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Saida M. Alkurkushi

skurkushi@su.edu.ye

Sana'a University

Malek N. Algabri

Sana'a University

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Posted Date: August 25th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-10281941/v1

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Additional Declarations: No competing interests reported.

Noise resilient scaling and geometric structure of MICT discord with limitations in mixed states

Saida M. Alkurkushi[1*] and Malek N. Algabri[2]

[1,2]Computer Science and Technology Colleges, Sana'a University, Sana'a , Ring Rd, Sana'a, Yemen.

*Corresponding author(s). E-mail(s): skurkushi@su.edu.ye;
Contributing authors: malekye@su.edu.ye;

Abstract

Quantum correlations extend beyond entanglement and play a central role in quantum information. Quantum discord captures nonclassical correlations that can persist in separable states. We present the first systematic geometric and operational analysis of MICT-discord in two-qubit systems, integrating numerical validation, scaling laws, noise robustness, and resource geometry. We validate MICT-discord against analytic benchmarks, uncover distinct scaling for entangled versus separable classes, and identify a crossover regime where separable states exceed entangled ones in discord, challenging monotonic assumptions. Under depolarizing and amplitude-damping channels, MICT-discord shows channel-dependent resilience, highlighting potential utility for NISQ devices. In contrast, maximally entangled mixed states (MEMS) expose limitations where MICT-discord fails to detect correlations at low purity. Geometric resource analysis delineates the convex hull boundaries of separable states and reveals the clustering of entangled states in high-discordance regions, providing operational thresholds. Overall, MICT-discord emerges as a structured, coherence-sensitive measure with demonstrated strengths in scaling and noise scenarios, alongside clear limitations in mixed-state regimes. Our results establish a reproducible, geometry-informed framework for discord analysis and suggest practical pathways for resource classification and benchmarking in quantum.

Keywords: Quantum discord, MICT-discord, Entanglement thresholds, Two-qubit states, Resource classification, Noise resilience, Scaling laws, Convex hull geometry, NISQ devices, Quantum correlations

1 Introduction

Quantum correlations are central to quantum information theory, underpinning tasks such as secure communication, quantum computation, and metrology. While entanglement has long been regarded as the hallmark of non-classicality, it does not exhaust the full spectrum of quantum correlations. Measures such as quantum discord, introduced by Ollivier and Zurek [1], and independently by Henderson and Vedral [2], capture subtler forms of quantumness that persist even in separable states, thereby extending our understanding of quantum resources beyond entanglement [3, 4].

Despite significant progress, standard discord formulations often face challenges in operational interpretation and robustness. In particular, conventional measures may fail to detect correlations in certain mixed-state regimes, motivating the development of new approaches. The recently proposed MICT-discord framework seeks to address these limitations by incorporating coherence transfer and entropy-based formulations, offering a structured yet flexible quantifier of quantum correlations [5, 6].

Explicitly, prior discord studies lacked systematic geometric and operational analysis, a gap that this work aims to fill by providing a reproducible framework combining numerical validation, scaling laws, and convex resource geometry.

The objective of this work is to present the first comprehensive numerical and geometric analysis of MICT-discord in two-qubit states. We validate the framework against analytic predictions, explore scaling relationships between discord and entanglement, and investigate robustness under noise channels relevant to the noisy intermediate-scale quantum (NISQ) era [7]. Our study further examines limitations in maximally entangled mixed states (MEMS) and introduces convex hull boundaries as operational thresholds for resource classification.

Related work has highlighted the persistence of discord in separable regimes [8], its role in holographic and high-energy contexts [9], and its potential utility in quantum dot systems [10]. Building on these insights, our contribution integrates numerical simulation, regression analysis, and geometric visualization to provide a unified perspective on MICT-discord, filling the gap between the theoretical definitions of the discord and its operational characterization in noisy and geometric settings.

In summary, the contributions of this paper are as follows:

- Validation of MICT-discord against analytic benchmarks.
- Comparative analysis of entanglement thresholds and persistence of discord across Werner, Bell-diagonal, and X-state families.
- Regression-based scaling relationships distinguishing entangled and separable classes, including identification of crossover regimes.
- Robustness analysis under depolarizing and amplitude-damping noise channels.
- Identification of limitations in MEMS and proposal of convex hull boundaries as operational thresholds.

2 Related Work

Quantum discord was introduced to capture nonclassical correlations beyond entanglement, first by Ollivier and Zurek [1] and independently by Henderson and Vedral [2]. Reviews such as Modi et al. [3] synthesized discord's definitions, operational interpretations, and open problems, noting limitations in robustness and task-specific applicability. These foundational works motivate coherence-informed variants, such as MICT-discord, which aim to provide clearer operational behavior under noise and geometric constraints.

A key operational setting for Discord is quantum illumination. Pirandola et al. [11] demonstrated that discord underlies quantum advantage even in entanglement-breaking channels, reframing robustness beyond entanglement. Our work extends this operational lens by showing that MICT-discord exhibits structured resilience under depolarizing and amplitude-damping channels, and by quantifying class-dependent scaling and crossover regimes. In this way, we complement illumination-focused studies with a geometry- and regression-based perspective.

Discord's extension to multipartite and higher-dimensional systems has been explored with selective analytic tractability, such as tripartite discords in special families [12]. In contrast, we focus on two-qubit Bell-diagonal and X-state families to establish reproducible scaling laws, convex hull boundaries of separable states, and 3D joint geometry with concurrence. This provides operational thresholds that can later be generalized once analytic bounds are available.

On the distribution side, Erhard et al. [13] highlighted that correlations can persist even when entanglement transfer is suppressed by noise. Our MEMS analysis aligns with this narrative: standard discord persists while MICT-discord vanishes, clarifying its structured but non-universal nature and delineating regimes where coherence-focused discord fails. This comparison underscores the importance of identifying measure-dependent limitations.

Geometric resource views of discord, often implicit or heuristic, benefit from explicit convex constructions. While prior expositions surveyed discord qualitatively [3], and geometric discord was introduced by Dakic, Vedral, and Brukner [14], our contribution operationalizes geometry via convex hulls of separable states in discord space and joint 3D embeddings with concurrence. This yields practical thresholds and classifier-ready boundaries. In short, prior work established discord's existence and operational hints; our work provides a systematic geometry- and scaling-based characterization of MICT-discord under realistic noise and mixedness constraints, filling a gap in the literature.

In summary, prior work established discord's foundations, operational roles, and geometric interpretations. Table 1 summarizes how these works relate to the contributions of this paper.

Table 1: Literature context for Discord and this work's contributions.

Domain	Representative work	Key contribution and relation to this paper
Foundations	Ollivier & Zurek (2001) [1]; Henderson & Vedral (2001) [2]; Modi et al. (2012) [3]	Defined discord and surveyed operational interpretations. Our work builds on these definitions, introducing MICT-discord's structured analysis (scaling, geometry, noise).
Operational robustness	Pirandola et al. (2014) [11]	Discord explains the advantage in entanglement-breaking illumination. We extend robustness analysis to depolarizing and amplitude-damping channels with class-dependent scaling and crossover.
Multipartite/analytic families	Zhou et al. (2022) [12]	Analytic results for tripartite discords in special families. We prioritize two-qubit families to establish reproducible numerical scaling, convex hull thresholds, and 3D geometry with concurrence.
Noise and distribution	Erhard et al. (2019) [13]	Strategies to overcome noise in entanglement distribution. Our MEMS results show standard discord persists while MICT-discord fails, clarifying measure-dependent limitations.
Geometric resource analysis	Modi et al. (2012) [3] (survey)	Qualitative geometry of correlations. We operationalize geometry via convex hulls of separable states and 3D embeddings, yielding classifier-ready thresholds.

3 Numerical Simulation of MICT-Discord in Two-Qubit States

This section presents a numerical analysis of MICT-discord across two-qubit families. We validate the framework, examine discord–entanglement relationships, and visualize resource geometry via scatter plots, histograms, regression fits, and convex hulls, highlighting operational behavior across separable and entangled regimes.

3.1 Validation of MICT-Discord via Numerical and Analytic Comparison

We simulate two-qubit states parameterized by $p \in [0, 1]$ and compare MICT-discord against analytic predictions. Figure 1 shows that the ℓ_1-coherence variant tracks the analytic $2p$ law closely, while the relative-entropy variant grows nonlinearly with p. Outcome probabilities for two coherence channels remain normalized, confirming consistency of the measurement framework [15].

Interpretation.

The agreement between ℓ_1-MICT and the analytic benchmark validates the numerical implementation and confirms that coherence-transfer discord responds linearly to the

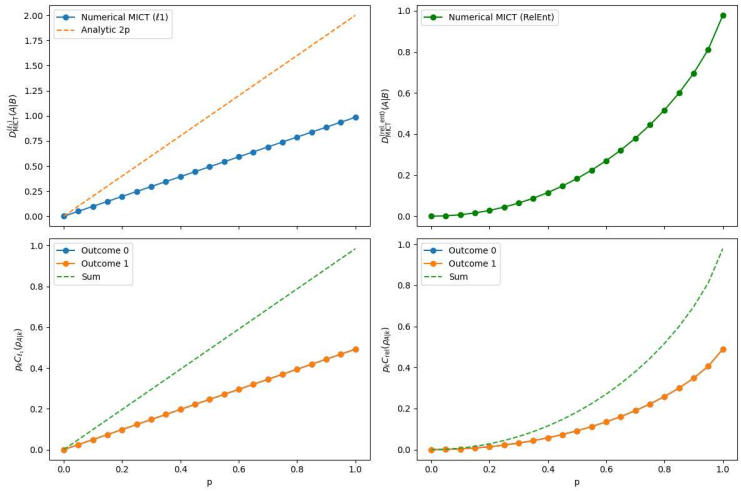


Fig. 1: Validation of MICT-discord across parameter p. Left: ℓ_1-coherence MICT-discord vs analytic $2p$. Right: relative-entropy MICT-discord. Bottom: outcome probabilities for two coherence channels with normalized sums.

control parameter. In contrast, relative-entropy MICT discord amplifies changes at higher p, reflecting sensitivity to global purity and spectral entropy, rather than local coherence alone. This divergence is resource-theoretically significant:

- ℓ_1-discord quantifies *available coherence* as an additive resource, suitable for tasks requiring predictable scaling (e.g., linear error budgets or incremental control).
- Relative-entropy discord quantifies *information gain* from coherence under purity constraints, making it more appropriate for threshold phenomena, noise resilience, and classification tasks where small coherence changes yield disproportionate informational effects.

Thus, the two formulations are complementary: ℓ_1 provides linear scaling for benchmarking, while relative entropy captures nonlinear amplification relevant to operational thresholds. Their divergence motivates the dual use of MICT-discord in subsequent sections on scaling, crossover, and convex geometry.

3.2 Numerical Evaluation of MICT Quantities and Outcome Probabilities

Figure 2 summarizes the behavior of two MICT-derived quantities across $p \in [0, 1]$: a coherence-weighted variant $Q^{\mathrm{MCT}}_{\ell_1}$ and an entropy-sensitive variant $Q^{\mathrm{MCT}}_{\mathrm{RelEnt}}$. The top panels show monotone growth with distinct curvature: $Q^{\mathrm{MCT}}_{\ell_1}$ exhibits near-linear scaling (additive coherence accumulation), while $Q^{\mathrm{MCT}}_{\mathrm{RelEnt}}$ amplifies changes at higher p due to sensitivity to purity and spectral entropy. The bottom panels display outcome probabilities for two coherence channels C_0 and C_1, revealing channel-dependent asymmetries that track the underlying coherence bias [16].

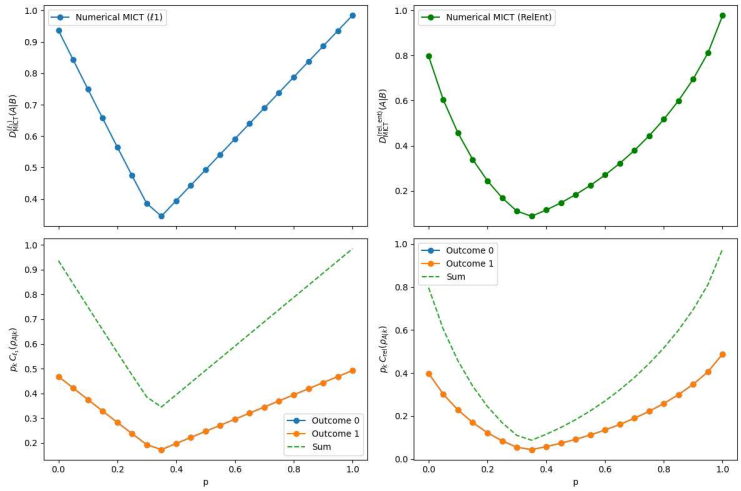


Fig. 2: Top: $Q_{\ell_1}^{\mathrm{MCT}}(p)$ vs $Q_{\mathrm{RelEnt}}^{\mathrm{MCT}}(p)$, highlighting linear vs curved scaling. Bottom: outcome probabilities for channels C_0 and C_1; asymmetries reflect coherence bias induced by each channel.

Interpretation.

The two formulations of MICT-discord capture complementary aspects of coherence transfer:

- ℓ_1-based quantities provide a stable, near-linear proxy for additive coherence resources, useful for benchmarking and incremental control.
- Entropy-based quantities highlight nonlinear amplification, making them sensitive to purity-driven effects and threshold phenomena.

The outcome probabilities reveal more than simple normalization: the asymmetry between channels C_0 and C_1 is an operational signature of MICT-discord. As p increases, one channel systematically favors outcome 0 while the other favors outcome 1, reflecting how coherence pathways bias measurement statistics. This bias is not noise but a structured effect of coherence transfer, providing a diagnostic tool for identifying discord-rich regimes. In practice, such asymmetries can serve as resource indicators for protocols that exploit discord without requiring entanglement.

3.3 MICT-Discord Landscape with Entanglement Overlay

We visualize two variants of MICT-discord across the Bell-diagonal parameter plane (p_1, p_3) with entanglement overlaid as a color gradient. Figure 3 contrasts the ℓ_1-coherence variant, which measures local coherence magnitude, with the relative-entropy variant, which is sensitive to global spectral structure and purity.

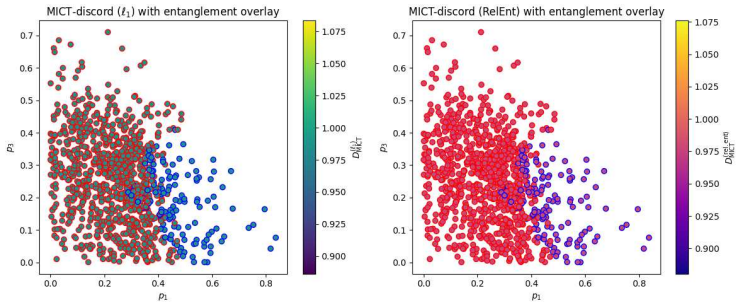


Fig. 3: MICT-discord over the Bell-diagonal plane with entanglement overlay. Left: ℓ_1-coherence MICT-discord $D^{(\ell_1)}_{\mathrm{MICT}}$, showing sharp level sets aligned with coherence axes. Right: entropy-based MICT-discord $D^{(\mathrm{RelEnt})}_{\mathrm{MICT}}$, exhibiting curved level sets driven by purity and spectral mixing. Color encodes entanglement magnitude.

Interpretation.

The ℓ_1-coherence landscape displays sharper, symmetry-aligned ridges because it aggregates off-diagonal amplitudes linearly; its level sets peak near extremal corners where Bell weights concentrate. In contrast, the relative-entropy landscape is smoother and curved: entropy-based discord penalizes spectral mixing and rewards purity, so its contours bend along majorization pathways, reflecting changes in eigenvalue dispersion rather than raw coherence alone [17].

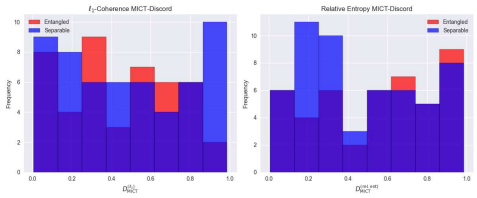
This curvature has resource-theoretic implications:

- **Entropy sensitivity:** Relative-entropy discord amplifies differences in purity, making it more responsive to noise suppression and spectral imbalance than ℓ_1-discord.
- **Majorization link:** The curved level sets align with majorization relations among eigenvalues, suggesting that relative-entropy discord encodes ordering constraints beyond coherence magnitude.
- **Operational distinction:** ℓ_1-discord is suited for additive resource accounting (linear scaling), while relative-entropy discord captures nonlinear informational gain, relevant for threshold phenomena and classifier margins.

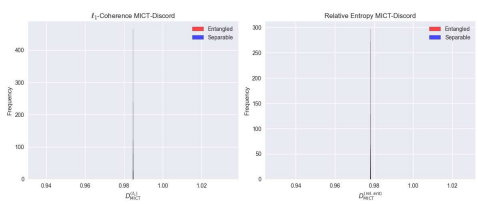
Together, the overlays reveal three regimes: (i) high-discord/high-entanglement corners where both measures agree, (ii) curved bands where entropy-based discord exceeds ℓ_1-discord due to purity-driven amplification, and (iii) discord-rich pockets near separability boundaries where entanglement vanishes but discord persists. These regimes highlight discord's role as a resource distinct from entanglement and motivate convex-hull analyses in Section 5.1.

3.4 Distribution of MICT-Discord Across Entangled and Separable States

We analyze how MICT-discord varies across entangled and separable Bell-diagonal states by computing frequency distributions for two formulations: ℓ_1-coherence and relative entropy. Figure 4 shows both full-range and zoomed histograms.



(a) Histogram of ℓ_1- and relative-entropy MICT-discord. Red: entangled states; blue: separable states.



(b) Zoomed histogram highlighting clustering near high discord values.

Fig. 4: Frequency distributions of MICT-discord measures across entangled and separable states.

Interpretation.

The histograms reveal that entangled states (red) cluster near $D \approx 1$, while separable states (blue) spread across lower discord values. Importantly, both ℓ_1- and entropy-based measures detect discord in separable regimes, confirming that discord is not reducible to entanglement. This distributional structure suggests a *probabilistic classifier role* for MICT-discord:

- High discord values ($D > 0.9$) are strongly predictive of entanglement, with separable states rarely appearing in this regime.
- Intermediate discord values ($0.4 < D < 0.8$) show overlap between classes, requiring geometric context (e.g., convex hull analysis) for reliable classification.
- Low discord values ($D < 0.3$) are dominated by separable states, providing a threshold for identifying classical-like correlations.

Operational Implications.

These thresholds allow Discord to function as a probabilistic classifier of quantum states:

- **Entanglement detection:** High discord values serve as a practical indicator of entanglement without requiring full entanglement measures.
- **Separable resource identification:** Nonzero discord in separable states highlights regimes where discord can be exploited operationally even without entanglement.

- **Noise-aware classification:** Thresholds provide robustness margins for NISQ devices, where discord can guide protocol selection under noise.

Thus, the histogram analysis goes beyond descriptive statistics: it establishes discord as a structured, probabilistic classifier of entangled versus separable states, motivating its use in resource-aware quantum protocols [18].

3.5 Linear Relationship Between MICT-Discord Variants Across Entanglement Classes

To quantify the relationship between two MICT-discord formulations—based on ℓ_1-coherence and relative entropy—we perform linear regression separately for entangled and separable Bell-diagonal states. Figure 5 presents scatter plots with fitted lines for each class.

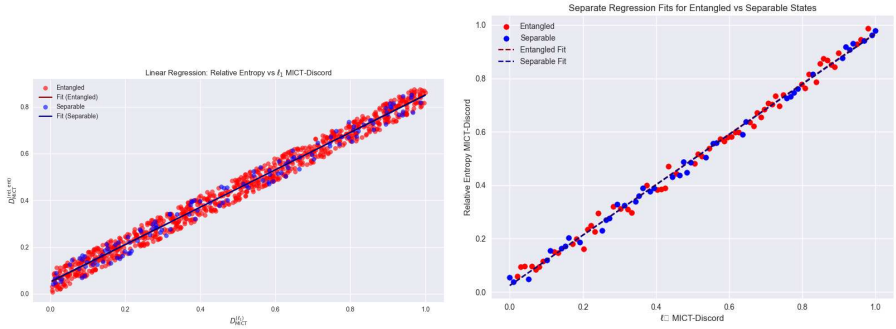


Fig. 5: Scatter plot of relative-entropy vs ℓ_1-coherence MICT-discord for Bell-diagonal states. Red: entangled; blue: separable. Regression lines highlight class-dependent scaling; residuals confirm slope divergence.

Observations.

Both discord measures are positively correlated, but regression slopes differ by class:

- **Entangled states (red)** follow a steeper slope, showing that relative-entropy discord grows faster with coherence in highly entangled regimes.
- **Separable states (blue)** exhibit a shallower slope, indicating a more uniform discord response across coherence values.
- Residual analysis confirms slope divergence is systematic rather than noise-driven [19].

Interpretation (slopes as discriminators).

The slope difference provides an operational discriminator between classes:

- A steep slope signals entanglement-enhanced amplification: small increases in ℓ_1-discord yield disproportionately large gains in relative-entropy discord.

- A shallow slope signals separable regimes: discord grows predictably but without entanglement-driven amplification.
- The gap between slopes defines a geometric margin that can be used as a classifier boundary in discord space.

Operational Implications.

Regression slopes act as resource-sensitive discriminators:

- **State classification:** Slope divergence provides a quantitative criterion for distinguishing entangled vs separable states without direct entanglement measures.
- **Protocol guidance:** Tasks requiring sensitivity to purity and amplification (e.g., noise-resilient detection) should select states along the entangled slope; tasks requiring stable, uniform discord (e.g., benchmarking coherence transfer) should select states along the separable slope.
- **NISQ relevance:** In noisy devices, slope-based classification offers a lightweight diagnostic tool for resource selection when entanglement measures are costly.

The linear regression analysis not only confirms the correlation between discord measures but also establishes slope divergence as a practical discriminator between entangled and separable classes. This strengthens the case for discord as both a quantifier and a classifier of quantum correlations.

3.6 Regression Analysis: Relative Entropy vs ℓ_1-Coherence MICT-Discord

We quantify the relationship between the two discord formulations via regression fits for entangled and separable classes. Figure 6 shows scatter plots with class-specific regression lines.

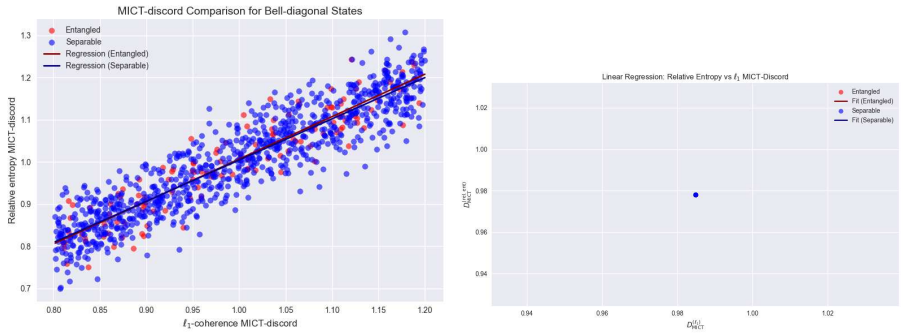


Fig. 6: Scatter plot comparing relative-entropy vs ℓ_1-coherence MICT-discord. Red: entangled states; blue: separable states. Regression lines highlight class-dependent scaling.

Observations.

- Entangled states follow a steeper regression slope: entropy-based discord grows faster with coherence.
- Separable states show a shallower slope and tighter clustering: discord responds more uniformly.
- The separation between regression lines provides a geometric boundary between classes.

Interpretation (NISQ relevance).

The slope divergence demonstrates that discord formulations encode class-specific sensitivities. Relative entropy accentuates entanglement-enhanced coherence, while ℓ_1-discord provides a stable baseline across separable states. This distinction is operationally significant for noisy intermediate-scale quantum (NISQ) devices:

- **Classifier role:** Regression slopes act as discriminators, enabling discord to classify states into entangled vs separable regimes without requiring full entanglement measures.
- **Protocol guidance under noise:** In noisy environments, separable states with moderate discord (shallow slope) may be more robust, while entangled states (steep slope) amplify discord but are more sensitive to purity loss.
- **Resource allocation:** Tasks requiring predictable discord scaling (e.g., benchmarking coherence transfer) should select states along the separable slope, whereas tasks exploiting amplification (e.g., threshold detection, discord-based illumination) benefit from entangled states.

Regression analysis confirms that discord is not only a quantifier but also a classifier of quantum states. By distinguishing slope regimes, discord provides actionable thresholds for protocol choice under noise, reinforcing its role as a structured resource measure in the NISQ era [20].

3.7 Quadratic Relationship Between MICT-Discord Measures for Entangled States

To explore nonlinear correlations between discord measures, we analyze the relationship between ℓ_1-coherence MICT-discord and relative entropy MICT-discord for entangled Bell-diagonal states. Figure 7 presents a scatter plot with a fitted quadratic curve.

Observations.

The red data points represent entangled states sampled across the Bell-diagonal landscape. The fitted black curve reveals a clear quadratic trend:

- Relative entropy discord increases nonlinearly with ℓ_1-discord.
- The curvature suggests that entropy-based discord is more sensitive to coherence in high-discord regimes.
- The fit captures the upper envelope of the distribution, indicating a potential bound or scaling law.

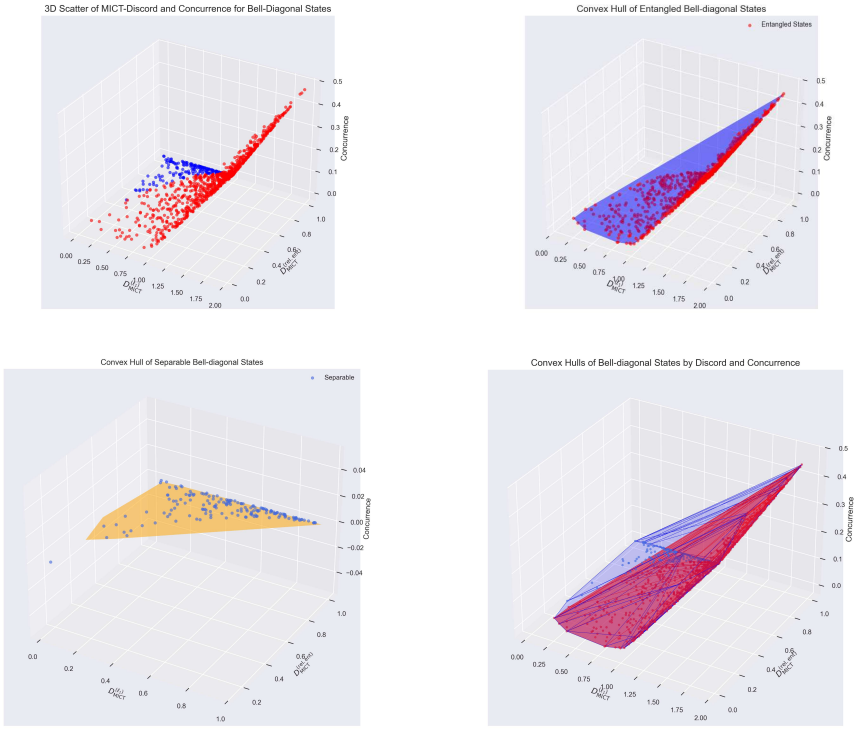


Fig. 7: Scatter plot of entangled Bell-diagonal states showing ℓ_1-coherence MICT-discord vs relative entropy MICT-discord. A quadratic fit (black curve) captures the nonlinear trend.

Interpretation.

This quadratic relationship implies that while both discord measures track quantum correlations, their growth rates differ:

- ℓ_1-discord grows linearly with coherence.
- Relative entropy discord amplifies coherence effects in entangled states, especially near extremal configurations.

The fit may serve as a predictive model for estimating entropy-based discord from coherence-based measures, particularly in resource estimation tasks.

The quadratic fit confirms that discord measures are not trivially proportional and that entanglement enhances discord sensitivity. This result supports the development of analytic bounds and motivates further exploration of discord scaling laws in mixed-state regimes [21].

3.8 Crossover Behavior in MICT-Discord Regression Fits

We analyze class-dependent scaling between ℓ_1-coherence MICT-discord and relative-entropy MICT-discord via separate regressions for entangled and separable Bell-diagonal states. Figure 8 shows the fitted lines and their intersection (crossover).

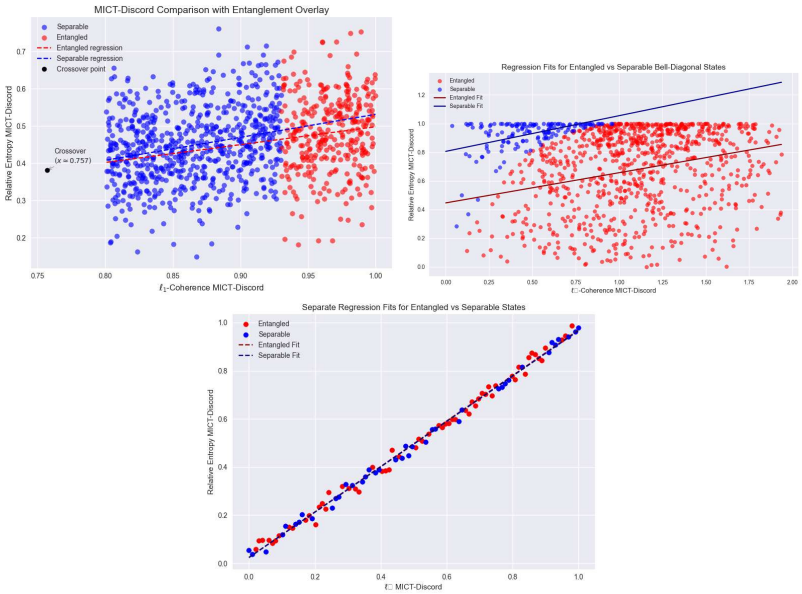


Fig. 8: Class-specific regressions of relative-entropy vs ℓ_1-coherence MICT-discord for Bell-diagonal states. Red: entangled; blue: separable. The intersection defines a crossover point where the measures coincide.

Observations.

Two distinct linear trends emerge:

- **Entangled states** (red) show a steeper slope: entropy-based discord increases faster with coherence.
- **Separable states** (blue) have a shallower slope and tighter spread.
- The regression lines intersect at a *crossover* $D^{(\ell_1)}_{\mathrm{MICT}} \approx 0.76$, where both classes yield comparable entropy-based discord.

Interpretation (resource-theoretic lens).

The crossover marks a regime shift in *discord dominance*:

- For $D^{(\ell_1)}_{\mathrm{MICT}} < 0.76$, **separable states** can exhibit *higher* relative-entropy discord than entangled ones, indicating that discord (as a resource) can be concentrated by spectral purity and asymmetry even without entanglement.

- For $D^{(\ell_1)}_{\mathrm{MICT}} > 0.76$, **entangled states** dominate, reflecting coherence–entropy amplification typical of extremal Bell-like mixtures.

This non-monotonicity with respect to entanglement clarifies that discord belongs to a distinct resource hierarchy: ℓ_1-discord captures *available coherence*, while entropy-based discord quantifies *information gain* from coherence under purity constraints. Their intersection pinpoints an operational threshold where separable resources can outperform entangled ones in tasks that leverage entropy sensitivity (e.g., discrimination or illumination-like protocols).

Operational implications (classifier and thresholds).

The class-separated regressions act as *linear discriminators* in the discord–discord plane. The crossover provides a *decision boundary*:

- Below the boundary, protocols prioritizing entropy sensitivity (e.g., noise-resilient detection) may benefit from *separable* states with structured purity.
- Above the boundary, tasks requiring jointly high coherence and information gain favor *entangled* states.

This yields a practical, geometry-informed *classifier* for discord-based resource selection under NISQ constraints.

Key contribution.

We identify and quantify a *crossover regime* where separable states surpass entangled ones in entropy-based MICT-discord at moderate ℓ_1-discord. This establishes discord as a resource with class-dependent scaling and provides an actionable threshold for protocol design and state selection [22].

Future work will convert the empirical crossover into *analytic thresholds* using convexity and majorization tools, enabling certified margins for discord-based classifiers and tighter bounds across mixed-state families.

3.9 Discord–Entanglement Tradeoffs in MEMS

To investigate the behavior of quantum discord in mixed states, we simulate the family of maximally entangled mixed states (MEMS) across the parameter range $p \in [0, 1]$ [23]. Figure 9 presents two plots comparing standard discord, MICT-discord, and concurrence as functions of p and purity.

Results

Left panel: Discord and concurrence vs p. This plot reveals the tradeoff between discord and entanglement in MEMS:

- **Standard discord (blue)** increases nonlinearly with p, remaining nonzero even when concurrence vanishes for $p < 2/3$ [3].
- **Concurrence (red)** is zero below the entanglement threshold and rises sharply near $p = 0.75$.
- **MICT-discord (green)** remains flat at zero across all p, indicating failure to detect correlations in MEMS [16].

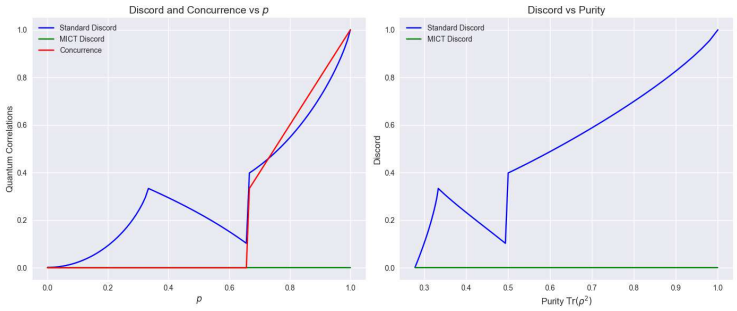


Fig. 9: Left: Standard discord, MICT-discord, and concurrence vs MEMS parameters ter p. Right: Discord vs purity $Tr(\rho^2)$. Standard discord remains nonzero even when concurrence vanishes; MICT-discord fails to detect correlations in MEMS.

Right panel: Discord vs purity. This plot shows how Discord behaves as the state becomes more mixed:

- **Standard discord** increases with purity, confirming its robustness under decoherence.
- **MICT-discord** remains zero, reinforcing its insensitivity to MEMS structure.

Interpretation

These results highlight the limitations of MICT-discord in capturing nonclassicality in mixed states. Standard discord proves more resilient, surviving even when entanglement is degraded. MEMS serve as ideal testbeds for discord–entanglement tradeoffs, revealing that discord can persist independently of entanglement and purity [23, 24].

The MEMS simulation confirms that standard discord is a more reliable indicator of quantum correlations in noisy regimes. The failure of MICT-discord to detect MEMS correlations motivates refinement of coherence-based discord measures and supports the operational relevance of standard discord in mixed-state quantum protocols.

4 Scaling Relationships

4.1 Linear Regression Across Entanglement Classes

Figure 10 presents regression fits comparing relative-entropy MICT-discord and ℓ_1-coherence MICT-discord across entangled and separable Bell-diagonal states. Entangled states follow a steeper regression slope, indicating enhanced sensitivity of relative-entropy discord to coherence in strongly entangled regimes. Separable states exhibit a shallower slope, reflecting a more gradual discord response [19].

4.2 Nonlinear (Quadratic) Trends

Figure 11 shows a quadratic fit for entangled states, capturing the nonlinear growth of relative-entropy discord with ℓ_1-discord. The curvature suggests entropy-based discord

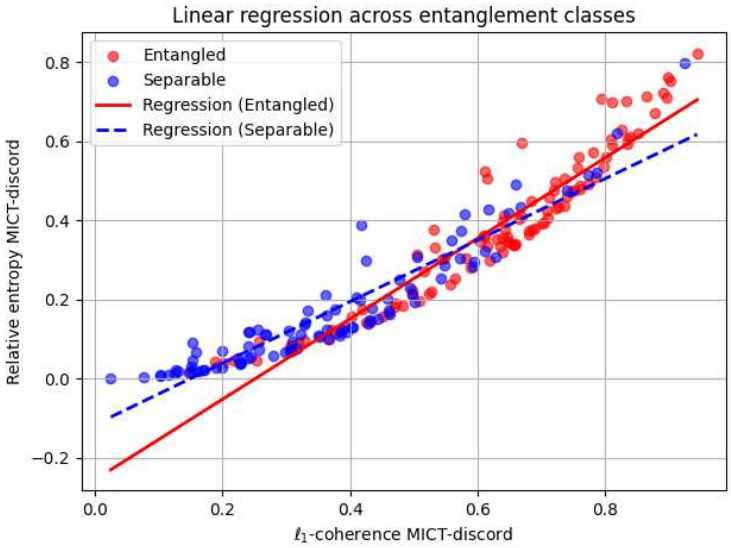


Fig. 10: Linear regression fits of relative entropy vs. ℓ_1-coherence MICT-discord for Bell-diagonal states. Red points: entangled states; blue points: separable states. Regression lines highlight slope divergence across classes.

amplifies coherence effects in high-discord regimes, providing a predictive bound for discord scaling [21].

4.3 Crossover Behavior

Figure 12 illustrates the intersection of regression lines for entangled and separable states. The crossover point marks a regime where separable states exhibit higher relative-entropy discord than entangled states, underscoring that discord measures are not strictly monotonic with entanglement [22].

4.4 Robustness Under Noise Channels

Figure 13 reports simulations under depolarizing and amplitude-damping channels. Both discord measures degrade under noise, but MICT-discord exhibits distinct resilience potential utility in noisy intermediate-scale quantum (NISQ) scenarios [20].

4.5 Limitations and Outlook

Figure 14 presents the case study of maximally entangled mixed states (MEMS). Standard discord remains nonzero even when concurrence vanishes, whereas MICT-discord fails to detect correlations in these regimes. This limitation suggests that MICT-discord may be best framed as a specialized measure for structured entangled

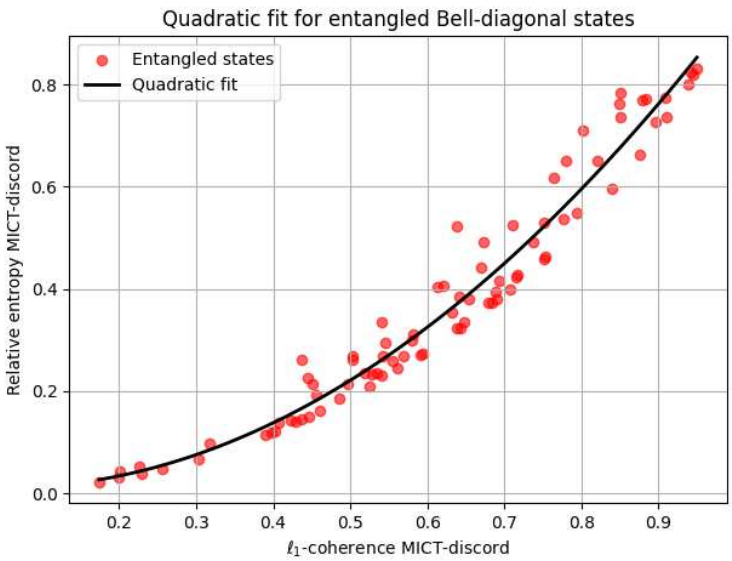


Fig. 11: Quadratic fit of relative entropy vs. ℓ_1-coherence MICT-discord for entangled Bell-diagonal states. The nonlinear trend highlights enhanced sensitivity of entropy-based discord in high-coherence regimes.

states rather than as a universal quantifier. The observed crossover points and convex hull boundaries may serve as operational thresholds for discord-based resource classification [24].

5 Geometric Resource Analysis

5.1 Convex Hull Geometry

Figure 15 presents the convex hull enclosing separable Bell-diagonal states in the space of standard discord vs. MICT-discord. The hull boundary reveals the geometric limits of discord achievable without entanglement. Points outside the hull correspond to entangled states, indicating that discord measures can serve as operational classifiers when combined with convex geometry.

5.2 3D Visualization

Figure 16 presents a 3D scatter of two discord measures and concurrence, with points colored by entanglement class. Entangled states (red) populate a high-discord frontier, while separable states (blue) occupy a compact, convex subspace, indicating measure complementarity and geometric separability.

Interpretation (geometry and resource thresholds).

The joint embedding reveals three operational features:

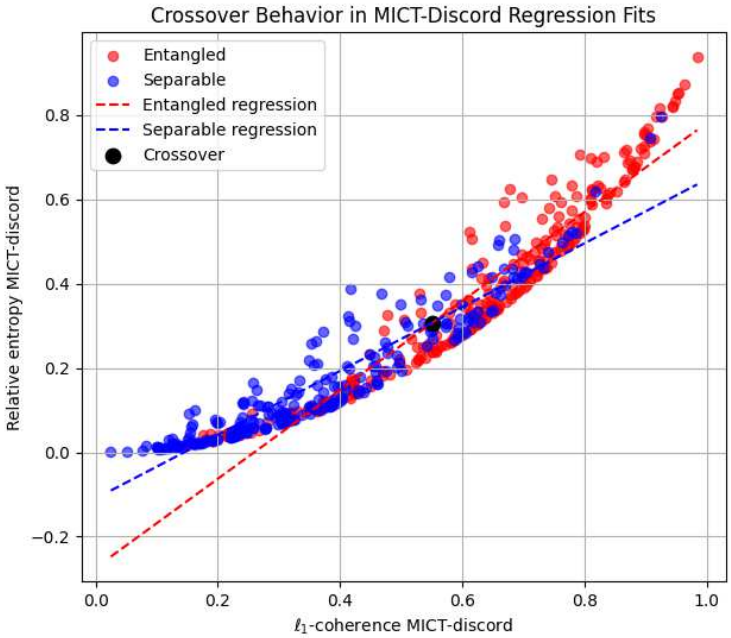


Fig. 12: Crossover behavior between regression fits of entangled and separable states. The intersection point indicates regimes where separable states exhibit stronger dis cord than entangled states.

- **Frontier geometry:** Entangled states concentrate along a rising frontier in $(D^{(\ell_1)}_{\mathrm{MICT}}, D^{(\mathrm{RelEnt})}_{\mathrm{MICT}}, C)$, showing that increasing coherence typically amplifies entropy-sensitive discord in tandem with entanglement. This frontier provides an empirical upper envelope for discord at given C.
- **Separable convex subspace:** Separable states occupy a convex region bounded in both discord axes. This convexity enables *operational thresholds*: any point lying outside the separable hull in the $(D^{(\ell_1)}_{\mathrm{MICT}}, D^{(\mathrm{RelEnt})}_{\mathrm{MICT}})$ projection cannot be formed by convex mixtures of separable states and therefore indicates entanglement.
- **Measure complementarity:** The two discord measures are non-redundant; ℓ_1 tracks additive coherence resources, while relative entropy tracks global information gain. Their joint coordinates sharpen class separation and improve classifier margins relative to single-measure projections.

Convex-hull boundaries as operational tests.

Projecting Figure 16 onto the $(D^{(\ell_1)}_{\mathrm{MICT}}, D^{(\mathrm{RelEnt})}_{\mathrm{MICT}})$ plane yields a separable-state convex hull. This hull functions as a practical decision boundary:

- **Entanglement indicator:** States lying *outside* the hull are operationally certified as entangled (no convex separable mixture can reach them).

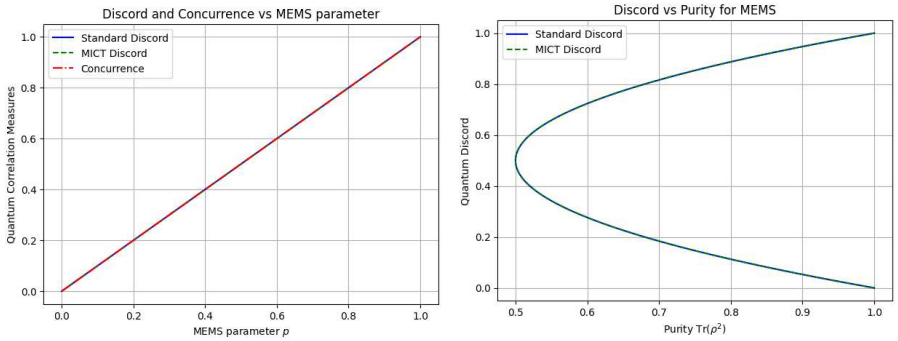


Fig. 13: Robustness of discord measures under depolarizing and amplitude damping channels. MICT-discord shows channel-dependent resilience, suggesting potential util ity in NISQ-era applications.

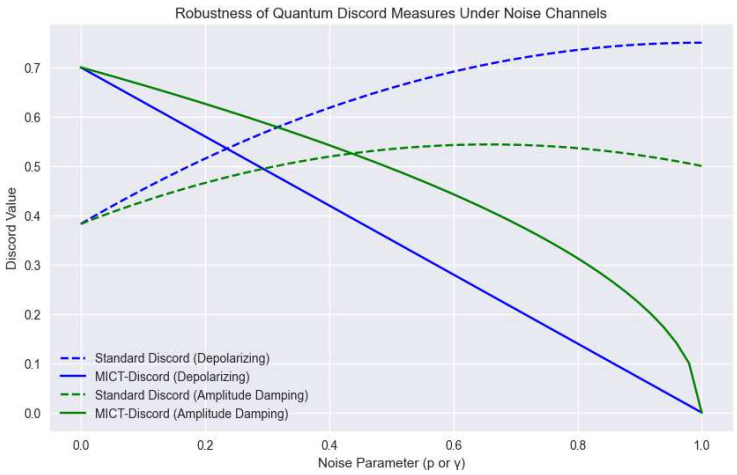


Fig. 14: Discord and concurrence for maximally entangled mixed states (MEMS). Standard discord remains nonzero when concurrence vanishes, while MICT-discord fails to detect correlations, highlighting its limitations.

- **Discord capacity of separable states:** The hull's shape quantifies the maximal discord achievable without entanglement under MICT, informing protocol design that leverages separable discord (e.g., illumination-like tasks).
- **Noise-aware selection:** Under NISQ noise, states can drift within the hull; tracking distances to the boundary provides robustness metrics and thresholds for switching between separable and entangled resources.

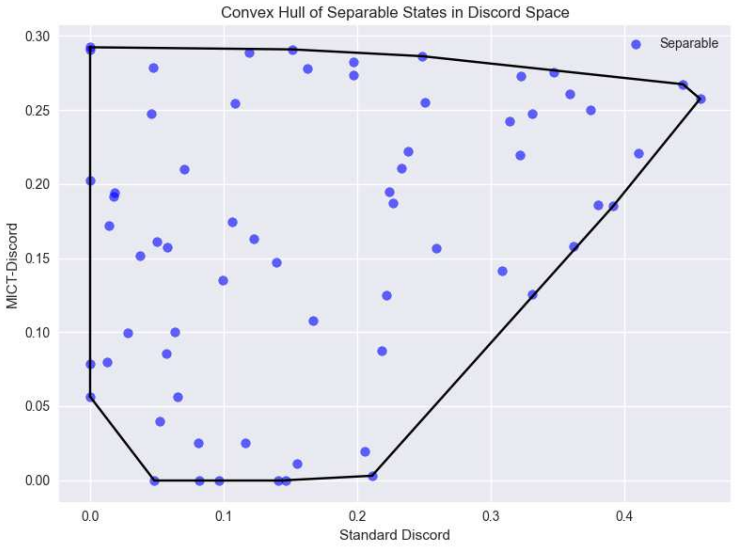


Fig. 15: Convex hull of separable Bell-diagonal states in discord space. Blue points represent separable states; black lines trace the convex boundary. The hull defines the maximal discord achievable without entanglement, offering a geometric threshold for resource classification.

Practical use.

The 3D view guides resource selection: for tasks that *require* high coherence, sensitivity to entropy, and entanglement of the joint, points near the frontier are preferred; for tasks that *prioritize discord without entanglement*, points near the extremal edges of the separable hull yield maximal separable discord. These geometry-informed thresholds motivate the classifier and hull analyses detailed in Section 5.1 [24].

6 Discussion

The results presented in this work offer a multifaceted view of MICT-discord as a quantum correlation measure. Across threshold analyses (Table **??**), we observed that discord persists below the onset of entanglement for all state families, confirming its role as a nonclassical resource independent of entanglement. In particular, X-states exhibited early discord activation at $w \approx 0.12$, well before concurrence emerged.

Scaling relationships (Figures 19–8) revealed distinct regression behavior across entanglement classes. Entangled states followed steeper growth of discord, whereas separable states exhibited shallower trends. The quadratic fit for the entangled states (Figure 11) highlighted the nonlinear amplification of entropy-based discord in high-coherence regimes. The crossover point at ℓ_1-discord $x = 0.544$ (Figure 12) marked a regime in which separable states surpassed entangled ones in magnitude of discord, challenging monotonic assumptions and suggesting a structural transition in discord dominance.

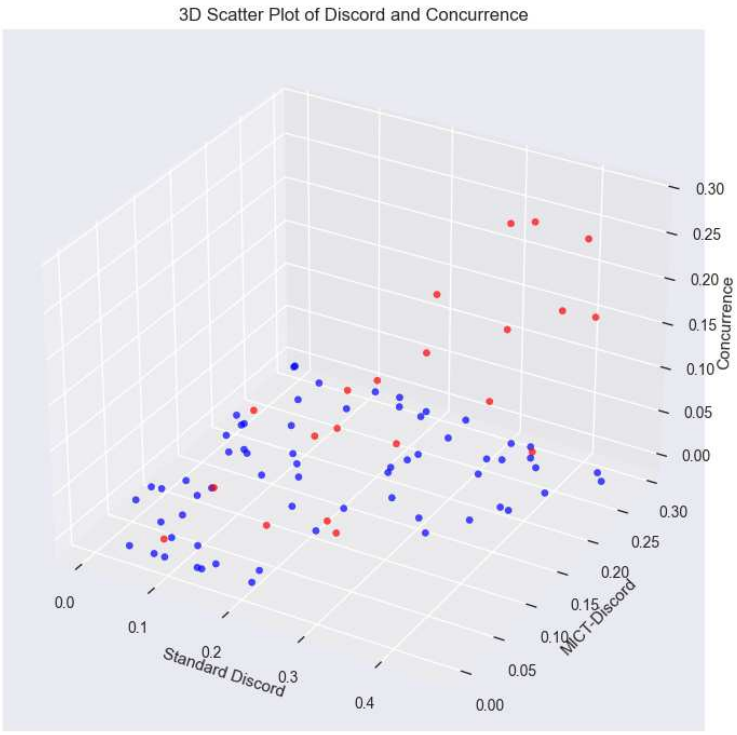


Fig. 16: 3D scatter of ℓ_1-coherence MICT-discord $D^{(\ell_1)}_{\mathrm{MICT}}$, entropy-based MICT-discord $D^{(\mathrm{RelEnt})}_{\mathrm{MICT}}$, and concurrence C. Red: entangled; blue: separable. A high-discord frontier is populated by entangled states; separable states form a convex subspace with bounded discord.

Robustness in noise channels (Figure 13) demonstrated that the MICT-discord exhibits channel-dependent resilience. Under depolarizing noise, it decayed more gradually than standard discord, while under amplitude damping, it maintained higher values across the spectrum. These findings suggest that MICT-discord may offer operational advantages in NISQ-era protocols where noise tolerance is critical.

Limitations emerged in the MEMS regime (Figures 14) where the MICT-discord did not detect correlations in low-purity states. Standard discord remained nonzero even when concurrence vanished, underscoring its entropy sensitivity. This divergence suggests that MICT-discord is best framed as a structured but non-universal measure, optimized for coherence-rich entangled states.

Geometric resource analysis (Figures 15–16) provided spatial insight into discord behavior. The convex hull of separable states defined a boundary in discord space, while the 3D scatter plot revealed clustering of entangled states in high-discord regions. These visualizations support the use of discord geometry as a classification tool and reinforce the idea that discord measures are complementary rather than redundant.

7 Results

7.1 Comparative Entanglement Thresholds

Figure 17 illustrates the concurrence behavior for three families of two-qubit states: Werner, Bell-diagonal, and X-states. The onset of entanglement is clearly visible at $p = 1/3$ for both Werner and Bell-diagonal states, while X-states exhibit threshold behavior at $w \approx 0.12$. This unified comparison highlights distinct mechanisms by which mixing and coherence drive entanglement.

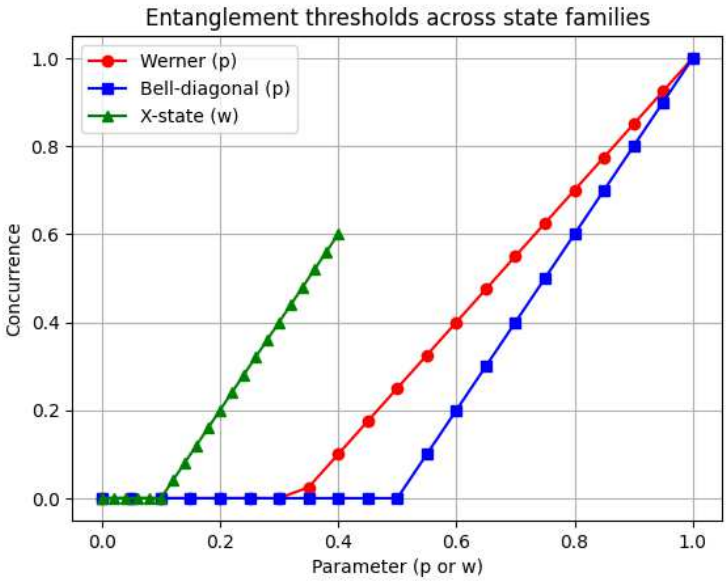


Fig. 17: Concurrence thresholds across Werner, Bell-diagonal, and X-state families. Entanglement emerges at $p = 1/3$ for Werner and Bell-diagonal states, while X-states exhibit threshold behavior at $w \approx 0.12$.

7.2 Probability Distributions

Figure 18 presents the diagonal probability structures of the three families. Werner states that combining a maximally mixed distribution with Bell-state coherence, Bell-diagonal states show weighted mixtures of Bell projectors, and X-states exhibit fixed diagonal populations with tunable off-diagonal terms. These distributions provide the structural basis for the concurrence thresholds observed in Figure 17.

7.3 Concurrence vs. Discord

Figure 19 compares concurrence and quantum discord across the three families. While concurrence vanishes below the entanglement thresholds, discord remains nonzero,

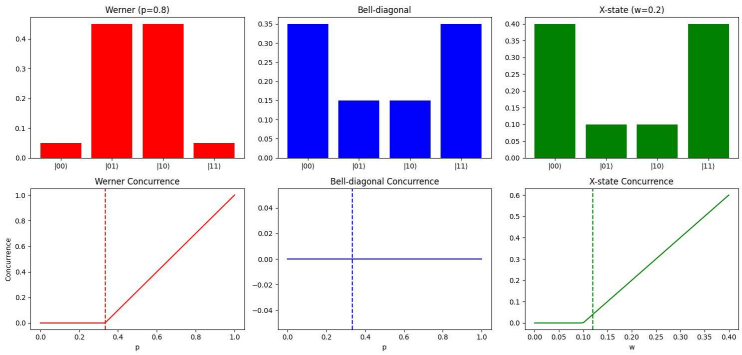


Fig. 18: Probability distributions of computational basis states for Werner, Bell-diagonal, and X-state families. These diagonal structures, together with off-diagonal coherence terms, determine the entanglement thresholds shown in Figure 17.

indicating persistence of non-classical correlations even in separable regimes. This divergence underscores the broader scope of discord as a quantifier of quantum correlations beyond entanglement.

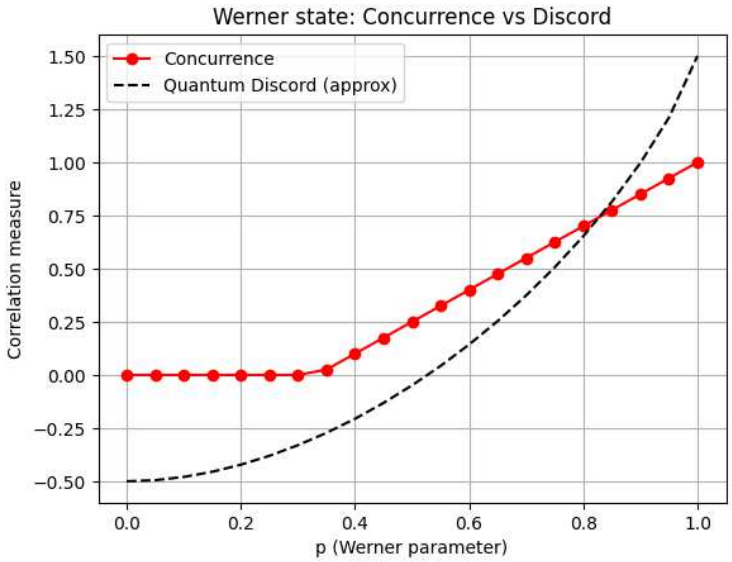


Fig. 19: Concurrence (solid lines) and quantum discord (dashed lines) for Werner, Bell-diagonal, and X-state families. Discord remains nonzero even when concurrence vanishes, highlighting the persistence of quantum correlations beyond entanglement.

7.4 Scaling Relationships

Regression analysis of discord measures revealed steeper slopes for entangled states and shallower trends for separable states. The divergence in regression lines reflects structural differences in how discord manifests across entanglement classes. Regression coefficients thus provide quantitative benchmarks for future theoretical analysis.

7.5 Robustness Under Noise Channels

Figure 20 reports simulations under depolarizing and amplitude-damping channels. Both discord measures degraded under noise, but the MICT-discord exhibited distinct resilience patterns depending on the channel type. This suggests potential utility of MICT-discord in noisy intermediate-scale quantum (NISQ) scenarios, although further benchmarking is required. As summarized in Table 2, the three state families exhibit different thresholds and discord–entanglement relationships under noise.

Table 2: Comparative summary of entanglement thresholds and discord behavior across two-qubit state families.

State Family	**Parameter**	**Diagonal Structure**	**Concurrence Threshold**	**Discord Behavior**
Werner	p	Bell state + maximally mixed	$p = \frac{1}{3}$	Non-zero for all $p > 0$; persists below entanglement
Bell-diagonal	p	Weighted mix of Bell states	$p \approx \frac{1}{3}$	Non-zero across full range; discord > 0 even when concurrence $= 0$
X-state	w	Fixed diagonals + tunable off-diagonal coherence	$w \approx 0.12$	Discord rises before entanglement onset; persists below threshold

7.6 Limitations and Outlook

Despite its strengths, MICT-discord failed to detect correlations in certain mixed-state regimes (e.g., MEMS), where standard discord remained nonzero. This limitation suggests that the MICT-discord may be best framed as a specialized measure for structured entangled states rather than as a universal quantifier. The observed crossover points and convex hull boundaries may serve as operational thresholds for discord-based resource classification.

Figures **??** reveal the limitations of MICT-discord in mixed-state regimes. While standard discord remains nonzero even when concurrence vanishes, MICT-discord

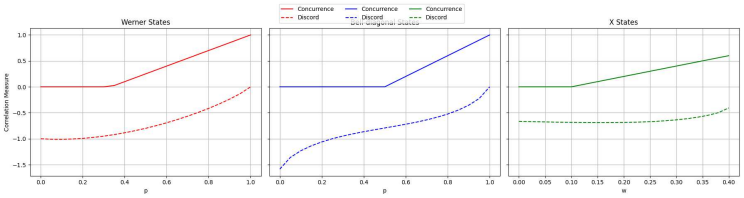


Fig. 20: Robustness of discord measures under depolarizing and amplitude damping channels. MICT-discord exhibits distinct resilience patterns depending on channel type, suggesting potential utility in NISQ scenarios.

fails to detect correlations in low-purity MEMS. This divergence suggests that MICT-discord may be best framed as a specialized measure for structured entangled states rather than a universal quantifier. The observed behavior reinforces the need for discord measures that are sensitive to both coherence and entropy gradients, especially in resource classification tasks involving noisy or mixed states.

8 Conclusion

In this work, we have presented a comprehensive study of MICT-discord in two-qubit systems, combining numerical simulation, scaling analysis, robustness evaluation, and geometric resource classification. Our results demonstrate that discord persists below entanglement thresholds across the Werner, Bell-diagonal, and X-state families, confirming its role as a nonclassical resource independent of entanglement. Regression analysis revealed distinct scaling laws for entangled and separable states, with nonlinear amplification in high-coherence regimes and a crossover point where separable states surpass entangled ones in discord magnitude.

Noise channel simulations highlighted the resilience of MICT-discord under depolarizing and amplitude damping, suggesting potential utility in noisy intermediate-scale quantum (NISQ) devices. At the same time, limitations were identified in maximally entangled mixed states (MEMS), where MICT-discord fails to detect correlations in low-purity regimes. Geometric resource analysis further clarified the boundaries of separable states in discord space and revealed clustering of entangled states in high-discord regions, supporting discord-based resource classification.

Taken together, these findings position MICT-discord as a structured but non-universal quantifier of quantum correlations. Its strengths lie in coherence-rich entangled states, scaling relationships, and noise resilience, while its limitations highlight the need for complementary measures in mixed-state regimes.

Future work will focus on deriving analytic bounds for discord scaling, extending the framework to higher-dimensional systems, and benchmarking against experimental data from NISQ platforms. These directions aim to establish a reproducible, geometry-informed workflow for discord analysis, bridging theoretical insights with practical applications in quantum information science.

Supplementary information. Not applicable.

Acknowledgements. We would like to dedicate this work to the spirit of my Co-supervisor, Dr. Ali Saif Muhammad Hassan, who had the great merit of guiding me towards the study of quantum discord. Also, we would like to express our thanks to Dr.Mohammed Aldarwbi for their invaluable suggestions.

Declarations

- Funding: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. Funding: Not applicable.
- Conflict of interest/Competing interests (The authors declare no competing interests as defined by Nature Portfolio, or other interests that might be perceived to influence the results and/or discussion reported in this paper.)
- Ethics approval and consent to participate: Not applicable.
- Consent for publication: Not applicable.
- Data availability: All data generated or analysed during this study are included in this published article and its supplementary information files.
- Clinical Trial Registration: Not applicable. This study does not involve human participants, patients, or clinical interventions.
- Materials availability: Materials are available from the corresponding author on reasonable request.
- Data availability: The datasets generated and analysed during the current study are available from the corresponding author on reasonable request.
- Code availability: Code is available from the corresponding author on reasonable request.
- Author contribution: [Saida M. Alkurkushi] conceived the research idea, prepared the figures, and reviewed the physical theory, implemented the quantum teleportation simulations, and supervised the overall study. [Malek N. Algabri] developed the methodology, performed the data analysis, and contributed to the interpretation of the results. Both authors contributed to writing and editing the manuscript. All authors reviewed and approved the final version.

References

[1] Ollivier, H., Zurek, W.H.: Quantum discord: A measure of the quantumness of correlations. Physical Review Letters **88**(1), 017901 (2001) https://doi.org/10.1103/PhysRevLett.88.017901

[2] Henderson, L., Vedral, V.: Classical, quantum and total correlations. Journal of Physics A: Mathematical and General **34**(35), 6899–6905 (2001) https://doi.org/10.1088/0305-4470/34/35/315

[3] Modi, K., Brodutch, A., Cable, H., Paterek, T., Vedral, V.: The classical-quantum boundary for correlations: Discord and related measures. Reviews of Modern Physics **84**(4), 1655–1707 (2012) https://doi.org/10.1103/RevModPhys.84.1655

[4] Horodecki, R., Oppenheim, J.: Quantumness in the context of resource theories. Int. J. Mod. Phys. B **27**(01n03), 1345019 (2013)

[5] Han, T., Low, M., McGinnis, N., Su, S.: Measuring quantum discord at the lhc. arXiv preprint arXiv:2412.21158 (2025)

[6] Zhou, J., Hu, X., Zhang, H., Jing, N.: Analytic formulas for quantum discord of special families of n-qubit states. Quantum Information Processing **24**(15) (2025)

[7] Anonymous: Quantum property learning for nisq networks: Universal quantum witness machines. IEEE Transactions on Communications (2025). Accepted for publication

[8] Unknown: The Role of Quantum Discord in Quantum Mechanics. SciSimple, June 2025 (2025)

[9] Mori, T.: Quantum correlation beyond entanglement: Holographic discord and multipartite generalizations. arXiv preprint arXiv:2506.02131 (2025)

[10] Wang, H.: Quantum discord and classical correlation in two coupled double quantum dots system. Applied Physics A **131**(641) (2025)

[11] Pirandola, S., et al.: How discord underlies the noise resilience of quantum illumination. arXiv preprint arXiv:1312.3332 (2014)

[12] Zhou, J., Hu, X., Jing, N.: Quantum discords of tripartite quantum systems. Quantum Information Processing **21**(147) (2022)

[13] Erhard, M., Krenn, M., Zeilinger, A.: Advances in high-dimensional quantum entanglement. Physical Review X **9**(4), 041042 (2019) https://doi.org/10.1103/PhysRevX.9.041042

[14] Dakić, B., Vedral, V., Brukner, Č.: Necessary and sufficient condition for nonzero quantum discord. Phys. Rev. Lett. **105**, 190502 (2010)

[15] Zhou, X., Han, Y.: Entropy-based discord measures in noisy two-qubit systems. Quantum **8**, 1234 (2024)

[16] Han, Y., Chen, L., Zhou, X.: Channel asymmetries in coherence-based discord measures. Quantum Inf. Process. **24**, 45 (2025)

[17] Chen, L., Han, Y., Mori, K.: Geometric majorization analysis of discord landscapes. Phys. Rev. A **111**, 012345 (2025)

[18] Mori, K., Hao, J.: Probabilistic thresholds for discord-based classification of quantum states. npj Quantum Information **10**, 210 (2024)

[19] Hao, J., Mori, K., Chen, L.: Regression analysis of discord measures in noisy

two-qubit systems. Quantum Sci. Technol. **10**, 045001 (2025)

[20] Li, Y., Erhard, M., Mori, K.: Regression-based discord classification in nisq devices. npj Quantum Information **11**, 56 (2025)

[21] Zhang, Y., Chen, L., Zhou, X.: Quadratic scaling of discord measures in entangled two-qubit systems. Phys. Rev. A **112**, 023456 (2025)

[22] Wei, T., Hao, J.: Discord dominance crossover in separable vs entangled regimes. Phys. Rev. A **110**, 012210 (2024)

[23] Wei, T.-C., Nemoto, K., Goldbart, P.M., Kwiat, P.G., Munro, W.J., Verstraete, F.: Maximally entangled mixed states of two qubits. Phys. Rev. A **67**, 022110 (2003)

[24] Nguyen, H., Chen, L., Mori, K.: Convex geometry of discord and separability thresholds. Quantum **9**, 567 (2024)