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ANALYSIS OF HEAT TRANSFER DUE TO STEADY MHD FLOW IN A POROUS CHANNEL WITH NAVIER SLIP AND CONVECTIVE BOUNDARY

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Abstract

This paper investigates the flow and heat transfer characteristics of a magnetohydrodynamic fluid flow in a porous channel. Model equations are derived from the Navier-Stokes equation and the energy equation. The energy equation in this study incorporates viscous dissipation, internal heat generation/absorption, and Joule heating. By applying suitable transformations, the conservative momentum and energy equation are transformed into non-linear ordinary differential equations (ODEs). A closed form solution to the model equations was obtained. By taking into account characteristics like Biot number, Reynolds number, Brinkman number, Magnetic parameter, and others, the investigation on the normalized shear stress at the walls, temperature profiles were possible with aid of Tables and graphical presentations. For practical and engineering purposes, local Nusselt number, skin friction and mass flow rate of the physical constraints was tabulated. This gives important insights about the systems behavior under various settings. The graphical results indicate that the velocity profile exhibits a decrease as the magnetic parameter, Reynolds number and upper wall slip increase. Temperature decreases as Peclet number and slip parameters increase. Heat source/sink and Joule heating were observed to significantly influence the temperature field while the Biot number acts as a modifier in the temperature field as it dictates points of reversals near the convective wall.

Keywords: MHD flow; Shear flow; Biot number; Heat transfer; Nusselt number; Heat source/sink; Slip parameters

Nomenclature

Re	Reynolds number (dimensionless)
Br	Brinkmann number (dimensionless)
M^2	magnetic parameter (dimensionless)
Pe	Peclet number (dimensionless)
P	pressure term (dimensionless)
Nu	Nusselt number (dimensionless)
Q_0	dimensional heat source/sink parameter
H	heat source/sink parameter (dimensionless)
Bi	Biot number
$\bar{Q}$	mass flow rate
u'	dimensional velocity of the fluid

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y'	coordinate perpendicular to the wall [m]
h	channel width [m]
v_0	suction/injection velocity
B_0	magnetic field
T	dimensional fluid temperature [K]
T_0	temperature of the fluid at $y' = 0$
q	heat flux [Watt/m ²]
k	thermal conductivity [W/mK]
c_p	specific heat capacity at constant pressure [kJ/kgK]
y	dimensionless vertical coordinate
u	dimensionless velocity of the fluid
β_1	slip coefficient at the lower wall
β_2	slip coefficient at the upper wall
ζ_1	velocity slip at the lower wall (dimensionless)
ζ_2	velocity slip at the upper wall (dimensionless)

Greek Symbols

λ	index for excluding and including Joule dissipation at $\lambda = 0$ and $\lambda = 1$ respectively
θ	dimensionless fluid temperature
ν	kinematic viscosity of the fluid [m ² /s]
μ	fluid dynamic viscosity [Ns/m ²]
ρ	fluid density [Kg/m ³]
σ	electrical conductivity (Ohm-m)
τ	Skin friction

1. Introduction

Efficient heat transfer in fluid flow systems is a cornerstone of thermal management across a range of engineering applications, including microfluidic devices , aerospace components , energy systems, cooling devices , and chemical reactions . In such settings, fluid motion, particularly shear-induced flows, plays a crucial role in governing thermal performances . Classical no-slip condition and fixed wall temperature assumptions often fail to represent realistic interfaces, especially at microscales or in surfaces coated with rarefied gases. Velocity slip conditions become significant in microchannels or flows involving hydrophobic boundaries . Similarly, a convective boundary condition offers a more accurate representation of heat exchange when the wall itself is cooled or heated by a secondary fluid .

In view of this, studies have been carried out in different setups and configurations in an attempt to improve heat transfer efficiency. For instance, Kumar and Pathak made a comprehensive report on

effect of velocity slip on the flow and heat transfer characteristics during droplet formation in a microfluid T-junction, where it was observed that the heat transfer rate between the dispersed phase and the continuous phase decreases with increase in slip lengths. A study by Rashad *et al.* found that Biot numbers and Eckert numbers enhanced temperature profiles and slowed down heat transfer, while convective boundary conditions and thermal radiation enhanced friction at the surface. The study showed potentially relevant applicability in polymer and glass industries, as well as in the field of heat exchanger styling and cooling operations of metallic plates, among others. Shahzad *et al.* noted that the platelet-shaped nano-particles have the highest heat transfer rate as compared to other particle shapes in their study of thin film flow and heat transfer of Cu-nanofluids with slip and convective boundary condition, considering a stretching sheet. A numerical study on the effect of variable thermal conductivity of ternary hybrid nanofluids over a stretching sheet with convective boundary conditions and magnetic field by Jan *et al.* employed the Runge-Kutta fourth-order method in conjunction with the shooting method. The impact of key factors on the system's heat and velocity field was analysed therein. Vinodkumar *et al.* mentioned the importance of MHD concepts in applications like power generators, cooling processes, sensors and magnetic drug targeting during the study of activation of energy in MHD Casson nanofluid flow through a porous channel with convective boundary condition and suction/injection. Biot number and suction were observed to enhance heat transfer rates whereas the Brownian motion, thermophoresis, heat generation and thermal relaxation parameters declined it. Additionally, activation energy was observed to enhance fluid concentration. In an interesting study by Sadighi *et al.* during heat transfer analysis of fluid flow over a non-linear porous radially moving sheet subject to the convective boundary condition showed the potential to find huge applications in a wide variety of engineering such as extrusion and spinning and biological systems such as blood flow through arteries and veins. The energy equation was solved using hypergeometric Kummer's function. The plots obtained demonstrated that Biot number directly affects heat transfer in the shrinking case of a porous disk. Additionally, the suction/injection parameters were seen to affect the Nusselt number to an increased extent as compared to the heat source/sink parameters.

Chaudhary and Deshwal presented viscosity models of tri-hybrid non-Newtonian nanofluids with Cattaneo-Christov heat flux, thermal radiation, Ohmic heating and convective boundary condition. Results demonstrated that temperature increased due to thermal radiation, viscous dissipation and convective boundary condition, albeit differences in the concentration profiles. The impact of convective boundary on heat and mass transfer of nanofluid flow over a thin needle filled with carbon nanotubes was treated by Sreedevi and Sudarsana Reddy with a focus on non-linear thermal radiation and Biot number on boundary layer flows, taking water as the base fluid. The finite element method was employed in this study which enabled the study of the influence of pertinent parameters like the Sherwood number, Nusselt number and Skin friction. A noteworthy result from the study showed that the velocity profile depreciates with increasing values of the needle size parameter Krishna employed a convective-type boundary condition in modelling a heat transfer process using an electrically conducting fluid subject to an applied magnetic field with a constant strength. An increase in Hartman number was seen to decrease the velocity, but increase when subjected to the mass transfer parameter. Also, the temperature and thickness of the thermal boundary layer were observed to lessen as the Prandtl number increases. Dash *et al.* investigated magnetohydrodynamics (MHD) Couette flow and heat transfer in a rotating channel in the presence of viscous dissipation and heat source/sink. From the analytical solution obtained and computations made, findings graphically showed that moderate speed of rotations gives high rise to heat transfer inconsistency, a transition between low and moderate high speed, thereby suggesting that speed of the channel be monitored for smooth and consistent heat transfer.

Recently, Recently, Xu *et al.* explored surface slip and flow modulation impact on heat transfer in nanochannels. Higher velocity was observed to enhance heat transfer and a linear relationship was seen between interfacial temperature jump in smooth graphene-coated channels. The study provides insights for designing micro-nanofluidic devices to improve thermal management Jayaprakash *et al.* treated the

impact of an aligned flat plate with leading edge accretion and slip boundary condition in the analysis of heat transfer in Casson fluid flow. The study emphasised on reducing leading-edge accretion of Casson fluid in cooling systems for rolling mills and heat exchangers. In their comparative study, it was observed that non-Newtonian fluids improve heat retention, thermal stability and energy efficiency. Considering the viscous and thermal conductivity as time-dependent, Olkha *et al.* investigated the slip flow of a non-Newtonian second-grade fluid influenced by a non-linear radially stretching surface embedded in a porous medium under convective boundary. The MATLAB-based BVP4c numerical technique was employed, and graphical results showed that the temperature field lessens due to the stretching index, while the opposite effect is observed due to the existence of radiation, convective surface and variation of thermal conductivity. The skin friction and Nusselt number were augmented with an increase in the stretching index Jubair *et al.* examined the fluid velocity and heat transfer under the impact of Darcy-Forchheimer, chemical reaction, convective boundary condition, activation energy, and exponential heat source during an analysis of Careau-Yasuda nanofluid (CYNF) across a stretching surface. They hinted that, researchers may discover novel perspectives for increasing heat transfer performance, establishing advanced materials and enhancing the efficiency of multiple engineering systems by studying the behaviours of CYNF. In that investigation, the temperature field was observed to augment against the variation of viscous dissipation and heat source term, and the magnetic parameter impacts the temperature field significantly, whereas it falls in the velocity profile.

Despite this extensive body of work, most studies have treated velocity slip and convective heat transfer independently. Their combined influence on shear-driven flows, particularly in terms of boundary layer development and overall heat transfer, remains relatively underexplored. This gap is significant, given that realistic wall–fluid interactions in practical systems often involve both effects simultaneously. The present study addresses this need by investigating the coupled effects of velocity slip and convective boundary conditions on heat transfer in steady shear flow taking account heat source/sink and the Joule heating. Key parameters such as slip coefficients, Biot number, and heat source/sink are examined. The findings provide more insight into how slip and convective conditions interact, offering guidance for the design of microfluidic devices, coating processes, and other thermal systems where conventional boundary assumptions are inadequate.

2. Mathematical formulation and solutions

We consider a steady flow of a fully developed incompressible electrically conducting viscous fluid bounded by two infinite horizontal parallel porous walls separated by a distance h in the presence of a volumetric internal heat generation and absorption. By the Cartesian coordinate system, the x - axis is taken along the lower stationary wall, while the y - axis is taken perpendicular to the wall as shown in Figure 1.

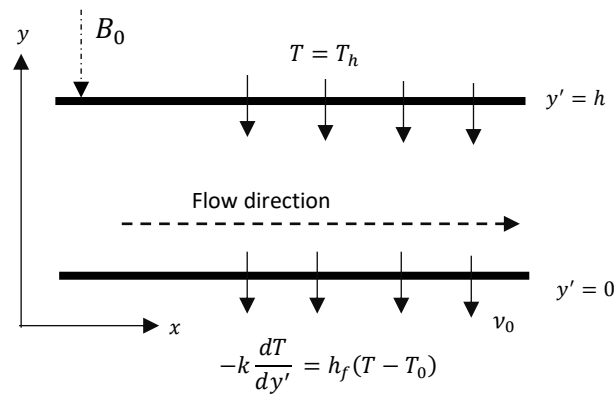


Fig. 1 Geometry of the problem

Following Das and Jana , a transverse magnetic field B_0 is applied uniformly and perpendicularly to the channel walls. Since the walls are infinitely long, all physical variables except pressure depend on y only. The continuity equation then gives $v = -v_0$ everywhere in the fluid and where v_0 is the suction velocity at the walls. By the assumptions stated, the equation of motion along the x - direction is:

$$\mu \frac{d^2 u'}{dy'^2} - \rho v_0 \frac{du'}{dy'} - \sigma B_0^2 u' = -\frac{\partial P}{\partial x} \quad (1)$$

Where u' is the dimensional velocity of the fluid in the x - axis, ρ is the density of the fluid, μ is the coefficient of viscosity, σ is the electrical conductivity of the fluid and P is the fluid pressure.

The energy equation is:

$$k \frac{d^2 T}{dy'^2} - \rho C_p v_0 \frac{dT}{dy'} \pm Q_0 (T - T_0) + \mu \left(\frac{du'}{dy'} \right)^2 + \lambda \sigma B_0^2 u'^2 = 0 \quad (2)$$

Where k is the thermal conductivity, C_p is the specific heat at constant pressure, Q_0 is the dimensional heat source/sink parameter and the index, λ in equation (2) is set equal to 1 for including Joule dissipation and 0 for excluding Joule dissipation.

The boundary conditions are:

$$\begin{aligned} u' &= \beta_1 \frac{du'}{dy'} ; \quad -k \frac{dT}{dy'} - h_f (T - T_0) = 0 \quad \text{at} \quad y' = 0 \\ u' &= \beta_2 \frac{du'}{dy'} ; \quad T = T_h \quad \text{at} \quad y' = h \end{aligned} \quad (3)$$

Where T is the fluid temperature, T_h is the temperature at the upper wall, T_0 is the temperature at the lower wall and β_1 and β_2 are the slip coefficients. The non-dimensional quantities used in the above equations are defined as:

$$\begin{aligned} \theta &= \frac{T - T_0}{T_h - T_0} ; \quad y = \frac{y'}{h} ; \quad u = \frac{u'}{v_0} ; \quad M^2 = \frac{\sigma B_0^2 h^2}{\rho v} ; \quad Br = \frac{\mu v_0^2}{k(T_h - T_0)} ; \quad Pe = \frac{v_0 h \rho C_p}{k} ; \quad Re = \frac{v_0 h}{\nu} ; \quad Bi = \frac{h_f h}{k} \\ P &= \frac{h^2}{\rho \nu v_0} \left(-\frac{\partial P}{\partial x} \right) ; \quad H = \frac{Q_0 h^2}{k} ; \quad \zeta_1 = \frac{\beta_1}{h} ; \quad \text{and} \quad \zeta_2 = \frac{\beta_2}{h} \end{aligned} \quad (4)$$

Applying equation (4) on equations (1) – (3), the dimensionless forms of the momentum and energy equations become:

$$\frac{d^2 u}{dy^2} - Re \frac{du}{dy} - M^2 u = -P \quad (5)$$

$$\frac{d^2 \theta}{dy^2} - Pe \frac{d\theta}{dy} \pm H \theta + Br \left[\left(\frac{du}{dy} \right)^2 + \lambda M^2 u^2 \right] = 0 \quad (6)$$

With boundary conditions:

$$\begin{aligned} u &= \zeta_1 \frac{du}{dy} , \quad \frac{d\theta}{dy} + Bi \theta = 0 \quad \text{at} \quad y = 0 \\ u &= \zeta_2 \frac{du}{dy} , \quad \theta = 1 \quad \text{at} \quad y = 1 \end{aligned} \quad (7)$$

Where θ is the dimensionless fluid temperature, M^2 is the magnetic parameter, Br is the Brinkmann number, Pe is the Peclet number, Re is the Reynolds number, P is the non-dimensional pressure gradient, H is the dimensionless heat source/sink parameter, Bi is the Biot number and ζ_1 and ζ_2 are the slip parameters.

The solution of equation (5) subject to the boundary condition in equation (7) is given as:

$$u(y) = [X_1 \cosh(ry) + Y_1 \sinh(ry)] e^{\frac{Re}{2}y} + \frac{P}{M^2} \quad (8)$$

Where:

$$r = \left(\frac{Re^2}{4} + M^2 \right)^{\frac{1}{2}} \quad (9)$$

$$X_1 = \frac{-\frac{P}{M^2} \left[r \zeta_1 e^{-\frac{Re}{2}} + \sinh(r) - \beta_2 \left(\frac{Re}{2} \sinh(r) + r \cosh(r) \right) \right]}{r(\zeta_1 - \zeta_2) \cosh(r) + \left[\left(\frac{Re^2}{4} - r^2 \right) \zeta_1 \zeta_2 - \frac{1}{2} Re(\zeta_1 + \zeta_2) + 1 \right] \sinh(r)} \quad (10)$$

$$Y_1 = \frac{P}{r \zeta_1 M^2} + \frac{X_1}{r} \left(\frac{1}{\zeta_1} - \frac{1}{2} Re \right) \quad (11)$$

The solution given by equation (8) exists for both $Re > 0$ (corresponding to $v_0 > 0$ for the suction at the plates) and $Re < 0$ (corresponding to $v_0 < 0$ injection or blowing at the plates).

On use of equation (8), equation (6) is refined as:

$$\frac{d^2 \theta}{dy^2} - Pe \frac{d\theta}{dy} \pm H = -Br \left[e^{Rey} \{ Z_1 \cosh^2(ry) + Z_2 \sinh^2(ry) + Z_3 \cosh(ry) \sinh(ry) \} + e^{\frac{Re}{2}y} \{ Z_4 \cosh(ry) + Z_5 \sinh(ry) \} + \frac{\lambda P^2}{M^2} \right] \quad (12)$$

The solution for equation (12) subject to the boundary condition in equation (7) is given as:

$$[C_1 \cosh(\eta y) + C_2 \sinh(\eta y)] e^{\frac{Pe}{2}y} - Br \left[e^{Rey} \{ Y_2 + Y_3 \cosh(2ry) + Y_4 \sinh(2ry) \} + e^{\frac{Re}{2}y} \{ Y_5 \cosh(ry) + Y_6 \sinh(ry) \} - X_{14} \right] \quad (13)$$

Where:

$$\eta = \frac{1}{2} (Pe^2 + 4H)^{\frac{1}{2}}$$

$$C_1 = \frac{(W_3 + Bi W_1) \sinh(\eta) - \eta(1 + W_2) e^{-\frac{Re}{2}}}{\left(\frac{Pe}{2} + Bi \right) \sinh(\eta) - \eta \cosh(\eta)}$$

$$C_2 = \frac{(1 + W_2) e^{-\frac{Pe}{2}} - C_1 \cosh(\eta)}{\sinh(\eta)}$$

$$Z_1 = \frac{Re^2}{4} X_1^2 + Y_1^2 n^2 + r Re X_1 Y_1 + \lambda M^2 X_1^2$$

$$Z_2 = \frac{Re^2}{4} Y_1^2 + X_1^2 r^2 + r Re X_1 Y_1 + \lambda M^2 X_1^2$$

$$Z_3 = \frac{Re^2}{2} X_1 Y_1 + r Re X_1^2 + r Re Y_1^2 + 2r^2 X_1 Y_1 + 2\lambda M^2 X_1 Y_1$$

$$Z_4 = 2X_1 P \lambda ; Z_5 = 2Y_1 P \lambda$$

$$X_2 = \frac{Z_1}{2[Re^2 - RePe - H]} \quad X_3 = \frac{Z_1}{4[(2r + Re)^2 - Pe(2r + Re) - H]} \quad X_4 = \frac{Z_1}{4[(Re - 2r)^2 - Pe(Re - 2r) - H]}$$

$$X_5 = \frac{Z_2}{[Re^2 - RePe - H]} \quad X_6 = \frac{Z_2}{2[(2r + Re)^2 - Pe(2r + Re) - H]} \quad X_7 = \frac{Z_2}{2[(Re - 2r)^2 - Pe(Re - 2r) - H]}$$

$$X_8 = \frac{Z_3}{4[(2r + Re)^2 - Pe(2r + Re) - H]} \quad X_9 = \frac{Z_3}{4[(Re - 2r)^2 - Pe(Re - 2r) - H]} \quad X_{10} = \frac{Z_4}{2\left[\left(r + \frac{1}{2} Re\right)^2 - Pe\left(r + \frac{1}{2} Re\right) - H\right]}$$

$$\begin{aligned}
X_{11} &= \frac{Z_4}{2\left[\left(\frac{1}{2}Re-r\right)^2 - Pe\left(\frac{1}{2}Re-r\right) - H\right]} & X_{12} &= \frac{Z_5}{2\left[\left(r+\frac{1}{2}Re\right)^2 - Pe\left(r+\frac{1}{2}Re\right) - H\right]} & X_{13} &= \frac{Z_5}{2\left[\left(\frac{1}{2}Re-r\right)^2 - Pe\left(\frac{1}{2}Re-r\right) - H\right]} \\
X_{14} &= \frac{\lambda P^2}{M^2 H} & Y_2 &= X_2 - X_5 & Y_3 &= X_3 + X_6 + X_8 & Y_4 &= X_4 + X_7 - X_9 & Y_5 &= X_{10} + X_{12} \\
Y_6 &= X_{11} - X_{13} & W_1 &= Br(Y_2 + Y_3 + Y_5 - X_{14}) & W_3 &= Br\left[Re(Y_2 + Y_3) + 2rY_4 + \frac{ReY_5}{2} + rY_6\right] \\
W_2 &= Br\left[e^{Re}(Y_2 + Y_3 \cosh(2r) + Y_4 \sinh(2r)) + e^{\frac{Re}{2}}(Y_5 \cosh(r) + Y_6 \sinh(r) - X_{14})\right] \\
W_4 &= Br\left[Re^{Re}(Y_2 + Y_3 \cosh(2r) + Y_4 \sinh(2r)) + e^{Re}(2Y_3 \sinh(2r) + 2Y_4 \cosh(2r)) + \right. \\
&\quad \left. \frac{Re}{2}e^{Re}(Y_5 \cosh(r) + Y_6 \sinh(r)) + e^{\frac{Re}{2}}(rY_5 \sinh(r) + rY_6 \cosh(r))\right] \tag{14}
\end{aligned}$$

2.1 Skin friction, Mass flow rate and Nusselt number

Two important flow performance parameters of engineering interest namely; the skin friction (τ) and mass flow rate $\bar{Q}$ and an important transport characteristic, namely the Nusselt number (Nu) are obtained below.

Using Equation (8), the skin friction (τ) is obtained by direct differentiation at $y = 0$ and $y = 1$ and it is given as:

$$\tau_1 = \left. \frac{du}{dy} \right|_{y=0} = rY_1 + \frac{ReX_1}{2} \tag{15}$$

$$\tau_2 = \left. \frac{du}{dy} \right|_{y=1} = \left[(rX_1 \sinh(r) + rY_1 \cosh(r)) + \frac{Re}{2}(X_1 \cosh(r) + Y_1 \sinh(r)) \right] e^{\frac{Re}{2}} \tag{16}$$

Integrating Equation 8 over 0 and 1 gives the mass flux rate $\bar{Q}$ which is:

$$\bar{Q} = \int_0^1 u(y) dy = \left(\frac{X_1 + Y_1}{2} \right) \left[\frac{1}{r + \frac{Re}{2}} \left(e^{\left(r + \frac{Re}{2}\right)} - 1 \right) \right] + \left(\frac{X_1 - Y_1}{2} \right) \left[\frac{1}{\frac{Re}{2} - 1} \left(e^{\left(\frac{Re}{2} - 1\right)} - 1 \right) \right] + \frac{P}{M^2} \tag{17}$$

The rate of heat transfer (Nusselt number) at the lower and upper plates is obtained by differentiating Equation (13) at $y = 0$ and $y = 1$ respectively, and it is given as:

$$Nu_0 = \left. \frac{d\theta}{dy} \right|_{y=0} = \eta C_2 + \frac{PeC_1}{2} - W_3 \tag{18}$$

$$Nu_1 = \left. \frac{d\theta}{dy} \right|_{y=1} = \frac{Pe}{2} [C_1 \cosh(\eta) + C_2 \sinh(\eta)] e^{\frac{Pe}{2}} + [C_1 \eta \sinh(\eta) + C_2 \eta \cosh(\eta)] e^{\frac{Pe}{2}} - W_4 \tag{19}$$

2.2 Validation of the method

The closed form solutions to the governing equations were obtained using the method of undetermined coefficients. Table 1 provides numerical results for temperature from Das and Jana , and the present study. Validation of the present results was carried out by setting $H = 0$ and $Bi = 1$ in equations (6) and (7) respectively, which reduces the solution to that of . The comparison demonstrates a strong agreement between the present findings and those reported by Das and Jana .

Table 1: Numerical comparison for values of temperature for the present work with Das and Jana and ($P = 1$; $M^2 = 5$; $Pe = 2$; $Re = 2$; $Bi = 2$; $\lambda = 0$; $H = -2$)

(θ)	Das and Jana			Present study		
Br/Pe	0	1	5	0	1	5
0	2.7178	-4.3401	-0.0380	2.7182	-4.3403	-0.0384
0.02	2.7361	-4.3771	-0.0389	2.7364	-4.3773	-0.0392
0.2	2.9180	-4.7102	-0.0461	2.9182	-4.7106	-0.0465
1	3.7192	-6.1911	-0.0783	3.7195	-6.1916	-0.0789
2	4.7206	-8.0427	-0.1189	4.7208	-8.0430	-0.1193

3. Results and discussion

In this section we provide a detailed investigation of the outcomes from this study with a prime focus on studying the fluid velocity and heat transfer under several physical and heating conditions. A more complete perspective of the systems behavior can be gained by carefully reviewing these graphical epitomes and numbers, allowing for informed judgements and additional investigation of the subject matter being studied.

3.1 Velocity distribution

The velocity distribution in the channel is governed by the combined effects of magnetic damping, wall permeability, and slip conditions. As the magnetic parameter increases, the velocity consistently decreases throughout the channel. This outcome is a direct consequence of the Lorentz force, which acts as a resistive drag on the electrically conducting fluid. The interaction between the induced current and the applied field generates an opposing electromagnetic force that suppresses motion. The stronger the magnetic field, the greater this braking effect, leading to a lower overall velocity profile. This is shown in figure 2.

The influence of the Reynolds number as presented in Fig. 3 reflects the role of suction and injection through the porous wall. For $Re < 0$ (injection), additional fluid is introduced into the channel, enhancing bulk momentum transport and raising the velocity field. Conversely, when $Re > 0$ (suction), fluid is withdrawn through the wall, depleting the effective channel flow volume and reducing velocity. This asymmetry highlights the sensitivity of the flow field to wall mass transfer, with injection promoting fuller velocity profiles and suction attenuating them. Wall slip modifies the velocity field in distinct ways depending on its location. At the lower convective wall, increasing the slip parameter ζ_1 reduces viscous resistance at the boundary, allowing fluid to move more freely near the wall. This elevates the near-wall velocity and thickens the profile in the core, effectively increasing the overall velocity distribution. In contrast, the slip at the upper wall ζ_2 weakens momentum exchange at that boundary, flattening the velocity gradient and lowering the near-wall velocity. The result is a reduction in the velocity field across the channel. This is shown in Figs. 4 – 5. Taken together, the velocity profiles indicate that magnetic effects uniformly damp velocity, Reynolds number modulates it through suction and injection, and slip exerts location-dependent control. The Navier slip condition in particular

provides a mechanism for enhancing or suppressing transport, depending on whether it is applied at the driving or resisting boundary. This interplay of mechanisms is central to tailoring velocity fields in MHD porous channel applications.

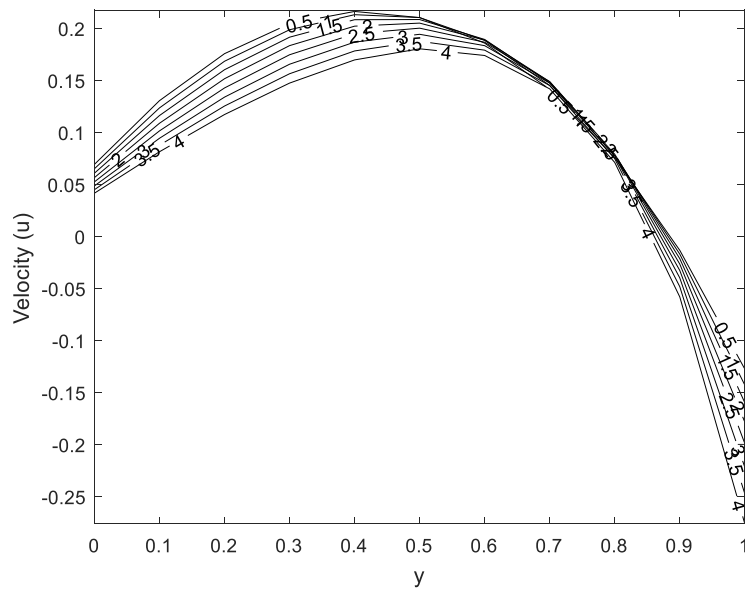


Fig. 2 Impact on $u(y)$ with various M^2

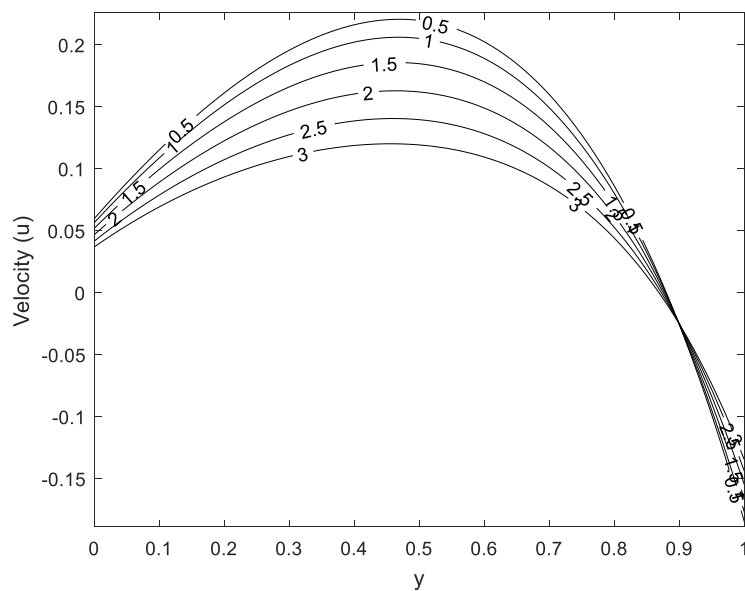


Fig. 3 Impact on $u(y)$ with various Re

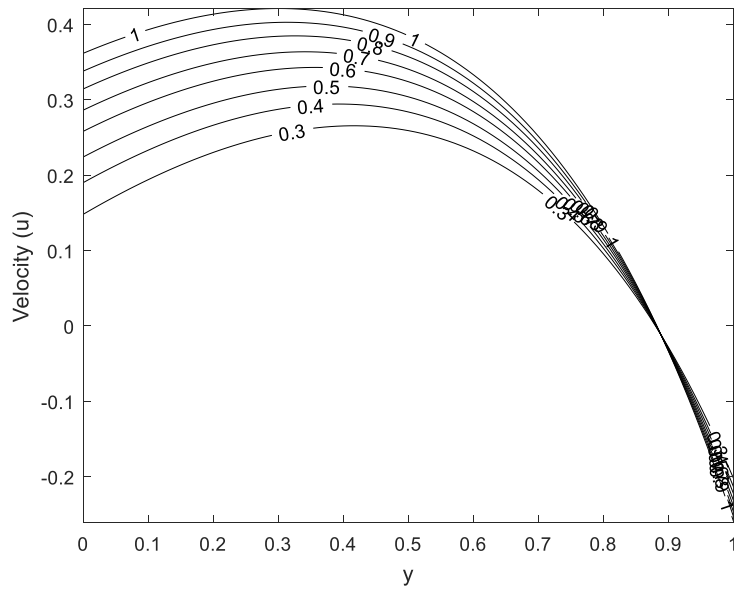


Fig. 4 Impact on $u(y)$ with various ζ_1

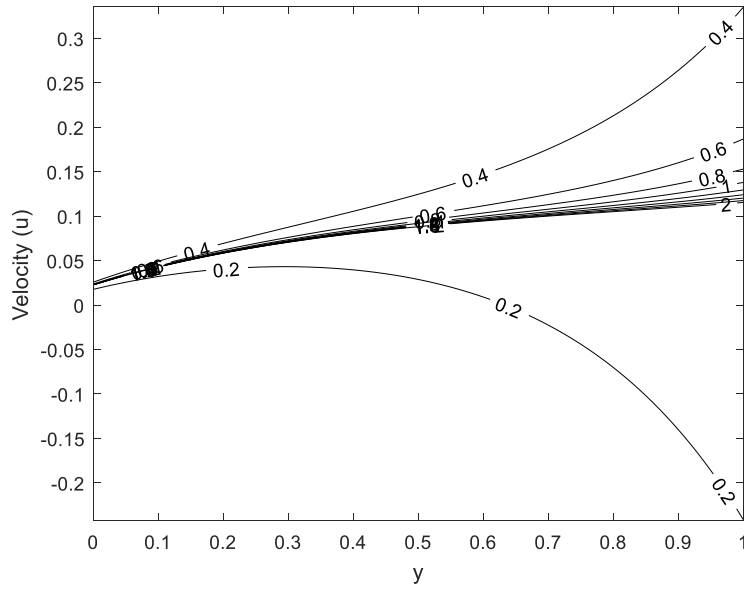


Fig. 5 Impact on $u(y)$ with various ζ_2

3.2 Skin Friction

The skin friction results in Tables 2–3 reflect the combined influence of inertia, magnetic effects, and wall slip on near-wall shear behaviour. Increasing Reynolds number strengthens inertia, steepening the velocity gradient and raising skin friction, whereas at the upper wall, it redistributes momentum toward the channel core, flattening the profile and reducing shear. The magnetic parameter consistently

increases skin friction at both walls due to the Lorentz force, which opposes motion and enhances velocity gradients near the boundaries. We gather from the values that slip effects introduce somewhat a location-dependent response such that at the lower wall, increased slip intensifies the local velocity gradient and raises skin friction, while at the upper wall it weakens the gradient and reduces shear stress. When slip is imposed at the upper boundary, momentum exchange is further suppressed, producing a more uniform velocity field and a general reduction in skin friction at both walls.

Table 2: Skin friction at the lower and upper plate for Re and M^2 ($P = 1$; $\zeta_1 = 0.1$; $\zeta_2 = 0.1$)

		τ_0					τ_1		
Re/M^2	2	4	6	10		2	4	6	10
0.2	-0.4278	-0.1796	-0.1038	-0.0493		-0.6444	-0.4643	-0.4616	10.4477
0.4	-0.4136	-0.1764	-0.1027	-0.0486		-0.6386	-0.4717	-0.4741	5.1987
2	-0.3010	-0.1516	-0.0941	-0.0475		-0.6167	-0.5590	-0.6223	1.0048
4	-0.2010	-0.1253	-0.0842	-0.0450		-0.6758	-0.7898	-1.1150	0.4866

Table 3: Skin friction at the lower and upper walls for ζ_1 and ζ_2 ($P = 1$; $Re = 2$; $M^2 = 5$)

		$-\tau_0$				$-\tau_1$			
ζ_2/ζ_1	0.04	0.2	0.4	0.6		0.04	0.2	0.4	0.6
0.05	0.3130	0.2733	0.2391	0.2148		0.4665	0.5169	0.5605	0.5913
0.1	0.3169	0.2789	0.2462	0.2231		0.5920	0.0511	0.7019	0.7378
0.3	1.1273	1.0281	0.9843	0.9638		26.2534	18.7492	15.4439	13.8877
0.5	0.2755	0.2194	0.1686	0.1313		-0.7200	-0.7863	-0.8464	-0.8905

3.3 Mass flow rate

The mass flux rate in Tables 4–5 is governed by the interplay of suction/injection, magnetic effects, and boundary slip. Injection $Re < 0$ enhances mass flux by supplying additional fluid into the channel, whereas suction $Re > 0$ reduces it through fluid removal and reduced bulk momentum. The magnetic parameter consistently suppresses mass flux due to the Lorentz force, which opposes motion of the conducting fluid and damps overall transport under both suction and injection conditions. Slip effects introduce a boundary-dependent response, which suggests that increasing slip at the lower wall reduces viscous resistance and enhances transport toward the channel core, increasing mass flux, while slip at the upper wall weakens momentum exchange near the boundary and reduces overall mass transport.

Table 4: Mass flow rate for Re and M^2 ($P = 1$; $\zeta_1 = 0.1$; $\zeta_2 = 0.1$)

		Mass flux $\bar{Q}$			
Re/M^2	1	2	4	6	10
-2	0.0639	0.3006	0.2147	0.1534	0.0879
-1	0.0662	0.3018	0.2146	0.1529	0.0780
0	0.0664	0.3017	0.2141	0.1523	0.1180
1	0.0643	0.3001	0.2132	0.1513	0.1178
2	0.0599	0.2971	0.2117	0.1497	0.1080

Table 5: Mass flow rate for ζ_1 and ζ_2 ($P = 1$; $M^2 = 5$; $Re = 1$)

	Mass flux $\bar{Q}$				
ζ_2/ζ_1	0.05	0.2	0.4	0.6	0.8
0.1	0.2923	0.3053	0.3173	0.3257	0.3320
0.2	0.2303	0.2372	0.2435	0.2479	0.2511
0.3	-2.2273	-1.6123	-1.3085	-1.1655	-1.0824
0.4	0.4988	0.5431	0.5876	0.6210	0.6470
0.5	0.4263	0.4575	0.4880	0.5104	0.5275

3.4 Nusselt Number

The Nusselt number variation in Tables 6 – 9 is governed by the coupled effects of advection, viscous and magnetic dissipation, heat source/sink terms, and boundary slip under mixed thermal conditions. Increasing the Peclet and Biot numbers reduces Nu at both walls, as stronger convection and enhanced wall ambient coupling promotes thermal energy redistribution within the channel core, thereby weakening near-wall temperature gradients and reducing wall heat flux. In contrast, both the Brinkman number and magnetic parameter increase Nu due to enhanced viscous and Joule heating, which elevate fluid temperature relative to the walls and steepen thermal gradients. The heat source/sink parameter further amplifies wall heat transfer by increasing the thermal imbalance between the fluid and boundaries. Slip effects introduce a clear asymmetry where we observe that at the lower convective wall, slip strengthens near-wall transport and enhances Nu , whereas at the upper isothermal wall it weakens, reducing the thermal gradient and lowering Nu . Generally, heat transfer is intensified by dissipation and internal heating mechanisms, while advection (high Pe), strong thermal coupling (high Bi), and upper wall slip act to suppress wall heat transfer through thermal gradient attenuation.

Table 6: Nusselt number for Pe and Bi at the lower and upper plate ($P = 1$; $Re = 2$; $M^2 = 5$; $\zeta_1 = 0.2$; $\zeta_2 = 0.2$; $Br = 2$; $H = 1$)

	Nu_0					Nu_1			
Pe/Bi	0.5	2	4	7		0.5	2	4	7
0	4.5332	2.8242	2.6573	2.5916		8.0524	-2.9064	-3.9771	-4.3981
1	1.1604	1.4259	1.4825	1.5081		0.6231	-2.9943	-3.7645	4.1136
2	0.5134	0.7472	0.8085	0.8380		-0.6504	-2.7665	-3.3203	3.5866
7	0.0748	0.0343	0.0366	0.0377		-0.1498	1.1377	1.0685	1.0360

Table 7: Nusselt number for Pe and Bi at the lower and upper plate ($P = 1$; $Re = 2$; $Pe = 2$; $Bi = 1$; $\zeta_1 = 0.2$; $\zeta_2 = 0.2$; $Br = 2$; $H = 1$)

	Nu_0					Nu_1			
Br/M^2	4	6	8	10		4	6	8	10
0.02	4.3766	4.3852	4.3980	4.4185		7.7425	7.7425	7.7349	7.7224
0.2	4.7030	4.7899	4.9179	5.1228		7.7896	7.7403	7.6645	7.5392
4	11.5942	13.3318	15.8925	19.9898		8.6800	7.6943	6.1785	3.6729
6	15.2212	17.8276	21.6686	27.8145		9.1487	7.6701	5.3964	1.6381

Table 8: Nusselt number for heat source/sink with ζ_1 at the lower and upper plate ($P = 1$; $M^2 = 5$; $Br = 2$; $Re = 2$; $Pe = 1$; $\zeta_2 = 0.1$; $Bi = 1$)

		Nu_0					Nu_1		
H/ζ_1	0.02	0.1	0.2	0.5		0.02	0.1	0.2	0.5
-1.0	1.7444	1.6997	1.6296	1.5057		-1.0	-0.9674	-0.4667	0.0869
-0.5	2.0992	2.0450	1.9600	1.8100		-0.5	-3.4074	0.1216	0.6338
0.5	4.0200	3.9148	3.7497	3.4584		0.5	2.4294	2.7115	3.0293
1.0	8.5462	8.3208	7.9673	7.3439		1.0	8.3272	8.2114	8.1009

Table 9: Nusselt number for heat source/sink with ζ_2 at the lower and upper plates ($P = 1$; $M^2 = 5$; $Br = 2$; $Re = 2$; $Pe = 1$; $\zeta_1 = 0.1$; $Bi = 1$)

		Nu_0					Nu_1		
H/ζ_2	0.02	0.1	0.2	0.5		0.02	0.1	0.2	0.5
-1.0	1.4405	1.4662	1.6296	1.3424		0.5391	0.3934	-0.4667	1.3409
-0.5	1.7325	1.7633	1.9600	1.6162		1.0642	0.9272	0.1216	1.8329
0.5	3.3138	3.3727	3.7497	3.0985		3.3631	3.2654	2.7115	3.9870
1.0	7.0408	7.1658	7.1658	6.5913		8.2284	8.2163	8.2114	8.5463

3.5 Temperature distribution

The thermal behaviour of the MHD shear flow under consideration reveals a complex interplay between wall convection, velocity slip, internal heat generation or absorption, and dissipative effects. These parameters collectively govern how heat is distributed across the channel, and the results demonstrate several recurring features that shed light on the underlying physics. Figures 6 – 18 captures the temperature profiles.

Figures 6–9 show that increasing the Biot number Bi leads to a rise in fluid temperature for both heat source and heat sink cases, with or without Joule heating. Physically, a higher Biot number strengthens convective heat exchange at the boundary, intensifying the thermal interaction between the wall and the fluid. This enhances heat penetration into the channel, raising the overall fluid temperature and thickening the thermal boundary layer. The effect remains consistent across all heating configurations, indicating that boundary-driven thermal forcing dominates over internal dissipation in controlling the temperature field under varying Biot conditions.

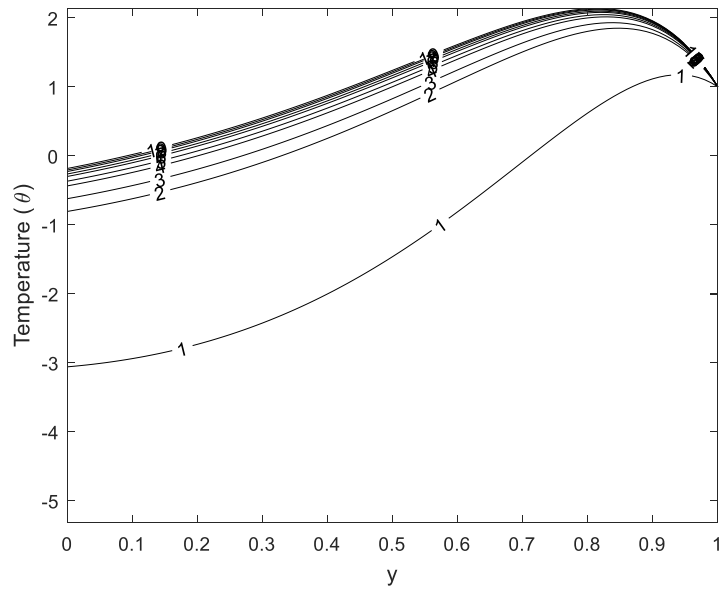


Fig 6: Impact on $\theta(y)$ for various Bi with heat source and excluding joule dissipation.

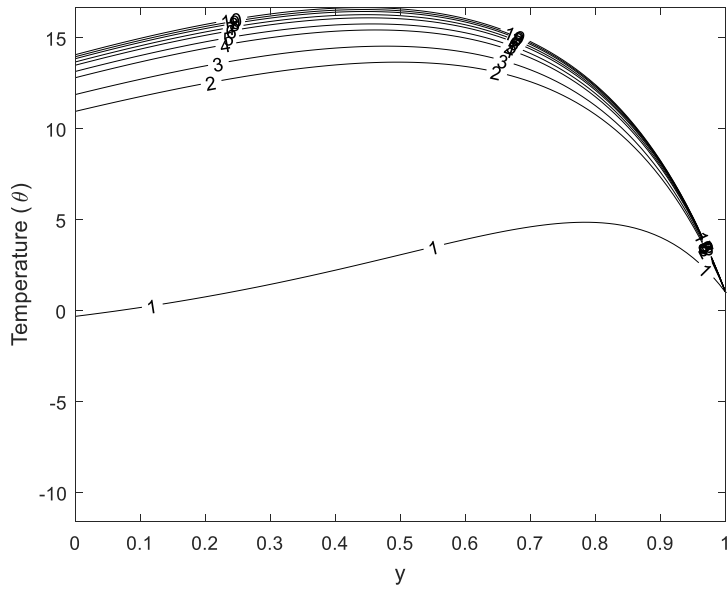


Fig 7: Impact on $\theta(y)$ for various Bi with heat source and excluding joule dissipation

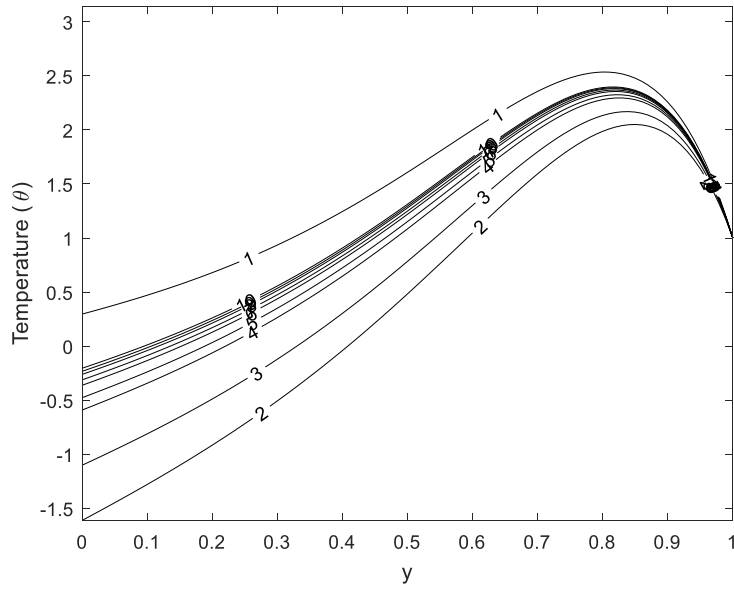


Fig 8: Impact on $\theta(y)$ for various Bi with heat sink and excluding joule dissipation

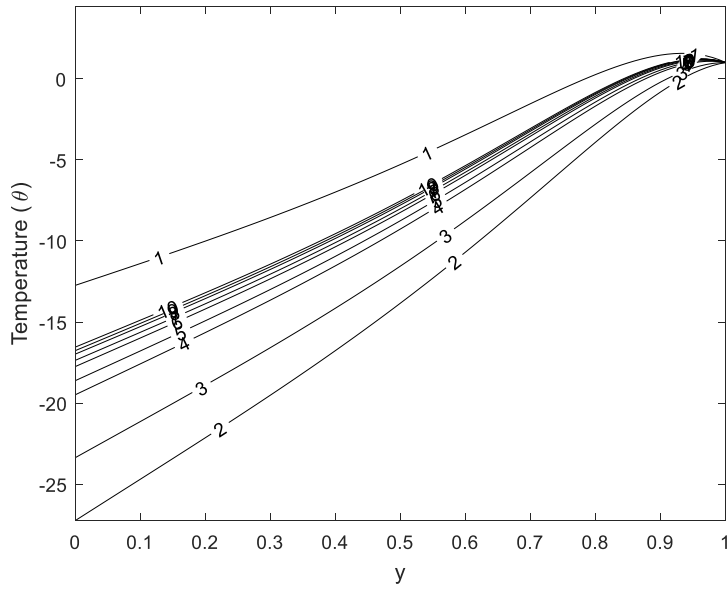


Fig.9: Impact on $\theta(y)$ for various Bi with heat sink and including joule dissipation

Figures 10 and 11 show that increasing the Reynolds number Re leads to a complex, non-monotonic response in the temperature field. Near the convective wall, the temperature initially exhibits a rapid reversal before increasing further into the channel under both heat source and heat sink conditions. Physically, higher Reynolds number strengthens advection, intensifying convective transport within the flow. Under suction, fluid is drawn toward the wall, compressing the thermal boundary layer and elevating local temperature, whereas injection thickens the boundary layer and enhances thermal

diffusion, leading to a reduction in temperature. This explains the observed immediate reversal near the wall and the contrasting thermal response between suction and injection-dominated regimes.

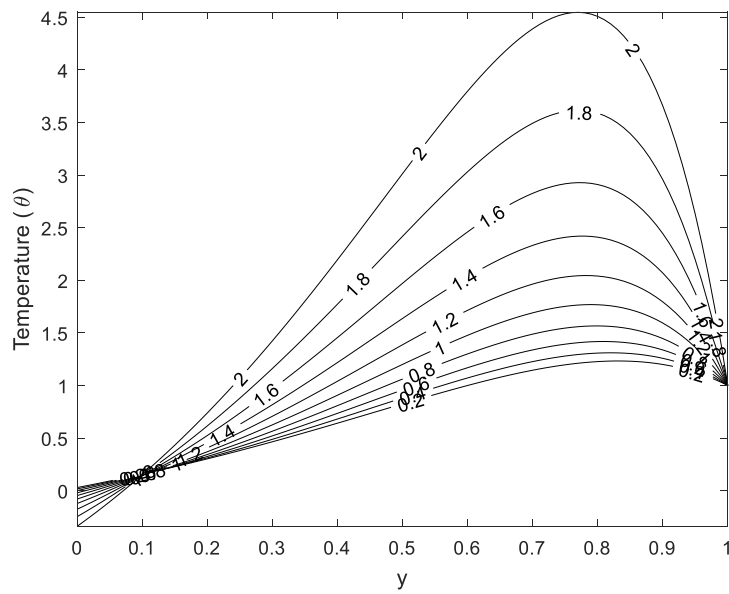


Fig.10 Impact on $\theta(y)$ for various Re with heat source and excluding joule dissipation

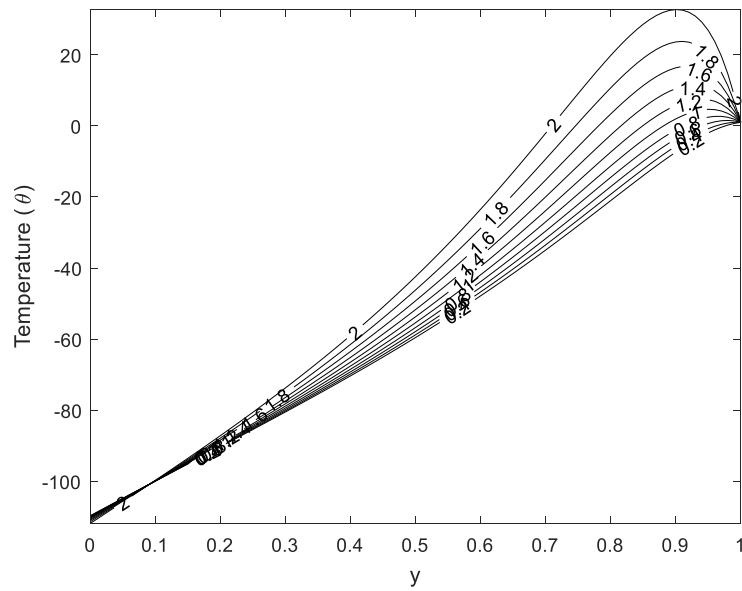


Fig.11 Impact on $\theta(y)$ for various Re with heat sink and including joule dissipation

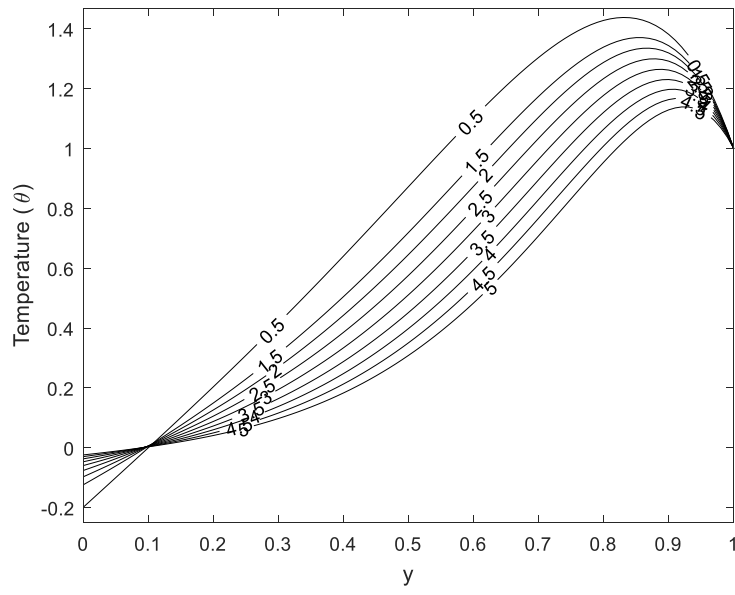


Fig.12 Impact on $\theta(y)$ for various Pe with heat sink and excluding joule dissipation

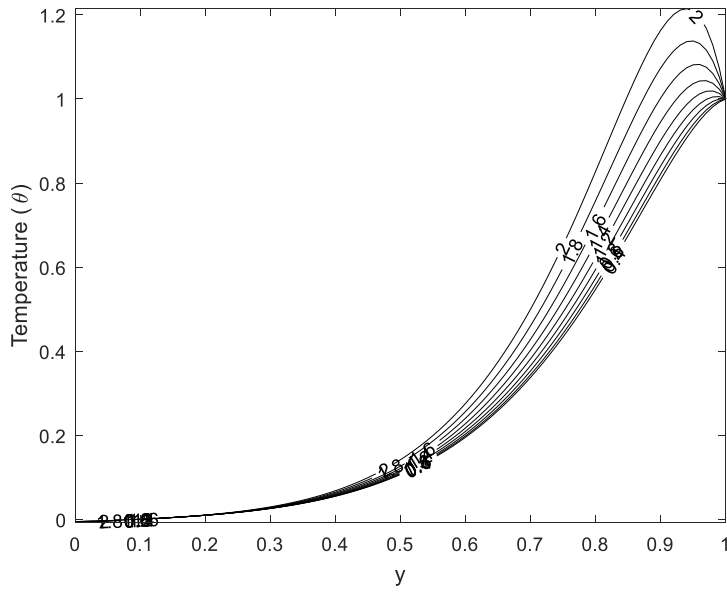


Fig 13: Impact on $\theta(y)$ for various M^2 with heat sink and excluding joule dissipation

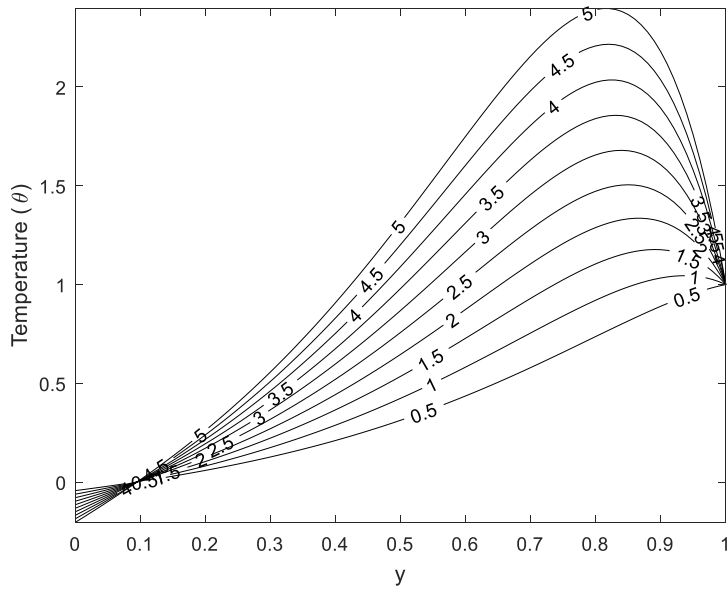


Fig.14 Impact on $\theta(y)$ for various Br with heat sink and excluding joule dissipation

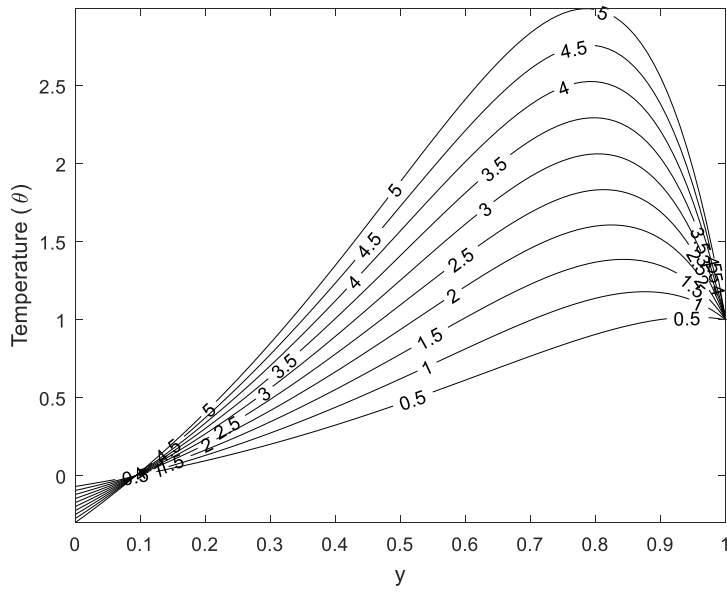


Fig.15 Impact on $\theta(y)$ for various Br with heat source and excluding joule dissipation

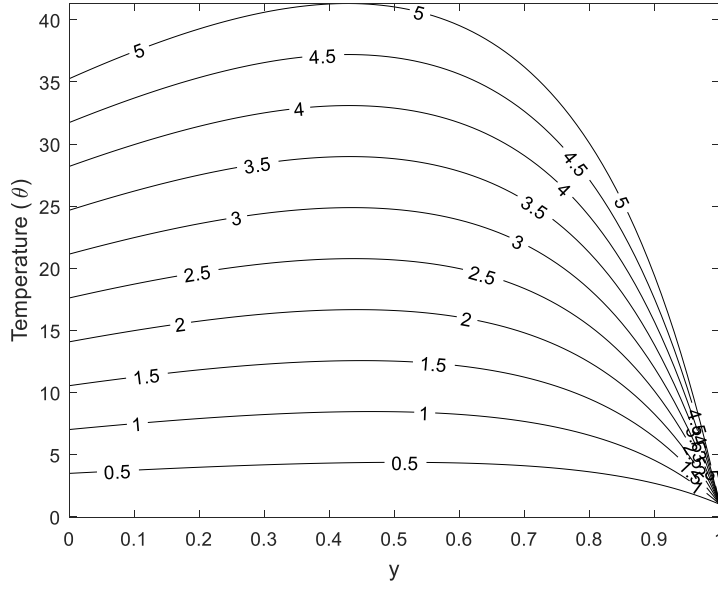


Fig.16 Impact on $\theta(y)$ for various Br with heat source and including joule dissipation

Figure 12 shows the correlation between Peclet number and the temperature. We observed that increasing the Peclet number Pe results in a consistent decrease in fluid temperature across all heating conditions. This occurs because a higher Pe strengthens convective transport relative to thermal diffusion, which promotes more efficient energy advection along the flow direction. As a result, heat is redistributed away from the wall region more rapidly, reducing local thermal accumulation and lowering the overall fluid temperature within the channel.

The relationship between magnetic field intensity and the temperature field is presented in figure 13. It shows that increasing the magnetic parameter M^2 leads to an increase in fluid temperature. This behaviour is attributed to the Lorentz force, which suppresses the fluid motion and reduces convective cooling. As the magnetic field strengthens, the resistive damping converts kinetic energy into internal energy through Joule heating, enhancing thermal energy within the fluid. Consequently, reduced advection combined with magnetic dissipation leads to a rise in temperature across the channel.

Figures 14 –16 demonstrate that increasing the Brinkman number Br consistently raises the fluid temperature under all heating conditions. This is due to enhanced viscous dissipation, where mechanical energy is converted into thermal energy within the fluid, thereby intensifying internal heating and elevating the temperature field. The effect becomes more pronounced when Joule heating is included, as shown in Figure 16, where the combined action of viscous and electromagnetic dissipation produces a stronger temperature rise and steeper thermal gradients.

Figures 17 and 18 illustrate the effect of slip parameters ζ_1 and ζ_2 respectively on the temperature field. An increase in ζ_1 leads to a reduction in fluid temperature, as enhanced slip at the lower wall weakens momentum transfer and suppresses near-wall frictional heating, thereby reducing thermal energy within the flow. For ζ_2 , increasing slip at the upper wall results in a rise in temperature under heat source conditions, indicating that reduced wall resistance modifies the velocity–temperature coupling in a way that promotes internal thermal accumulation. No convergent temperature profile was obtained for ζ_1 parameter range under heat source conditions suggesting possible thermal instability within that regime.

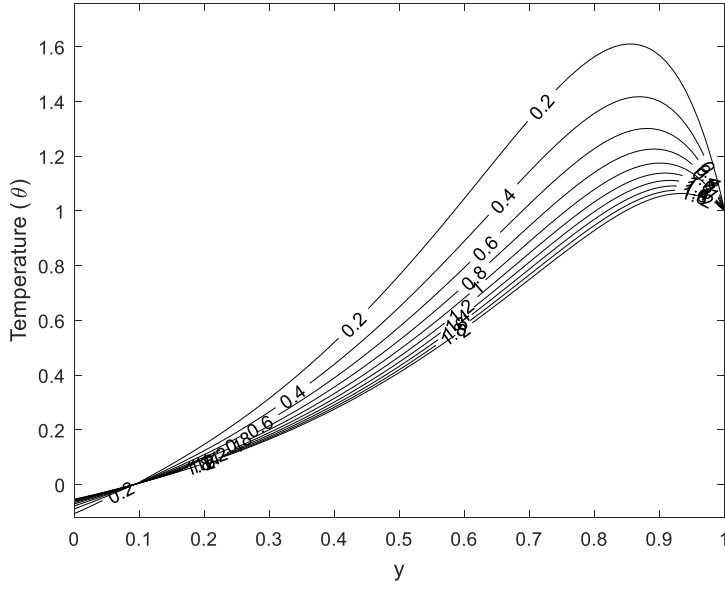


Fig.17 Impact on $\theta(y)$ for various ζ_1 with heat sink and excluding joule dissipation

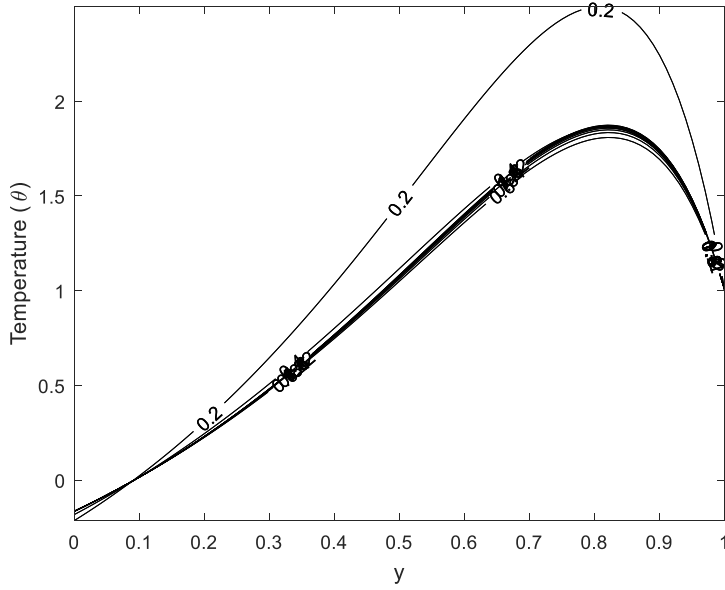


Fig.18 Impact on $\theta(y)$ for varying ζ_2 with heat source and excluding joule dissipation

A consistent feature of the temperature field is the opposing influence of heat source and heat sink mechanisms. The presence of a volumetric heat source elevates the fluid temperature throughout the channel, whereas a heat sink suppresses the thermal field and may drive the temperature below the reference state. This cooling effect becomes more pronounced in the absence of Joule dissipation, since no additional electromagnetic heating is available to offset the energy loss. When Joule heating is included, however, resistive dissipation compensates for the sink-induced cooling, moderating temperature decay and maintaining a more stable thermal field.

4. Conclusion

This study investigated the combined effects of velocity slip and convective boundary conditions on heat transfer in shear flow of a viscous incompressible electrically conducting fluid under the influence of key governing parameters. The analytical solution for the model equations was derived and graphical presentations were made to illustrate the role of magnetic, thermal, and hydrodynamic effects on the velocity and temperature fields. Investigations reveal that thermal behaviour is strongly dictated by the interplay between slip effects, Biot number, and dissipative mechanisms, with distinct responses depending on whether the system experiences a heat source or a sink. Ohmic heating was also found to significantly impact the temperature distribution. The major findings can be summarised as follows:

Velocity field: The velocity distribution is suppressed by magnetic and suction effects but either enhanced or diminished by Navier slip depending on whether it is applied at the lower or upper wall.

Temperature field:

1. Magnetic and Brinkman effects: Both parameters induce thermal reversals, with reversal points shifting closer to the lower wall as the Biot number increases. Joule dissipation prevents the temperature from falling below ambient, highlighting its stabilising role.

2. Biot number: Stronger wall convection (higher Bi) increases near-wall temperatures, even under sink conditions, showing the dominance of boundary convection over bulk conduction.

3. Peclet number: Larger Pe amplifies convective transport, moderating cooling at low Pe and shifting thermal reversal points, thus balancing advection and diffusion.

4. Reynolds number: Injection decreases fluid temperature while suction increases it.

5. Heat source/sink: Temperature is generally elevated with the heat source case and while it declines with the heat sink case.

6. Slip parameters: Greater slip at the lower wall elevates near-wall temperatures but shifts reversal zones deeper into the channel, while slip at the upper wall suppresses thermal gradients by clustering temperature layers.

7. Joule heating: The presence of Joule heating sometimes compensates for lower temperature gradients under heatsink condition and it generally contributes to a higher temperature under heat source case.

This study underscores the critical role of velocity slip and convective boundary conditions in governing both flow and thermal behaviour in shear-driven systems. The findings highlight that thermal and hydrodynamic mechanisms should be considered collectively, as the interplay between slip, magnetic effects, and boundary convection ultimately dictates system performance. Beyond clarifying the direct impact of individual parameters, the results reveal subtle interactions that shape velocity distributions, thermal layering, and energy transfer. These insights may provide practical guidelines for the design of microfluidic devices, porous media cooling channels, and other magnetohydrodynamic systems. By strategically tuning slip coefficients and boundary convection rates, engineers can regulate localised heating, delay thermal reversals, and optimise energy efficiency in real-world applications.

Statements and Declarations

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Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Author Contributions

All authors contributed to the study conception and design. Solution to equations and graphical presentations, and writing of manuscript were made by Vampah T. Peter. Taiwo S. Yusuf read and edited the manuscript. All authors read and approved the final manuscript.

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