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Integrated Collaborative Learning and Polya Problem Solving Model for Enhancing Pre Service Teachers Critical Thinking through the Mediating Roles of Student Engagement and Attitudes and the Moderating Role of Prior Knowledge

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Abstract

This study examined the effectiveness of integrating collaborative learning with Polya's problem-solving approach in enhancing pre-service teachers' critical thinking skills in mathematics, with attitudes and engagement as mediators and prior knowledge as a moderator. Guided by a pragmatist paradigm, the study employed mixed-methods quasi-experimental design. A purposive sample of 390 pre-service mathematics teachers participated in the quantitative phase, while qualitative data were obtained from a subset of participants. Quantitative data were collected using a critical thinking achievement test from simultaneous equation and structured questionnaires, and analyzed using PROCESS Macro Model 4 to test mediation and moderation effects. Qualitative data were gathered through open-ended responses and classroom-based evidence and analyzed thematically following Braun and Clarke's six-phase approach. The results revealed that the integrated instructional approach significantly improved pre-service teachers' critical thinking skills. Attitudes and engagement were found to play significant mediating roles, while prior knowledge significantly moderated the effectiveness of the intervention. Qualitative findings supported the quantitative results, showing that collaborative engagement, positive attitudes, and sustained participation contributed to higher-order thinking. This study contributes to the literature by integrating collaborative learning with Polya's problem-solving approach within a single instructional framework and empirically testing both mediating and moderating mechanisms.

Introduction

The persistent demand for mathematically competent, reflective, and analytically minded teachers has positioned the development of critical thinking skills as a fundamental goal of mathematics education, particularly within teacher education institutions responsible for preparing future mathematics educators [Yuanita, Zulnaidi & Zakaria, 2018]. Critical thinking enables learners to analyze mathematical situations logically, evaluate alternative solutions, justify reasoning processes, and apply mathematical knowledge to unfamiliar and real-life contexts [Bicer, Lee, Perihan, Capraro & Capraro, 2020; Andrade & Ching, 2020; Asoma et al., 2022; Dolapcioglu & Doğanay, 2022]. In contemporary mathematics education, emphasis is increasingly placed on learners' ability to engage in reflective reasoning, interpretation, problem-solving, and decision-making rather than merely reproducing memorized procedures. Consequently, teacher education programs are under increasing pressure to adopt pedagogical approaches that move beyond procedural teaching toward instructional models that actively engage pre-service teachers in meaningful problem solving and collaborative knowledge construction [Febrianti, Prasetyo, Hayu, Chongbang & Kumar, 2023]. Thus, critical thinking is not only essential for pre-service teachers' professional competence but also fundamental to their future ability to facilitate higher-order thinking among learners in mathematics classrooms.

Despite this educational expectation, evidence from classroom assessments and instructional observations indicates that many pre-service teachers demonstrate weak critical thinking abilities when solving mathematical word problems, particularly simultaneous equation word problems. Although these learners have been exposed to algebraic concepts throughout their academic journey, many continue to experience considerable difficulty interpreting mathematical situations, analyzing relationships among

variables, and applying appropriate solution strategies to real-life mathematical problems. As a result, many pre-service teachers rely heavily on memorization of procedures rather than analytical reasoning and reflective thinking when solving mathematical tasks. This persistent weakness limits their ability to justify solutions logically, evaluate alternative methods, and engage in higher-order mathematical reasoning expected of future mathematics teachers. Geisler, Rach and Rolka [2023] observed that weak reasoning and limited conceptual engagement often constrain learners' ability to construct meaningful mathematical understanding. Similarly, empirical studies continue to show that many pre-service teachers experience challenges in reflective problem-solving despite exposure to formal mathematics instruction [Ghunaimat & Alawneh, 2025; Ateeq, 2023].

Simultaneous equation word problems were selected in this study because they naturally require learners to engage in interpretation, analytical thinking, logical reasoning, representation, and reflective problem-solving, all of which are central components of critical thinking. Such problems demand that learners understand problem contexts, identify relevant variables, formulate simultaneous equations correctly, apply suitable solution methods, and evaluate the validity of solutions obtained. However, many pre-service teachers experience serious difficulties in solving such problems effectively. For example, learners frequently struggle to solve problems such as:

The perimeter of a rectangle is 64 cm. The length is 8 cm more than the width. Find the area of the rectangle.

Although the problem appears relatively straightforward, many pre-service teachers fail to define variables appropriately, formulate correct simultaneous equations, and select suitable solution procedures such as substitution or elimination. Studies by Yagan [2021], Su [2023], Damrongpanit [2019], and Johari et al. [2020] further reveal that learners often experience computational errors, weak algebraic manipulation skills, and difficulties translating verbal information into mathematical representations. Additionally, Aliyu et al. [2023] and Manurung, Hutapea and Suanto [2024] found that reading and interpretation difficulties significantly affect learners' ability to solve simultaneous equation word problems successfully.

Several classroom-related and instructional factors appear to contribute to these persistent difficulties among pre-service teachers. From the researcher's classroom observations as a mathematics lecturer, many learners demonstrate low engagement during mathematical problem-solving lessons, particularly when tasks require interpretation, reflective reasoning, and higher-order thinking. Learners frequently become passive during classroom discussions, avoid participation in collaborative activities, and show limited persistence when mathematical tasks become cognitively demanding. In addition, many pre-service teachers exhibit negative attitudes toward word problems, perceiving them as confusing, difficult, and anxiety-provoking. Such attitudes reduce learners' confidence, willingness to participate, and motivation to attempt challenging mathematical tasks. Furthermore, differences in prior knowledge significantly influence learners' ability to define variables, connect mathematical concepts, and apply algebraic procedures appropriately. Learners with weak foundational knowledge often struggle to interpret simultaneous equation problems and participate effectively in reflective reasoning activities. According to Lestari and Yudhanegara [2024], prior knowledge serves as a cognitive foundation that enables learners to retrieve and apply previously acquired concepts during problem-solving. Similarly, Hanifah et al. [2025],

Chan et al. [2023], and Khontee and Poonputta [2025] emphasized that inadequate prior knowledge constrains learners' participation in collaborative reasoning and higher-order thinking processes. These classroom realities suggest that instructional strategies capable of improving engagement, attitudes, and prior knowledge are necessary for enhancing critical thinking among pre-service teachers.

In response to these instructional challenges, collaborative learning and Polya's problem-solving approach have increasingly been recognized as effective pedagogical approaches for promoting meaningful mathematics learning and higher-order thinking. Collaborative learning enables learners to externalize ideas, negotiate meaning, share reasoning, and co-construct mathematical understanding through peer interaction and dialogue [Geisler et al., 2023]. Rooted in Vygotsky's [1978] social constructivist theory, collaborative learning emphasizes that knowledge develops through social interaction and guided participation. Likewise, Polya's problem-solving approach provides learners with a structured cognitive framework involving understanding the problem, devising a plan, carrying out the plan, and looking back. This systematic process supports analytical reasoning, reflective thinking, and metacognitive awareness during mathematical problem-solving [Ghunaimat & Alawneh, 2025]. However, previous studies often examine collaborative learning and Polya's approach independently, despite growing recognition that integrating both approaches may strengthen learners' reasoning, reflection, and engagement simultaneously [Sanchez-Lima et al., 2026; Applebaum, 2026; Milagros et al., 2026]. Consequently, integrating collaborative learning with Polya's problem-solving approach may provide a more comprehensive instructional model capable of fostering critical thinking among pre-service teachers.

The effectiveness of such integrated instructional approaches may further depend on learner-related factors such as engagement, attitudes, and prior knowledge. Research increasingly demonstrates that engagement functions as a psychological mechanism through which instructional strategies influence learning outcomes and critical thinking development [Aabeyir et al., 2025; Ahmed, 2018]. Similarly, attitudes toward mathematics influence learners' confidence, persistence, motivation, and willingness to engage in problem-solving activities [Febrianti et al., 2023; Yagan, 2021; Geisler et al., 2023]. Prior knowledge also determines learners' readiness to participate in collaborative reasoning and apply systematic problem-solving strategies effectively [May, 2024; Lee et al., 2021]. Although these variables have individually been linked to mathematics learning outcomes, they are rarely examined simultaneously within integrated pedagogical frameworks designed to develop critical thinking among pre-service teachers. Moreover, empirical studies investigating the mediating roles of engagement and attitudes alongside the moderating role of prior knowledge within collaborative Polya-based instructional environments remain limited, particularly within mathematics teacher education contexts in developing countries. Therefore, the present study seeks to address these gaps by investigating the effect of an integrated collaborative learning and Polya problem-solving approach on pre-service teachers' critical thinking development while examining the mediating roles of engagement and attitudes and the moderating role of prior knowledge.

Source: Field Data (2025), Fig. 1: Conceptual Framwork for the Study

Research objectives.

1. To examine whether preservice teachers attitude mediates the effect of the integrated method on critical thinking in problem-solving.
2. To determine whether preservice teachers' engagement mediate the effect of the integrated method on critical thinking in problem-solving.
3. To determine whether preservice teachers' previous knowledge moderate the effect of the integrated method on critical thinking in problem solving.

Methodology

This study was grounded in a pragmatist research paradigm, which prioritizes the selection of research methods based on their effectiveness in addressing the research problem rather than adherence to a single epistemological perspective. Consistent with this paradigm, a mixed-methods quasi-experimental design was adopted, where quantitative data were collected and analyzed first to determine the effect of the instructional intervention, followed by qualitative data to provide deeper explanations of the observed outcomes. The quantitative phase examined whether there was a statistically significant difference in critical thinking achievement between pre-service teachers exposed to the integrated teaching method and those taught using conventional instruction. Given the highly significant quantitative results, a qualitative phase was subsequently conducted to explore participants' experiences and interpretations of the instructional approach, thereby providing contextual explanations for the observed effects.

The study was conducted within mathematics teacher education programs in Ghana, where pre-service teachers are prepared to teach mathematics at the basic education level. The context of the study reflects ongoing national efforts to improve mathematics instruction through innovative pedagogical approaches that promote critical thinking and problem-solving. Within this setting, the integrated instructional method combining collaborative learning with the problem-solving framework of George Pólya was implemented to examine its influence on pre-service teachers' learning experiences and critical thinking development.

The study targeted a population of 15,600 pre-service teachers enrolled in mathematics education programs. Using the Yamane formula $n = \frac{N}{1+N(e^2)}$ with a 5% margin of error, a representative sample of 390 participants was determined. Participants were selected using purposive sampling based on their relevance to the research objectives and their exposure to the instructional intervention. The sample was divided into two independent groups: a treatment group that received instruction through the integrated collaborative learning and problem-solving approach, and a control group that was taught using conventional instructional methods.

The instructional intervention consisted of an integrated teaching method that combined collaborative learning with the structured stages of problem-solving proposed by George Pólya. In the treatment group, pre-service teachers worked collaboratively to analyze mathematical problems, devise solution strategies, implement solutions, and reflect on their reasoning processes. Instructional activities were designed to encourage discussion, peer interaction, and reflective thinking, thereby supporting the development of critical thinking skills. In contrast, the control group received conventional teacher-centered instruction that emphasized explanation and individual practice. The intervention took a period of six weeks as follows:

Week 1 – Understanding the Problem

- Introduce simultaneous equations through real-life contexts (e.g., budgeting, mixture problems).
- Discuss the meaning of variables, coefficients, and constants.
- Students work collaboratively to identify knowns and unknowns in sample problems.
- Group discussion on how to interpret equations and the relationships between them.

Week 2 – Devising a Plan (Strategy Selection)

- Present different methods for solving simultaneous equations: substitution, elimination, and graphical approach.
- Students, in small groups, choose an appropriate method for a set of practice problems.
- Encourage justification of method choice using reasoning and prior knowledge.
- Teacher facilitates discussion, highlighting advantages and limitations of each method.

Week 3 – Implementing the Plan (Solving Equations)

- Students apply the selected methods to solve simultaneous equations collaboratively.
- Focus on step-by-step execution: substitution of variables, combining equations, checking solutions.
- Groups verify solutions and discuss any errors or alternative approaches.

Week 4 – Reflection and Checking (Looking Back)

- Students review solutions and verify correctness using substitution into original equations.
- Compare solutions obtained by different methods within the group.
- Teacher prompts discussion: “Could another method be more efficient? What patterns do you notice?”

Week 5 – Application to Real-Life Problems

- Students tackle contextual problems requiring simultaneous equations (e.g., cost and revenue, distance-time problems).
- Groups translate real-life scenarios into equations and solve collaboratively.
- Emphasize reasoning and critical thinking, asking students to explain why each step is taken.

Week 6 – Consolidation and Presentation

- Groups present solutions to assigned problems to the whole class.
- Peer evaluation: students ask questions, suggest alternative methods, and discuss reasoning.
- Teacher facilitates reflection on learning strategies, problem-solving approaches, and critical thinking skills developed during the 6-week intervention.

Data were collected using three primary instruments: a mathematics achievement test, close-ended questionnaire and open-ended questionnaire. The mathematics achievement test was designed to measure pre-service teachers' critical thinking in problem-solving, focusing on reasoning, analysis, evaluation, and application rather than rote memorization. The test content was centered on simultaneous equations and was structured to reflect the stages of problem-solving, ensuring alignment with the instructional approach used in the treatment group. The open-ended questionnaire was administered during the qualitative phase to capture participants' perceptions of collaborative learning, their engagement with the problem-solving stages, and their perceived development of critical thinking skills.

The content validity of both instruments was established through expert review by specialists in mathematics education. Reliability of the mathematics achievement test was determined through a pilot study, which produced acceptable reliability coefficients, including (critical thinking = 0.96; attitudes = 0.83; engagement = 0.79; prior knowledge = 0.87).

The data collection process followed the sequence of the explanatory mixed-methods design. First, the quantitative phase was conducted, during which the mathematics achievement test was administered to both the treatment and control groups to measure critical thinking outcomes. After the quantitative results indicated highly significant differences, qualitative data were collected using an open-ended questionnaire to explore participants' perceptions and experiences of the integrated instructional method.

Data were analyzed using PROCESS Macro (Model 4) with 5,000 bootstrap samples to estimate direct, indirect, and total effects, and to examine theoretically motivated moderation effects via mean-centred interaction terms. Bootstrapping supported robust inference without reliance on normality assumptions, consistent with current methodological standards in mathematics education research. Ethical principles of informed consent, voluntary participation, and confidentiality were observed throughout.

Ethical standards were strictly observed throughout the study. Participants provided informed consent prior to their involvement, and confidentiality of their responses was maintained. Participation was voluntary, and respondents were informed of their right to withdraw from the study at any stage without penalty. These procedures ensured that the research adhered to accepted ethical principles in educational research.

Results

Participants Information

The demographic characteristics of the respondents are presented in Table 4.1. Most pre-service teachers were aged 18–22 years, representing 50.3% (n = 196), followed by 23–25 years at 37.2% (n = 145), and 26–33 years at 12.6% (n = 49). This reflects typical teacher education entry in Ghana, while also showing flexibility for mature entrants and in-service teachers. Gender distribution was nearly balanced, with males at 51.5% (n = 201) and females at 48.5% (n = 189), consistent with current enrolment trends influenced by policies promoting gender equity. Regarding program of study, 60.5% (n = 236) were enrolled in

Mathematics with ICT (JHS), and 39.5% (n = 154) in Mathematics (Primary). This aligns with Ghana's emphasis on strengthening mathematics instruction through ICT integration, where JHS students are exposed to more advanced content and pedagogy. Overall, the sample is contextually appropriate and supports meaningful analysis

Table 1
represents Participants Information

Demographic Variable	Categories	Frequency (n)	Percentage (%)
Age	18—22 years	196	50.3
	23—25 years	145	37.2
	26—33 years	49	12.6
Total		390	100
Gender	Male	201	51.5
	Female	189	48.5
Total		390	100
Program of Study	Mathematics with ICT (JHS)	236	60.5
	Mathematics (Primary)	154	39.5
Total		390	100
Source: Field Data (2025)			

Descriptive Statistics

The means and standard deviations in Table 2 show students' performance across pre- and post-intervention stages for both groups. Student Engagement (SE) had a pre-intervention mean of 2.157 (SD = 0.386), indicating low engagement consistent with teacher-centred practices in Ghana, with a small SD showing similar initial levels. Post-intervention, SE increased to 4.096 (SD = 0.696), reflecting improved engagement through learner-centred strategies, with slightly higher variability. Student Attitude (SA) improved from 2.145 (SD = 0.571) to 3.808 (SD = 0.901), indicating more positive attitudes and greater variation in responses. Prior Knowledge (PK) rose from 8.170 (SD = 1.200) to 14.61 (SD = 3.457), showing notable gains, though variability increased. Critical Thinking (CT) increased from 8.092 (SD = 1.058) to 10.241 (SD = 3.201), with higher variability suggesting differing levels of cognitive engagement. Skewness values were positive, ranging from 0.153 (PK) to 0.592 (CT), with SE (0.195), SA (0.192), and PK (0.153) approximately symmetric, while CT showed slight skewness. Kurtosis values were negative, from 0.978 (CT) to 1.465 (SE), indicating platykurtic distributions. Since all values fall within acceptable limits for large samples (N = 390), the data are suitable for parametric analyses, and overall mean increases confirm the effectiveness of the intervention.

Table 2
Descriptive Statistics

Variable	N	Mean	St. Deviation	Kurtosis		Skewness	
Student Engagement (SE)	195	2.157	0.386	1.465	0.247	0.195	0.124
	195	4.096	0.696				
Student Attitude (SA)	195	2.145	0.571	1.278	0.247	0.192	0.124
	195	3.808	0.901				
Prior Knowledge (PK)	195	8.170	1.200	1.235	0.247	0.153	0.124
	195	14.61	3.457				
Critical Thinking (CT)	195	8.092	1.058	0.978	0.247	0.592	0.124
	195	10.241	3.201				
Source: Field Data (2025)							

Pre-Test Results

An independent samples t-test was conducted to examine whether there was a significant difference in the pre-test scores of critical thinking between the experimental group and the control group prior to the intervention. The experimental group had a mean score of *M = 2.157* with a standard deviation of *SD = 0.386*, while the control group had a mean score of *M = 2.096* with a standard deviation of *SD = 0.397*. The t-test results indicated that the difference in pre-test scores between the two groups was not statistically significant, *t(388) = 42.561, p = .230*, suggesting that both groups were comparable at baseline. The effect size, calculated using Cohen's d, was *d = 0.023*, indicating a negligible difference between the groups. The mean difference was 0.061 points, with a 95% confidence interval ranging from *− 1.82737* to *2.05161*, suggesting that the true difference in pre-test scores between the groups could plausibly fall anywhere in this interval.These results suggest that the experimental and control groups were equivalent in terms of critical thinking prior to the intervention, providing a solid baseline for evaluating the effects of the integrated collaborative learning and Polya problem-solving approach. Therefore, any significant differences observed in the post-test scores can be confidently attributed to the intervention rather than pre-existing disparities between the groups.

Table 3
Pre-Test Results in the Effect of Integrated Method on Critical Thinking

Group	*M*	*SD*	*F*	*sig*	*t*	*df*	*p*	*Cohen's d*	*95% CI*	
									L	*U*
2	*2.157*	*0.386*	*42.561*	*0.230*	*34.010*	*388*	*0.305*	*0.023*	*-1.82737*	*2.05161*
1	*2.096*	*0.397*								

Source: Field Data (2025)

Preservice Teachers Attitude Mediates the the Integrated Method and Critical Thinking

A simple mediation analysis was conducted and presented in Table 3 using PROCESS Model 4 (Hayes, 2018) to examine whether pre-service teachers attitude mediated the relationship between group (instructional condition) and critical thinking. As shown in Table 3, the total effect of group on critical thinking was statistically significant, β = 6.44, SE = 0.26, t = 24.58, p < .05, indicating that students in the treatment group demonstrated significantly higher critical thinking scores than those in the control group.

The direct effect of group on student attitude was also statistically significant, β = 5.79, SE = 0.41, t = 14.14, p < .05, suggesting that the instructional intervention substantially improved students' attitudes. Furthermore, student attitude significantly predicted critical thinking, β = 0.40, SE = 0.07, t = 5.59, p = .05. When student attitude was included in the model, the direct effect of group on critical thinking remained significant, β = 5.35, SE = 0.43, t = 12.53, p < .05, indicating partial mediation.

The indirect effect of group on critical thinking through student attitude was statistically significant, β = 1.09, with a 95% bootstrap confidence interval that did not include zero [0.16, 2.23]. This result confirms that student attitude partially mediated the relationship between instructional group and critical thinking outcomes.

After the quantitative results followed by the qualitative results in Fig. 1, which were generated to complement, deepen the understanding and explain the reasons behind the quantitative results reported earlier. The qualitative data were analyzed using Braun and Clarke's (2006) thematic analysis, a flexible and systematic approach that enabled the identification and interpretation of meaningful patterns within participants' responses.

The analysis followed the six phases (familiarization with the data, generating initial codes, searching for themes, reviewing themes, defining and naming themes, producing the report / narrative interpretation) proposed by Braun and Clarke, allowing themes to emerge both from the data itself and in relation to the study's theoretical framework and research objectives. This approach was particularly useful in capturing participants lived experiences, perceptions, and interpretations of the instructional practices implemented

in the study. As a result, the qualitative results provide rich insights into how and why the intervention influenced learning-related outcomes.

The themes presented in figure reflect participants' views on teaching and learning processes, engagement, attitudes, and critical thinking development within the instructional context. To enhance clarity and credibility, direct excerpts from participants' responses are included where appropriate to illustrate each theme. the qualitative results offer contextual depth and explanatory power, thereby strengthening the interpretation of the study's findings within the Ghanaian educational context.

The analysis shows that the integrated teaching model directly enhances pre-service teachers' (PSTs) critical thinking through both structured cognitive processes and collaborative social interactions. Polya's problem-solving steps and task decomposition provide a framework for analyzing problems, planning solutions, and making reasoned decisions, while collaborative activities encourage teamwork, idea sharing, and joint problem-solving. Participants emphasized that these mechanisms support critical thinking even without additional mediators, highlighting the model's intrinsic effectiveness. For instance, R4 noted that the model *"helps pre-service teachers to analyze data fast and give accurate answers to problems or tasks given on time,"* illustrating how the structured approach improves efficiency and reasoning. Similarly, other participants described the approach as offering *"a structured cognitive framework and a socially interactive environment, which together facilitate critical thinking in mathematics,"* confirming that both cognitive and social dimensions are integral to fostering analytical skills.

The qualitative data further reveal that pre-service teachers' attitudes play a key mediating role in translating the model into enhanced critical thinking outcomes. Positive attitudes—interest, confidence, persistence, and openness to collaboration—enable learners to engage deeply with mathematical tasks and apply Polya's steps thoughtfully. Participants explicitly linked attitudinal shifts to critical thinking gains: R1 stated that *"the mediation role of pre-service teacher's attitude improving positive attitude can be a channel to developing critical thinking skills,"* while R3 emphasized that *"positive attitudes strengthen the impact of collaborative learning with Polya problem-solving skills."* Others highlighted related effects, with R5 noting that *"a positive attitude makes them more open to collaboration and Polya steps,"* and R7 describing how structured peer problem-solving fosters *"greater confidence, interest, and persistence, encouraging deeper analysis, evaluation, and reflection."* Collectively, R2, R6, R8, and R9 reinforce that attitudes act as a psychological bridge, explaining how the instructional model affects engagement, interest, and ultimately, higher-order reasoning in mathematics. These findings underscore that fostering positive learner dispositions is critical for translating instructional strategies into effective critical thinking development.

Table 4
Represents Total, Direct, and Indirect Effects of Group on Critical Thinking via Student Attitude (PROCESS Model 4

Effect	paths	β	SE	t	Sig(2-tail)	95% CI	
						lower	upper
Total	Group - -> Critical thinking (CT)	6.441	0.262	24.579	0.000	5.926	6.956
Direct	Group - - >Students Attitude (SA)	5.787	0.409	14.138	0.000	4.982	6.592
Direct	Group– >CT	5.350	0.427	12.534	0.000	4.511	6.189
Direct	SA - -> CT	0.396	0.675	5.587	0.001	0.932	1.724
indirect	Group - - > SA - -> CT	1.091	0.523	---	0.000	0.162	2.225
Source: Field Data (2025)							

Preservice Teachers' Engagement Mediate between the Integrated Method and Critical Thinking in Problem-Solving.

A simple mediation analysis was conducted again, using PROCESS Model 4 (Hayes, 2018) to examine whether student engagement mediated the relationship between group (instructional condition) and critical thinking. As presented in Table 4, the total effect of group on critical thinking was statistically significant, β = 6.44, SE = 0.26, t = 24.58, p < .05, indicating that students exposed to the instructional intervention demonstrated substantially higher critical thinking scores than those in the control group.

The direct effect of group on student engagement was also statistically significant, β = 4.42, SE = 0.57, t = 7.75, p < .05, suggesting that the instructional approach effectively promoted students' active participation and involvement in learning. In addition, student engagement significantly predicted critical thinking, β = 0.78, SE = 0.10, t = 7.96, p < .05, indicating that higher engagement was associated with improved critical thinking skills. When student engagement was included in the model, the direct effect of group on critical thinking remained statistically significant, β = 5.30, SE = 0.58, t = 9.08, p < .05, demonstrating partial mediation.

The indirect effect of group on critical thinking through student engagement was statistically significant, β = 2.03, with a 95% bootstrap confidence interval that did not include zero [0.38, 3.56]. This result confirms that student engagement partially mediated the relationship between instructional group and critical thinking outcomes. The results suggest that, beyond the direct influence of the instructional approach, increased student engagement serves as a key mechanism through which critical thinking skills are enhanced.

The qualitative data results demonstrate how preservice teachers' engagement mediates the effect of the integrated method on their critical thinking skills. Participants emphasized that active participation,

sustained focus, and reflective involvement during problem-solving tasks helped them internalize Polya's steps and improve understanding. One participant stated, "*When we actively participate, I understand the problem better because we discuss in groups,"* highlighting how engagement reinforced learning. Another noted, *"Following the Polya steps with my peers keeps me focused and thinking deeper,"* reflecting the importance of structured participation.

Collaboration played a significant role in strengthening understanding. Participants reported that peer discussions and idea sharing clarified difficult concepts: *"Working with classmates allows us to explore different approaches and clarify concepts we couldn't understand on our own,"* and *"Sharing our strategies in groups helps me understand better,"* showing that collaborative learning supports engagement and knowledge construction.

Again, engagement fostered higher-order thinking and critical problem-solving. Participants reflected on the benefits of observing peers and persisting through challenges: *"Being engaged helps me follow the Polya steps carefully and solve problems more effectively,"* and *"Observing how my peers approach problems allow me to think more critically and creatively,"* demonstrating that active engagement strengthens critical thinking.

These qualitative insights clearly explain the reasons behind the exciting quantitative results, which showed that the integrated method significantly increased critical thinking scores. The results suggest that the method's effectiveness is transmitted through enhanced engagement, rather than occurring independently, confirming the mediation role of preservice teachers' engagement in linking collaborative learning and Polya problem-solving to higher-order critical thinking.

Table 5
Total, Direct, and Indirect Effects of Group on Critical Thinking via Student Engagement (PROCESS Model 4)

Effect	**paths**	**β**	**SE**	**t**	**Sig(2-tail)**	**95% CI**	
						lower	**upper**
Total	Group - -> Critical thinking	6.441	0.262	24.579	.000	5.926	6.956
Direct	Group - - > Critical thinking (CT)	5.300	0.584	9.075	.000	4.152	6.448
Direct	Group- - > Students engagement (SE)	4.415	0.570	7.748	.000	3.294	5.535
Direct	SE - -> CT	0.781	0.981	0.796	.000	1.148	2.710
indirect	Group - - > SE- -> CT	2.026	0.805	---	.000	0.379	3.561
Source: Field Data (2025)							

Preservice Teachers' Previous Knowledge Moderate the Effect of The Integrated Method on Critical Thinking in Problem Solving.

To examine whether the effect of the independent variable on critical thinking (CT) was moderated, a regression analysis was conducted with the interaction term (INTERACT) included as a predictor in Tables 6 and 7. The ANOVA results indicated that the overall regression model was statistically significant, $F(1, 388) = 541.10$, $p < .05$, demonstrating that the model explained a substantial proportion of the variance in critical thinking scores.

The regression coefficients further revealed that the interaction term significantly predicted critical thinking, $B = 1.02$, $SE = 0.04$, $\beta = 0.76$, $t (388) = 23.26$, $p < .05$, with a 95% confidence interval of [0.934, 1.106]. This indicates a significant moderation effect, meaning that the relationship between the independent variable and critical thinking varies depending on the level of the moderator. Specifically, the positive coefficient suggests that as the moderator increases, the impact of the independent variable on critical thinking becomes stronger, highlighting the conditional nature of the effect.

These results suggest that the inclusion of the moderator enhances understanding of the factors influencing students' critical thinking outcomes, showing that the effectiveness of the instructional intervention depends on contextual or learner-related variables. The results underscore the importance of considering interaction effects in educational research, particularly in the Ghanaian context, where learner engagement and instructional conditions may jointly influence critical thinking development.

Figure 8 illustrates the interaction effect of the Collaborative Polya Problem-Solving Approach (CPPSA) and prior knowledge on pre-service teachers' critical thinking (CT). The graph compares critical thinking scores before and after the implementation of CPPSA for three levels of prior knowledge: high (+ 1 SD), mean, and low (− 1 SD). Before the implementation of CPPSA, participants with high, mean, and low prior knowledge had nearly identical critical thinking scores (approximately 6.1), indicating that there were no substantial differences in critical thinking among the three groups at the baseline. This suggests that the groups started from a relatively similar level of critical thinking before the intervention. After the implementation of CPPSA, critical thinking scores increased across all three groups, demonstrating that the intervention had a positive effect regardless of participants' prior knowledge. However, the magnitude of improvement differed considerably according to the level of prior knowledge. Participants with high prior knowledge exhibited the greatest improvement in critical thinking, with their mean score increasing from approximately 6.1 to 9.2. The steep slope of the green line indicates that CPPSA was most effective for learners who entered the programme with strong prior mathematical knowledge. Participants with average prior knowledge also experienced a substantial improvement, with their critical thinking score increasing from approximately 6.1 to 7.8. Although the increase was smaller than that of the high prior knowledge group, the positive slope demonstrates that CPPSA effectively enhanced critical thinking among learners with moderate prior knowledge. In contrast, participants with low prior knowledge showed only a modest improvement, with their critical thinking score increasing from approximately 6.1 to 6.7. The relatively flat slope of the red line suggests that although CPPSA benefited these learners, the gains in critical thinking were considerably smaller than those observed among participants with higher levels of prior knowledge.

The differing slopes of the three lines indicate a moderation (interaction) effect. Specifically, the effectiveness of CPPSA in improving critical thinking depended on the learners' level of prior knowledge. As prior knowledge increased, the positive effect of CPPSA on critical thinking became stronger. The non-parallel nature of the lines provides graphical evidence that prior knowledge moderated the relationship between CPPSA and critical thinking.

The qualitative findings in Fig. 2 further help to explain the quantitative results. Participants reported that their engagement and learning experiences were influenced not only by the instructional approach but also by factors such as peer collaboration, prior knowledge, and classroom support. These insights align explain the significant moderation effect observed in the quantitative analysis, suggesting that the impact of the instructional intervention on critical thinking varies depending on students' engagement and contextual factors. Themes such as active participation, collaborative problem-solving, and supportive learning environments provide rich contextual explanations for why certain students benefited more from the intervention, illustrating the conditional nature of the effect observed in the regression analysis.

The results indicate that prior knowledge significantly moderates the relationship between the integrated instructional model (collaborative learning with Polya's problem-solving approach) and pre-service teachers' critical thinking skills. Participants reported that learners with strong prior knowledge can better understand and apply Polya's problem-solving strategies, actively engage in discussions, and make meaningful contributions to group problem-solving tasks, thereby enhancing the model's effectiveness. For example, one participant stated, *"Pre-service teachers with higher prior knowledge might benefit more from these approaches because they can better understand and apply the strategies" (R2)*, while another noted, "*High prior knowledge… engage more deeply in collaborative discussions and apply Polya steps effectively. Their existing schemas allow them to ask better questions" (R8)*. Conversely, learners with limited prior knowledge may struggle to contribute effectively, potentially slowing critical thinking development; however, collaborative interactions allow peers to scaffold understanding: "*Those with limited prior knowledge may struggle to contribute effectively, which can slow down critical thinking development. However, collaborative learning allows peers to scaffold each other" (R3*). Additionally, prior knowledge helps learners connect new concepts to existing schemas, build confidence, and approach problem-solving systematically: *"It gives more understanding of concepts, content. It also helps to gain confidence wherever problem is before you" (R4, R7*). Overall, these results suggest that the strength of the model's effect on critical thinking depends on learners' existing mathematical knowledge, confirming prior knowledge as a key moderating factor that enhances or constrains the benefits of collaborative learning with Polya's approach.

Table 7
ANOVA for Regression of Critical Thinking (CT) on Interaction Term (INTERACT)

Predictor	B	SE	β	t	sig	95%CI (Lower,Upper)	
Constant	6.269	0.258	---	24.258	.000	5.761	6.777
Interaction term	1,020	0.044	0.763	23.262	.000	0.934	1.106
Source: Field Data (2025)							

Discussion of Findings

Firstly, the findings of the study demonstrate that the integrated teaching method (collaborative learning + Polya's problem-solving approach) exerts a significant influence on pre-service teachers' critical thinking both directly and indirectly through attitudes. The significant total effect, together with meaningful direct and indirect pathways, confirms that the instructional model enhances critical thinking not merely as a pedagogical technique but through its capacity to reshape learners' affective orientations toward mathematics. This result aligns with established evidence that affective dispositions mediate the relationship between instructional conditions and higher-order cognitive outcomes [Chan et al., 2023; Kamber Hamzić et al., 2025; Khontee & Poonputta, 2025]. Meta-analytic findings showing strong effects of collaborative problem solving on critical thinking dispositions and attitudinal tendencies [Acharya et al., 2022] provide further empirical support for the plausibility of attitudes as a central mechanism linking pedagogy and reasoning development, particularly in mathematics education contexts.

Qualitative evidence strengthens this interpretation by illustrating how positive dispositions toward the instructional approach functioned as a psychological conduit for critical thinking development. Participants (*R1,4,7,8*) consistently emphasized that favorable attitudes enhanced openness to collaboration and acceptance of Polya's structured steps, thereby increasing receptivity to cognitive challenge. These accounts suggest that attitudes are not peripheral emotional responses but play an enabling role in shaping how learners engage with mathematical reasoning processes. By fostering a supportive emotional and cognitive climate, the integrated approach appears to facilitate deeper engagement with problem-solving tasks, supporting the view that affective readiness is essential for the effective enactment of higher-order thinking in mathematics classrooms [Hourigan, Leavy, & Carroll, 2024; Gómez-Chacón et al., 2024].

The themes of interest, motivation, and task engagement further clarify how attitudinal shifts translated into sustained cognitive involvement. Participants(*R2,5,7,9*) reported increased willingness to participate in discussions, invest effort, and collaboratively explore complex problems, suggesting that motivation acted as a reinforcing mechanism that sustained engagement over time. This finding aligns with theoretical perspectives that emphasize the role of interest and engagement in transforming instructional opportunities into meaningful learning experiences [Febrianti, Prasetyo, Hayu, Chongbang, and Kumar, 2023**]**. In this sense, ICP appears to promote conditions under which learners actively co-construct

understanding, rather than passively apply procedures, thereby supporting critical thinking through dialogue, reflection, and shared reasoning [Kiwanuka et al., 2020].

Moreover, the emergence of confidence and persistence as salient outcomes highlights the close interdependence between affective and cognitive processes in mathematical problem solving. Participants' accounts indicate that confidence enabled learners to persist through challenging tasks, reflect on solution strategies, and evaluate outcomes more critically(R6,8,9). These findings resonate with prior research demonstrating that attitudes, motivation, and confidence interact to support sustained reasoning and higher-order thinking [Khontee & Poonputta, 2025; Hanifah, Budayasa, & Sulaiman, 2025]. Collectively, the results underscore the pedagogical importance of designing mathematics instruction that deliberately integrates collaborative discourse with structured problem-solving heuristics, supported by intentional scaffolding and reflective questioning, to ensure that positive attitudes serve as effective pathways for the development of critical thinking among pre-service teachers.

Secondly, the findings provide robust evidence that engagement significantly mediates the relationship between the integrated collaborative learning and Polya's problem-solving approach (ICP) and pre-service teachers' critical thinking. The significant total, direct, and indirect effects indicate that ICP enhances critical thinking not only through its instructional structure but also through its capacity to foster active learner engagement. This finding supports prior work in mathematics education showing that engagement functions as a crucial mechanism linking instructional design to higher-order cognitive outcomes [Damrongpanit, 2019; Geisler, Rach, & Rolka, 2023; Günhan, 2020]. The mediation effect suggests that instructional strategies alone are insufficient; rather, their effectiveness depends on the extent to which learners are cognitively, behaviorally, and emotionally involved in mathematical activity.

Qualitative findings deepen this interpretation by illustrating how engagement operates as a mediating mechanism in practice. Participants consistently emphasized that ICP influenced their critical thinking indirectly through increased participation, sustained involvement, and reflective engagement. Engagement was described as a "bridge" that translated collaborative problem-solving activities into meaningful reasoning experiences, enabling learners to follow Polya's stages more deliberately and learn from peer explanations. These accounts highlight engagement as an explanatory construct that clarifies how and why ICP works, rather than merely whether it works. Such evidence aligns with constructivist assumptions that learning emerges through active participation and shared meaning-making within social contexts.

The themes further reveal that collaborative learning enhances conceptual understanding, which in turn sustains engagement and supports critical thinking. Peer interaction, discussion, and joint exploration of solution strategies enabled learners to clarify misconceptions and internalize problem-solving processes. These findings are consistent with Bruner's Theory of Cognitive Representation [1966] and the work of Author et al., [2025], they emphasize progression from concrete experiences to abstract reasoning. By engaging learners in discussion, representation, and reflection, ICP creates conditions that promote deeper cognitive processing and support the development of analytical and evaluative skills. Engagement thus functions not only as a motivational factor but also as a cognitive process that facilitates conceptual understanding and reasoning.

Furthermore, the results demonstrate that sustained and reflective engagement strengthens critical thinking by promoting persistence, exposure to multiple perspectives, and evaluative dialogue. Participants linked active involvement and persistence directly to higher-order thinking, suggesting that engagement enables learners to analyze problems more deeply, justify solutions, and evaluate alternative strategies. This finding is consistent with research showing that engagement in structured collaborative tasks enhances critical thinking and problem-solving performance (Hanifah et al., 2024; Gyamerah, 2025; Author et al., 2025]. Collectively, the findings underscore t[at engagement is not merely an outcome of effective instruction but a central mediating process through which collaborative learning integrated with Polya's problem-solving approach fosters critical thinking in mathematics teacher education.

Thirdly, quantitative findings confirm a significant moderating effect of pre-service teachers' (PSTs) prior knowledge on the relationship between the integrated collaborative learning and Polya problem-solving approach (ICP) and critical thinking (CT). The interaction term ($\beta = 0.763$, $t = 23.262$, $p < 0.05$) indicates that PSTs with higher prior knowledge derive greater cognitive benefits from ICP, enhancing analytical reasoning and reflective problem-solving skills. These results corroborate the work of Author, author, and Author (2024) and align with prior studies in mathematics education showing that learners' existing knowledge structures amplify the effects of active instructional strategies [May, 2024; Lee et al., 2021; Khontee & Poonputta, 2025]. The findings suggest that prior knowledge functions as a cognitive amplifier, enabling PSTs to engage more effectively with collaborative and structured problem-solving activities, thereby promoting deeper reasoning outcomes.

Qualitative evidence further elucidates the mechanisms underlying this moderation. Participants consistently reported that "*prior knowledge guided their problem-solving processes, enhanced comprehension, and supported confident contributions during collaborative discussions(R2,4,6,7)".* Illustratively, "*PSTs described connecting new ideas to existing schemas, systematically applying Polya's steps, and critically evaluating peers' solutions"* (*R1,2,4,5,7,9*). This demonstrates that prior knowledge not only scaffolds understanding but also strengthens engagement and reflective thinking, enabling learners to co-construct knowledge and participate meaningfully in collaborative problem-solving tasks. These qualitative insights reinforce findings from Febrianti, Prasetyo, Hayu, Chongbang, and Kumar [2023], highlighting the centrality of cognitive readiness in facilitating higher-order thinking.

The moderation effect is theoretically grounded in Piaget's Cognitive Development Theory [1952], Bruner's Theory of Cognitive Representation [1966], and Vygotsky's Social Constructivism [1978]. Piaget emphasizes that learners' existing cognitive schemas determine their ability to assimilate and accommodate new information, supporting systematic problem-solving. Bruner's framework highlights how prior knowledge facilitates transitions from concrete to abstract reasoning, enabling learners to internalize Polya's problem-solving stages. Social Constructivism further explains how PSTs with richer prior knowledge contribute more effectively to collaborative dialogue within the Zone of Proximal Development (ZPD), promoting co-construction of knowledge and the development of critical thinking. Collectively, these theories explain why learners with stronger prior knowledge benefit more from ICP.

Practically, the findings underscore the importance of assessing and leveraging PSTs' prior knowledge in mathematics teacher education. Differentiated instructional design that accounts for cognitive readiness allows learners to engage fully in collaborative problem-solving, apply Polya's steps systematically, and develop robust critical thinking skills. Empirical studies support this approach, demonstrating that learners with high prior knowledge exhibit greater conceptual gains and reasoning ability when engaged in structured, inquiry-based instructional strategies [May, 2023; Lee et al., 2021; Khontee & Poonputta, 2025]. Overall, these results highlight that ICP's effectiveness is contingent upon learners' cognitive preparedness, integrating affective and engagement mechanisms with prior knowledge to optimize critical thinking development in pre-service mathematics teachers [Siddiqi, 2018; Yuanita, Zulnaidi, & Zakaria, 2018; Lee, Byun, Alagic, & Liu, 2021].

Practical Implications

The findings have several implications for mathematics teacher education:

1. Instructional Design: Teacher educators should integrate collaborative learning with Polya's problem-solving stages to foster critical thinking.
2. Prior Knowledge Activation: Diagnostic assessment of Pre-service teachers' prior knowledge should guide task complexity and scaffolding.
3. Fostering Engagement and Attitude: Structured discussions, reflective questioning, and small-group activities should be embedded to maximize cognitive and affective benefits.

Limitations

While the study yielded highly significant findings, several limitations should be noted: The research context was limited to a specific cohort of PSTs, which may limit generalizability. Measures of engagement and attitudes relied partially on self-report instruments, introducing potential response bias. The study did not explore long-term retention of critical thinking skills beyond the intervention period.

Recommendations for Future Research

Conduct longitudinal studies to examine sustained effects of ICP on critical thinking. Explore additional moderators such as motivation, technological knowledge, or epistemological beliefs. Compare effects across different educational contexts and cultural settings. Investigate how ICP impacts actual classroom teaching practice and student outcomes once PSTs enter the profession.

Conclusion

The study provides strong empirical evidence that integrating teaching method (collaborative learning + Polya's problem-solving approach) significantly enhances pre-service teachers' critical thinking in mathematics problem-solving. Attitudes and engagement mediate this effect, while prior knowledge

strengthens it, demonstrating the value of a theoretically grounded, learner-centered, and scaffolded approach to teacher preparation. These findings reinforce the relevance of constructivist and cognitive development theories in guiding effective instructional design for mathematics education.

Declarations

Ethical Approval

This study was conducted in accordance with the ethical standards governing research involving human participants. Ethical approval was obtained from Akenten Appiah-Menka University of Skill Training and Entrepreneurial Development [AAMUSTED] ethics review committee before data collection commenced (Approval No. AAMUSTED/IERC/2025/035]. All procedures performed in this study complied with institutional and national ethical guidelines for educational research.

Accordance Statement

The authors confirm that this research was conducted in accordance with the principles of the Declaration of Helsinki (1964) and its subsequent amendments, as well as applicable institutional and national ethical standards for research involving human participants.

Consent to Participate

Informed consent was obtained from all participants prior to their participation in the study. Participants were informed about the purpose of the research, the voluntary nature of their participation, their right to withdraw from the study at any time without penalty, and the confidentiality of the information they provided.

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Author Contribution

DDI conceived and designed the study, collected and analyzed the data, interpreted the findings, and drafted the manuscript. YDA supervised the study, contributed to the research design and methodology, and critically reviewed and revised the manuscript. AE contributed to the data analysis, interpretation of results, and manuscript review and editing. All authors read and approved the final manuscript.

Acknowledgment:

The author acknowledges the participants who responded during the data collection process

Data Availability

1. Data availability statementThe datasets generated and/or analysed during the current study are not publicly available because they contain information that could compromise the privacy and confidentiality of the research participants. However, the datasets are available from the corresponding author upon reasonable request.

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Table

Table 6 is not available with this version.

Figures

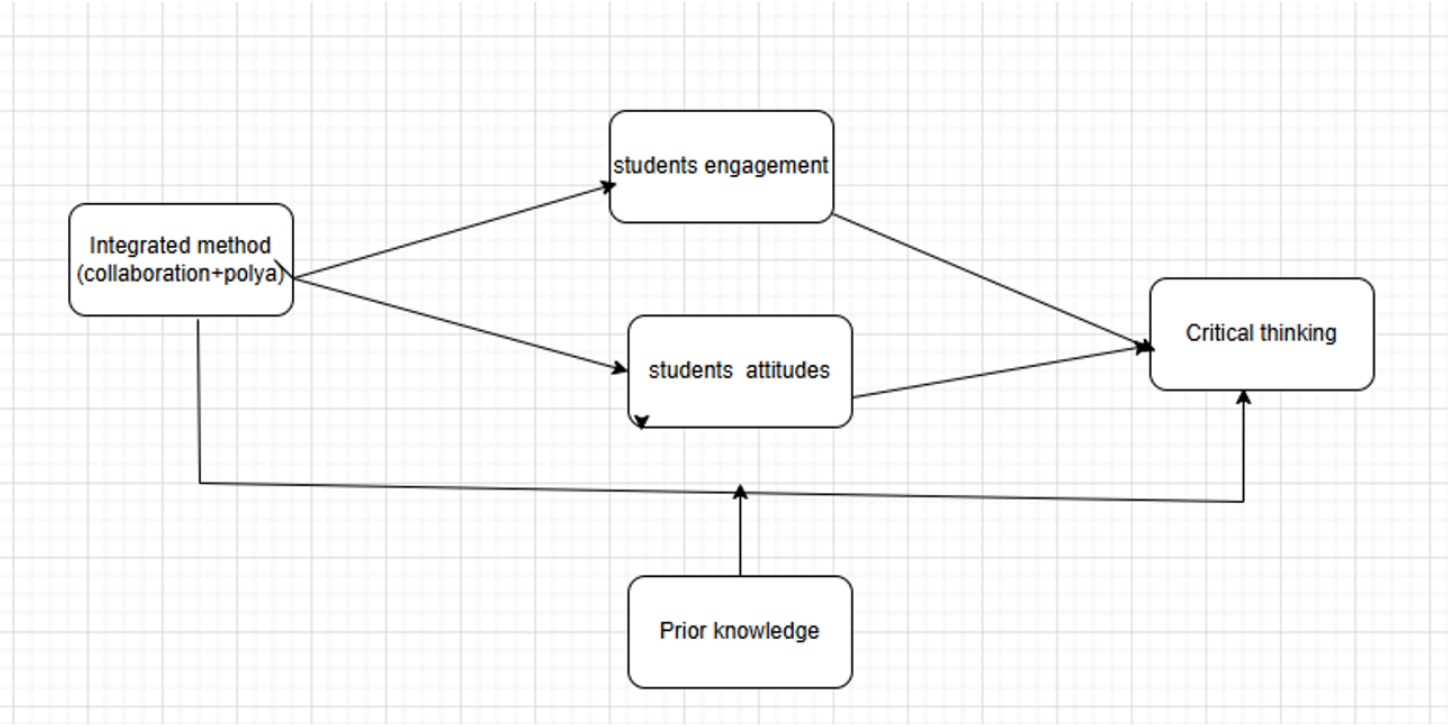


Figure 1

Conceptual Framwork for the Study

Source: Field Data (2025),

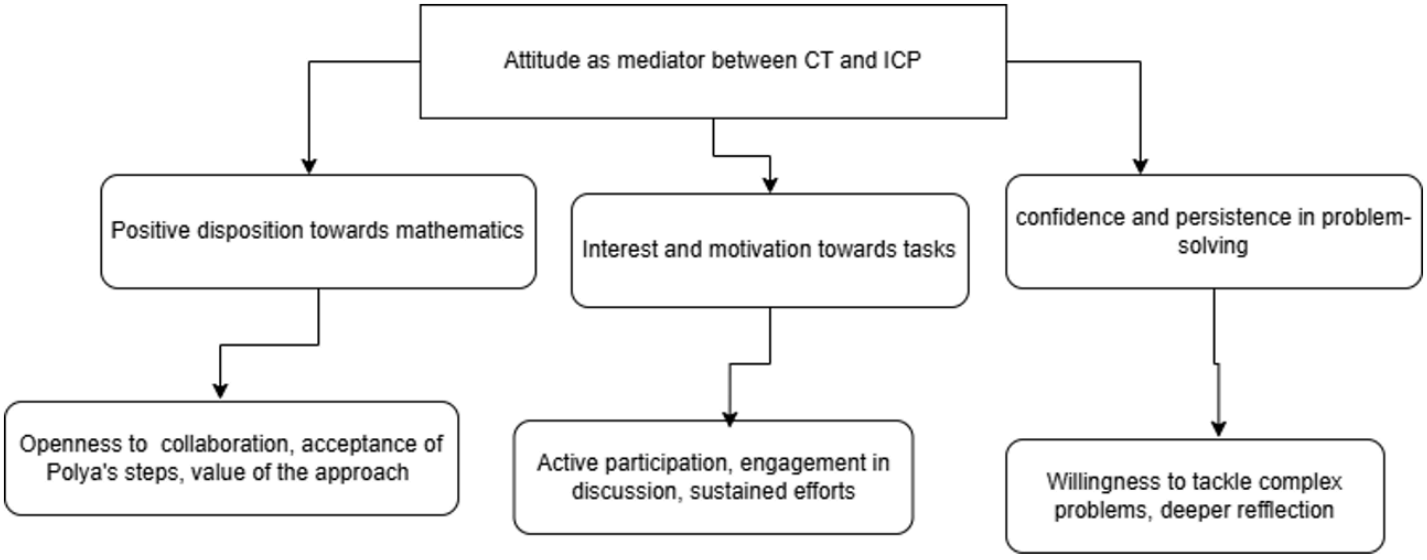


Figure 2

Preservice Teachers Attitude Mediates the Integrated Method and Critical Thinking.

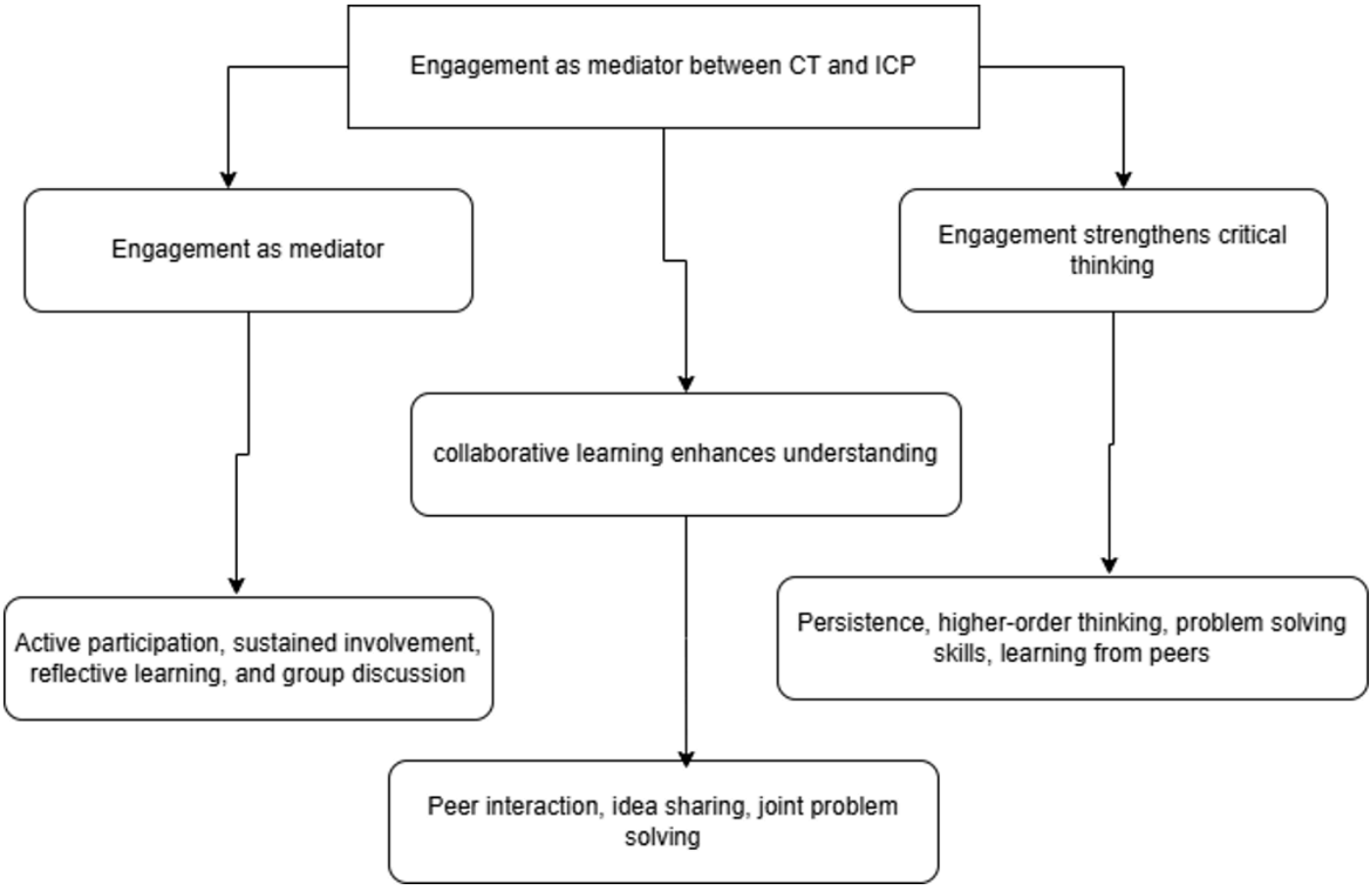


Figure 3

Direct, and Indirect Effects of Group on Critical Thinking via Student Engagement

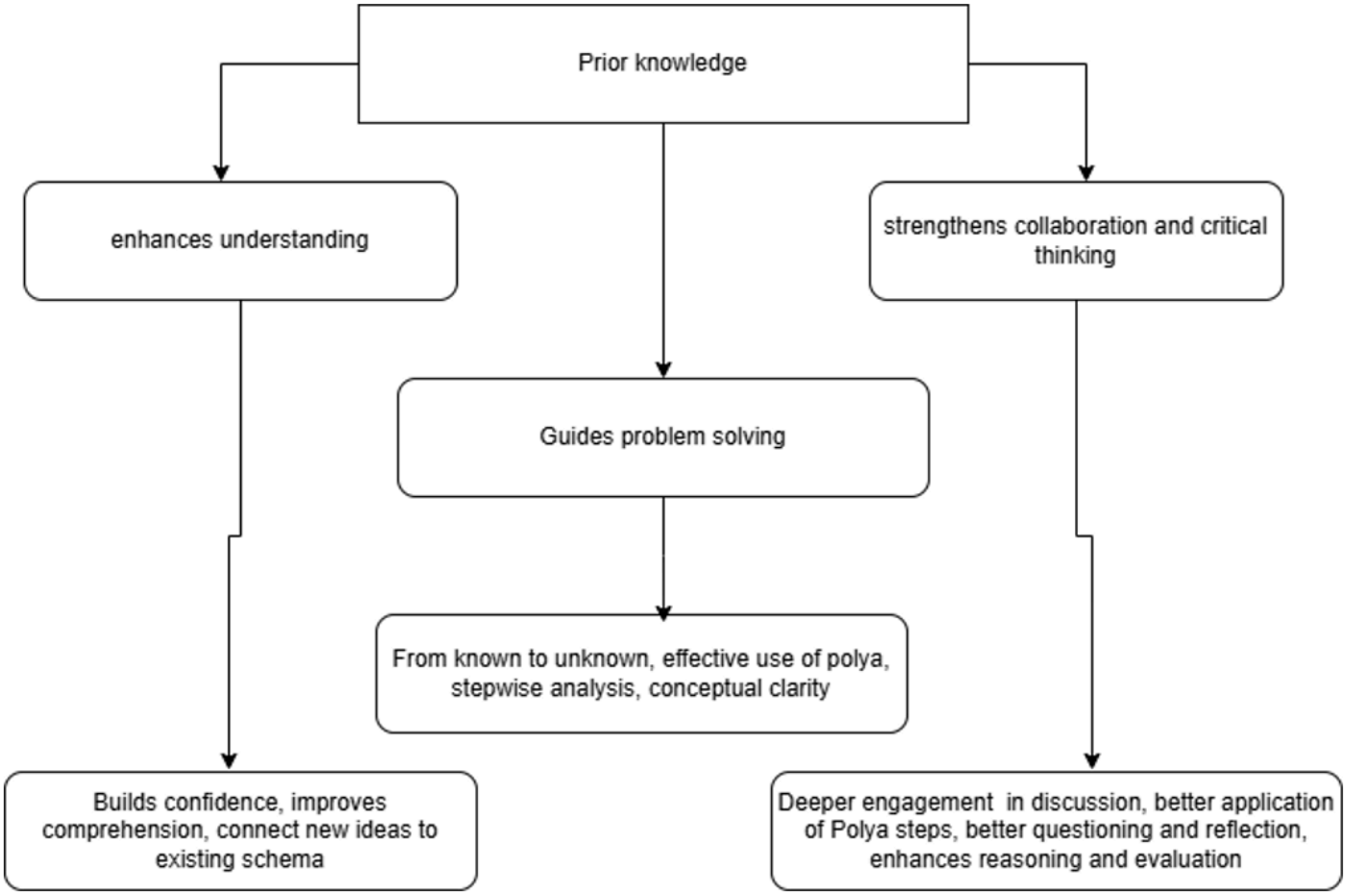


Figure 4

previous knowledge moderates the effect of the integrated method on critical thinking in problem solving

Source: Field Data (2025)

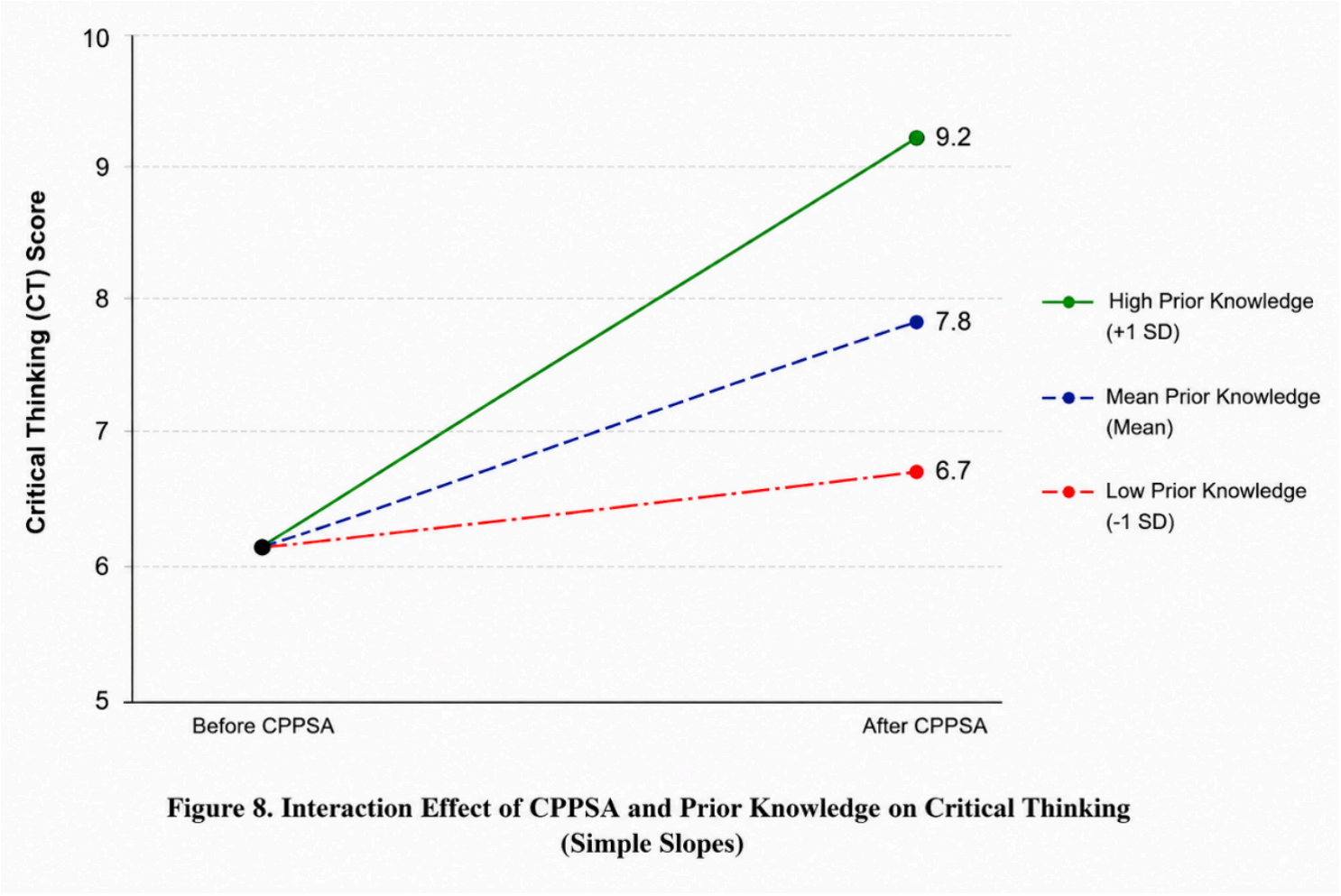


Figure 5

Source: Field Data (2025)