Theoretical Ecology: Supplementary Information to Modeling the Interactive Effects of Predation and Infection in Food-Subsidized Wildlife Populations

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S1. Parameterization

We applied our model to a well-studied host-pathogen system in North America, conjunctivitis caused by the bacterial pathogen *Mycoplasma gallisepticum* in House Finches (*Haemorhous mexicanus*). We utilized published field and laboratory experiments, and continent-wide citizen science studies, to parameterize our model (described below for each parameter).

Immigration (λ)

Utilizing the equilibrium equation $0 = \lambda – mN$, we set $N = 16$ based on literature value ranges of wintering finches in the eastern United States (Hosseini et al. 2004; Sydenstricker et al. 2006; Moyers et al. 2018; Birds of the World 2022). With $m = 0.03$ day^{-1} (see below), $\lambda = 0.48$ birds per day.

Emigration (m)

Jennelle (2007) estimated 0.15-0.3 as the range for weekly probabilities of House Finch emigration from a local study area. Thus, we calculated the daily probability of emigration from a single backyard using the equation $1-(1-x)^{\frac{1}{7}}$, where x was 0.15 and 0.3. This gave a range of 0.0229-0.0497 day^{-1}. Another study (Davis 2008) said that most birds captured in the fall were seen for up to 3 months, with birds staying in the yard for an average of 79.5 days. Using the equation $\frac{1}{\text{days}}$, we found the range of emigration values to be 0.0111-0.0126 day^{-1}. Therefore, the overall range is 0.0111-0.0497 day^{-1}. Solving for the midpoint value and rounding, the emigration rate is $m = 0.03$ day^{-1}.

Natural Mortality (μ)

Birds of the World (2022) gave an overwinter survival range of 39%-63%, while Hosseini et al. (2004) and Hurtado (2008) both used a 50% survival chance over a year. From these values, we used the following equation: $e^{-(\mu * \text{days})}$ = survival probability. We obtained 0.00190-0.00623 as the probability of natural mortality per day. Solving for the midpoint value and rounding, $\mu = 0.004$ day^{-1}.

Predation Rate (π)

Lindström (1989) examined finches in Sweden and found a ∼10% chance of predation per bird over 1.5 months. Using the equation in the paper to calculate an individual finch's chance of being predated by a hawk per day in a flock of size 10-100, we obtained $\pi = 0.002$ day^{-1}.

Increased Predation Risk (θ)

Birds of the World (2022) stated the high wild bird disease mortality was likely due to increased predation, and Kollias et al. (2004) described infected birds as lethargic with reduced activity. Additionally, Adelman et al. (2017) reported reduced anti-predator behaviors in infected House Finches. This information suggested $\theta > 1$. However, without more information, we decided to use $\theta = 1$ (no increased predation) and $\theta = 3$ (high increased predation).

Disease Mortality (v)

Fleming-Davies et al. (2018) lists the maximum disease mortality rate as 0.25 day^{-1}, which is our high virulence value. Their minimum disease induced mortality rate (used in numerical sensitivity analysis) was 0.01 day^{-1}, which is our low virulence value.

Transmission (Contact * Susceptibility = $\alpha\delta$)

Williams et al. (2014) reports transmission rates (measured in bird^{-1}day^{-1}) of 0.003 and 0.005 for the low virulence strain. We took the average value to find a transmission rate of 0.004 bird^{-1}day^{-1}. Likewise, the authors reported transmission rates of 0.014 and 0.034 for the high virulence strain. We took the average value to find a transmission rate of 0.024

bird^{-1}day^{-1}. To split transmission into contact and susceptibility, we set contact rates (α) equal to one contact per bird per day.

Recovery (γ)

Williams et al. (2014) reports recovery rates (measured in day^{-1}) of 0.061 and 0.111 for the low virulence strain. We took the average value to find $\gamma = 0.086$ day^{-1}. Likewise, the authors report transmission rates of 0.000 and 0.006 for the high virulence strain. We took the average value to find $\gamma = 0.003$ day^{-1}.

S2. Minimum Predation Rate (π_{min}) Calculation

S2.1. Model Equations

The following differential equations track the number of Susceptible (S) and Infected (I) hosts through time, as well as the cumulative number of hosts dying (C) from infection, predation, or natural causes. The parameters that can be influenced by food subsidy effects have a subscript "F".

$\frac{dS}{dt} = \lambda_F - m_F S - \mu_F S - \pi_F S - \alpha_F \delta_F SI + \gamma_F I$

$\frac{dI}{dt} = \alpha_F \delta_F SI - \mu_F I - \theta \pi_F I - v_F I - \gamma_F I$

$\frac{dC}{dt} = \mu_F S + \pi_F S + \mu_F I + \theta \pi_F I + v_F I$

S2.2. Pathogen Basic Reproduction Numbers

The pathogen basic reproduction number (R_0) represents the number of new infections caused by one infectious individual introduced to a wholly susceptible population (Anderson 1982). The expression for pathogen outbreak potential can be derived by setting $\frac{dS}{dt} = 0, I = 0$ and solving for S_{eq}, substituting S_{eq} into the following inequality ($\frac{dI}{dt} > 0$), and rearranging.

In the absence of predators, the expression for pathogen basic reproduction number is:

$R_{0np} = \frac{\lambda_F \alpha_F \delta_F}{(\mu_F + v_F + \gamma_F)(m_F + \mu_F)}$

In the presence of predators, the expression for pathogen basic reproduction number is:

$R_{0p} = \frac{\lambda_F \alpha_F \delta_F}{(\mu_F + v_F + \gamma_F + \theta \pi_F)(m_F + \mu_F + \pi_F)}$

S2.3. Minimum Predation Rates

An R_0 value below one indicates the pathogen is unable to spread and be sustained within a population. To understand under what conditions predators can prevent pathogen invasion, we set $R_{0p} = 1$ and solved for π (which we call the minimum predation rate or π_{min}) for each parameter altered by food subsidy. Then, for each parameter altered by food subsidy, we found the simplest equation that explained the qualitative dependence of π_{min} on food subsidy effect size (F). Note that π_{min} is never negative in our model. Instead, $\pi_{min} = 0$ when $R_{0np} < 1$ (i.e., food subsidy alone is sufficient to prevent pathogen invasion).

Immigration: $\lambda_F = F\lambda$

$$R_{0p} = 1 = \frac{F\lambda\alpha\delta}{(\mu+v+\gamma+\theta\pi_{min})(m+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+\gamma}{2\theta} + \frac{\mu+m}{2}) + \sqrt{(\frac{\mu+v+\gamma}{2\theta} - \frac{\mu+m}{2})^2 + \frac{F\lambda\alpha\delta}{\theta}}$$

At large food subsidy effect sizes, π_{min} behaves like $\sqrt{F}$.

Emigration: $m_F = \frac{m}{F}$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{(\mu+v+\gamma+\theta\pi_{min})(\frac{m}{F}+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+\gamma}{2\theta} + \frac{\mu+\frac{m}{F}}{2}) + \sqrt{(\frac{\mu+v+\gamma}{2\theta} - \frac{\mu+\frac{m}{F}}{2})^2 + \frac{\lambda\alpha\delta}{\theta}}$$

As $F \to \infty$, $\frac{1}{F^2} \to 0$ faster than $\frac{1}{F} \to 0$. Additionally, while it is true that π_{min} behaves like $-\frac{1}{F} + \sqrt{C_1 - \frac{C_2}{F}}$ (where C_1 and C_2 are constants) at large food subsidy effect sizes, we observe that $\sqrt{C_1 - \frac{C_2}{F}} \approx \sqrt{C_1}$ when F is large. Thus, we will say that π_{min} behaves like $-\frac{1}{F}$ for simplicity.

Natural Mortality: $\mu_F = \frac{\mu}{F}$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{(\frac{\mu}{F}+v+\gamma+\theta\pi_{min})(m+\frac{\mu}{F}+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\frac{\mu}{F}+v+\gamma}{2\theta} + \frac{\frac{\mu}{F}+m}{2}) + \sqrt{(\frac{\frac{\mu}{F}+v+\gamma}{2\theta} - \frac{\frac{\mu}{F}+m}{2})^2 + \frac{\lambda\alpha\delta}{\theta}}$$

Using a similar argument for π_{min} as we did when food subsidy decreased emigration, we will say that π_{min} behaves like $-\frac{1}{F}$ for simplicity.

Predation Success: $\pi_F = F\pi$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{(\mu+v+\gamma+F\theta\pi_{min})(m+\mu+F\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+\gamma}{2F\theta} + \frac{\mu+m}{2F}) + \sqrt{(\frac{\mu+v+\gamma}{2F\theta} - \frac{\mu+m}{2F})^2 + \frac{\lambda\alpha\delta}{F^2\theta}}$$

At large food subsidy effect sizes, π_{min} behaves like $\frac{1}{F}$.

Disease-Induced Mortality: $v_F = \frac{v}{F}$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{(\mu+\frac{v}{F}+\gamma+\theta\pi_{min})(m+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+\frac{v}{F}+\gamma}{2\theta} + \frac{\mu+m}{2}) + \sqrt{(\frac{\mu+\frac{v}{F}+\gamma}{2\theta} - \frac{\mu+m}{2})^2 + \frac{\lambda\alpha\delta}{\theta}}$$

Using a similar argument for π_{min} as we did when food subsidy decreased emigration, we will say that π_{min} behaves like $-\frac{1}{F}$ for simplicity.

Contact: $\alpha_F = F\alpha$

$$R_{0p} = 1 = \frac{F\lambda\alpha\delta}{(\mu+v+\gamma+\theta\pi_{min})(m+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+\gamma}{2\theta} + \frac{\mu+m}{2}) + \sqrt{(\frac{\mu+v+\gamma}{2\theta} - \frac{\mu+m}{2})^2 + \frac{F\lambda\alpha\delta}{\theta}}$$

At large food subsidy effect sizes, π_{min} behaves like $\sqrt{F}$.

Susceptibility: $\delta_F = \frac{\delta}{F}$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{F(\mu+v+\gamma+\theta\pi_{min})(m+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+\gamma}{2\theta} + \frac{\mu+m}{2}) + \sqrt{(\frac{\mu+v+\gamma}{2\theta} - \frac{\mu+m}{2})^2 + \frac{\lambda\alpha\delta}{F\theta}}$$

At large food subsidy effect sizes, π_{min} behaves like $\frac{1}{\sqrt{F}}$.

Recovery: $\gamma_F = F\gamma$

$$R_{0p} = 1 = \frac{\lambda\alpha\delta}{(\mu+v+F\gamma+\theta\pi_{min})(m+\mu+\pi_{min})}$$

$$\Rightarrow \pi_{min} = -(\frac{\mu+v+F\gamma}{2\theta} + \frac{\mu+m}{2}) + \sqrt{(\frac{\mu+v+F\gamma}{2\theta} - \frac{\mu+m}{2})^2 + \frac{\lambda\alpha\delta}{\theta}}$$

While it is true that π_{min} behaves like $-F + \sqrt{F^2 - F}$ at large food subsidy effect sizes, we observe that $\sqrt{F^2 - F} = \sqrt{F(F-1)} \approx \sqrt{F^2}$ at large F. Furthermore, we note that $\lim_{F\to\infty}(-F + \sqrt{F^2}) - \lim_{F\to\infty}(-F + \sqrt{F^2 - F}) = 0 - (-\frac{1}{2}) = \frac{1}{2}$, which means the behavior at large F differs by a constant. Thus, we will say that π_{min} behaves like $-F + \sqrt{F^2}$ (i.e., π_{min} has negative slope when F is negative and π_{min} is zero when F is nonnegative) for simplicity.

S3. Daily Death Difference (DDD) Calculation

S3.1. Model Equations

$\frac{dS}{dt} = \lambda_F - m_F S - \mu_F S - \pi_F S - \alpha_F \delta_F SI + \gamma_F I$

$\frac{dI}{dt} = \alpha_F \delta_F SI - \mu_F I - \theta \pi_F I - v_F I - \gamma_F I$

$\frac{dC}{dt} = \mu_F S + \pi_F S + \mu_F I + \theta \pi_F I + v_F I$

S3.2. Solving for Equilibrium

We set the equations modeling Susceptible (S) and Infected (I) hosts equal to zero (i.e, $\frac{dS}{dt} = 0, \frac{dI}{dt} = 0$) and solved for S and I at equilibrium (S_{eq}, I_{eq}) in the presence and absence of disease and predators.

Disease, Predators:

$0 = \lambda_F - m_F S_{eq} - \mu_F S_{eq} - \pi_F S_{eq} - \alpha_F \delta_F S_{eq} I_{eq} + \gamma_F I_{eq}$
$0 = \alpha_F \delta_F S_{eq} I_{eq} - \mu_F I_{eq} - \theta \pi_F I_{eq} - v_F I_{eq} - \gamma_F I_{eq}$

Thus,

$\alpha_F \delta_F S_{eq} I_{eq} = \mu_F I_{eq} + \theta \pi_F I_{eq} + v_F I_{eq} + \gamma_F I_{eq}$
$\Rightarrow \alpha_F \delta_F S_{eq} = \mu_F + \theta \pi_F + v_F + \gamma_F$
$\Rightarrow S_{eq} = \frac{\mu_F + \theta \pi_F + v_F + \gamma_F}{\alpha_F \delta_F}$

$(\alpha_F \delta_F S_{eq} - \gamma_F) I_{eq} = \lambda_F - (m_F + \mu_F + \pi_F) S_{eq}$
$\Rightarrow (\alpha_F \delta_F \frac{\mu_F + \theta \pi_F + v_F + \gamma_F}{\alpha_F \delta_F} - \gamma_F) I_{eq} = \lambda_F - (m_F + \mu_F + \pi_F) \frac{\mu_F + \theta \pi_F + v_F + \gamma_F}{\alpha_F \delta_F}$
$\Rightarrow (\mu_F + \theta \pi_F + v_F) I_{eq} = \lambda_F - (m_F + \mu_F + \pi_F) \frac{\mu_F + \theta \pi_F + v_F + \gamma_F}{\alpha_F \delta_F}$
$\Rightarrow I_{eq} = \frac{\lambda_F}{(\mu_F + \theta \pi_F + v_F)} - \frac{(m_F + \mu_F + \pi_F)(\mu_F + \theta \pi_F + v_F + \gamma_F)}{\alpha_F \delta_F (\mu_F + \theta \pi_F + v_F)}$

Disease, No Predators:

$\pi_F \equiv 0$

Thus,

$S_{eq} = \frac{\mu_F + v_F + \gamma_F}{\alpha_F \delta_F}$
$I_{eq} = \frac{\lambda_F}{(\mu_F + v_F)} - \frac{(m_F + \mu_F)(\mu_F + v_F + \gamma_F)}{\alpha_F \delta_F (\mu_F + v_F)}$

No Disease, Predators:

$I_{eq} = 0$
$0 = \lambda_F - m_F S_{eq} - \mu_F S_{eq} - \pi_F S_{eq}$

Thus,

$(m_F + \mu_F + \pi_F)S_{eq} = \lambda_F$
$\Rightarrow S_{eq} = \frac{\lambda_F}{m_F+\mu_F+\pi_F}$

$\frac{dC}{dt} = \mu_F S_{eq} + \pi_F S_{eq} + \mu_F I_{eq} + \theta\pi_F I_{eq} + v_F I_{eq}$

No Disease, No Predators:

$\pi_F \equiv 0$

Thus,

$S_{eq} = \frac{\lambda_F}{m_F+\mu_F}$
$I_{eq} = 0$

S3.3. Host Deaths Per Day

We substituted the equilibrium values of Susceptible (S_{eq}) and Infected (I_{eq}) hosts (found above) into the equation for daily host deaths ($\frac{dC}{dt}$) to find the number of host deaths per day at equilibrium in the presence and absence of disease and predators.

$\frac{dC}{dt} = \mu_F S_{eq} + \pi_F S_{eq} + \mu_F I_{eq} + \theta\pi_F I_{eq} + v_F I_{eq}$

Disease, Predators:

$S_{eq} = \frac{\mu_F+\theta\pi_F+v_F+\gamma_F}{\alpha_F\delta_F}$
$I_{eq} = \frac{\lambda_F}{(\mu_F+\theta\pi_F+v_F)} - \frac{(m_F+\mu_F+\pi_F)(\mu_F+\theta\pi_F+v_F+\gamma_F)}{\alpha_F\delta_F(\mu_F+\theta\pi_F+v_F)}$

Thus,

$\frac{dC}{dt} = (\mu_F + \pi_F)\frac{\mu_F+\theta\pi_F+v_F+\gamma_F}{\alpha_F\delta_F} + (\mu_F + \theta\pi_F + v_F)(\frac{\lambda_F}{(\mu_F+\theta\pi_F+v_F)} - \frac{(m_F+\mu_F+\pi_F)(\mu_F+\theta\pi_F+v_F+\gamma_F)}{\alpha_F\delta_F(\mu_F+\theta\pi_F+v_F)})$
$\Rightarrow \frac{dC}{dt} = \frac{(\mu_F+\pi_F)(\mu_F+\theta\pi_F+v_F+\gamma_F)}{\alpha_F\delta_F} + \lambda_F - \frac{(m_F+\mu_F+\pi_F)(\mu_F+\theta\pi_F+v_F+\gamma_F)}{\alpha_F\delta_F}$
$\Rightarrow \frac{dC}{dt} = \lambda_F - \frac{m_F(\mu_F+\theta\pi_F+v_F+\gamma_F)}{\alpha_F\delta_F}$

Disease, No Predators:

$\pi_F \equiv 0$
$S_{eq} = \frac{\mu_F+v_F+\gamma_F}{\alpha_F\delta_F}$
$I_{eq} = \frac{\lambda_F}{(\mu_F+v_F)} - \frac{(m_F+\mu_F)(\mu_F+v_F+\gamma_F)}{\alpha_F\delta_F(\mu_F+v_F)}$

Thus,

$\frac{dC}{dt} = \mu_F\frac{\mu_F+v_F+\gamma_F}{\alpha_F\delta_F} + (\mu_F + v_F)(\frac{\lambda_F}{(\mu_F+v_F)} - \frac{(m_F+\mu_F)(\mu_F+v_F+\gamma_F)}{\alpha_F\delta_F(\mu_F+v_F)})$
$\Rightarrow \frac{dC}{dt} = \frac{\mu_F(\mu_F+v_F+\gamma_F)}{\alpha_F\delta_F} + \lambda_F - \frac{(m_F+\mu_F)(\mu_F+v_F+\gamma_F)}{\alpha_F\delta_F})$

$\Rightarrow \frac{dC}{dt} = \lambda_F - \frac{m_F(\mu_F+v_F+\gamma_F)}{\alpha_F\delta_F}$

No Disease, Predators:

$S_{eq} = \frac{\lambda_F}{m_F+\mu_F+\pi_F}$
$I_{eq} = 0$

Thus,

$\frac{dC}{dt} = \frac{\lambda_F(\mu_F+\pi_F)}{m_F+\mu_F+\pi_F}$

No Disease, No Predators:

$\pi_F \equiv 0$
$S_{eq} = \frac{\lambda_F}{m_F+\mu_F}$
$I_{eq} = 0$

Thus,

$\frac{dC}{dt} = \frac{\lambda_F\mu_F}{m_F+\mu_F}$

S3.4. Daily Death Difference (DDD) Calculation

To quantify the effect predators have on cumulative host mortality, we solved for the daily death difference:

DDD = $\frac{dC}{dt}$ at equilibrium with predators - $\frac{dC}{dt}$ at equilibrium without predators

Where positive values indicate host deaths per day increase in the presence of predators and negative values indicate host deaths per day decrease in the presence of predators.

Immigration: $\lambda_F = F\lambda$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} increase to infinity.
λ_F**:** $R_{0p} < R_{0np} < 1$

DDD = $(\frac{F\lambda(\mu+\pi)}{m+\mu+\pi}) - (\frac{F\lambda\mu}{m+\mu})$
$\Rightarrow$ DDD = $\frac{F\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$

Thus, DDD is always positive and increases linearly (slope = $\frac{\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$) as food subsidy effect size (F) increases.

λ_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{F\lambda(\mu+\pi)}{m+\mu+\pi}) - (F\lambda - \frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = \frac{m(\mu+v+\gamma)}{\alpha\delta} - \frac{F\lambda m}{m+\mu+\pi}$

Thus, DDD always decreases linearly (slope $= -\frac{\lambda m}{m+\mu+\pi}$) as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{(\mu+v+\gamma)(m+\mu+\pi)}{\lambda\alpha\delta}$.

λ_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (F\lambda - \frac{m(\mu+\theta\pi+v+\gamma)}{\alpha\delta}) - (F\lambda - \frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and does not change with food subsidy effect size (F).

Emigration: $m_F = \frac{m}{F}$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} increase and saturate at parameter-dependent values that may or may not be greater than 1.

$R_{0p} \to \frac{\alpha\delta\lambda}{(\mu+\pi)(\mu+v+\gamma+\theta\pi)}$ and $R_{0np} \to \frac{\alpha\delta\lambda}{\mu(\mu+v+\gamma)}$ as $F \to \infty$

m_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{\frac{m}{F}+\mu+\pi}) - (\frac{\lambda\mu}{\frac{m}{F}+\mu})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(\frac{m}{F}+\mu+\pi)(m+F\mu)}$

$\frac{DDD}{dF} = \lambda m(\frac{\mu+\pi}{(m+F\mu+F\pi)^2} - \frac{\mu}{(m+F\mu)^2})$
$\Rightarrow \frac{DDD}{dF} = 0$ when $F = \frac{m}{(\mu^2+\mu\pi)^{\frac{1}{2}}}$

Thus, DDD is always positive and tends to zero as food subsidy effect size (F) increases. If $F = \frac{m}{(\mu^2+\mu\pi)^{\frac{1}{2}}} > 1$, then DDD will reach a maximum at $\text{DDD} = \frac{\lambda\pi(\mu^2+\mu\pi)^{\frac{1}{2}}}{((\mu^2+\mu\pi)^{\frac{1}{2}}+\mu+\pi)((\mu^2+\mu\pi)^{\frac{1}{2}}+\mu)}$.

m_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{\frac{m}{F}+\mu+\pi}) - (\lambda - \frac{\frac{m}{F}(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = m(\frac{\mu+v+\gamma}{F\alpha\delta} - \frac{\lambda}{m+F\mu+F\pi})$

$\frac{DDD}{dF} = m(\frac{\lambda(\mu+\pi)}{(m+F\mu+F\pi)^2} - \frac{\mu+v+\gamma}{F^2\alpha\delta})$
$\Rightarrow \frac{DDD}{dF} = 0$ when $F = \frac{m((\mu+\pi)(\mu+v+\gamma)+(\alpha\delta\lambda(\mu+\pi)(\mu+v+\gamma))^{\frac{1}{2}})}{(\mu+\pi)(\alpha\delta\lambda-(\mu+\pi)(\mu+v+\gamma))}$

Thus, DDD tends to zero as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{m(\mu+v+\gamma)}{\alpha\delta\lambda-(\mu+\pi)(\mu+v+\gamma)}$. If $F = \frac{m((\mu+\pi)(\mu+v+\gamma)+(\alpha\delta\lambda(\mu+\pi)(\mu+v+\gamma))^{\frac{1}{2}})}{(\mu+\pi)(\alpha\delta\lambda-(\mu+\pi)(\mu+v+\gamma))} > 1$, then DDD will reach a minimum. Note that F when DDD reaches a minimum is larger than F when DDD

passes through zero.

m_F: $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{\frac{m}{F}(\mu+\theta\pi+v+\gamma)}{\alpha\delta}) - (\lambda - \frac{\frac{m}{F}(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{F\alpha\delta}$

Thus, DDD is always negative and tends to zero as food subsidy effect size (F) increases.

Natural Mortality: $\mu_F = \frac{\mu}{F}$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} increase and saturate at parameter-dependent values that may or may not be greater than 1.

$R_{0p} \to \frac{\alpha\delta\lambda}{(m+\pi)(v+\gamma+\theta\pi)}$ and $R_{0np} \to \frac{\alpha\delta\lambda}{m(v+\gamma)}$ as $F \to \infty$

μ_F: $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\frac{\mu}{F}+\pi)}{m+\frac{\mu}{F}+\pi}) - (\frac{\lambda\frac{\mu}{F}}{m+\frac{\mu}{F}})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(m+\frac{\mu}{F}+\pi)(m+\frac{\mu}{F})}$

$\frac{DDD}{dF} = \lambda\mu m(\frac{1}{(Fm+\mu)^2} - \frac{1}{(\mu+Fm+F\pi)^2}) > 0$

Thus, DDD is always positive and increases towards $\text{DDD} = \frac{\lambda\pi}{m+\pi}$ as food subsidy effect size (F) increases.

μ_F: $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\frac{\mu}{F}+\pi)}{m+\frac{\mu}{F}+\pi}) - (\lambda - \frac{m(\frac{\mu}{F}+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = m(\frac{\mu+Fv+F\gamma}{F\alpha\delta} - \frac{F\lambda}{Fm+\mu+F\pi})$

$\frac{DDD}{dF} = -\mu m(\frac{1}{(F^2\alpha\delta)} + \frac{\lambda}{(\mu+Fm+F\pi)^2}) < 0$

Thus, DDD decreases towards $\text{DDD} = \frac{m(v+\gamma)}{\alpha\delta} - \frac{\lambda m}{m+\pi}$ as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{\mu((m+v+\gamma+\pi)+((m+v+\gamma+\pi)^2+4(\alpha\delta\lambda-(m+\pi)(v+\gamma)))^{\frac{1}{2}})}{2(\alpha\delta\lambda-(m+\pi)(v+\gamma))}$.

μ_F: $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{m(\frac{\mu}{F}+\theta\pi+v+\gamma)}{\alpha\delta}) - (\lambda - \frac{m(\frac{\mu}{F}+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and does not change with food subsidy effect size (F).

Predation Success: $\pi_F = F\pi$

As food subsidy effect size (F) increases, R_{0np} remains unchanged while R_{0p} decreases towards zero.

π_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+F\pi)}{m+\mu+F\pi}) - (\frac{\lambda\mu}{m+\mu})$
$\Rightarrow \text{DDD} = \frac{F\lambda\pi m}{(m+\mu+F\pi)(m+\mu)}$

$\frac{DDD}{dF} = \frac{\lambda\pi m}{(m+\mu+F\pi)^2} > 0$

Thus, DDD is always positive and increases towards $\text{DDD} = \frac{\lambda m}{m+\mu}$ as food subsidy effect size (F) increases.

π_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+F\pi)}{m+\mu+F\pi}) - (\lambda - \frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = m(\frac{\mu+v+\gamma}{\alpha\delta} - \frac{\lambda}{m+\mu+F\pi})$

$\frac{DDD}{dF} = \frac{\lambda\pi m}{(m+\mu+F\pi)^2} > 0$

Thus, DDD increases towards $\text{DDD} = \frac{m(\mu+v+\gamma)}{\alpha\delta}$ as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{\lambda\alpha\delta-(m+\mu)(\mu+v+\gamma)}{\pi(\mu+v+\gamma)}$.

π_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{m(\mu+F\theta\pi+v+\gamma)}{\alpha\delta}) - (\lambda - \frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -F\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and decreases linearly (slope $= -\frac{m\theta\pi}{\alpha\delta}$) with food subsidy effect size (F).

Disease-Induced Mortality: $v_F = \frac{v}{F}$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} increase and saturate at parameter-dependent values that may or may not be greater than 1.

$R_{0p} \to \frac{\alpha\delta\lambda}{(m+\mu+\pi)(\mu+\gamma+\theta\pi)}$ and $R_{0np} \to \frac{\alpha\delta\lambda}{(m+\mu)(\mu+\gamma)}$ as $F \to \infty$

v_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\frac{\lambda\mu}{m+\mu})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$

Thus, DDD is always positive and does not change with food subsidy effect size (F).

v_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\lambda - \frac{m(\mu+\frac{v}{F}+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = m(\frac{\mu+\frac{v}{F}+\gamma}{\alpha\delta} - \frac{\lambda}{m+\mu+\pi})$

$\frac{DDD}{dF} = -\frac{mv}{F^2\alpha\delta} < 0$

Thus, DDD decreases towards $\text{DDD} = m(\frac{\mu+\gamma}{\alpha\delta} - \frac{\lambda}{m+\mu+\pi})$ as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{v(m+\mu+\pi)}{\lambda\alpha\delta-(\mu+\gamma)(m+\mu+\pi)}$.

v_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{m(\mu+\theta\pi+\frac{v}{F}+\gamma)}{\alpha\delta}) - (\lambda - \frac{m(\mu+\frac{v}{F}+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and does not change with food subsidy effect size (F).

Contact: $\alpha_F = F\alpha$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} increase to infinity.
α_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\frac{\lambda\mu}{m+\mu})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$

DDD is always positive and does not change with food subsidy effect size (F).

α_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\lambda - \frac{m(\mu+v+\gamma)}{F\alpha\delta})$
$\Rightarrow \text{DDD} = \frac{m(\mu+v+\gamma)}{F\alpha\delta} - \frac{\lambda m}{m+\mu+\pi}$

$\frac{DDD}{dF} = -\frac{m(\mu+v+\gamma)}{F^2\alpha\delta} < 0$

Thus, DDD decreases towards $\text{DDD} = -\frac{\lambda m}{m+\mu+\pi}$ as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{(\mu+v+\gamma)(m+\mu+\pi)}{\lambda\alpha\delta}$.

α_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{m(\mu+\theta\pi+v+\gamma)}{F\alpha\delta}) - (\lambda - \frac{m(\mu+v+\gamma)}{F\alpha\delta})$

$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{F\alpha\delta}$

$\frac{DDD}{dF} = \frac{m\theta\pi}{F^2\alpha\delta} > 0$

Thus, DDD is always negative and increases towards zero with food subsidy effect size (F).

Susceptibility: $\delta_F = \frac{\delta}{F}$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} decrease to zero.
δ_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\frac{\lambda\mu}{m+\mu})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$

Thus, DDD is always positive and does not change with food subsidy effect size (F).

δ_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\lambda - F\frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = F\frac{m(\mu+v+\gamma)}{\alpha\delta} - \frac{\lambda m}{m+\mu+\pi}$

Thus, DDD increases linearly (slope = $\frac{m(\mu+v+\gamma)}{\alpha\delta}$) as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{\lambda\alpha\delta}{(\mu+v+\gamma)(m+\mu+\pi)}$.

δ_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - F\frac{m(\mu+\theta\pi+v+\gamma)}{\alpha\delta}) - (\lambda - F\frac{m(\mu+v+\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -F\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and decreases linearly (slope = $-\frac{m\theta\pi}{\alpha\delta}$) with food subsidy effect size (F).

Recovery: $\gamma_F = F\gamma$

As food subsidy effect size (F) increases, both R_{0p} and R_{0np} decrease toward zero.
γ_F**:** $R_{0p} < R_{0np} < 1$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\frac{\lambda\mu}{m+\mu})$
$\Rightarrow \text{DDD} = \frac{\lambda\pi m}{(m+\mu+\pi)(m+\mu)}$

Thus, DDD is always positive and does not change with food subsidy effect size (F).

γ_F**:** $R_{0p} < 1 < R_{0np}$

$\text{DDD} = (\frac{\lambda(\mu+\pi)}{m+\mu+\pi}) - (\lambda - \frac{m(\mu+v+F\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = \frac{m(\mu+v+F\gamma)}{\alpha\delta} - \frac{\lambda m}{m+\mu+\pi}$

Thus, DDD increases linearly (slope = $\frac{m\gamma}{\alpha\delta}$) as food subsidy effect size (F) increases. DDD passes through zero when $F = \frac{\lambda\alpha\delta-(\mu+v)(m+\mu+\pi)}{\gamma(m+\mu+\pi)}$.

γ_F**:** $1 < R_{0p} < R_{0np}$

$\text{DDD} = (\lambda - \frac{m(\mu+\theta\pi+v+F\gamma)}{\alpha\delta}) - (\lambda - \frac{m(\mu+v+F\gamma)}{\alpha\delta})$
$\Rightarrow \text{DDD} = -\frac{m\theta\pi}{\alpha\delta}$

Thus, DDD is always negative and does not change with food subsidy effect size (F).