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Economic Complexity and Environmental Impact using a Neural-Network Embedded Semiparametric Mixture of Experts Model

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Abstract

The environmental Kuznets curve (EKC) postulates an inverted-U relationship between environmental quality and economic performance. Recent studies examine the EKC hypothesis across groups of countries (e.g., developing and OECD) by considering the impact of economic complexity (a measure of economic performance) on CO_2 emissions (a measure of environmental quality). However, most of the studies impose, possibly, restrictive assumptions on the data: (1) all countries are assumed to follow the same developmental path and hence EKC; (2) the functional relationship between CO_2 emissions and economic complexity is assumed to have a known parametric form. In this paper, we propose to relax these assumptions. First, we allow for the possibility that countries may follow different developmental paths by using the mixture of experts (MoE) approach. This gives rise to multiple EKCs. The MoE approach further allows us to endogenously identify different developmental paths and thus accounts for unobserved heterogeneity. Moreover, this approach allows us to probabilistically classify each country into any one of the obtained developmental paths. Second, we assume that the functional relationship between CO_2 emissions and economic complexity is represented by a nonparametric (unknown) function of economic complexity. This allows the EKC the flexibility to take any form based on the data. Therefore, each expert regression model is a partially linear model (PLM). In contrast

to existing PLMs, we allow the nonparametric part to be a multivariate function of any dimension. For model estimation, we propose a maximum likelihood estimation procedure through a hybrid algorithm that combines feed-forward neural networks (NN) with the classical expectation-conditional-maximization (ECM) algorithm, termed the ECM-NN algorithm. We use a simulation study to demonstrate the performance of this estimation procedure across various scenarios with different dimensions of the nonparametric function. Finally, we apply the proposed methods to a cross-country panel dataset of 64 countries spanning 2003–2022 and find two developmental paths in which the EKC holds.

Keywords: Environmental Kuznets Curve (EKC), Economic complexity, Expectation-conditional-maximization (ECM), Mixture of experts, Neural networks, Partially linear models

JEL Classification: C14 , C33 , C45 , Q56

1 Introduction

The nexus between economic development and environmental sustainability has long been a concern in environmental economics, and the traditional framework that anchors this is the Environmental Kuznets Curve (EKC) hypothesis. According to the EKC hypothesis, in the early stages of development, economies tend to prioritise increased production and income growth at the expense of environmental sustainability and other developmental concerns. Consequently, economic expansion at this stage is often associated with rising environmental degradation (see, for example, Dasgupta et al. 2002; Dinda 2004; Stern 2004). However, as income levels increase and greater importance is placed on environmental quality and public health, environmental degradation declines with higher levels of income – perhaps due to adoption of cleaner and more environmentally friendly technologies by producers – hence the bell-shaped Kuznets curve relationship between economic activity and environmental quality (Selden and Song 1994; Shafik 1994; Grossman and Krueger 1995). Meanwhile, growing concern about climate change and its associated risks has further heightened interest in understanding the environmental implications of economic development over the past few decades. In other words, the adverse effects associated with ozone layer depletion and rising greenhouse gas emissions have intensified international concerns about climate change. Subsequently, global initiatives such as the Paris Agreement have emphasised the need to limit global warming to 1.5°C above the preindustrial levels. To achieve this target, it is believed that the greenhouse gas emissions are expected to peak before 2025 and decline significantly by 2030[1]. Thus, identifying the conditions under which economic development can be reconciled with environmental sustainability remains a central policy concern.

Notwithstanding the plethora of empirical studies testing the EKC hypothesis, most studies have continued to employ the gross domestic product (GDP) and/or per capita GDP as the main indicator(s) of economic development (see, for example,

[1] https://unfccc.int/process-and-meetings/the-paris-agreement/.

Bennedsen et al. 2023). Although GDP remains a useful proxy for measuring aggregate economic activity, it is increasingly seen as an incomplete indicator of development. In particular, GDP captures only the scale of economic output and is limited in providing the useful information required to understand the underlying productive capacities, technological sophistication, innovation capacity, and knowledge accumulation that spur long-run economic transformation. In addition, it is not out of place to argue that countries with similar levels of GDP may differ significantly in their productive structures and technological prowess. Therefore, GDP-based measures may not adequately capture the multidimensional nature of development relevant to understanding environmental outcomes. These limitations, therefore, elicit important questions: (i) can alternative measures of economic activity provide a superior explanatory power in examining the nexus between economic development and environmental quality? (ii) can these alternative measures capture dimensions of productive structure that are overlooked by aggregate and/or traditional measures such as GDP?

In response to these concerns, a growing body of literature has focused on developing an index of economic complexity as a more robust alternative measure of economic development. Economic complexity seeks to capture the technical know-how embedded in production processes by examining the diversity and sophistication of the outputs a country produces (Hidalgo and Hausmann 2009; Albeaik et al. 2017; Hidalgo 2021; Balland et al. 2022; Inoua 2023; Stojkoski et al. 2023). Unlike the traditional income-oriented proxy (GDP), variations in economic structures, especially in terms of productive diversity and innovative capacity encompassed in economic complexity, provide deeper insights into the nature of modern economic development and its implications for environmental sustainability.

The increasing availability of complexity measures has birthed a strand of literature evaluating the empirical relationships between economic complexity and environmental fundamentals, including green growth (Stojkoski et al. 2023), carbon emissions (Agozie et al. 2022; Ansari 2022; Hamrouni et al. 2025), energy transition (Adekoya et al. 2023; Bakhsh et al. 2024), environmental degradation (Aluko et al. 2023), and sustainable development (Zhang et al. 2024a,b). Improvements in economic complexity are associated with greater technological advancement and increased adoption of innovative production techniques driven by research and development. These developments are essential for reducing dependence on carbon-intensive production inputs and lowering the global carbon footprint. Hence, improvements in economic complexity may reduce carbon-intensive production activities and support environmental sustainability. While the foregoing studies provide essential insights into the environmental implications of economic complexity, they primarily examine average relationships and implicitly assume that the sample countries follow a common environmental (development) path.

We argue that this methodological path may be too narrow, as countries differ considerably across factors such as institutional quality, environmental regulation, technological know-how, and industrial and energy composition, among other components of development. These differences suggest that the nexus between development and environmental quality may not be sufficiently characterised by a single EKC applicable to all countries. Rather, multiple latent environmental development pathways

may exist, with distinct EKC dynamics observed across different groups of countries. Ignoring this important heterogeneity may bias estimates, lead to misleading policy implications, and, by extension, limit our understanding of the environmental implications of economic development. It is important to note that earlier studies have highlighted the importance of heterogeneity in the relationship between economic development and environmental quality (Bertinelli and Strobl 2005; Dijkgraaf and Vollebergh 2005; Azomahou et al. 2006); however, relatively little attention has been given to modelling latent or unobserved heterogeneity within the EKC framework.

To address this heterogeneity, mixture of experts (MoE) models have been proposed. The MoE models combine finite mixture modelling with probabilistic classification, allowing observations to be assigned to different latent regimes according to estimated probabilities (see Jacobs et al. 1991; Jordan and Jacobs 1994). Thus, instead of imposing a single relationship across the sample, the MoE model allows various sub-samples to exhibit distinctive behavioural patterns. This flexibility has led to successful applications in various studies where latent heterogeneity is often encountered (see, for example, Alfo et al. 2008; Frühwirth-Schnatter et al. 2012; Konte 2018; Sawadogo and Ouoba 2024; Patel 2025). Put differently, the MoE models provide a probabilistic mechanism for detecting heterogeneity across groups and simultaneously estimating group-specific relationships.

However, most existing mixture of experts applications assume that the expert regression functions follow known parametric specifications. Such assumptions may be problematic when the underlying functional relationship is unknown or highly nonlinear, as is often the case in an EKC-related study such as this. In the context of the EKC hypothesis, imposing a specific functional form may constrain the shape of the relationship and potentially mask important nonlinearities, multiple turning points, or regime-specific dynamics. Consequently, there is a need for more flexible modelling frameworks that allow the data to determine the shape of the relationship while simultaneously accounting for unobserved heterogeneity.

Semiparametric mixture of regression models provides a promising solution to this challenge. By combining parametric and nonparametric components within a finite mixture framework, these models offer substantially greater flexibility than conventional parametric approaches. In particular, semiparametric mixtures allow complex nonlinear relationships to be modelled without imposing restrictive assumptions regarding functional form, while simultaneously accommodating latent heterogeneity across observational units. Despite these advantages, their application in economics remains relatively limited, particularly within environmental economics and EKC research. Given the increasing recognition of heterogeneity and nonlinearity in environment-economic development relationships, semiparametric mixture models have considerable potential for advancing empirical investigations in this area. A notable contribution in this direction is provided by (Bennedsen et al. 2023). Nevertheless, as detailed in the subsequent paragraphs, our study departs from theirs in several key respects. Most importantly, we focus on economic complexity rather than GDP as the primary indicator of development. In addition, we propose a hybrid Expectation Conditional Maximization Neural Network (ECM–NN) estimation procedure that combines machine-learning techniques with likelihood-based estimation to

enhance computational efficiency and flexibility. We further introduce a probabilistic classification framework that allocates countries to latent regimes according to their estimated membership probabilities, thereby yielding richer insights into cross-country heterogeneity.

Against this background, this study proposes a semiparametric mixture of experts framework for investigating the Environmental Kuznets Curve using economic complexity as a measure of economic development. Specifically, we employ a mixture of partially linear experts model (Hwang et al. 2025) in which the nonparametric component of each expert regression is specified as a function of economic complexity. This framework allows the relationship between economic complexity and carbon emissions to vary across latent regimes, thereby accommodating both nonlinearity and unobserved heterogeneity. By avoiding restrictive functional-form assumptions, the model permits the data to determine the shape of the environmental-development relationship within each latent group while simultaneously providing probabilistic classification of countries.

To estimate the proposed model, we develop a hybrid maximum-likelihood estimation procedure that combines neural networks with the traditional expectation conditional maximisation (ECM) algorithm (Meng and Rubin 1993), referred to as the ECM-NN algorithm. The proposed estimation strategy exploits neural networks' approximation capabilities while preserving the desirable statistical properties of likelihood-based estimation. Furthermore, unlike conventional partially linear models, the proposed framework accommodates multivariate nonparametric functions of arbitrary dimension, thereby substantially enhancing modelling flexibility. The performance of the proposed ECM-NN algorithm is evaluated through a comprehensive Monte Carlo simulation study.

We demonstrate the practical application of the proposed method by examining the nexus between economic complexity and carbon emissions using the economic complexity index (ECI) dataset developed by (Stojkoski et al. 2023). The index evaluates countries based not merely on the volume of exports, but on the diversity and sophistication of the products they are capable of producing and exporting. Economies that manufacture a wide range of specialised and less easily replicable products are assigned higher complexity scores, reflecting the extent of productive capabilities and embedded knowledge within the economy. While economic complexity is measured along four dimensions of trade flows (ECI Trade), patents (ECI Technology), research publications (ECI Research), and software employment (ECI Software), with each dimension representing a distinct channel of productive knowledge, this study primarily relies on ECI Trade in examining the Environmental Kuznets Curve (EKC) hypothesis. The choice of ECI Trade is motivated by the fact that trade-based complexity captures the productive structure of economies through internationally observable economic activities, thereby providing a closer proxy for aggregate economic activity than the other ECI variants. Furthermore, ECI Trade remains the dominant measure in the empirical literature on economic complexity and environmental quality, including studies examining the EKC hypothesis (See, for example, Laverde-Rojas and Correa 2021; Bucher et al. 2023; Duran et al. 2026).

This paper makes the following contributions:

(1) Extending the EKC literature by explicitly accounting for unobserved heterogeneity and allowing for the existence of multiple latent environmental-development trajectories rather than imposing a common EKC pathway across all countries.
(2) Contributing to the growing economic complexity literature by employing economic complexity, rather than GDP or per capita GDP, as the main indicator of economic development in studying the EKC hypothesis.
(3) Introducing a semiparametric mixture of experts framework to the EKC literature, thereby providing a flexible and data-driven approach for modelling heterogeneous nonlinear relationships. To the best of our knowledge, this is the first application of a semiparametric mixture of regressions model to the study of the EKC.
(4) Developing and implementing a hybrid ECM-NN estimation procedure that combines machine learning techniques with likelihood-based estimation to improve computational efficiency and flexibility.
(5) Proposing a probabilistic classification framework that assigns countries to latent regimes based on estimated membership probabilities, thereby facilitating a better understanding of heterogeneity in environmental and economic development dynamics across countries.

Based on a cross-country panel dataset of 64 countries spanning 2003 - 2022, our findings reveal two distinct groups of countries. By fitting nonparametric functions of economic complexity (specifically ECI Trade) for both groups, evidence consistent with the EKC hypothesis is observed in the relationship between ECI and environmental quality (CO_2 emissions).

The remainder of the paper is structured as follows. Section 2 presents the methodological framework and estimation procedure. Section 3 highlights a simulation study to demonstrate the performance of the proposed methods. Section 4 presents and discusses the results of an empirical application of the proposed methods to a real cross-country panel dataset of 64 countries over the period 2003–2022. Finally, Section 5 concludes the paper and gives directions for future research.

2 Methodology

In this section, we present the methodology for our empirical study.

2.1 Partially linear model

Our main assumption in this paper is that economic complexity has an unknown nonlinear impact on the environment. Therefore, our baseline model is the following partially linear model

$$y = g(\mathbf{z}) + \mathbf{x}^\top \boldsymbol{\beta} + \epsilon, \tag{1}$$

where y is our response variable represented by CO_2 emissions. The variable $\mathbf{z}$ measures the economic complexity. The function $g(\cdot)$ is an unknown but smooth (hence, nonparametric) function of $\mathbf{z}$. The vector $\mathbf{x}$ includes measurements of the control variables. The error term ϵ is assumed to be normally distributed with mean 0 and variance

σ^2 and also independent of the covariates $\mathbf{z}$ and $\mathbf{x}$ (Konte 2018).
The key term in model (1) is $g(\mathbf{z})$, which tells us the marginal impact of changes in economic complexity on the environment. As already mentioned, we assume that this impact is nonlinear, similar to the impact of economic growth on the environment, which tends to exhibit an inverted U-shaped form popularly known as the environmental Kuznets curve (EKC) (Dinda 2005).
Model (1) assumes that all countries follow the same EKC or developmental path. Moreover, the model assumes that the impact of the normally distributed factors with mean 0 and variance σ^2 (Konte 2018) and of $\mathbf{x}$ on the environment, y, is the same across countries. These assumptions may be questionable in practice, as countries behave differently. Therefore, a model that accounts for multiple developmental paths may provide a better fit than model (1). We now introduce a probabilistic extension of model (1) that enables testing for multiple developmental paths. Moreover, it allows us to test whether the effect of economic complexity on the environment is heterogeneous and depends on the developmental path a given country follows.

2.2 Mixture of partially linear experts

Mixture of experts (MoE) models provide a flexible framework for modelling unobserved heterogeneity in regression (Jacobs et al. 1991). This modelling framework is suitable for detecting latent or unobserved growth regimes or developmental paths without any prior assumptions (Konte 2018). Let K denote the number of latent groups and assume that the error terms are independent and identically distributed from the normal distribution, then the MoE model is given by

$$f(y|\mathbf{X}=\mathbf{x},\mathbf{T}=\mathbf{t}) = \sum_{k=1}^{K} \pi_k(\mathbf{t}) f_k(y|m_k(\mathbf{x}),\sigma_k^2), \tag{2}$$

where $m_k(\mathbf{x}) = \mathbb{E}\{y|\mathbf{X}=\mathbf{x}\}$ is the expert regression function of the k^{th} latent component with a $p-$dimensional vector of covariates, $\mathbf{x}\in\mathbb{R}^p$; $f_k(\cdot|\mu_k,\sigma_k^2)$ is the normal density function with mean μ_k and variance σ_k^2. Finally, $\pi_k(\mathbf{t})$ is the mixing probability function, typically modelled as

$$\pi_k(\mathbf{t};\boldsymbol{\alpha}) = \frac{\exp\{\mathbf{t}^\top\boldsymbol{\alpha}_k\}}{\sum_{k=1}^{K}\exp\{\mathbf{t}^\top\boldsymbol{\alpha}_\ell\}}, \tag{3}$$

where $\boldsymbol{\alpha}_k$ is the $(q+1)-$dimensional coefficient vector associated with $(1,\mathbf{t})\in\mathbb{R}^{q+1}$, the design vector of concomitant variables, and $\boldsymbol{\alpha}=(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\ldots,\boldsymbol{\alpha}_{K-1})$, with $\boldsymbol{\alpha}_k = \mathbf{0}$ for model identifiability. The function (3) satisfies the conditions $\pi_k(\mathbf{t}) > 0$ and $\sum_{k=1}^{K}\pi_k(\mathbf{t}) = 1$, for all $\mathbf{t}$ and $k = 1,2,\ldots,K$.
Assuming that each expert regression function takes an equivalent form to model (1) defined as

$$\mathbb{E}\{y|\mathbf{X}=\mathbf{x},\mathbf{Z}=\mathbf{z}\} = m_k(\mathbf{x},\mathbf{z}) = g_k(\mathbf{z}) + \mathbf{x}^\top\boldsymbol{\beta}_k. \tag{4}$$

Then the mixture of partially linear experts (MoPLE) (Hwang et al. 2025) is obtained as

$$f\left(y|\mathbf{X}=\mathbf{x},\mathbf{T}=\mathbf{t},\mathbf{Z}=\mathbf{z}\right)=\sum_{k=1}^{K}\pi_k(\mathbf{t};\boldsymbol{\alpha})f_k\left(y|m_k(\mathbf{x},\mathbf{z}),\sigma_k^2\right), \tag{5}$$

where $m_k(\mathbf{x},\mathbf{z})$ is defined as (4). In model (5), the functions, $g_k(\cdot)$, and coefficients $\boldsymbol{\beta}_k$ may differ across the different regimes $k=1,2,\ldots,K$. This model accounts for the possibility of latent (unobserved) factors, which may lead countries to follow different developmental paths. In model (5), the function (3) quantifies the probability that a given country belongs to the k^{th} latent group given the covariates $\mathbf{t}$. Note that the belongingness of each country to a specific latent group is not allowed to switch over time, as in Konte (2018). Furthermore, note that the covariates $\mathbf{t}$ and $\mathbf{z}$ can be part of $\mathbf{x}$ or different.
When $\pi_k(\mathbf{t}_i)=\pi_k$ for all $\mathbf{t}_i$, then model (5) is the mixture of partially linear models (MoPLM) proposed by Wu and Liu (2017):

$$f\left(y|\mathbf{X}=\mathbf{x},\mathbf{T}=\mathbf{t},\mathbf{Z}=\mathbf{z}\right)=\sum_{k=1}^{K}\pi_k f_k\left(y|m_k(\mathbf{x},\mathbf{z}),\sigma_k^2\right). \tag{6}$$

This model is useful as a first estimate to determine whether the data have distinct latent groups, in which the marginal effects of the covariates differ across groups.
Note that, in order to avoid the curse-of-dimensionality (Xue et al. 2025), the models (5) and (6) were developed for the case where $\mathbf{z}\in\mathbb{R}^Q$ is one-dimensional, that is $Q=1$. In this paper, we extend the model to accommodate the case of multi-dimensional $\mathbf{z}$. Hence $\mathbf{z}=(z_1,z_2,\ldots,z_r)\in\mathbb{R}^r$, where $r>>1$.

2.3 Model estimation

To estimate model (5), we make use of the maximum likelihood estimation via the expectation-conditional-maximization (ECM) (Meng and Rubin 1993) as in Hwang et al. (2025). In contrast to Hwang et al. (2025), who used the Nadaraya-Watson estimator to estimate the nonparametric functions $g_k(\cdot)$, $k=1,2,\ldots,K$, we propose an estimation procedure similar to Xue et al. (2025) and make use of a machine learning procedure to estimate the nonparametric functions. This approach allows us to fit nonparametric functions of any dimension without negatively influencing the performance of the estimators, as is the case for the Nadaraya-Watson estimator. In other words, this approach overcomes the curse-of-dimensionality. Next, we outline the proposed hybrid estimation procedure for model (5) and (6). Since (6) is a special case of (5), the discussion will be about (5).
Consider a random sample $\{(\mathbf{x}_i,\mathbf{t}_i,\mathbf{z}_i,y_i):i=1,2,\ldots,n\}$ from the model (5), the log-likelihood is given by

$$\ell\{\boldsymbol{\theta},\boldsymbol{g}\}=\sum_{i=1}^{n}\log\left[\sum_{i=1}^{K}\pi_k(\mathbf{t}_i;\boldsymbol{\alpha})f_k(y_i|g_k(\mathbf{z}_i)+\mathbf{x}_i^\top\boldsymbol{\beta}_k,\sigma_k^2)\right], \tag{7}$$

where $\boldsymbol{\theta} = (\boldsymbol{\alpha}; \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_K; \sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2)$ is the vector of all the model parameters and $\boldsymbol{g} = (g_1(\cdot), g_2(\cdot), \ldots, g_K(\cdot))$ contains all the nonparametric functions.
We maximize $\ell(\boldsymbol{\theta}, \boldsymbol{g})$ using a hybrid algorithm that combines neural networks (Bishop 2006) with the expectation-conditional-maximization (ECM) algorithm (Meng and Rubin 1993), which we call the ECM-NN algorithm. We first define the latent indicator variable $\mathbf{v}_i = (v_{i1}, v_{i2}, \ldots, v_{iK})$, for $i = 1, 2, \ldots, n$, where $v_{ik} = 1$ if the i^{th} country belongs to the k^{th} regime or 0 otherwise. Given the sample $\{(\mathbf{x}_i, \mathbf{t}_i, \mathbf{z}_i, y_i, \mathbf{v}_i) : i = 1, 2, \ldots, n\}$, we define the complete-data log-likelihood as

$$\ell_c(\boldsymbol{\theta}, \boldsymbol{g}) = \sum_{i=1}^{n} \sum_{k=1}^{K} v_{ik} \log \left[\pi_k(\mathbf{t}_i; \boldsymbol{\alpha}) f_k(y_i | g_k(\mathbf{z}_i) + \mathbf{x}_i^\top \boldsymbol{\beta}_k, \sigma_k^2) \right] \tag{8}$$

The proposed ECM-NN algorithm starts with appropriate initial values $\boldsymbol{g}^{(0)}$ and $\boldsymbol{\theta}^{(0)}$ and then iterates between the expectation (E-) step and two conditional maximization (CM-) steps. In the E-step, we calculate the conditional expected value of $\ell_c\{\boldsymbol{\theta}, \boldsymbol{g}\}$, denoted by $Q(\boldsymbol{\theta}, \boldsymbol{g} | \boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$, given the observed data and the current estimates $(\boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$. In the first CM-step (CM-Step 1), we maximize $Q(\boldsymbol{\theta}, \boldsymbol{g} | \boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$ with respect to $\boldsymbol{\theta}$ by holding $\boldsymbol{g}$ fixed at $\boldsymbol{g}^{(\text{old})}$. In CM-step 2, we maximize $Q(\boldsymbol{\theta}, \boldsymbol{g} | \boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$ by holding $\boldsymbol{\theta}$ fixed at $\boldsymbol{\theta}^{(\text{new})}$ from CM-step 1.

E-Step

The conditional expected value of $\ell_c\{\boldsymbol{\theta}, \boldsymbol{g}\}$ is given by

$$Q(\boldsymbol{\theta}, \boldsymbol{g} | \boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})}) = \sum_{i=1}^{n} \sum_{k=1}^{K} \gamma_{ik}^{(\text{new})} \log \left[\pi_k(\mathbf{t}_i; \boldsymbol{\alpha}) f_k(y_i | g_k(\mathbf{z}_i) + \mathbf{x}_i^\top \boldsymbol{\beta}_k, \sigma_k^2) \right], \tag{9}$$

where $\gamma_{itk}^{(\text{new})}$ is the expected value of v_{ik} given by

$$\gamma_{ik}^{(\text{new})} = \mathbb{E}\{v_{ik} | \mathbf{x}_i, \mathbf{t}_i, \mathbf{z}_i, y_i\} = \frac{\pi_k(\mathbf{t}_i; \boldsymbol{\alpha}^{(\text{old})}) f_k\left(y_i | g_k^{(\text{old})}(\mathbf{z}_i) + \mathbf{x}_i^\top \boldsymbol{\beta}_k^{(\text{old})}, \sigma_k^{2(\text{old})}\right)}{\sum_{\ell=1}^{K} \pi_\ell(\mathbf{t}_i; \boldsymbol{\alpha}^{(\text{old})}) f_\ell\left(y_i | g_\ell^{(\text{old})}(\mathbf{z}_i) + \mathbf{x}_i^\top \boldsymbol{\beta}_\ell^{(\text{old})}, \sigma_\ell^{2(\text{old})}\right)}.$$

CM-Step 1

In the first CM step, we fix $\boldsymbol{g}$ at $\boldsymbol{g}^{(\text{old})}$ and maximize $Q(\boldsymbol{\theta}, \boldsymbol{g} | \boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$ with respect to $\boldsymbol{\theta}$ to obtain

$$\boldsymbol{\beta}_k^{(\text{new})} = (\mathbf{X}^\top \mathbf{W}_k \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{W}_k (\mathbf{y} - \boldsymbol{g}_k), \tag{10}$$

$$\sigma_k^{2(\text{new})} = \frac{\sum_{i=1}^{n} \gamma_{ik}^{(\text{new})} (y_{i,t} - \mathbf{x}_i^\top \boldsymbol{\beta}_k^{(\text{new})} - g_k^{(\text{old})}(\mathbf{z}_{i,t}))^2}{\sum_{i=1}^{n} \gamma_{itk}^{(\text{new})}}, \tag{11}$$

where $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$ is an $n \times p$ matrix of covariates, $\mathbf{y} = (y_1, \ldots, y_n)$ and $\mathbf{W}_k = \text{diag}\{\gamma_{1k}^{(\text{new})}, \ldots, \gamma_{nk}^{(\text{new})}\}$.
Note that the likelihood estimator of $\boldsymbol{\alpha}$ does not have a closed-form expression. Thus, for maximizing $Q(\boldsymbol{\theta}, \boldsymbol{g}|\boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$ with respect to $\boldsymbol{\alpha}$ we make use of the Newton-Raphson algorithm. Given an appropriate starting value $\boldsymbol{\alpha}^{(0)}$, at the s^{th} iteration of the Newton-Raphson algorithm, we update $\boldsymbol{\alpha}$ as follows

$$\boldsymbol{\alpha}^{(s)} = \boldsymbol{\alpha}^{(s-1)} - \mathbf{H}^{-1}[\boldsymbol{\alpha}^{(s-1)}]\mathbf{g}[\boldsymbol{\alpha}^{(s-1)}], \tag{12}$$

where $\mathbf{H}[\boldsymbol{\alpha}^{(s-1)}]$ and $\mathbf{g}[\boldsymbol{\alpha}^{(s-1)}]$ are the Hessian matrix and gradient vector, respectively, evaluated at $\boldsymbol{\alpha}^{(s-1)}$. The value of $\boldsymbol{\alpha}^{(\text{new})}$ is taken as the optimal value of $\boldsymbol{\alpha}$ at convergence of the updating rule in (12). For the MoPLM (6), the mixing probability π_k, for $k = 1, 2, \ldots, K$, is updated as follows

$$\pi_k^{(new)} = \frac{\sum_{i=1}^{n} \gamma_{ik}^{(new)}}{n}.$$

Let $\boldsymbol{\theta}^{(\text{new})}$ be the vector of the updated parameters.

CM-Step 2

In the second CM step, we fix $\boldsymbol{\theta}$ at $\boldsymbol{\theta}^{(\text{new})}$ and update $g_k(\cdot)$, for $k = 1, 2, \ldots, K$, making use of the two-layer feed forward neural network (FNN) as defined in Bishop (2006) and Mambondimumwe et al. (2026). First, note that the nonparametric function can be defined as

$$g_k(\mathbf{z}) = \mathbb{E}\{y - \mathbf{x}^\top \boldsymbol{\beta}_k | \mathbf{z}\}. \tag{13}$$

Thus, $g_k(\mathbf{z})$ is a regression function with response variable $y - \mathbf{x}^\top \boldsymbol{\beta}_k$ and covariate $\mathbf{z}$. Let $r = y - \mathbf{x}^\top \boldsymbol{\beta}_k$ and take $\mathbf{z}$ as an input. The two-layer FNN is given by

$$\begin{aligned} y(\mathbf{z}, \mathbf{w}_k) &= f^{(2)}\left(w_{k0}^{(2)} + \sum_{j=1}^{M} w_{kj}^{(2)} f^{(1)}\left(w_{k0}^{(1)} + w_{k1}^{(1)} z_1 + \cdots + w_{kQ}^{(1)} z_Q\right)\right) \\ &=: g_k(\mathbf{z}), \end{aligned} \tag{14}$$

where $w_{k0}^{(1)}$ and $w_{k0}^{(2)}$ are the bias parameters; and $w_{kj}^{(1)}$ and $w_{kj}^{(2)}$, for $j = 1, 2, \ldots, M$, are the weight parameters. The functions $f^{(1)}(\cdot)$ and $f^{(2)}(\cdot)$ are the hidden layer and output layer activation functions, respectively. Finally, M is the number of units in the hidden layer. We use the ReLU activation function (Nair and Hinton 2010) in the hidden layer and the identity activation function in the output layer. The vector $\mathbf{w}_k$ holds all the bias and weight parameters for the k^{th} growth regime.
The weights $\mathbf{w}_k$ can be obtained by maximizing $Q(\boldsymbol{\theta}, \boldsymbol{g}|\boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$ after substituting (14) into $Q(\boldsymbol{\theta}, \boldsymbol{g}|\boldsymbol{\theta}^{(\text{old})}, \boldsymbol{g}^{(\text{old})})$. This is equivalent to minimizing the following weighted

sum-of-squared error loss function

$$L(\mathbf{w}_k) = \frac{1}{2}\sum_{i=1}^{n} \gamma_{ik}^{(\text{new})} ||r_i - y(\mathbf{z}_i, \mathbf{w}_k)||^2. \tag{15}$$

Let $\mathbf{w}_k$, for $k = 1, 2, \ldots, K$, be the vector of weights and biases obtained from minimizing (15). Then, the updated nonparametric functions are given by $g_k^{(\text{new})}(\mathbf{z}) = y(\mathbf{z}, \mathbf{w}_k^{(\text{new})})$, for $k = 1, 2, \ldots, K$.
We repeat the above E- and CM-steps until convergence.

2.4 Choosing the number of latent growth regimes, K

In practice, the number of growth regimes, K, is unknown and must be estimated before fitting the model to the data. This is a model selection problem, typically solved using a data-driven approach. In this paper, we propose to use the information criteria: Akaike information criterion (AIC) (Akaike 1974), Bayesian information criterion (BIC) (Schwarz 1978), integrated classification likelihood (ICL) (Biernacki et al. 2000) and PanIC (Nguyen 2024). We estimate model (5), for $K = 1, 2, \ldots, K_{\max}$, where $K_{\max}$ is the largest possible value of K we consider. We calculate AIC(K), BIC(K), ICL(K) and PanIC(K) for all these values of K. The optimal value of K, denoted by $\hat{K}$, is given by

$$\hat{K} = \min_{K=1,2,\ldots,K_{\max}} \text{IC}(K), \tag{16}$$

where IC can be any of the four information criteria.

2.5 Model-based clustering

Once the model (5) has been estimated, we can allocate each data point into one of the K estimated regimes based on the posterior probability

$$\begin{aligned} P(i^{th} \text{ data point belongs to } k^{th} \text{ regime}) &= \hat{\eta}_{ik} \\ &= \frac{\pi_k(\mathbf{t}_i; \hat{\boldsymbol{\alpha}}) f_k(y_i | \hat{g}_k(z_i) + \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}_k, \hat{\sigma}_k^2)}{\sum_{\ell=1}^{K} \pi_\ell(\mathbf{t}_i; \hat{\boldsymbol{\alpha}} f_\ell(y_i | \hat{g}_\ell(z_i) + \mathbf{x}_i^\top \hat{\boldsymbol{\beta}}_\ell, \hat{\sigma}_\ell^2)}. \end{aligned} \tag{17}$$

The i^{th} country is assigned to the k^{th} if the probability $\hat{\eta}_{ik}$ is larger than the probability $\hat{\eta}_{jk}$ for all $k \neq j$.

3 Simulation study

In this section, we conduct a simulation study to demonstrate the performance of the proposed estimation procedure. More specifically, we compare the performance of the ECM-NN algorithm with the estimation procedure of Hwang et al. (2025) for fitting the MoPLE model (5) using local kernel estimation via the ECM algorithm (henceforth, ECM-Kernel). The main purpose of the simulation study is to show that

the proposed ECM-NN algorithm performs well even for the case when each $g_k(\mathbf{z})$ is a high-dimensional multivariate function. This is in contrast to the kernel approaches, which are known to experience a decline in performance in high-dimension (Fan and Gijbels 1996), a phenomenon famously known as the curse-of-dimensionality (Bishop 2006). We consider three scenarios of varying dimensions for $\mathbf{z}$:

Scenario 1 In this scenario, we consider the simple case of $K = 2$ univariate functions:

$$g_1(z) = -3z, \qquad \text{and} \qquad g_2(z) = 3z,$$

$\mathbf{z} = z$, where z is generated from the uniform distribution on the interval $(0, 1)$.
Scenario 2 In this scenario, we consider $K = 2$ three (3)-dimensional functions:

$$g_1(\mathbf{z}) = 2\sin(2\pi z_1) + 3z_2^2 + \exp(-2z_3), \quad \text{and} \quad g_2(\mathbf{z}) = 4\cos(\pi z_1 z_2) + 2\sqrt{z_3} + z_1 z_3,$$

$\mathbf{z} = (z_1, z_2, z_3)$, where z_j, for $j = 1, 2, 3$, is generated from the uniform distribution on the interval $(0, 1)$.
Scenario 3 In this scenario, we consider $K = 2$ five (5)-dimensional functions:

$$\begin{aligned} g_1(\mathbf{z}) &= 5\sin(\pi z_1 z_2) + 2z_3^3 - 3z_4^2 + \exp(z_5), \qquad \text{and} \\ g_2(\mathbf{z}) &= 4\cos(2\pi z_1) + \frac{3}{1 + \exp(-4z_2)} + 2z_3 z_4 + \log(z_5 + 2), \end{aligned}$$

$\mathbf{z} = (z_1, z_2, z_3, z_4, z_5)$, where z_j, for $j = 1, 2, 3, 4, 5$, is generated from the uniform distribution on the interval $(-1, 1)$.

In each scenario, we generate 100 samples of sizes $n = 100, 500, 1000$ and 2000 from a $K = 2$ regime MoPLE model (5). In all the scenarios, we generate the covariates $(\mathbf{x}, \mathbf{t})$ such that $\mathbf{t} = (1, x)^\top$ and $\mathbf{x} = x$, where x is generated from the uniform distribution on the interval $(-1, 1)$. And we use the parameter values given in Table 1. We calculate the accuracy of the estimated model parameters using the mean squared errors (MSE) and BIAS of the parameters as

$$\text{MSE}(\hat{\theta}_k) = (\hat{\theta}_k - \theta_k)^2, \quad \text{and} \quad \text{BIAS}(\hat{\theta}_k) = (\hat{\theta}_k - \theta_k), \tag{18}$$

where $\hat{\theta}_k$ and θ_k are the estimated and true parameter values, respectively. To measure the goodness of the fitted functions $\hat{g}_k$, for $k = 1, 2$, we calculate the root MSE (RMSE)

$$\left(\frac{1}{n} \sum_{k=1}^{K} \sum_{i=1}^{n} \left(g_k(\mathbf{z}_i) - \hat{g}_k(\mathbf{z}_i) \right)^2 \right)^{1/2}, \tag{19}$$

where $g_k(\mathbf{z})$ and $\hat{g}_k(\mathbf{z})$ are the true and fitted functions, respectively. Figures 1–3 give the RMSE values of the fitted nonparametric functions $\hat{g}_k(\cdot)$, for $k = 1, 2$, obtained using the proposed ECM–NN estimation procedure and the ECM–Kernel procedure of Hwang et al. (2025), for Scenarios 1–3, respectively. The figures show that ECM–NN outperforms ECM–Kernel, especially when $\mathbf{z}$ is high-dimensional (Scenario 3). Table

Table 1: Parameter values used in the simulations

	Parameters		
Regime 1	$\boldsymbol{\alpha}_1 = (0, 10)$	$\beta_1 = 5$	$\sigma_1^2 = 0.25$
Regime 2	$\boldsymbol{\alpha}_2 = (0, 0)$	$\beta_2 = -1$	$\sigma_2^2 = 0.25$

2 presents the fits for the remaining model parameters across the three scenarios. The results from the table further confirm the effectiveness of the proposed ECM–NN for estimating the nonparametric function in high-dimensional settings.

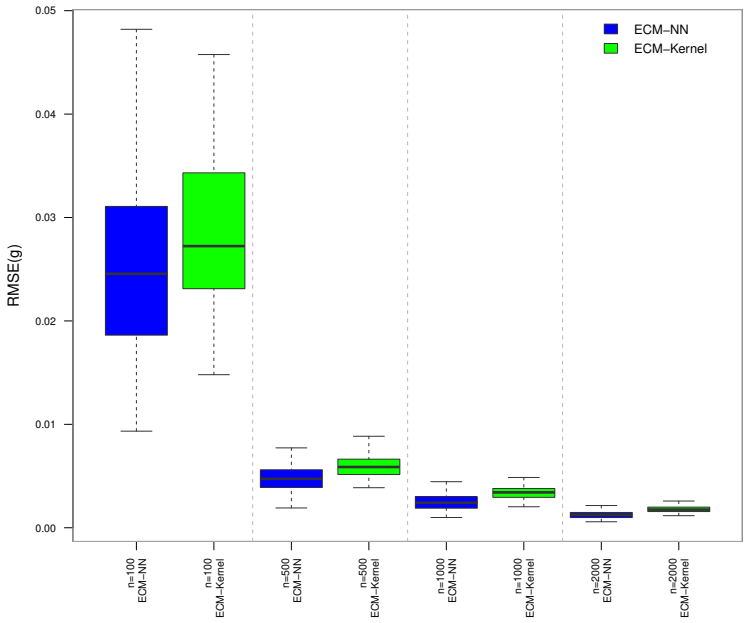


Fig. 1: The RMSE of the fitted nonparametric functions $\hat{g}_k(\cdot)$, for $k = 1, 2$, in Scenario 1 using the ECM-NN procedure and the ECM-Kernel procedure for $n = 100, 500, 1000$ and 2000

4 Application

In this section, we apply the proposed methods to a real-world dataset. We consider a balanced cross-country panel dataset comprising 64 countries over the period 2003–2022. To smooth short-term fluctuations and capture medium-term dynamics, the annual data are transformed into four non-overlapping 5-year panel datasets of these

Table 2: MSE and BIAS values calculated for comparing ECM-NN and ECM-Kernel based on Scenarios 1–3 calculated over the 100 replications

			$n = 100$		$n = 500$		$n = 1000$		$n = 2000$	
Scenario	Parameter		ECM-Kernel	ECM-NN	ECM-Kernel	ECM-NN	ECM-Kernel	ECM-NN	ECM-Kernel	ECM-NN
Scenario 1	β_1	Bias	0.073	0.093	0.027	0.013	−0.006	0.0024	−0.0035	−0.0031
		MSE	0.062	0.298	0.010	0.010	0.006	0.0057	0.0029	0.0028
	β_2	Bias	0.046	−0.013	0.010	0.013	−0.011	0.006	0.0045	0.0096
		MSE	0.063	0.064	0.011	0.010	0.005	0.0048	0.0023	0.0023
	σ_1^2	Bias	0.024	0.028	0.003	0.005	−0.0007	0.0008	0.0011	0.0017
		MSE	0.003	0.003	0.001	0.001	0.0002	0.0002	0.0001	0.0001
	σ_2^2	Bias	0.022	−0.002	0.006	0.007	0.0012	0.0027	0.0014	0.0022
		MSE	0.003	0.060	0.001	0.001	0.0003	0.0003	0.0001	0.0001
	$\alpha_{0,1}$	Bias	−0.060	−0.016	0.022	0.038	0.0094	0.0117	−0.0013	−0.0052
		MSE	0.443	0.452	0.055	0.055	0.0267	0.0267	0.0096	0.0096
	$\alpha_{1,1}$	Bias	−0.853	−0.884	−0.583	−0.555	−0.3141	−0.271	−0.0016	0.0087
		MSE	8.485	10.687	2.367	2.349	0.8585	0.8372	0.5251	0.5182
Scenario 2	β_1	Bias	0.753	−0.099	0.104	0.054	−0.013	0.057	−0.057	0.051
		MSE	1.941	0.135	0.122	0.019	0.018	0.011	0.008	0.006
	β_2	Bias	−0.278	−0.066	−0.027	−0.018	−0.074	−0.021	−0.034	−0.008
		MSE	0.414	0.100	0.037	0.013	0.014	0.007	0.005	0.004
	σ_1^2	Bias	0.113	−0.043	0.049	−0.116	0.078	−0.131	0.062	−0.089
		MSE	0.017	0.015	0.008	0.024	0.007	0.037	0.004	0.008
	σ_2^2	Bias	0.062	−0.007	0.074	−0.095	0.032	−0.247	0.044	−0.088
		MSE	0.012	0.003	0.008	0.016	0.004	0.122	0.002	0.016
	$\alpha_{0,1}$	Bias	0.110	−0.007	0.030	0.037	0.081	0.051	0.029	0.060
		MSE	0.350	0.385	0.059	0.065	0.038	0.041	0.013	0.018
	$\alpha_{1,1}$	Bias	2.066	−1.537	0.505	−0.668	1.122	−0.540	0.358	−0.444
		MSE	18.152	11.892	3.763	2.564	3.166	1.592	1.174	0.670
Scenario 3	β_1	Bias	2.510	0.878	2.514	0.402	2.529	0.377	2.352	0.315
		MSE	6.592	1.002	6.629	0.183	6.585	0.153	5.688	0.106
	β_2	Bias	−1.869	−0.100	−1.612	−0.008	−2.540	0.159	−2.328	−0.003
		MSE	4.317	0.348	3.087	0.013	6.567	0.043	5.602	0.009
	σ_1^2	Bias	0.237	−0.519	0.245	−0.596	0.244	−1.181	0.242	−0.658
		MSE	0.057	1.678	0.060	2.361	0.059	5.686	0.058	3.636
	σ_2^2	Bias	0.249	−0.294	0.242	−0.447	0.243	−1.471	0.245	−0.616
		MSE	0.062	0.101	0.058	0.215	0.059	8.340	0.060	0.899
	$\alpha_{0,1}$	Bias	0.466	0.101	−0.097	−0.122	0.065	−0.003	0.162	−0.066
		MSE	0.515	0.641	0.155	0.058	0.143	0.046	0.057	0.016
	$\alpha_{1,1}$	Bias	8.217	−0.594	8.811	2.278	8.225	2.113	8.056	1.717
		MSE	67.778	14.641	77.709	5.729	67.753	4.618	64.977	3.150

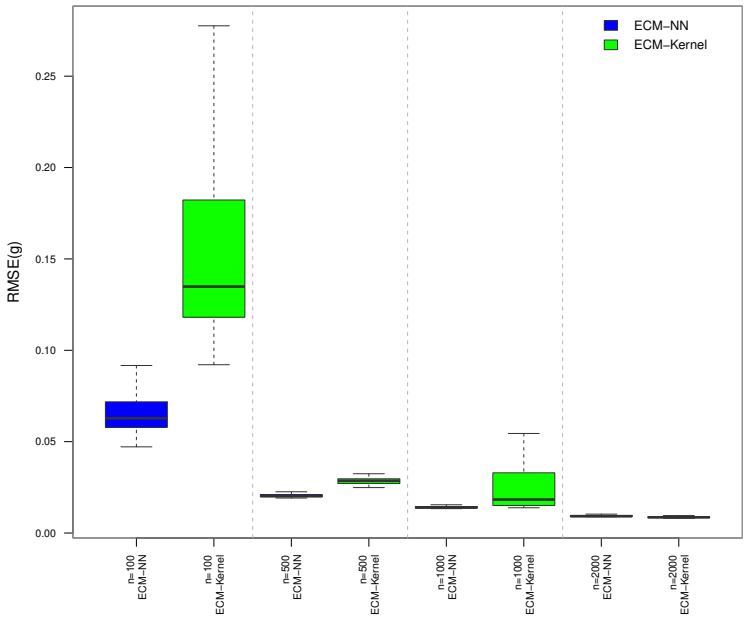


Fig. 2: The RMSE of the fitted nonparametric functions $\hat{g}_k(\cdot)$, for $k = 1, 2$, in Scenario 2 using the ECM-NN procedure and the ECM-Kernel procedure for $n = 100, 500, 1000$ and 2000

64 countries. Consequently, our response variable, $y_{i,t}$, is the average annual CO_2 emissions of the i^{th} country calculated over the t^{th} 5-year period. Meanwhile, our main covariate $z_{i,t}$, economic complexity, is proxied by the trade economic complexity index (ECI), denoted by `eci_trade`. The CO_2 emissions data was taken from the Our World In Data database[2] and the economic complexity data was taken from the Observatory of Economic Complexity[3].

The study sets out to investigate the extent to which variations in trade complexity explain differences in carbon emissions across different countries. To minimize omitted variable bias, the analysis further considers a range of theoretically relevant controls, including energy, macroeconomic, developmental, demographic, and fiscal variables. These variables are also averaged across the same 5-year intervals to ensure consistency with the dependent variable structure. The energy-related variables include fossil fuel consumption and renewable energy generation. These variables capture the contrasting implications of energy composition for environmental quality. While greater renewable energy adoption is expected to reduce carbon emissions, higher fossil fuel consumption is anticipated to intensify environmental degradation.

[2] https://ourworldindata.org/.
[3] https://oec.world/en/rankings/eci/hs6/hs96?tab=ranking.

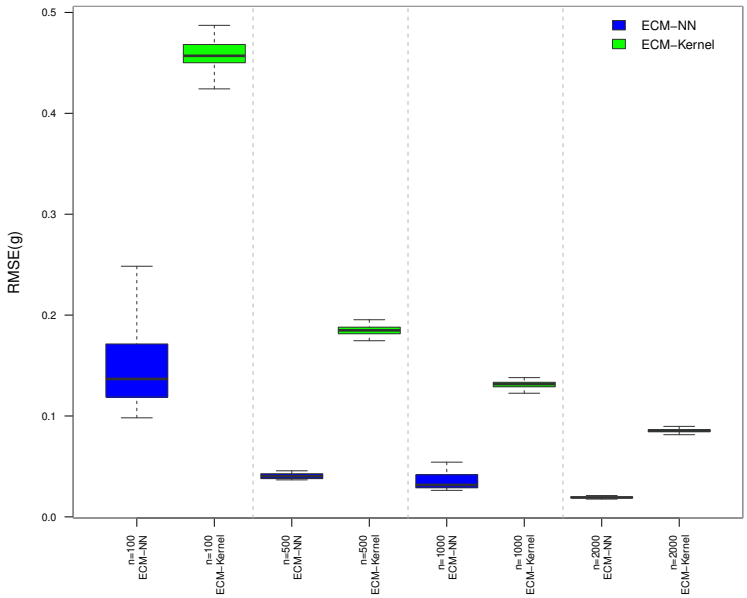


Fig. 3: The RMSE of the fitted nonparametric functions $\hat{g}_k(\cdot)$, for $k = 1, 2$, in Scenario 3 using the ECM-NN procedure and the ECM-Kernel procedure for $n = 100, 500, 1000$ and 2000

Meanwhile, the developmental and demographic controls include the Human Development Index (HDI) and population growth rate. Consistent with the Environmental Kuznets Curve (EKC) hypothesis, higher levels of human development are expected to contribute to lower carbon emissions through increased environmental awareness, improved living standards, and stronger institutional capacity for environmental protection. Population growth, however, may have either positive or negative effects on emissions, depending on the dominant energy structure of the economy. Where population expansion is accompanied by increased adoption of renewable energy, the effect on emissions may be limited; otherwise, reliance on fossil-based energy sources may amplify carbon emissions as the population grows. Furthermore, the macroeconomic variables comprise GDP growth and foreign direct investment (FDI) inflows, measured as a percentage of GDP. Economic growth may increase carbon emissions due to higher production and energy demand, particularly in economies dependent on carbon-intensive activities. The environmental implications of FDI inflows, however, depend largely on the sectors attracting investment, especially whether investments are directed towards renewable or non-renewable energy-intensive industries. In addition, the study incorporates military expenditure (as a percentage of GDP) as a fiscal factor. Higher military spending may exacerbate environmental degradation due to the environmental consequences associated with warfare, military operations,

and geopolitical tensions. These additional data save the macro factors (GDP growth and FDI inflows) are sourced from the Our World In Data database as well as the World Development Indicators database[4]. Table 3 gives the descriptive statistics of the variables used in this study.

Overall, while the study acknowledges only the potential effects of these control variables on carbon emissions, the primary focus remains on examining the relationship between economic complexity and environmental outcomes (measured by CO_2 emissions) within the framework of the EKC hypothesis.

4.1 Is there more than one regime? Results from the MoPLM.

We estimate the semiparametric mixture of experts model by assuming that the countries in our sample data may belong to multiple unknown regimes. Moreover, we assume that the relationship between the environment (as measured by CO2 emissions) and economic complexity (as measured by a country's trade complexity) is nonlinear and differs across regimes. Following previous studies (e.g., Konte (2018), Wang and Lee (2022), and Lee and He (2022)) that find evidence for two or three growth regimes or developmental paths, we search for the optimal number of regimes by fitting the model with $K = 1$ to $K = 3$. The optimal number of regimes is chosen using the criterion (17). To test whether or not there exists multiple regimes in the data such that the marginal impact of trade complexity differs across different regimes, we fit the MPLM (6) with $y = \texttt{ln(co2_emissions)}$, $\mathbf{x} = (\texttt{hdi}, \texttt{gdp_g}, \texttt{pop_g}, \texttt{fdi_in}, \texttt{mil_spend}, \texttt{fossil_fuel}, \texttt{renewable})^\top$ and $\mathbf{z} = (\texttt{eci_trade})$. Table 4 gives the goodness-of-fit results from fitting the MoPLE model for $K = 1$ to $K = 3$. Based on all the information criteria, the optimal value of K is 2. This suggests that our sample data can be best explained by a model with $K = 2$ regimes. In Table 5, we give the results of the best-fitted model with $K = 2$ regimes. Table 6 presents the classification of the countries into the two regimes using the procedure presented in Section 2.5. The table includes the classification probability of each country into each of the two regimes. A country is classified into a regime based on the regime with the highest probability. Based on the table, we note that about 15% of the countries have been classified in the first regime (Regime 1), and the remaining 85% in the second regime. We can see that regime 1 is made up of countries such as China (with probability: 1), the United States (1), India (1), Japan (1) and Germany (0.63). These are the most highly industrialized countries whose industries still rely heavily on fossil fuels, and also contribute significantly to global CO_2 emissions[5]. On the other hand, regime 2 comprises countries such as South Africa (0.96), Sweden (1), and Norway (1), among others, whose contributions to global carbon emissions are lower than those of the former.

Turning now to the fitted models in both regimes. We first focus on our main term in the model, $g_k(\cdot)$, for $k = 1, 2$. We can see from Figure 4a and 4b that the marginal effect of trade complexity on CO_2 emissions is nonlinear in both regimes. Moreover, the fitted function in both regimes has an inverted U shape (see also, Agozie et al.

[4] https://databank.worldbank.org/source/world-development-indicators.
[5] See: https://www.worldometers.info/co2-emissions/co2-emissions-by-country/.

Table 3: Descriptive Statistics of the variables

Variable	Description	Mean	Median	Std. Dev.	Min	Max	Skewness	Kurtosis	N
`eci_trade`	Economic Complexity Index (trade-based), measuring the productive knowledge embedded in a country's export basket	0.6045	0.6611	0.8274	−1.5308	2.1649	−0.3200	−0.7135	256
`ln(co2_emissions)`	Natural log of total CO_2 emissions (kt), capturing greenhouse gas output	18.7877	18.5247	1.3736	16.2611	23.1203	0.6667	0.3222	256
`hdi`	Human Development Index, a composite measure of life expectancy, education, and per-capita income (0–1 scale)	0.8213	0.8352	0.1023	0.4788	0.9676	−0.8560	0.4203	256
`gdp_g`	Annual GDP growth rate (%), measuring the rate of economic expansion or contraction	3.0436	2.7347	3.0485	−13.6711	21.4706	0.2398	8.6031	256
`pop_g`	Annual population growth rate (%), capturing demographic change over time	0.8220	0.7728	1.0598	−2.0309	6.4432	1.3454	5.6957	256
`fdi_in`	Inward foreign direct investment (% of GDP), reflecting net inflows of investment from abroad	3.8674	2.4572	6.1684	−15.5192	38.4125	3.1818	13.7249	256
`military_spending`	Military expenditure (% of GDP), indicating the share of national output devoted to defence	2.1251	1.5266	1.5998	0.2534	10.3710	2.3511	6.6756	256
`fossil_fuel`	Total fossil fuel energy consumption (TWh), encompassing coal, oil, and natural gas use	1802.5123	520.2610	4491.1326	48.1035	34 802.1908	5.0453	27.2925	256
`renewable`	Total renewable energy consumption (TWh), including solar, wind, hydro, and other clean sources	50.6539	10.1676	134.7564	0.0000	1278.2512	5.7756	41.9257	256

Note: All statistics are computed on panel-averaged observations ($N = 256$ country-period units). `ln(co2_emissions)` is the natural logarithm of raw CO_2 emissions in kt. Skewness and excess kurtosis are reported.

Table 4: Goodness-of-fit results across three different values of K for the fitted MoPLM. The best value in each column is shown in **bold**.

Number of regimes (K)	AIC	BIC	ICL	PanIC
1	489.4868	517.8482	517.8482	495.9321
2	**198.4132**	**258.6812**	**278.6841**	**212.1094**
3	331.5423	423.7169	546.0171	352.4895

2022; Ansari 2022). More specifically, in regime 1, CO_2 emissions increase as trade complexity increases up until a point (around 0). Beyond this point, further increases in trade complexity decrease CO_2 emissions. In regime 2, CO_2 emissions increase as trade complexity increases up until a point (around -0.5). Beyond this point, further increases in trade complexity have no significant upward effect on CO_2 emissions. This is in line with the EKC hypothesis. Furthermore, these results are consistent with the hypothesis that the marginal effect of trade complexity on CO_2 emissions differs across regimes. We can see that the turning points between the two regimes are different.

Turning now to the other control variables included in the model, our results reveal that the human development index (`hdi`) has a negative and significant effect in both regimes. Meanwhile, the growth rates of GDP (`gdp_g`) and the population (`pop_g`) have the opposite effects in both regimes. More specifically, `gdp_g` has a positive and insignificant effect in regime 1, whereas it has a negative and significant effect in regime 2. On the other hand, `pop_g` has a negative and significant effect in regime 1, whereas it has a positive and significant effect in regime 2. Foreign direct investment inflows (`fdi_in`) have a negative and significant effect in regime 1—contrary to (Agozie et al. 2022), whereas they have a positive and insignificant effect in regime 2. Military spending (`mil_spending`), on the other hand, has an insignificant effect in both regimes, while fossil fuel consumption (`fossil_fuel`) has a positive and significant effect in both regimes (Ansari 2022). The positive influence of fossil fuel is larger in regime 2 than in regime 1, as shown by the magnitude of the coefficients. Finally, renewable energy consumption (`renewable`) has a significant negative effect in regime 1, whereas in regime 2, it has an insignificant negative effect (Ansari 2022).

These results carry important implications for reducing global carbon footprints. Specifically, improvements in human development can contribute to lower carbon emissions as shown across the two regimes (see also, Zhu and Li 2017; Zhang et al. 2024b). This effect may operate through channels such as increased environmental awareness resulting from better education, improved living standards associated with enhanced health outcomes, and stronger institutional capacity for environmental protection. On the contrary, higher fossil fuel consumption in both regimes contributes to increased carbon emissions (Alam et al. 2016; Ali and Seraj 2022). Although, as expected, the magnitude of this effect is larger in regime 2, which comprises mostly developing countries that continue to rely heavily on fossil fuels for their various energy needs. Meanwhile, the results regarding renewable energy adoption indicate that it remains

an effective option for reducing greenhouse gas emissions (see also, Ahmed and Shimada 2019; Zhang et al. 2024b) and by extension, supporting efforts to limit global warming to 1.5°C.

As expected, control variables such as population growth and FDI have different impacts on carbon emissions across the two regimes. We find a significant negative effect of population growth on carbon emissions in regime 1, whereas the effect is positive and significant in regime 2 (Alam et al. 2016; Thi et al. 2024). Most countries in regime 2 are developing countries characterized by relatively high population growth rates compared to those in regime 1. In fact, countries such as the United States, in regime 1, have implemented policies to moderate population growth through stricter immigration controls. These results also affirm the fossil consumption results, which show that fossil fuel use imposes a larger positive effect on carbon emissions in regime 2 than in regime 1. This suggests that population expansion in regime 2 is associated with greater reliance on fossil fuel-based energy consumption, reflecting the differing levels of economic development across the two regimes.

Finally, as previously argued, the environmental implications of FDI inflows depend largely on the level of a country's development. Our results show that while FDI inflows reduce carbon footprint in the first regime, they heighten emissions in the second regime (Liu et al. 2021). This finding is consistent with concerns that less-developed economies may attract pollution-intensive activities, whereas more-developed economies tend to benefit from cleaner technologies and stricter environmental regulations. It may also help explain the rationale behind policies such as the Carbon Border Adjustment Mechanism (CBAM) pursued by the EU extraction of this regime (regime 2), particularly in relation to imports originating from these countries.

4.2 Important variables for classifying countries into regimes

We now estimate the MoPLE model (5) to identify informative variables for classifying countries into different regimes. The regime to which a country belongs determines its developmental path. Thus, we want to identify the determinants of the developmental paths of the two regimes. Let $\mathbf{t} = (\texttt{hdi}, \texttt{gdp_g}, \texttt{renewable})^\top$ be a vector of concomitant variables that we consider. Research has shown that renewable energy consumption (`renewable`) and economic growth (`gdp_g`) both positively affect economic development. Moreover, changes in the standards of living (measured by `hdi`) are a good indicator of economic development (Suri et al. 2011; Ntanos et al. 2018; Ahmed and Shimada 2019; Zhang et al. 2024b). Table 8 gives the estimated $K = 2$ regimes MoPLE model. Note that, as with the fitted MoPLM, the data still supports a $K = 2$ regimes model as shown in 7. The coefficients of the covariates $\mathbf{x}$ have the same interpretation as in the MoPLM.

We now turn to the coefficients of the concomitant variables. The coefficient for `hdi` is negative, which indicates that the higher the living standards of a given country, the more likely the country belongs to regime 2. The coefficient for `gdp_g` is negative, which indicates that the higher the rate of economic growth for a given country, the more likely that the country belongs to regime 2. However, this effect is not statistically significant (the confidence interval includes a zero). Finally, the coefficient

Table 5: Parameter estimates for the $K = 2$ regime MoPLM. 95% confidence intervals are given in brackets.

	Regime 1	Regime 2
Variable	Estimate [95% CI]	Estimate [95% CI]
`hdi`	−0.2123 [−0.2731, −0.1570]	−0.2019 [−0.2581, −0.1247]
`gdp_g`	0.0206 [−0.0098, 0.0684]	−0.0909 [−0.1344, −0.0357]
`pop_g`	−0.2244 [−0.2806, −0.1800]	0.0653 [0.0164, 0.1185]
`fdi_in`	−0.4744 [−0.5224, −0.4258]	0.0019 [−0.0426, 0.0505]
`mil_spend`	0.0170 [−0.0290, 0.0773]	−0.0447 [−0.0967, 0.0019]
`fossil_fuel`	0.4053 [0.3628, 0.4357]	4.1566 [3.9886, 4.3107]
`renewable`	−0.1412 [−0.1701, −0.1052]	−0.0553 [−0.1519, 0.0327]
Number of countries	8	56
σ^2	0.0046	0.0836

for `renewable` is positive, indicating that the higher a country's level of renewable energy consumption, the more likely it is to belong to regime 1. Finally, Figures 4c and 4d show the fitted nonparametric functions of `eci_trade` in regime 1 and regime 2, respectively. The figures show that the EKC still holds. Moreover, the EKC in each regime has a different turning point, as was the case for the fitted MoPLM.

5 Conclusion

In this paper, we propose a flexible approach for studying the environmental Kuznets curve (EKC) for cross-country panel datasets using a semiparametric mixture of experts (MoE) model. This approach has some advantages over traditional approaches: (1) we can detect whether or not there are groups of countries each with its own unique developmental path (or regime) and, consequently, EKC; (2) we can classify each country into its own regime using a probabilistic procedure; and (3) we can determine which variables most effectively separate countries into different regimes. To estimate the model, we proposed a maximum likelihood estimation procedure via a hybrid algorithm that combines neural networks (NN) with the classical expectation-conditional maximization (ECM), termed the ECM-NN algorithm. A simulation study showed the good performance of the estimation procedure, especially when the non-parametric part of the model is high-dimensional. For a real-world data application, we considered data on 64 countries for the period 2003–2022. We used CO_2 emissions to measure environmental impact and economic complexity, in particular trade complexity, to measure economic performance. Our results showed that the data can be

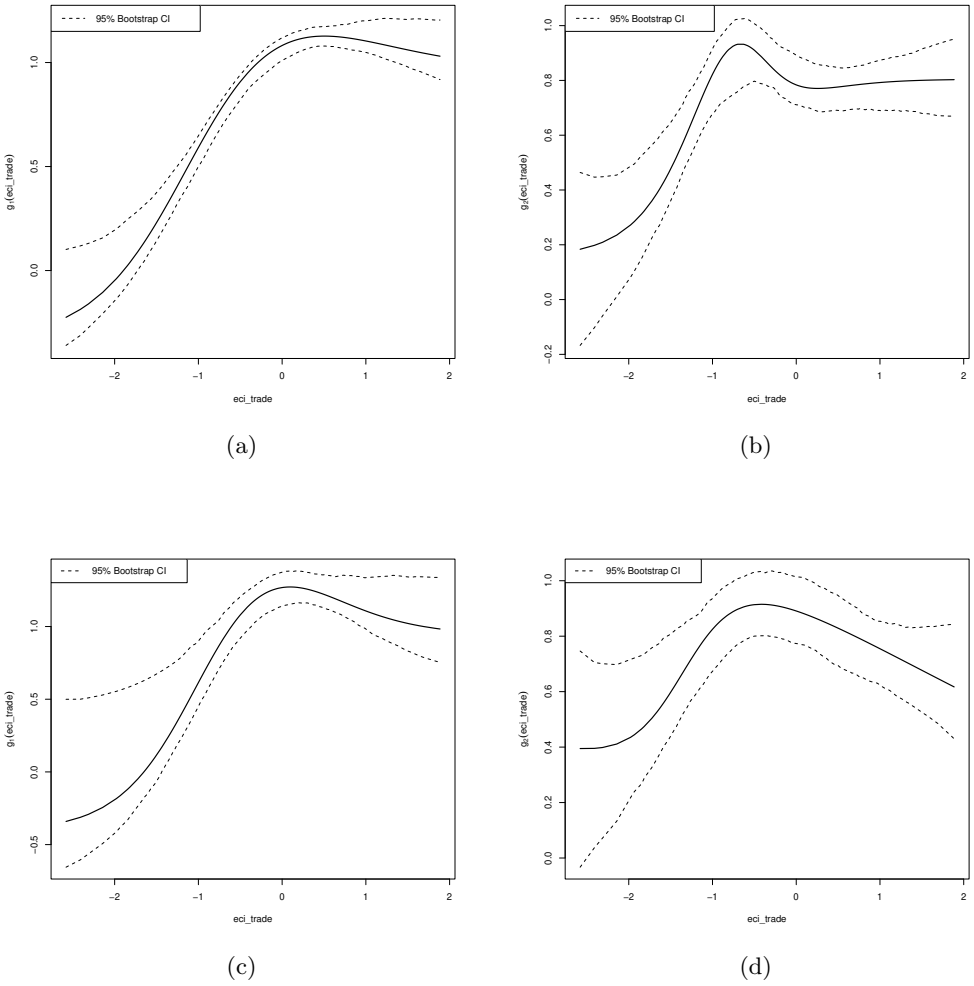


Fig. 4: Fitted nonparametric functions of `eci_trade` for the first regime (solid black line; left panel) and the second regime (solid black line; right panel) based on the fitted MoPLM (top panel) and the MoPLE (bottom panel)

explained by a model with two regimes, each having its own unique EKC. About 15% of the countries (including the USA, China and India) were classified into the first regime and the rest (including France, Spain and South Africa) were classified into the second regime. We showed, in each of these regimes, evidence consistent with the EKC hypothesis. We also found that the marginal effects of some control variables differ across the two regimes. For instance, the population growth rate has a negative effect in the first regime, whereas it has a positive effect in the second regime. Meanwhile, the adoption of renewable energy significantly reduced the carbon footprint across the

Table 6: Classifying the countries into different regimes. The value in the table represents the probability that the country belongs to the regime. The larger the probability the more likely the country belongs in the regime.

Country	Regime 1	Regime 2	Country	Regime 1	Regime 2
Algeria	0.46	0.54	Kuwait	0.00	1.00
Argentina	0.00	1.00	Lithuania	0.00	1.00
Australia	0.14	0.86	Malaysia	0.00	1.00
Austria	0.00	1.00	Mexico	0.00	1.00
Azerbaijan	0.00	1.00	Morocco	0.00	1.00
Bangladesh	0.00	1.00	Netherlands	0.00	1.00
Belarus	0.00	1.00	New Zealand	0.00	1.00
Belgium	0.00	1.00	Norway	0.00	1.00
Brazil	0.25	0.75	Oman	0.09	0.91
Bulgaria	0.00	1.00	Pakistan	0.00	1.00
Canada	0.00	1.00	Peru	0.00	1.00
Chile	0.00	1.00	Philippines	0.00	1.00
China	1.00	0.00	Poland	0.00	1.00
Colombia	0.00	1.00	Portugal	0.00	1.00
Croatia	0.00	1.00	Romania	0.00	1.00
Czechia	0.00	1.00	Russia	1.00	0.00
Denmark	0.00	1.00	Saudi Arabia	0.46	0.54
Ecuador	0.00	1.00	Singapore	0.33	0.67
Egypt	0.00	1.00	Slovakia	0.00	1.00
Finland	0.00	1.00	Slovenia	0.00	1.00
France	0.00	1.00	South Africa	0.04	0.96
Germany	0.64	0.36	South Korea	0.00	1.00
Greece	0.00	1.00	Spain	0.00	1.00
Hungary	0.08	0.92	Sri Lanka	0.00	1.00
India	1.00	0.00	Sweden	0.00	1.00
Indonesia	0.31	0.69	Switzerland	0.00	1.00
Iran	0.78	0.22	Thailand	0.00	1.00
Ireland	0.00	1.00	Turkey	0.00	1.00
Israel	0.00	1.00	Ukraine	0.00	1.00
Italy	0.00	1.00	United Kingdom	0.13	0.87
Japan	1.00	0.00	United States of America	1.00	0.00
Kazakhstan	0.63	0.37	Venezuela	0.00	1.00

two regimes. Moreover, we identified the human development index and renewable energy consumption as determinants of countries' classification into the two regimes. The effectiveness of the methodology proposed in this paper encourages the extension and/or development of similar methodological innovations for studying unobserved heterogeneity in other economic phenomena, such as the environmental Phillips curve hypothesis (Kashem and Rahman 2020).

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Table 7: Goodness-of-fit results across three different values of K for the fitted MoPLE. The best value in each column is shown in **bold**.

Number of regimes (K)	AIC	BIC	ICL	PanIC
1	489.4817	517.8431	517.8431	495.927
2	**173.1307**	**254.6698**	266.2052	**191.661**
3	360.879	495.5958	**210.7167**	391.4942

Table 8: Parameter estimates for the $K = 2$ regime MoPLE. 95% confidence intervals are given in brackets.

	Regime 1	Regime 2
Variable	Estimate [95% CI]	Estimate [95% CI]
Expert Model		
`hdi`	-0.0403 $[-0.1287, 0.0150]$	-0.1640 $[-0.2380, -0.0815]$
`gdp_g`	0.0834 $[0.0286, 0.1588]$	-0.0820 $[-0.1317, -0.0300]$
`pop_g`	-0.1091 $[-0.2308, -0.0265]$	0.0784 $[0.0310, 0.1300]$
`fdi_in`	-0.4347 $[-0.7058, -0.1396]$	0.0094 $[-0.0342, 0.0546]$
`mil_spend`	-0.0490 $[-0.1392, 0.0713]$	-0.0631 $[-0.1143, -0.0166]$
`fossil_fuel`	0.4147 $[0.3421, 0.4599]$	4.0349 $[3.7872, 4.2818]$
`renewable`	-0.1608 $[-0.2065, -0.0986]$	0.2068 $[-0.0945, 0.4713]$
Gating Model		
Intercept	$-2.4913[-39.7936, -1.7563]$	
`hdi`	$-1.2255[-17.5455, -0.3043]$	
`gdp_g`	$-0.2711[-2.5435, 0.9586]$	
`renewable`	$5.6606[4.1186, 95.9665]$	
Number of countries	8	56
σ^2	0.0112	0.0975

No. 150170. The opinions expressed and conclusions arrived at are those of the authors and are not necessarily to be attributed to the NRF.

Data Availability:

The data are publicly available.

Declaration of Competing Interest:

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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