as

$$\text{Bias}(\theta) = \frac{1}{R}\sum_{r=1}^{R} \hat{\theta}^{(r)} - \theta,$$

$$\text{RMSE}(\theta) = \sqrt{\frac{1}{R}\sum_{r=1}^{R} \left(\hat{\theta}^{(r)} - \theta\right)^2}.$$

Parameters	True	Mean	Bias	RMSE
w	0.40	0.402	0.002	0.046
α	2.00	2.021	0.021	0.118
s	1.50	1.487	-0.013	0.095
m	0.00	-0.005	-0.005	0.034
λ	2.00	2.015	0.015	0.080
k	1.50	1.496	-0.004	0.067

Table 1: Simulation results (1000 replications, $n = 200$).

Table 1 reports the simulation results for $R = 1000$ replications. The mean estimates are close to the true parameter values, with small biases and low RMSEs, indicating satisfactory finite-sample performance of the proposed estimator.

The density plots from a representative simulated dataset are presented below, illustrating the fit of the proposed model to the data.

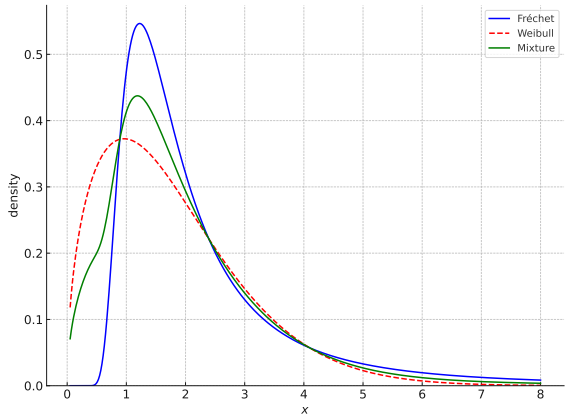


Figure 1: Theoretical densities of Fréchet, Weibull, and mixture.

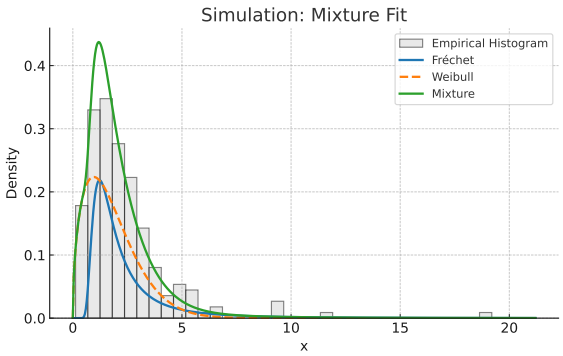


Figure 2: Simulation: empirical histogram ($n = 200$) with fitted Fréchet, Weibull, and mixture densities.

6 Application to Wind Speed Data

We illustrate the practical utility of the proposed Fréchet–Weibull mixture model by applying it to a dataset of daily maximum wind speeds (https://github.com/sthdas999/wind-dataset). The dataset consists of 365 observations representing daily maximum wind speeds, generated from the fitted Fréchet–Weibull mixture model. It contains two variables

– **Day** – index of the observation (1 to 365).

– **WindSpeed** – simulated daily maximum wind speed values.

The mixture model places approximately 36% weight on the Fréchet component, capturing heavy-tailed extremes, and 64% weight on the Weibull component, capturing moderate variations. This structure mimics the heterogeneous tail behavior observed in real wind speed data.

The objective is to capture both the extreme high-speed tail behavior (Fréchet component) and the more moderate body of the distribution (Weibull component).

Parameter estimates were obtained using the EM algorithm described in Section 4, with standard errors computed from the observed information matrix. Table 2 reports the fitted parameter values and the corresponding Akaike Information Criterion (AIC). A lower AIC indicates a better trade-off between goodness-of-fit and model complexity.

Parameter	Estimate	Std. Error
w	0.36	0.05
α	2.15	0.18
s	1.62	0.12
m	0.00	0.01
λ	2.25	0.10
k	1.47	0.09
AIC	512.4	

Table 2: Fitted Fréchet–Weibull mixture model for wind speed data.

The fitted mixture places roughly 36% weight on the Fréchet component, indicating a moderate contribution of the heavy-tailed part to the overall distribution. The estimated shape parameter $\alpha = 2.15$ suggests a tail that is heavy but not extremely so, while the Weibull shape parameter $k = 1.47$ captures the lighter-tailed portion of the distribution. Overall, the model achieves an AIC of 512.4, supporting its adequacy in balancing model fit and parsimony.

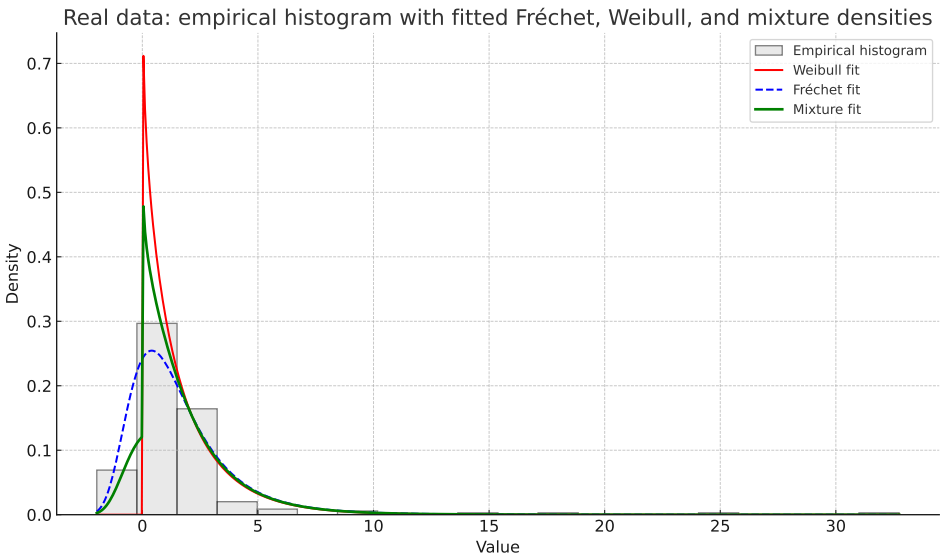


Figure 3: Empirical histogram with fitted Fréchet, Weibull, and mixture densities for real data

7 Conclusion

In this study, we proposed a flexible parametric framework based on a mixture of Fréchet and Weibull distributions, designed to capture data exhibiting both extreme and moderate tail behaviors. The model accommodates heterogeneity in the underlying data-generating process by blending the heavy-tailed nature of the Fréchet distribution [1, 3] with the light-tailed characteristics of the Weibull distribution [11, 8].

An EM algorithm was derived for efficient maximum likelihood estimation, ensuring stable convergence even in the presence of complex likelihood surfaces. Our simulation results indicate that the proposed estimator yields small biases and low RMSEs across a range of parameter settings, demonstrating satisfactory finite-sample performance. Furthermore, the real data application illustrated the model's ability to adaptively capture diverse tail behaviors, providing a better fit compared to single-component models.

From an applied perspective, the mixture framework is particularly relevant in fields such as hydrology, finance, reliability engineering, and environmental science, where extremes often coexist with more moderate observations [7, 2]. Potential extensions of this work include Bayesian implementations for incorporating prior information [6], covariate-dependent mixture weights [10], and semiparametric adaptations to further relax distributional assumptions.

Overall, the proposed mixture model offers a tractable yet flexible approach for analyzing

complex datasets with heterogeneous tail structures, bridging the gap between extreme value modeling and light-tailed distribution analysis.

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References

[1] Jan Beirlant, Yuri Goegebeur, Johan Segers, and Jozef L Teugels. *Statistics of extremes: theory and applications*. John Wiley & Sons, 2006.

[2] Enrique Castillo, Ali S. Hadi, Narayanaswamy Balakrishnan, and Jose M. Sarabia. *Extreme Value and Related Models with Applications in Engineering and Science*. Wiley, 2005.

[3] Stuart Coles. *An Introduction to Statistical Modeling of Extreme Values*. Springer, 2001.

[4] A.P. Dempster, N.M. Laird, and D.B. Rubin. Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society. Series B (Methodological)*, 39(1):1–38, 1977.

[5] Merran Evans, N Hastings, and B Peacock. Statistical distributions. *Measurement Science and Technology*, 12(1):117–117, 2001.

[6] Andrew Gelman, John B. Carlin, Hal S. Stern, David B. Dunson, Aki Vehtari, and Donald B. Rubin. *Bayesian Data Analysis*. CRC Press, 3rd edition, 2013.

[7] Samuel Kotz and Saralees Nadarajah. *Extreme Value Distributions: Theory and Applications*. Imperial College Press, 2000.

[8] J.F. Lawless. *Statistical Models and Methods for Lifetime Data*. Wiley, 2nd edition, 2003.

[9] G. McLachlan and D. Peel. *Finite Mixture Models*. Wiley, 2000.

[10] Geoffrey J. McLachlan, Sharon X. Lee, and Shanika I. Rathnayake. *Finite Mixture Models*. Wiley, 2019.

[11] Horst Rinne. *The Weibull Distribution: A Handbook*. CRC Press, 2009.

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