

Adaptive Tail Correction for Extreme Value Modelling: A Power Log Hybrid Extension of the GEV Distribution

Arslan Khalid

arslan.khalid999@gmail.com

International Islamic University Islamabad

Ishfaq Ahmed

International Islamic University Islamabad

Ibrahim Mufrah Almanjahie

King Khalid University

Research Article

Keywords: extreme value theory, generalized extreme value distribution, PLH-GEV, survival transformation, tail attenuation, return levels

Posted Date: July 7th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10244324/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.
[Read Full License](#)

Additional Declarations: No competing interests reported.

Adaptive Tail Correction for Extreme Value Modelling: A Power Log Hybrid Extension of the GEV Distribution

Arslan Khalid^{1*}, Ishfaq Ahmed¹, Ibrahim Mufrah Almanjahie²

¹ Department of Mathematics and Statistics, International Islamic University Islamabad

² Department of Mathematics, King Khalid University, Saudi Arabia

*Corresponding author: Arslan Khalid

Email: arslan.khalid999@gmail.com

ORCID:

Arslan Khalid: <https://orcid.org/0009-0003-3794-067X>

Ishfaq Ahmed: <https://orcid.org/0000-0002-1124-7485>

Ibrahim Mufrah Almanjahie: <https://orcid.org/0000-0002-4651-3210>

Abstract

Extreme value modelling is widely used in finance, insurance, hydrology, environmental risk, and disaster management, where interest focuses on rare tail events rather than average behaviour. The generalized extreme value distribution is a standard model for extremes, but its survival tail can be too rigid in finite empirical samples. This paper proposes a Power Log Hybrid extension of the generalized extreme value distribution, denoted PLH-GEV, by applying a flexible transformation directly to the baseline survival function. The model contains the classical generalized extreme value distribution as a limiting case and introduces an interpretable survival tail correction governed by an effective attenuation ratio. The corresponding density, admissibility conditions, quantile equations, return level formulation, and grid profile maximum likelihood estimation strategy are presented. The model is evaluated using four empirical datasets: S&P 500 negative daily returns, an S&P 500 extreme loss subset, Danish fire insurance losses, and Fort Collins annual maximum precipitation. The results show that PLH-GEV substantially improves tail fit and extreme quantile accuracy in the financial and insurance datasets, while the Fort Collins precipitation data return the neutral generalized extreme value case. These findings support PLH-GEV as an adaptive survival tail correction rather than a distributional extension that automatically improves every dataset.

Keywords: extreme value theory; generalized extreme value distribution; PLH-GEV; survival transformation; tail attenuation; return levels.

Mathematics Subject Classification (2020): 62G32; 62E15; 62F10; 62P05; 62P12

1. Introduction

Extreme observations are central to many applied problems. In finance, a risk manager is interested in rare market crashes. In insurance, an analyst is concerned with catastrophic claims. In hydrology, engineers estimate rainfall levels that may be exceeded only rarely. In such cases, the main question is not what happens on average, but what can happen under rare and severe conditions.

Extreme value theory provides the statistical foundation for modelling such events. The generalized extreme value (GEV) distribution is one of the most widely used models because it unifies the Frechet, Gumbel, and Weibull limiting families for maxima (Fisher and Tippett, 1928; Gnedenko, 1943; Gumbel, 1958; Jenkinson, 1955; Coles, 2001). The GEV model is widely used to estimate high quantiles, exceedance probabilities, and return levels. For example, the 100 year return level in hydrology or the 0.999 loss quantile in finance are both tail quantities derived from fitted extreme value models.

However, the theoretical justification of the GEV distribution is asymptotic. In real datasets, samples are finite and may be affected by volatility clustering, dependence, structural changes, reporting thresholds, mixture behaviour, nonstationarity, and sampling variation. These features may cause the empirical tail to deviate from the fitted GEV tail. If the fitted survival curve lies above the empirical survival curve in the far tail, the model overestimates exceedance probabilities and produces excessively large return levels. If the fitted curve lies below the empirical survival curve, the model underestimates risk. Both forms of tail mismatch are practically important because small differences in survival probability can produce large changes in high quantiles.

This study proposes a Power Log Hybrid extension of the GEV distribution, denoted PLH-GEV. The central idea is to keep the GEV distribution as the baseline extreme value model while applying an interpretable transformation to the GEV survival function. The survival function is used because it directly represents exceedance probability. If X is a loss variable, then $S(x) = P(X > x)$ is the probability that the loss exceeds x . This is the quantity of direct interest in extreme risk analysis.

The paper makes four contributions. First, it defines a new survival transformed PLH-GEV distribution. Second, it derives the corresponding density and states the validity conditions under which the transformed survival function defines a proper probability distribution. Third, it introduces a reparameterization that automatically enforces the positive attenuation subclass used in empirical work. Fourth, it evaluates the model on four datasets with different tail behaviours, showing both strong PLH-GEV improvement cases and a control case in which GEV remains preferred.

2. Classical GEV Model

Let X denote an extreme observation, such as an annual maximum rainfall amount, an insurance loss, or a transformed negative financial return. The generalized extreme value distribution has cumulative distribution function

$$G(x; \mu, \sigma, \xi) = \exp \left[- \left\{ 1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right\}^{-1/\xi} \right],$$

where

$$1 + \xi \left(\frac{x - \mu}{\sigma} \right) > 0, \quad \sigma > 0.$$

Here, $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ is the scale parameter, and $\xi \in \mathbb{R}$ is the shape parameter. Positive ξ gives a heavy tailed Frechet type model, $\xi = 0$ corresponds to the Gumbel limiting case, and negative ξ gives a bounded Weibull type upper tail.

The corresponding GEV survival function is

$$S_G(x) = 1 - G(x; \mu, \sigma, \xi).$$

For a given threshold x , $S_G(x)$ estimates the fitted probability that the observation exceeds x . High quantiles are defined through

$$G(q_p) = p,$$

or equivalently

$$S_G(q_p) = 1 - p.$$

Thus, $q_{0.999}$ is the value exceeded with probability 0.001. This interpretation is central in risk management because it connects fitted distributions to rare event thresholds.

3. Proposed PLH-GEV Survival Transformation

Let

$$S = S_G(x).$$

The proposed PLH-GEV survival function is

$$S_{PLH}(x) = S_G(x) \left[1 + \frac{\alpha}{\lambda} \{S_G(x)^\lambda - 1\} \right]^\beta,$$

where $\lambda > 0$ and $\beta > 0$. Define

$$A(S) = 1 + \frac{\alpha}{\lambda} (S^\lambda - 1).$$

Then the proposed survival function can be written compactly as

$$S_{PLH}(x) = S_G(x) A(S_G(x))^\beta.$$

The cumulative distribution function is therefore

$$F_{PLH}(x) = 1 - S_{PLH}(x).$$

The model modifies the baseline GEV survival probability by multiplying it by a PLH correction factor. This correction factor is governed mainly by the ratio α/λ . When $\alpha = 0$, the correction factor is one and the model reduces exactly to the classical GEV distribution. When α approaches λ , the transformation strongly attenuates the far tail survival probability.

4. Density Derivation and Validity

The PLH-GEV density is obtained from

$$f_{PLH}(x) = -\frac{d}{dx}S_{PLH}(x).$$

Since

$$S_{PLH}(x) = S_G(x)A(S_G(x))^\beta,$$

and

$$\frac{dS_G(x)}{dx} = -f_G(x),$$

where $f_G(x)$ is the GEV density, differentiation gives

$$f_{PLH}(x) = f_G(x)A(S)^{\beta-1}[A(S) + \alpha\beta S^\lambda],$$

where $S = S_G(x)$. This expression shows that PLH-GEV keeps the baseline GEV density and multiplies it by a survival dependent correction factor.

For the transformed model to be valid, the survival function must be non-negative and non-increasing, and the density must be non-negative. The key terms are

$$A(S) = 1 + \frac{\alpha}{\lambda}(S^\lambda - 1)$$

and

$$B(S) = A(S) + \alpha\beta S^\lambda.$$

A sufficient and interpretable admissible region is

$$\lambda > 0, \quad \beta > 0,$$

with

$$\max(-\lambda, -1/\beta) < \alpha < \lambda.$$

Under this condition, $A(S) > 0$ and $B(S) > 0$ for $0 < S < 1$. The empirical part of this paper uses an even safer positive attenuation subclass, described below, which automatically gives $0 \leq \alpha < \lambda$.

5. Reparameterization and Tail Attenuation Interpretation

Direct estimation of α , β , and λ can be unstable if constraints are not enforced. Therefore, the empirical implementation uses reparameterization

$$\rho = 1 - \exp(-\eta/\beta),$$

and

$$\alpha = \lambda\rho,$$

with

$$\eta \geq 0, \quad \beta > 0, \quad \lambda > 0.$$

This automatically gives

$$0 \leq \rho < 1$$

and therefore

$$0 \leq \alpha < \lambda.$$

The parameter ρ is the effective attenuation ratio. The parameter η is a computational strength parameter that affects the model through ρ . Using $\alpha = \lambda\rho$, the PLH factor becomes

$$A(S) = 1 + \rho(S^\lambda - 1) = 1 - \rho + \rho S^\lambda.$$

Thus, the ratio between PLH-GEV survival and GEV survival is

$$\frac{S_{PLH}(x)}{S_G(x)} = [1 - \rho + \rho S_G(x)^\lambda]^\beta.$$

In the body of the distribution, where $S_G(x)$ is not close to zero, the transformation can remain close to the GEV baseline. In the far upper tail, where $S_G(x) \rightarrow 0$, the ratio approaches

$$\frac{S_{PLH}(x)}{S_G(x)} \rightarrow (1 - \rho)^\beta = \exp(-\eta).$$

This result gives η a simple interpretation. When $\eta = 0$, the asymptotic survival ratio is one and PLH-GEV reduces to GEV. As η increases, the far tail survival ratio decreases. Once η is large, further increases may have little practical effect because the ratio is already close to zero. For this reason, eta boundary behaviour must be interpreted through the effective attenuation ρ , the survival curve, the information criteria, and the extreme quantiles rather than through the numerical value of η alone.

6. Quantile Function and Return Levels

For a probability level p , the PLH-GEV quantile q_p satisfies

$$F_{PLH}(q_p) = p.$$

Equivalently,

$$S_{PLH}(q_p) = 1 - p.$$

Let

$$u = S_G(q_p).$$

Then u solves

$$u \left[1 + \frac{\alpha}{\lambda} (u^\lambda - 1) \right]^\beta = 1 - p.$$

After solving this equation numerically for u , the quantile is recovered from the inverse GEV survival function:

$$q_p = G^{-1}(1 - u).$$

For a return period T , the corresponding probability level is

$$p_T = 1 - \frac{1}{T}.$$

The PLH-GEV return level is therefore

$$x_T = Q_{PLH} \left(1 - \frac{1}{T} \right).$$

Equivalently, one solves

$$u \left[1 + \frac{\alpha}{\lambda} (u^\lambda - 1) \right]^\beta = \frac{1}{T}$$

and then applies the inverse GEV survival quantile.

7. Estimation Strategy and Effective Eta Rule

For observations $x_1, x_2, \dots, x_n$, the PLH-GEV log-likelihood is

$$\ell(\mu, \sigma, \xi, \eta, \beta, \lambda) = \sum_{i=1}^n \log f_{PLH}(x_i).$$

Using the density derived above,

$$\ell = \sum_{i=1}^n [\log f_G(x_i) + (\beta - 1) \log A(S_i) + \log \{A(S_i) + \alpha \beta S_i^\lambda\}],$$

where

$$S_i = S_G(x_i).$$

The model contains three baseline GEV parameters (μ, σ, ξ) and three PLH transformation parameters (η, β, λ) . To avoid unstable direct six parameter optimization, a grid profile likelihood strategy is used. For each candidate (η, β, λ) on the grid, ρ and α are computed, the GEV parameters are estimated by maximum likelihood conditional on the PLH parameters, and the fitted model is evaluated using AIC, BIC, KS, AD, return levels, and extreme quantile errors.

A central diagnostic issue is the interpretation of η . Large values of η do not always imply a materially different fitted distribution because the mapping from η to ρ is saturating. Therefore, this paper uses an effective eta rule: among all grid fits within two AIC units of the minimum PLH-GEV AIC, the smallest η is selected. This conservative rule avoids reporting an unnecessarily large eta value when a smaller eta gives practically the same fit.

8. Empirical Data and Diagnostics

The empirical evaluation uses four datasets that represent distinct types of extreme tail behaviour. For compact tables and figures, the labels SP500 negative, SP500 extreme, Danish fire, and Fort Collins are used.

First, S&P500 negative daily returns are transformed into positive loss magnitudes. If r_t denotes a daily log return, the loss variable is defined as

$$X_t = -r_t \quad \text{for } r_t < 0.$$

Thus, larger values of X_t correspond to more severe market losses. This dataset is used as the main financial loss application.

Second, an S&P500 extreme loss subset is used as a sensitivity analysis. This subset focuses on the more severe part of the S&P500 negative return distribution and tests whether PLH-GEV still improves the fitted tail when the analysis is restricted to extreme losses.

Third, Danish fire insurance losses represent large positive insurance claims. This dataset is valuable because it provides a non-financial heavy loss application in which extreme claims and high return levels are practically meaningful.

Fourth, the Fort Collins annual maximum precipitation series represents an environmental block maxima example. It is used as a control case because the ordinary GEV model is already expected to be competitive for annual maxima.

The models are compared using AIC, BIC, the Kolmogorov-Smirnov statistic (KS), and the Anderson-Darling statistic (AD). AIC and BIC measure overall likelihood fit while penalizing model complexity. KS measures the largest distance between empirical and fitted distribution functions. AD is especially important for this study because it gives more weight to tail discrepancies. Extreme quantile and return level comparisons are also reported because improvements in fit statistics are only practically useful if they translate into better tail estimates.

9. Main Empirical Results

9.1 Goodness of fit comparison

Table 1 presents the main goodness of fit results for the four datasets. Smaller AIC, BIC, KS, and AD values indicate better fit.

Table 1. Goodness of fit comparison of GEV and PLH-GEV.

Dataset	Model	AIC	BIC	KS	AD
SP500 negative	GEV	-70093.24	-70071.96	0.0426	35.1200
	PLH-GEV	-70710.50	-70667.95	0.0310	15.4400
SP500 extreme	GEV	-3162.40	-3150.11	0.0411	1.1500
	PLH-GEV	-3180.77	-3156.20	0.0307	0.5565
Danish fire	GEV	6790.84	6807.88	0.0281	2.7950
	PLH-GEV	6726.90	6760.99	0.0262	1.5202
Fort Collins	GEV	1136.9631	1144.7786	0.0402	0.1976
	PLH-GEV	1142.9631	1158.5941	0.0402	0.1976

The results show clear dataset specific behaviour. For S&P500 negative daily returns, PLH-GEV improves AIC by 617.26 units and BIC by 595.99 units. KS decreases from 0.0426 to 0.0310, and AD decreases from 35.1200 to 15.4400. This is a strong improvement in both overall fit and tail sensitive fit. For the S&P500 extreme loss subset, the improvement remains positive but smaller: AIC decreases by 18.37 units, BIC by 6.09 units, KS decreases from 0.0411 to 0.0307, and AD decreases from 1.1500 to 0.5565. For Danish fire losses, the improvement is again substantial, with AIC decreasing by 63.94 units, BIC by 46.89 units, KS decreasing from 0.0281 to 0.0262, and AD decreasing from 2.7950 to 1.5202. By contrast, Fort Collins precipitation shows no empirical improvement under the effective eta rule; GEV and PLH-GEV coincide in KS, AD, quantiles, and return levels, while AIC and BIC penalize PLH-GEV for its extra flexibility. Figure 1 shows that the eta profile stabilizes quickly in all four datasets, while Figure 2 shows how the fitted survival curves behave relative to the empirical tail.

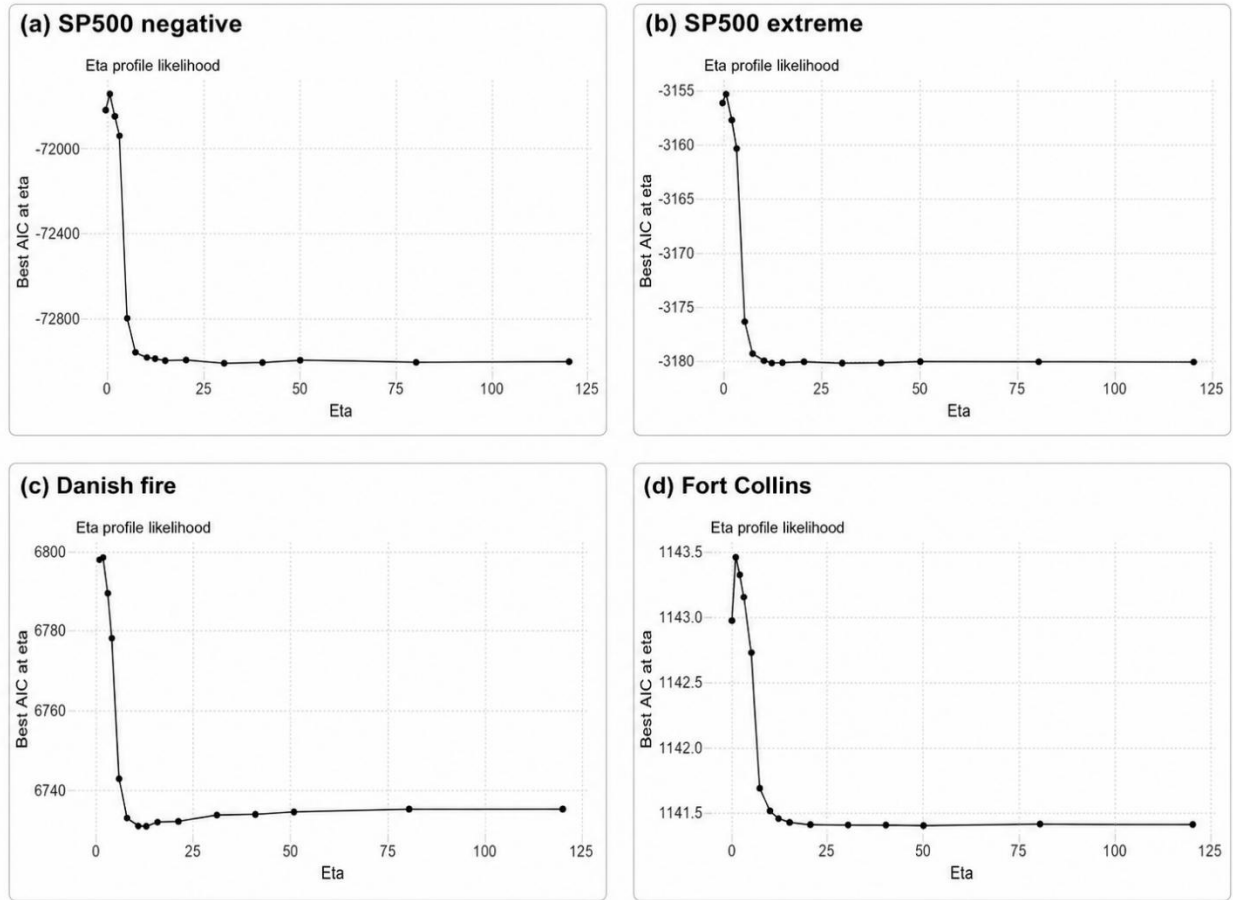


Fig. 1 Eta profile likelihood panels for the four datasets. Each panel plots the best AIC achieved at each eta value. In all four cases the profile improves sharply at small eta and then stabilizes, supporting the effective eta rule used in estimation.

9.2 Effective PLH parameters and tail behaviour

Table 2 reports the effective PLH-GEV transformation parameters selected by the effective eta rule.

The S&P500 negative return dataset selects a strong near saturated PLH correction, with $\eta = 30$, $\beta = 3$, $\lambda = 12$, effective $\alpha = 11.999455$, and $\rho = 0.999955$. This indicates that the far tail of the fitted GEV survival curve requires very strong attenuation. The S&P500 extreme loss subset also selects a strong correction, with $\eta = 7$, $\beta = 1$, $\lambda = 20$, effective $\alpha = 19.981762$, and $\rho = 0.999088$, though the gain is milder than in the full negative return series. Figure 3 confirms that both S&P500 applications generate tail ratio curves well below one over the upper tail, meaning that PLH-GEV systematically attenuates the survival probability assigned by the ordinary GEV model.

The Danish fire loss dataset behaves differently. It selects $\eta = 10$, $\beta = 30$, $\lambda = 2$, effective $\alpha = 0.566937$, and $\rho = 0.283469$.

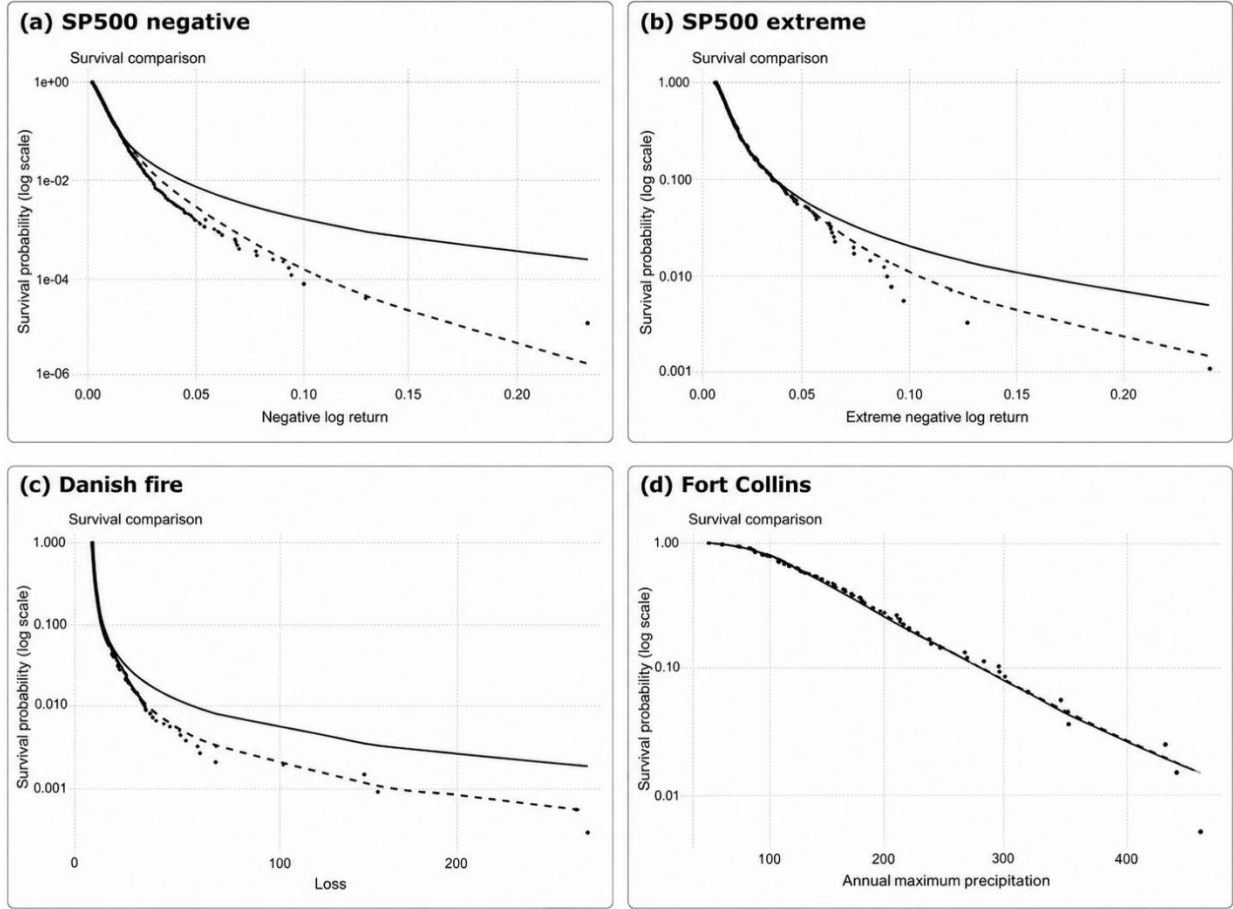


Fig. 2 Empirical survival comparison for the four datasets. Points denote the empirical survival curve, the dashed line denotes the fitted GEV survival, and the solid line denotes the fitted PLH-GEV survival. The greatest practical gains occur when the GEV fit overstates the far survival tail.

Table 2. Effective PLH-GEV transformation parameters.

Dataset	eta	beta	lambda	rho	alpha	Interpretation
SP500 negative	30	3	12	0.999955	11.999455	Strong, near saturated
SP500 extreme	7	1	20	0.999088	19.981762	Strong but finite
Danish fire	10	30	2	0.283469	0.566937	Moderate and stable
Fort Collins	0	1	0.5	0.000000	0.000000	Neutral GEV case

This is not a near boundary solution in terms of ρ . It indicates moderate but meaningful survival tail correction. This is important because it shows that PLH-GEV is not useful only through near saturated attenuation; it can also improve fit through an interior correction regime.

The Danish tail ratio panel in Figure 3 shows a persistent but smoother downward adjustment than the two S&P500 cases.

Fort Collins precipitation provides the cleanest control example. The effective rule selects $\eta = 0$, which gives $\rho = 0$, $\alpha = 0$, and therefore

$$S_{PLH}(x) = S_G(x).$$

The model therefore collapses to the ordinary GEV case. This confirms that PLH-GEV does not force unnecessary tail modification when the empirical evidence does not support it. In Figure 3, the Fort Collins tail ratio panel is essentially a horizontal line at one, which visually verifies that no practical correction is being applied.

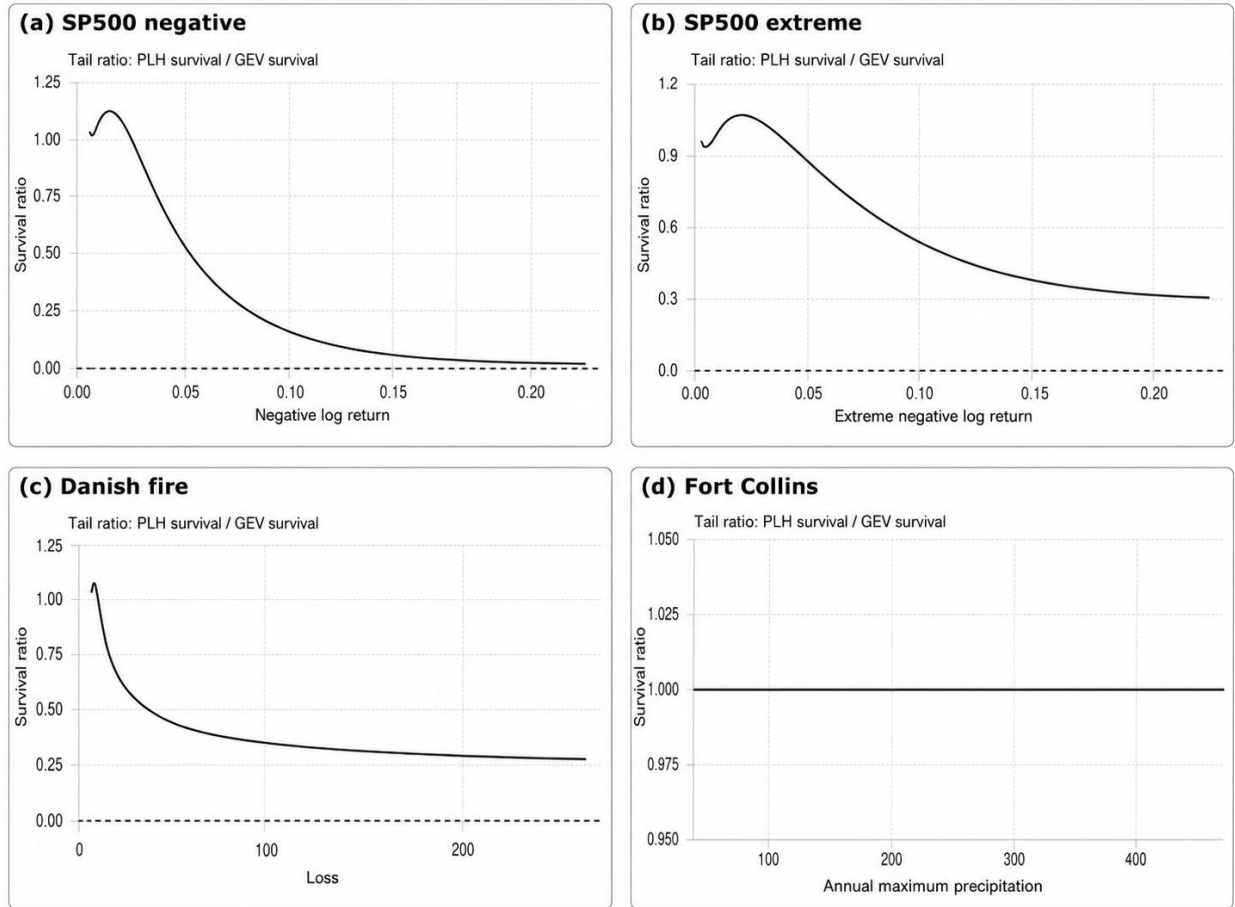


Fig. 3 Tail ratio panels for the four datasets. The plotted ratio $S_{PLH}(x)/S_{GEV}(x)$ summarizes the implied tail correction. Ratios substantially below one indicate attenuation of the GEV tail, whereas a flat ratio near one indicates collapse to the GEV baseline.

9.3 Extreme quantile performance

Table 3 compares the empirical, GEV, and PLH-GEV 0.999 quantiles. This probability level corresponds to a threshold exceeded with probability 0.001, or approximately once in one thousand comparable observations.

The extreme quantile results strongly support the adaptive interpretation. For S&P500 negative returns, the GEV 0.999 quantile is 0.1918, while the empirical value is 0.0773. PLH-GEV gives 0.0815, which is much closer. For S&P500 extreme losses, GEV again overestimates the far tail quantile, whereas PLH-GEV reduces the error substantially. For Danish fire losses, GEV gives 364.11 compared with the empirical value 148.53, while PLH-GEV gives 128.24. In Fort Collins, PLH-GEV is identical to GEV under the effective fit, so both models have the same high quantile error.

Figure 4 reports the absolute error across the extreme quantile levels $p = 0.90, 0.95, 0.99, 0.995$, and 0.999. The pattern is clear: PLH-GEV sharply reduces error in the three datasets where the GEV tail is too heavy, whereas Fort Collins again shows no practical distinction because both fits collapse to the same return level structure.

Table 3. Extreme quantile comparison at $p = 0.999$.

Dataset	Empirical $q_{0.999}$	GEV $q_{0.999}$	PLH-GEV $q_{0.999}$	GEV absolute error	PLH-GEV absolute error
SP500 negative	0.0773	0.1918	0.0815	0.1145	0.0042
SP500 extreme	0.2290	0.6362	0.2959	0.4072	0.0669
Danish fire	148.5302	364.1149	128.2452	215.5846	20.2950
Fort Collins	463.00	845.8579	845.8579	382.8579	382.8579

9.4 Return level interpretation

Return levels translate tail probabilities into practically interpretable thresholds. Table 4 compares the 1000 observation return level for GEV and PLH-GEV.

Table 4. Return level comparison at $T = 1000$.

Dataset	GEV return level	PLH-GEV return level	PLH-GEV / GEV
SP500 negative	0.1918	0.0815	0.4249
SP500 extreme	0.6362	0.2959	0.4651
Danish fire	364.11	128.24	0.3522
Fort Collins	845.8579	845.8579	1.0000

For S&P500 negative daily returns, the PLH-GEV return level is approximately 42 percent of the GEV return level at $T = 1000$. For S&P500 extreme losses, the ratio is about 47 percent. For Danish fire losses, it is about 35 percent. In Fort Collins precipitation, the ratio is exactly one under the effective eta rule. Figure 5 extends this comparison across the full set of return periods and shows how the divergence grows as T moves deeper into the tail.

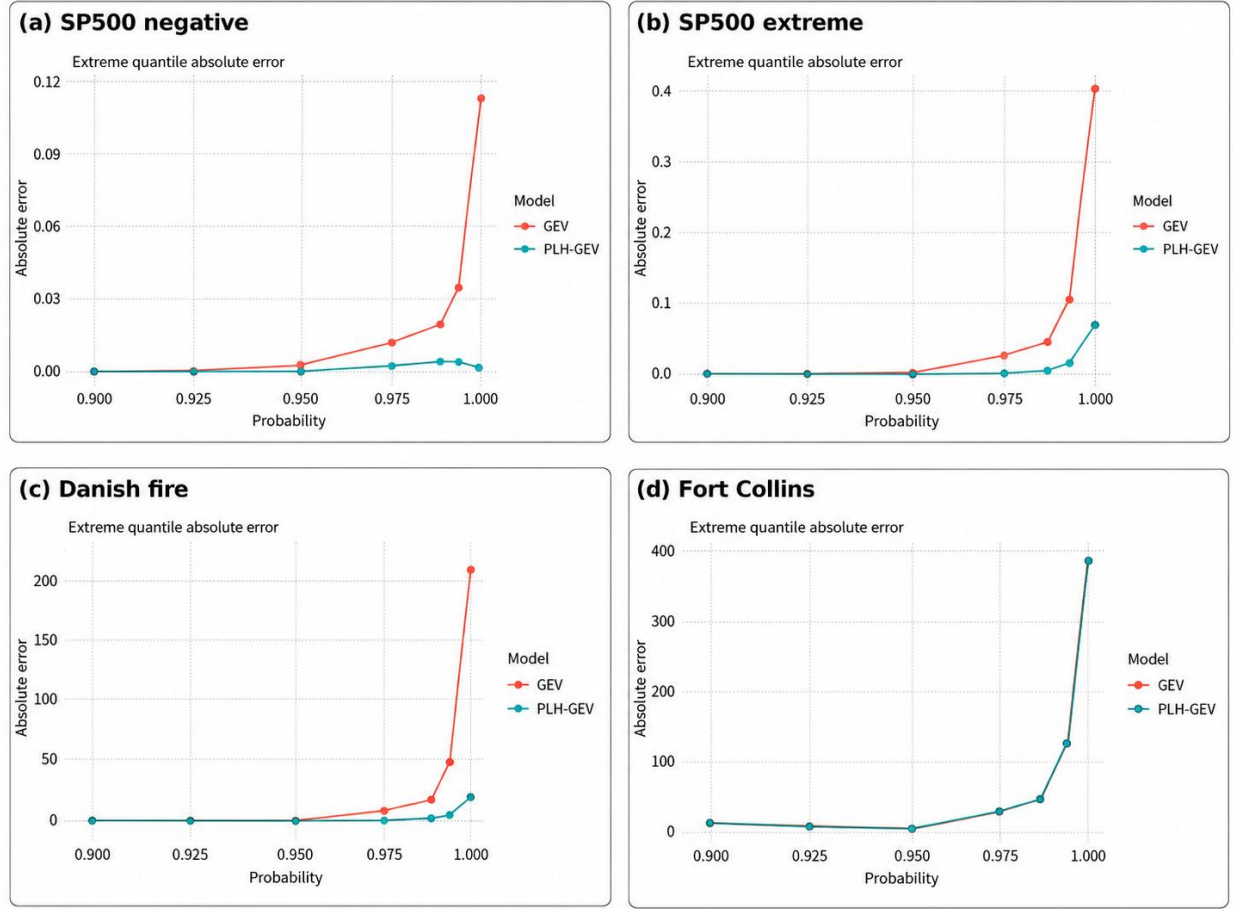


Fig. 4 Extreme quantile absolute error across probability levels 0.90, 0.95, 0.99, 0.995 and 0.999. Smaller values indicate better agreement with the empirical extreme quantiles. The S&P500 and Danish datasets show clear reductions in the most extreme region.

10. Dataset Specific Discussion

10.1 S&P500 negative daily returns

The S&P500 negative return results provide the strongest evidence for PLH-GEV. The ordinary GEV fit is useful as a baseline, but it assigns excessive mass to the far survival tail. This produces high return levels and a large 0.999 quantile error. PLH-GEV corrects this by selecting a strong survival tail attenuation. The effective parameters $\eta = 30$, $\beta = 3$, $\lambda = 12$, $\rho = 0.999955$, and $\alpha = 11.999455$ imply a near saturated positive attenuation regime, meaning that the fitted model moves very close to the strongest positive correction allowed by the chosen parameterization.

This should not be interpreted as a defect by itself. The η profile behaviour discussed earlier shows that the fit stabilizes after sufficient η ; the model does not obtain unlimited improvement

from arbitrarily increasing η . More importantly, the large η value is accompanied by a major improvement in AIC, BIC, KS, AD, and extreme quantile accuracy. In practical language, the data indicate that ordinary GEV substantially overstates the far tail size of the most severe negative return losses, and PLH-GEV provides the required attenuation.

10.2 S&P500 extreme loss subset

The S&P500 extreme loss subset acts as a sensitivity check. The result remains favourable to PLH-GEV, but the improvement is smaller than in the full negative return dataset. AIC improves by 18.37 units and BIC by 6.09 units, while the AD statistic is reduced from 1.1500 to 0.5565. At $p = 0.999$, the GEV estimate is 0.6362 compared with the empirical value 0.2290, whereas PLH-GEV gives 0.2959.

This behaviour is reasonable. When the analysis is already restricted to extreme losses, the GEV model has less body information to reconcile with the tail, and part of the extreme structure is already built into the subset. Nevertheless, the PLH transformation still reduces the excessive far tail prediction and improves the highest quantile estimate. Therefore, the extreme subset supports the robustness of the PLH-GEV idea without being presented as a fully independent second financial dataset.

10.3 Danish fire insurance losses

The Danish fire insurance data are particularly important because they show strong PLH-GEV improvement without relying on a near boundary ρ estimate. PLH-GEV reduces AIC from 6790.84 to 6726.90 and BIC from 6807.88 to 6760.99. The AD statistic falls from 2.7950 to 1.5202, indicating a substantial improvement in tail sensitive fit.

The 0.999 quantile comparison is practically meaningful. The empirical 0.999 quantile is 148.53. GEV gives 364.11, overestimating the far tail loss level by 215.58. PLH-GEV gives 128.24, with an absolute error of only 20.29. The effective parameters $\eta = 10$, $\beta = 30$, $\lambda = 2$, $\rho = 0.283469$, and $\alpha = 0.566937$ indicate moderate, stable attenuation. This dataset therefore provides a balanced success case: PLH-GEV improves the tail substantially while remaining away from the near saturated ρ regime.

10.4 Fort Collins annual maximum precipitation

Fort Collins precipitation gives the opposite message. The ordinary GEV model is already adequate, and the effective PLH-GEV fit selects $\eta = 0$. This gives $\rho = 0$, $\alpha = 0$, and therefore exactly the ordinary GEV survival function. Consequently, the log likelihood, KS, AD, return levels, and quantiles are the same under GEV and PLH-GEV, while AIC and BIC penalize PLH-GEV because of the larger parameter count.

This result strengthens the article rather than weakening it. A flexible model that always appears to win may simply be overfitting. Fort Collins demonstrates that PLH-GEV behaves adaptively: when the GEV tail is already sufficient, the effective rule returns the neutral case, and the model does not force an artificial tail correction.

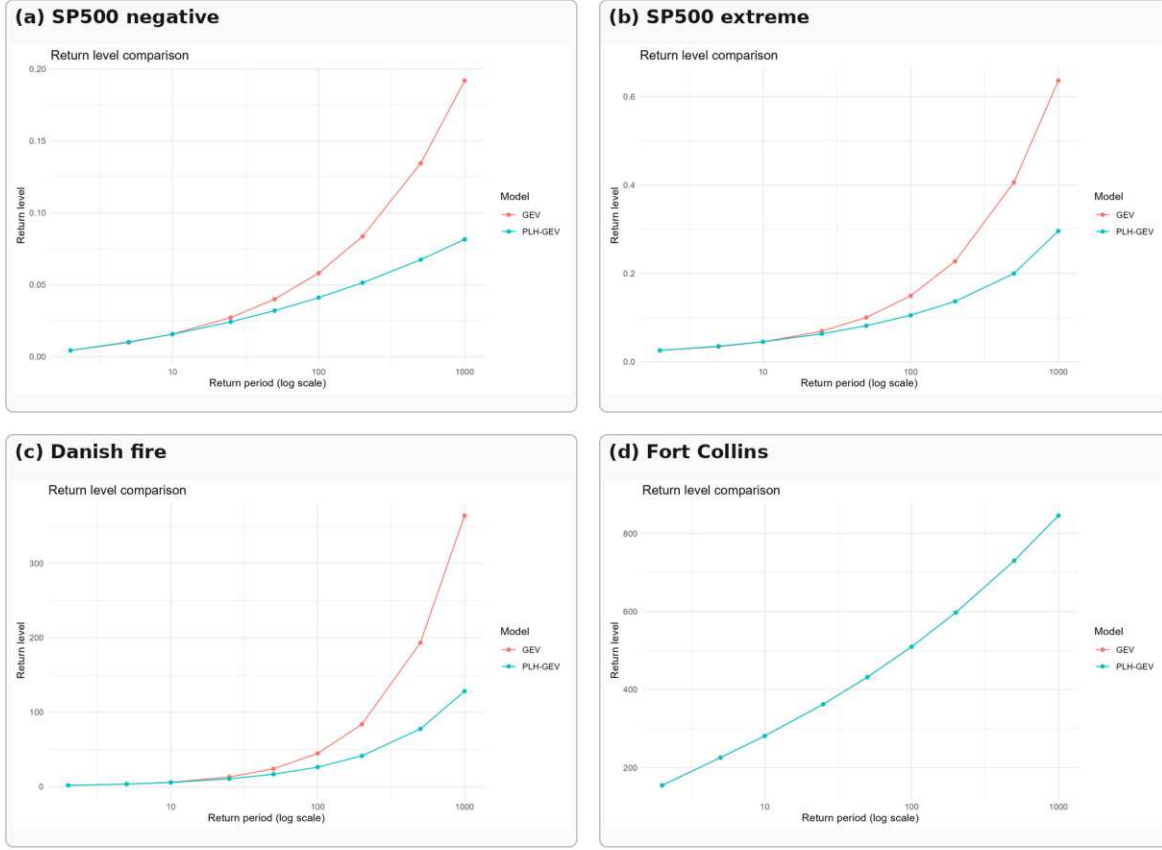


Fig 5. Return level comparison across return periods. Divergence between the GEV and PLH-GEV curves indicates the practical impact of the PLH correction, whereas overlap indicates that PLH-GEV effectively reduces to the classical GEV fit.

11. General Interpretation of PLH-GEV Behaviour

The four datasets together suggest that PLH-GEV should be interpreted as an adaptive survival tail correction rather than a universal replacement for GEV. The S&P500 negative return data require strong attenuation. The S&P500 extreme subset still benefits from attenuation, but the improvement is smaller. The Danish fire loss data require a moderate and stable correction. The Fort Collins precipitation data does not require PLH correction at all.

This pattern is exactly what one would want from a useful distributional extension. The model does not simply add parameters to improve every dataset. Instead, it changes behaviour according to the empirical tail. When the GEV survival curve is too heavy relative to the empirical tail, PLH-GEV attenuates it. When GEV is already adequate, PLH-GEV collapses to the GEV case.

The effective attenuation ratio ρ is central to this interpretation. Values close to zero indicate little or no correction. Values close to one indicate strong tail attenuation. The parameter η is useful computationally, but it should always be interpreted through ρ , the survival ratio

behaviour, and the resulting quantiles. Large η values do not necessarily imply numerical pathology because ρ saturates.

12. Conclusion

This paper proposed a Power Log Hybrid extension of the generalized extreme value distribution using a survival function transformation. The resulting PLH-GEV model preserves the GEV baseline while introducing an interpretable adaptive correction to the survival tail. The main mathematical results include the transformed survival function, the derived density, admissible parameter conditions, a validity preserving reparameterization, quantile and return level equations, and a grid profile likelihood estimation strategy.

The empirical evidence shows that PLH-GEV is adaptive. For S&P500 negative daily returns, PLH-GEV gives a strong improvement over GEV and substantially reduces the 0.999 quantile error. For the S&P500 extreme loss subset, PLH-GEV remains better than GEV but with a more moderate improvement. For Danish fire insurance losses, PLH-GEV provides a strong and stable improvement without a near boundary attenuation ratio. For Fort Collins annual maximum precipitation, the effective PLH-GEV model collapses to the ordinary GEV case and is penalized by AIC and BIC.

The main conclusion is therefore not that PLH-GEV always dominates GEV. Rather, PLH-GEV provides a flexible and interpretable mechanism for correcting survival tail mismatch when such mismatch is present. When no correction is needed, the model can return to the GEV baseline. This makes PLH-GEV a promising adaptive framework for extreme tail modelling in finance, insurance, hydrology, and related applied fields.

Statements and Declarations

Funding

No funds, grants, or other support was received for conducting this study.

Competing interests

The authors have no relevant financial or non financial interests to disclose.

Author contributions

Arslan Khalid contributed to the conceptualization, methodology, model development, data analysis, interpretation of results, and original draft preparation. Ishfaq Ahmed contributed to methodology, supervision, interpretation, and manuscript review. Ibrahim Mufrah Almanjahie contributed to validation, interpretation, and manuscript review. All authors read and approved the final manuscript.

Data availability

The datasets analysed in this study are publicly available from the sources cited in the manuscript. The processed datasets and replication files used to generate the tables and figures are available from the corresponding author upon reasonable request.

Code availability

The R code used for estimation, diagnostics, tables, and figures is available from the corresponding author upon reasonable request.

Ethics approval

Not applicable.

Consent to participate

Not applicable.

Consent for publication

Not applicable.

References

Akaike H (1974) A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19:716–723. <https://doi.org/10.1109/TAC.1974.1100705>

Anderson TW, Darling DA (1952) Asymptotic theory of certain goodness-of-fit criteria based on stochastic processes. *The Annals of Mathematical Statistics* 23:193–212. <https://doi.org/10.1214/aoms/1177729437>

Beirlant J, Goegebeur Y, Segers J, Teugels J (2004) *Statistics of extremes: theory and applications*. Wiley, Chichester. <https://doi.org/10.1002/0470012382>

Coles S (2001) *An introduction to statistical modeling of extreme values*. Springer, London. <https://doi.org/10.1007/978-1-4471-3675-0>

Davison AC, Smith RL (1990) Models for exceedances over high thresholds. *Journal of the Royal Statistical Society: Series B* 52:393–442. <https://doi.org/10.1111/j.2517-6161.1990.tb01796.x>

de Haan L, Ferreira A (2006) *Extreme value theory: an introduction*. Springer, New York. <https://doi.org/10.1007/0-387-34471-3>

Embrechts P, Klüppelberg C, Mikosch T (1997) *Modelling extremal events: for insurance and finance*. Springer, Berlin. <https://doi.org/10.1007/978-3-642-33483-2>

Fisher RA, Tippett LHC (1928) Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Mathematical Proceedings of the Cambridge Philosophical Society* 24:180–190. <https://doi.org/10.1017/S0305004100015681>

Gnedenko B (1943) Sur la distribution limite du terme maximum d’une série aléatoire. *Annals of Mathematics* 44:423–453. <https://doi.org/10.2307/1968974>

Gumbel EJ (1958) *Statistics of extremes*. Columbia University Press, New York. <https://doi.org/10.7312/gumb92958>

Jenkinson AF (1955) The frequency distribution of the annual maximum or minimum values of meteorological elements. *Quarterly Journal of the Royal Meteorological Society* 81:158–171. <https://doi.org/10.1002/qj.49708134804>

Katz RW, Parlange MB, Naveau P (2002) Statistics of extremes in hydrology. *Advances in Water Resources* 25:1287–1304. [https://doi.org/10.1016/S0309-1708\(02\)00056-8](https://doi.org/10.1016/S0309-1708(02)00056-8)

Pickands J III (1975) Statistical inference using extreme order statistics. *The Annals of Statistics* 3:119–131. <https://doi.org/10.1214/aos/1176343003>

Schwarz G (1978) Estimating the dimension of a model. *The Annals of Statistics* 6:461–464. <https://doi.org/10.1214/aos/1176344136>

Smith RL (1985) Maximum likelihood estimation in a class of nonregular cases. *Biometrika* 72:67–90. <https://doi.org/10.1093/biomet/72.1.67>