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Mapping Students' Learning Difficulties in Geometric Proof Writing: A Three-Domain Analysis of Conceptual Understanding, Logical Reasoning, and Deductive Structure

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Abstract

This study examined the levels of understanding and learning difficulties of Grade 9 students in geometric proof-writing based on their written proof solutions. Specifically, the study focused on conceptual understanding, logical reasoning, and the structure of deductive proof. A descriptive qualitative-supported research design complemented by quantitative descriptive analysis was employed involving 47 Grade 9 students from Sarah Jane Ferrer High School during the School Year 2025–2026. Data were gathered using a researcher-developed diagnostic test and constructed-response proof-writing assessment validated by mathematics education experts. Students' written proofs were analyzed using analytic rubrics and thematic content analysis. Findings revealed that most students were classified under the Beginning level across all domains. Students' written proofs commonly reflected fragmented conceptual understanding, weak logical sequencing, unsupported justifications, and incomplete deductive structures. The study further revealed that difficulties in conceptual understanding contributed to fragmented logical reasoning and weak proof organization. Overall, the findings highlight the need for scaffolded and reasoning-oriented instructional approaches to strengthen students' geometric proof-writing competence.

Keywords. geometric proof-writing; conceptual understanding; logical reasoning; deductive proof structure; mathematics education

Introduction

Geometry plays an important role in mathematics education because it develops students' reasoning, justification, and deductive thinking abilities. Among the various competencies in geometry, proof-writing is considered one of the most cognitively demanding tasks because it requires learners to integrate conceptual understanding, logical reasoning, and the formal organization of deductive arguments. Writing geometric proofs involves more than obtaining correct answers; it requires students to justify mathematical statements systematically using definitions, postulates, properties, and theorems.

Despite its importance, many students continue to experience difficulties in geometric proof-writing. Previous studies have shown that learners commonly struggle with conceptual interpretation, logical sequencing, and the organization of deductive structures during proof construction. According to van Hiele and van Hiele-Geldof, many students remain at lower levels of geometric thinking where they recognize figures visually but experience difficulty engaging in formal deductive reasoning. Similarly, Harel and Sowder (1998) explained that students' proof schemes often remain empirical or procedural rather than deductive, indicating limited conceptual processing of geometric relationships. Stylianides (2007) further emphasized that proof-writing requires coherent logical reasoning, valid justification, and deductive continuity.

Several studies have identified recurring difficulties in geometry proof-writing. Miyazaki et al. (2016) reported that students frequently experience difficulties in recognizing proof structures, organizing deductive sequences, and aligning statements with appropriate justifications. Likewise,

Del Mundo (2018) observed that students struggle to maintain coherent deductive progression in proof construction, while Perez and Ramirez (2020) noted that many learners rely on intuition and procedural manipulation rather than formal deductive reasoning. In the Philippine context, students' difficulties in proof-writing are further associated with weak prerequisite knowledge, fragmented conceptual understanding, and limited exposure to structured proof-writing activities (Martinez, 2021).

Although several studies have examined instructional approaches in geometry, fewer studies have focused on students' actual written proof solutions as direct evidence of understanding and learning difficulties. The analysis of written proofs provides important insights into how students interpret geometric concepts, organize deductive reasoning, and construct formal proof structures. Through written proof analysis, recurring conceptual, logical, and structural difficulties may be identified systematically based on observable student outputs.

Hence, the present study examined the levels of understanding and learning difficulties of Grade 9 students in geometric proof-writing based on their written proof solutions. Specifically, the study focused on conceptual understanding, logical reasoning, and the structure of deductive proof in order to identify recurring patterns of difficulties experienced by students during proof construction.

Objective

This study aimed to examine the levels of understanding and learning difficulties of Grade 9 students in writing basic geometric proofs based on their written proof solutions in terms of conceptual understanding, logical reasoning, and the structure of deductive proof.

Specifically, the study aimed to:

1. determine the levels of understanding of Grade 9 students in writing basic geometric proofs in terms of:
    a. conceptual understanding,
    b. logical reasoning, and
    c. the structure of deductive proof; and
2. identify the learning difficulties experienced by students in writing basic geometric proofs as reflected in their written solutions to selected proof-based problem-solving tasks.

Framework

This study is anchored on three interrelated domains essential in geometric proof-writing: conceptual understanding, logical reasoning, and the structure of deductive proof. These domains served as the basis for examining the levels of understanding and learning difficulties of Grade 9 students as reflected in their written proof solutions. The framework assumes that successful proof-writing requires the integration of conceptual knowledge, coherent deductive reasoning, and properly organized proof structures.

Conceptual understanding is grounded in the Proof Schemes Framework of Harel and Sowder (1998), which explains how students justify mathematical statements through empirical, procedural, or deductive reasoning. According to the framework, students who rely primarily on memorized procedures or visual observations often experience difficulty constructing formally justified proofs. In the present study, this framework provided the basis for analyzing how students recognized, interpreted, and connected geometric concepts within proof-writing tasks.

Logical reasoning is anchored on the framework of proof and proving proposed by Stylianides (2007), which emphasizes deductive validity, logical sequencing, coherence of arguments, and proper mathematical justification in proof construction. In this study, logical reasoning refers to students' ability to organize deductive relationships coherently, justify proof steps appropriately, and establish logically valid conclusions within their written proofs.

The structure of deductive proof is grounded in the Theory of Geometric Thinking developed by van Hiele and van Hiele-Geldof (1986). The theory explains that students progress through hierarchical levels of geometric thinking, beginning from visual recognition and advancing toward formal deduction. Learners who remain at lower levels often experience difficulty organizing statements, reasons, and deductive relationships into coherent proof structures. This framework guided the analysis of students' ability to construct organized and complete deductive proofs.

Collectively, these theoretical perspectives explain the cognitive demands involved in geometry proof-writing. Conceptual understanding supports the interpretation of geometric relationships, logical reasoning sustains deductive validity and coherence, and proof structure organizes statements and justifications into complete mathematical arguments. Together, these domains provided the theoretical basis for analyzing students' written proof solutions and identifying recurring learning difficulties in geometric proof-writing.

Methodology

Research Design

This study employed a descriptive research design supported by qualitative analysis to examine the levels of understanding and learning difficulties of Grade 9 students in geometric proof-writing. The study focused on analyzing students' written proof solutions in terms of conceptual understanding, logical reasoning, and the structure of deductive proof. The design was appropriate because the study aimed to describe and interpret students' proof-writing performance and identify recurring conceptual, logical, and structural difficulties reflected in their written outputs rather than evaluate the effectiveness of an instructional intervention (Creswell & Creswell, 2018).

Quantitative descriptive analysis was utilized to determine students' levels of understanding across the three domains using rubric-based scores derived from their written proofs. Frequency counts, percentages, and mean scores were computed to classify students' performance levels. Qualitative analysis, meanwhile, involved the examination and annotation of students' written proof responses to identify recurring misconceptions, logical breakdowns, structural inconsistencies, and proof-writing difficulties (Fraenkel et al., 2019).

Participants

The participants of the study consisted of 47 Grade 9 students enrolled at Sarah Jane Ferrer High School during the School Year 2025–2026. The participants were purposively selected because they had prior exposure to foundational Geometry concepts and were currently engaged in lessons involving deductive reasoning and proof-writing activities. The study focused on foundational proof-writing competencies, including conditional reasoning, valid and invalid arguments, properties of equality, and basic geometric postulates.

Instrument

Two researcher-developed instruments were utilized in the study: a diagnostic test and a proof-writing assessment composed of constructed-response geometric proof tasks. The diagnostic test identified students' prerequisite gaps related to conditional reasoning, valid and invalid arguments, properties of equality, and geometric postulates. The proof-writing assessment served as the primary source of data and consisted of four proof tasks requiring students to construct deductive proofs using appropriate statements, reasons, definitions, postulates, and theorems. Performance-based proof tasks are appropriate for assessing higher-order mathematical reasoning because they allow students to demonstrate conceptual understanding, deductive reasoning, and structural organization within authentic mathematical contexts (Brookhart, 2013).

Both instruments underwent expert validation and obtained excellent content validity results, with Item-Level Content Validity Index (I-CVI) values ranging from 0.88 to 1.00 and an overall Scale-Level Content Validity Index (S-CVI/Ave) of 0.97 (Polit & Beck, 2006). The diagnostic test also demonstrated excellent reliability, obtaining a KR-20 coefficient of 0.92 during pilot testing and 0.95 during confirmatory testing.

Three analytic scoring rubrics were utilized to evaluate students' written proofs: the Conceptual Understanding Rubric, the Logical Reasoning Rubric, and the Structure of Deductive Proof Rubric. These rubrics were grounded on the frameworks of Harel and Sowder (1998), Stylianides (2007), and van Hiele and van Hiele-Geldof (1986), respectively. Each rubric used a four-level analytic scale: Beginning (0), Developing (1), Proficient (2), and Excellent (3).

Data Gathering Procedure

Prior to data collection, permission to conduct the study was secured from the school administration. The researcher then coordinated with the mathematics teachers regarding the schedule and administration of the assessment. The proof-writing assessment was administered during regular class sessions using standardized instructions to ensure consistency. Students were given sufficient time to analyze the geometric situations and construct complete written proofs independently.

After administration, students' written proof solutions were collected, organized, and evaluated using the analytic rubrics. The responses were repeatedly examined across the domains of conceptual understanding, logical reasoning, and proof structure. Annotation and content analysis were likewise conducted to identify recurring misconceptions, fragmented reasoning patterns,

incomplete justifications, invalid deductive transitions, and structural inconsistencies reflected in students' written proofs (Fraenkel et al., 2019).

Data Analysis

Quantitative data derived from the rubric scores were analyzed using descriptive statistics, including frequency counts, percentages, and mean scores. Mean scores were interpreted using the following rubric-based scale: 0.00–0.49 (Beginning), 0.50–1.49 (Developing), 1.50–2.49 (Proficient), and 2.50–3.00 (Excellent). This criterion-referenced interpretation allowed the classification of students' proof-writing performance according to predefined standards (Popham, 2017).

Qualitative data from students' written proof solutions were analyzed through thematic and content analysis. The analysis focused on recurring learning difficulties related to conceptual interpretation, logical sequencing, deductive justification, misuse of geometric properties, fragmented proof structures, and inconsistencies in aligning statements and reasons. The integration of quantitative and qualitative findings provided a comprehensive analysis of students' levels of understanding and learning difficulties in geometric proof-writing.

Result and Discussion

Levels of Understanding of Grade 9 Students in Writing Basic Geometric Proofs

This section presents the levels of understanding of Grade 9 students in writing basic geometric proofs based on their written proof solutions. The analysis focused on three domains essential in proof-writing: conceptual understanding, logical reasoning, and the structure of deductive proof. Students' responses were evaluated using analytic rubrics, and the findings were interpreted using descriptive statistics and qualitative examination of students' written proofs.

Conceptual Understanding in Writing Basic Geometric Proofs

Conceptual understanding refers to students' ability to recognize, explain, represent, and connect geometric concepts within deductive proof-writing. Guided by the Proof Schemes Framework of Harel and Sowder (1998), this domain was examined through conceptual recognition, conceptual explanation, mathematical representation, and conceptual linkage.

Table 1 and figure 1 show that both the groups demonstrated generally low levels of conceptual understanding during the assessment. Most students were classified under the Beginning level across all indicators. In the group A, 59% of the students were classified under the Beginning level in conceptual recognition, 68% in conceptual explanation and mathematical representation, and 73% in conceptual linkage. Similarly, in the group B, 56% of the students were classified under the Beginning level in conceptual recognition, 64% in conceptual explanation and conceptual linkage, and 68% in mathematical representation. Both groups obtained a class average score of 0.34, interpreted as Beginning level.

Table 1. Levels of Conceptual Understanding of Grade 9 Students in Writing Basic Geometric Proofs

Indicator	Maximum Possible Score (per indicator)	Control Group (Pre-test) n=22			Experimental Group (Pre-test) n=25		
		Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation	Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation
Conceptual Recognition	3.00	0.38	0.39	Beginning	0.38	0.38	Beginning
Conceptual Explanation	3.00	0.32	0.36	Beginning	0.31	0.36	Beginning
Mathematical Representation	3.00	0.34	0.40	Beginning	0.35	0.40	Beginning
Conceptual Linkage	3.00	0.32	0.40	Beginning	0.33	0.39	Beginning
Overall (Average)	**3.00**	**0.34**	**0.39**	**Beginning**	**0.34**	**0.38**	**Beginning**

Descriptive Interpretation.. *Excellent Level [2.50-3.00]; Proficient Level [1.50-2.49]; Developing Level [0.50-1.49]; Beginning Level [0.00-0.49].*

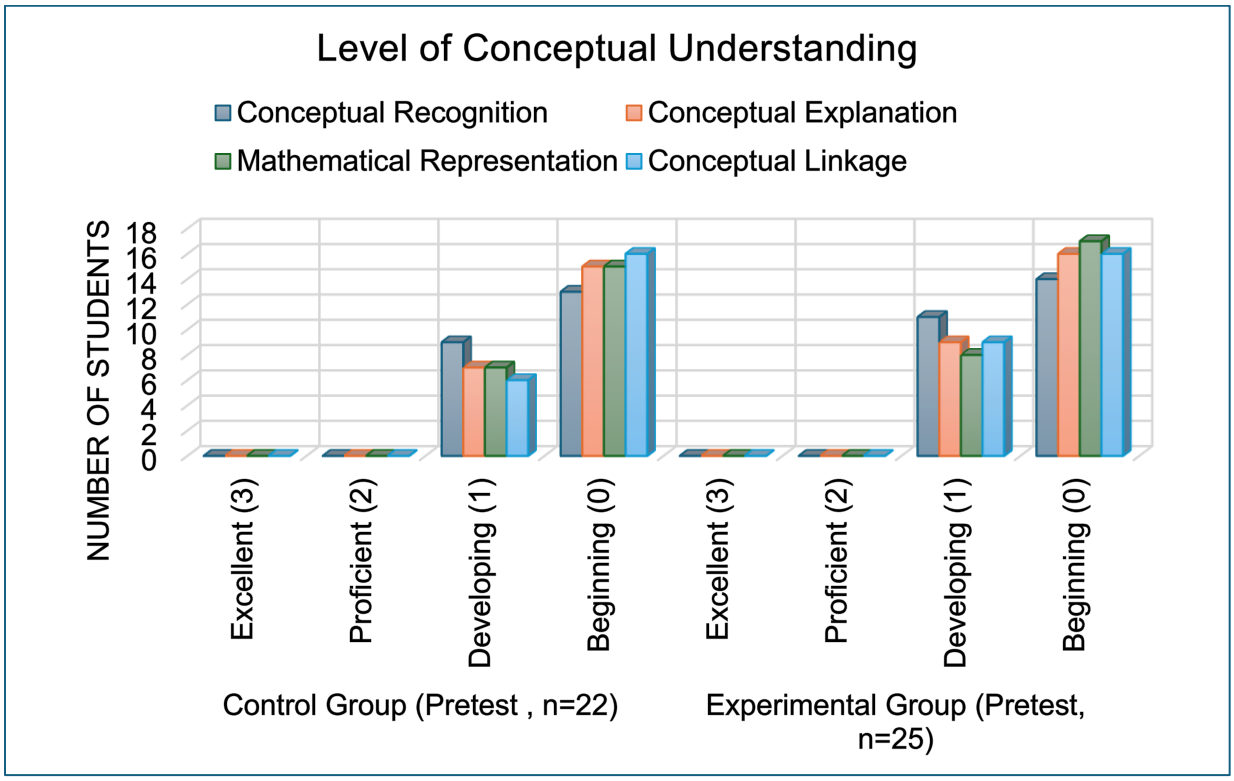


Figure 3. Students' Levels of Conceptual Understanding Across the Four Indicators for Both Groups During the Pretest

The findings indicate that students experienced difficulties in recognizing and connecting geometric concepts, explaining their applicability within proofs, and representing mathematical relationships accurately. Students' written proofs commonly reflected fragmented explanations,

weak conceptual continuity, improper mathematical representations, and incomplete deductive integration.

Although some students demonstrated emerging recognition of geometric concepts and symbolic representations, their conceptual understanding remained largely procedural and insufficiently grounded for coherent proof construction. These findings support previous studies showing that students often rely on intuitive and procedural reasoning rather than conceptually grounded deductive thinking during proof-writing (Perez & Ramirez, 2020; Ordiz & Mecate, 2022; Gagani, 2023).

Logical Reasoning in Writing Basic Geometric Proofs

Logical reasoning refers to students' ability to initiate, organize, justify, and validate deductive reasoning within written geometric proofs. Guided by the framework of proof and proving proposed by Stylianides (2007), this domain was examined through deductive initiation, logical sequencing, justificatory reasoning, and deductive validity. Table 2 reveals that both groups demonstrated generally weak logical reasoning across all indicators during the assessment.

Table 2. Levels of Logical Reasoning of Grade 9 Students in Writing Basic Geometric Proofs

Indicator	Maximum Possible Score (per indicator)	**Control Group (Pre-test) n=22**			**Experimental Group (Pre-test) n=25**		
		Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation	Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation
Deductive Initiation	3.00	0.32	0.35	Beginning	0.32	0.39	Beginning
Logical Sequencing	3.00	0.26	0.24	Beginning	0.28	0.34	Beginning
Justificatory Reasoning	3.00	0.30	0.35	Beginning	0.29	0.33	Beginning
Deductive Validity	3.00	0.28	0.33	Beginning	0.29	0.35	Beginning
Overall (Average)	**3.00**	**0.29**	**0.32**	**Beginning**	**0.30**	**0.35**	**Beginning**

Descriptive Interpretation. *Excellent Level [2.50-3.00]**;** Proficient Level [1.50-2.49]; Developing Level [0.50-1.49]; Beginning Level [0.00-0.49].*

Also in figure 2, in the group A, 68% of the students were classified under the Beginning level in deductive initiation, while 73% were classified under the Beginning level in logical sequencing, justificatory reasoning, and deductive validity. Similarly, in the group B, 68% of the students were classified under the Beginning level in deductive initiation, 72% in logical sequencing, and 64% in justificatory reasoning and deductive validity. The class average scores of both groups were interpreted under the Beginning level.

The findings indicate that students experienced difficulty initiating deductive reasoning, organizing logical proof sequences, providing valid justifications, and constructing logically valid conclusions. Students' written proofs commonly reflected incomplete deductive progression, unsupported logical transitions, missing intermediate reasoning steps, and weakly justified

statements. Several responses merely restated given information or presented isolated mathematical statements without establishing coherent deductive relationships.

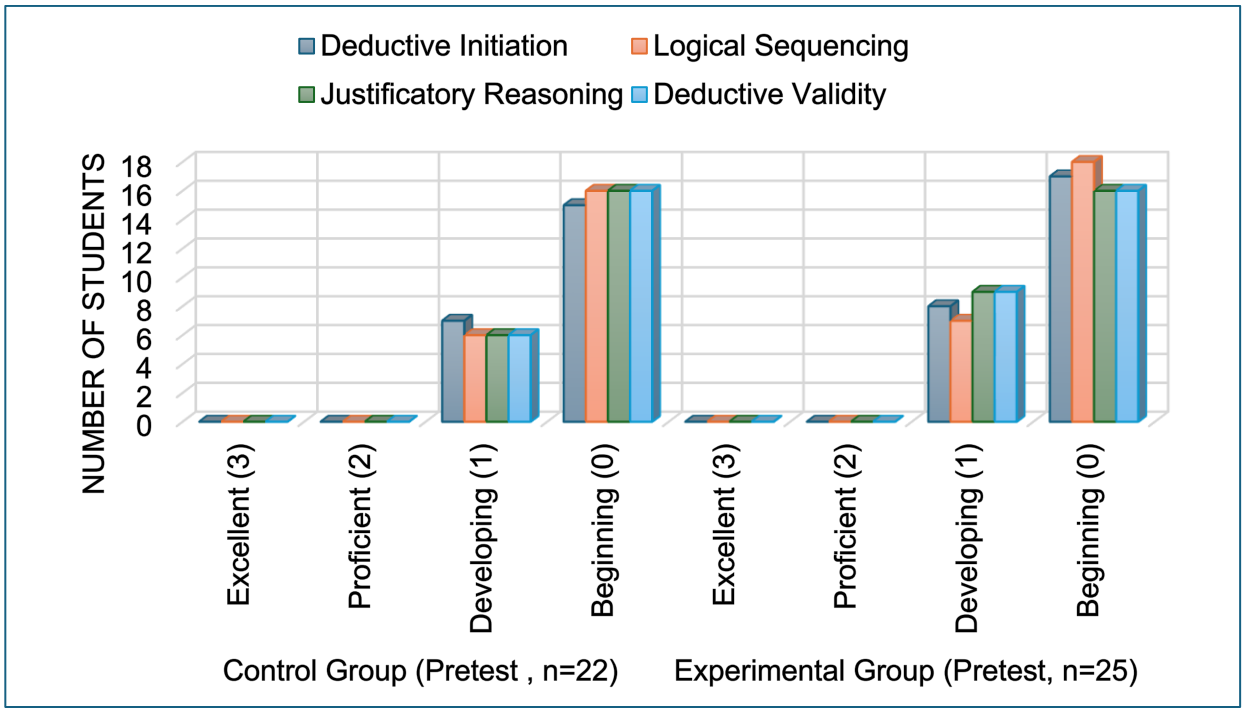


Figure 2. Students' Levels of Logical Reasoning Across the Four Indicators for Both Groups During the Pretest

Although some students demonstrated partial awareness of geometric relationships, their deductive progression frequently remained inconsistent, weakly connected, and logically incomplete. These findings suggest that students relied more on procedural manipulation and isolated reasoning than on formally connected deductive arguments. Similar observations were reported by Perez and Ramirez (2020), Ordiz and Mecate (2022), and Gagani (2023), who emphasized that students often demonstrate fragmented deductive reasoning and weak logical continuity during proof-writing. Overall, the findings indicate insufficient logical reasoning skills necessary for coherent geometric proof construction.

Structure of Deductive Proof in Writing Basic Geometric Proofs

The structure of deductive proof refers to students' ability to organize, align, and complete the formal components of deductive proofs. Grounded in the Theory of Geometric Thinking of van Hiele and van Hiele-Geldof (1986), this domain was examined through structural awareness, sequential organization, component alignment, and structural completeness.

Table 3 and figure 3 show that both groups demonstrated very low levels across all indicators during the assessment. In the group A, 77% of the students were classified under the Beginning level in structural awareness, 91% in sequential organization, 82% in component alignment, and 100% in structural completeness. Similarly, in the group B, 80% of the students were classified

under the Beginning level in structural awareness, 92% in sequential organization, 84% in component alignment, and 100% in structural completeness.

Table 3. Levels of Structure of Deductive Proof of Grade 9 Students in Writing Basic Geometric Proofs

Indicator	Maximum Possible Score (per indicator)	Control Group (Pre-test) n=22			Experimental Group (Pre-test) n=25		
		Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation	Mean Score (M)	Standard Deviation (SD)	Descriptive Interpretation
Structural Awareness	3.00	0.11	0.21	Beginning	0.32	0.13	Beginning
Sequential Organization	3.00	0.07	0.16	Beginning	0.28	0.09	Beginning
Component Alignment	3.00	0.10	0.20	Beginning	0.29	0.11	Beginning
Structural Completeness	3.00	0.03	0.09	Beginning	0.29	0.04	Beginning
Overall (Average)	**3.00**	**0.08**	**0.16**	**Beginning**	**0.30**	**0.09**	**Beginning**

Descriptive Interpretation. *Excellent Level [2.50-3.00]**;** Proficient Level [1.50-2.49]; Developing Level [0.50-1.49]; Beginning Level [0.00-0.49].*

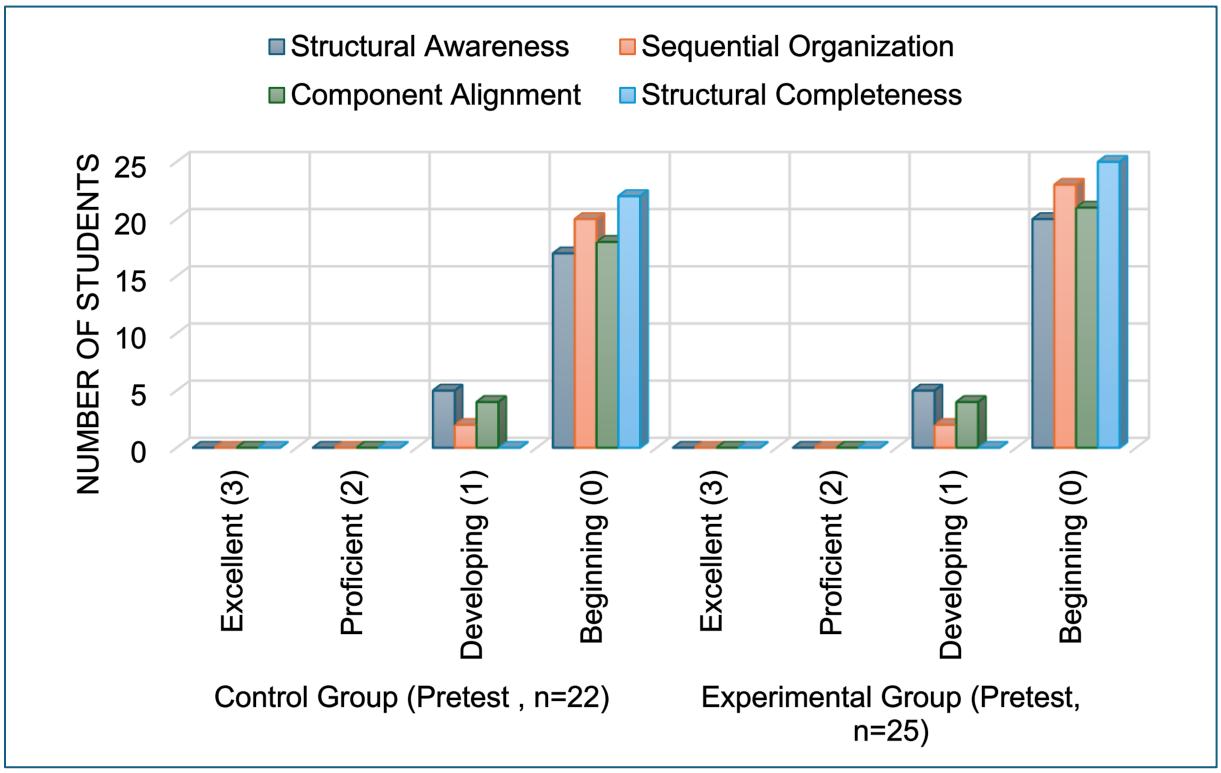


Figure 3. Students' Levels of Structure of Deductive Proof Across the Four Indicators for Both Groups During the Pretest

The findings suggest that students struggled not only in recognizing formal proof structures but also in organizing deductive sequences, aligning statements with corresponding reasons, and sustaining deductive continuity from the givens toward the conclusion. Students' written proofs commonly reflected fragmented proof arrangements, incomplete statement–reason structures, missing intermediate steps, weak alignment between statements and justifications, and undeveloped conclusions. Weak structural awareness and component alignment were evident in students' difficulty organizing proof steps logically and connecting statements with appropriate justifications.

Although some students demonstrated partial awareness of proof organization, the deductive sequence frequently lacked continuity and coherence, resulting in incomplete and structurally weak proofs. These findings support previous studies reporting that students often experience difficulty organizing proof steps and constructing coherent deductive structures (Del Mundo, 2018; Stylianides, 2018; Anwar & Goedhart, 2019). Overall, the findings indicate insufficient structural understanding necessary for formal geometric proof-writing.

Collectively, the findings across the three domains reveal that Grade 9 students demonstrated generally weak proof-writing competence in Geometry. Weak conceptual understanding contributed to fragmented logical reasoning and incomplete proof structures, resulting in deductively weak and poorly organized proofs. The findings highlight the interconnected nature of conceptual understanding, logical reasoning, and proof structure, emphasizing the need for instructional approaches that explicitly develop these domains simultaneously in geometry proof-writing.

Learning Difficulties in Geometry Proof-Writing

Building upon the findings on students' levels of understanding in conceptual understanding, logical reasoning, and the structure of deductive proof, this section examines the learning difficulties reflected in students' written proof solutions. The analysis identified recurring patterns of misconceptions, fragmented reasoning, unsupported justifications, and incomplete proof structures that contributed to students' weak proof-writing performance.

Learning Difficulties in Conceptual Understanding

Conceptual understanding serves as the foundation of proof construction because it involves students' ability to recognize, explain, and connect geometric definitions, properties, postulates, and relationships within deductive proofs. The analysis of students' written proof solutions revealed recurring conceptual difficulties, including unsupported restatements of givens, misuse of justifications, disconnected conceptual relationships, incomplete conceptual development, and confusion in mathematical representation.

These manifestations suggest that students' difficulties were rooted not only in procedural weaknesses but also in fragmented conceptual understanding. Table 4 presents the conceptual understanding learning difficulties identified from students' written proof solutions, particularly in

conceptual recognition, conceptual explanation, and conceptual linkage, which affected students' ability to construct meaningful and conceptually grounded deductive proofs.

Table 4. Conceptual Understanding Learning Difficulties in Geometry Proof-writing

Conceptual Understanding Indicator	Observed Patterns in Students' Proof Solutions	Identified Learning Difficulty	Plate and Annotation References
Conceptual Recognition	Given-only responses, given-only justification, and restatement without conceptual extension, and Partial conceptual recognition	Lack of Fundamental Conceptual Understanding	Plate 10–1; Plate 11–1; Plate 12–3; Plate 13–1
Conceptual Linkage	Failure to proceed beyond given, no conceptual progression, incomplete conceptual development, Incomplete and fragmented conceptual structure, and correct conclusion with weak conceptual flow	Weak Conceptual Linkage of Geometric Relationships	Plate 10–2; Plate 11–2; Plate 12–2a–2b; Plate 10–5; and, Plate 13–5
Conceptual Explanation	Conclusion stated without conceptual basis, irrelevant conceptual step, vague/non-mathematical justification, invalid justification, Procedural application without full conceptual grounding; and Conceptual uncertainty	Misconception and Misapplication of Geometric Concepts	Plate 10–3; Plate 11–3; Plate 12–1; Plate 12–5; Plate 13–3; Plate 13–6
Conceptual Linkage / Conceptual Explanation	Misaligned statement–concept relationships, unclear conceptual relationships, isolated conceptual recognition, appropriate but isolated concept use, and conceptual inconsistency	Weak Conceptual Integration	Plate 10–4; Plate 11–4; Plate 12–4; Plate 13–2; Plate 13–4

Students' difficulties in conceptual recognition were evident in responses characterized by unsupported restatements of givens and weak conceptual progression toward the conclusion. Several students merely copied portions of the problem without transforming the given information into meaningful geometric relationships necessary for proof construction. Although some responses reflected partial recognition of geometric concepts and properties, these ideas frequently remained isolated, weakly connected, and insufficiently developed to support coherent deductive arguments.

The annotated responses in Plates 1–4 further revealed recurring weaknesses in conceptual explanation and conceptual linkage, where students inserted vague justifications, presented fragmented geometric relationships, and failed to establish meaningful conceptual continuity throughout the proof process. These findings suggest that students possessed fragmented conceptual understanding and weak conceptual schemas necessary for meaningful proof construction, relying more on superficial recognition and memorized procedures than on conceptually grounded deductive reasoning.

Furthermore, the findings indicate that students' conceptual understanding remained largely procedural rather than conceptually grounded. Although some responses contained partially correct statements or conclusions, the conceptual relationships connecting proof components frequently remained weak, isolated, and inconsistently developed. Several students also

demonstrated conceptual uncertainty through irrelevant information and inconsistent justifications.

Plate 1. Annotated Student Responses Demonstrating Weak Conceptual Recognition and Incomplete Conceptual Progression

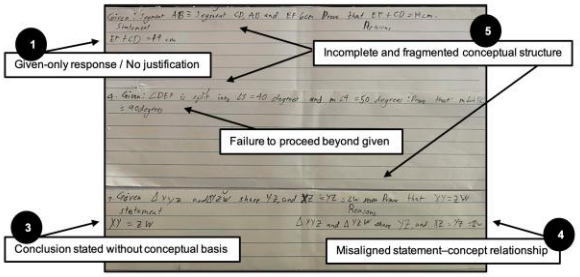


Plate 2. Annotated Student Responses Demonstrating Weak Conceptual Explanation and Weak Conceptual Linkage

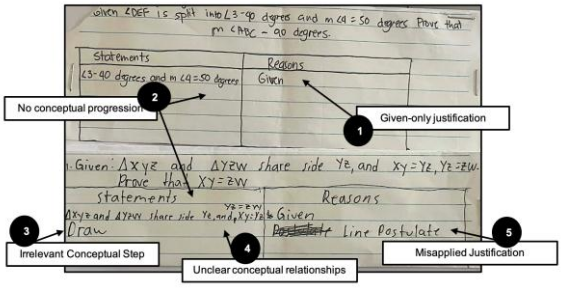


Plate 3. Annotated Student Responses Demonstrating Weak Conceptual Explanation and Incomplete Conceptual Development

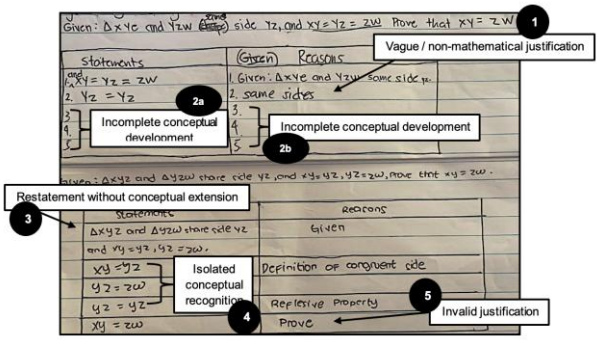
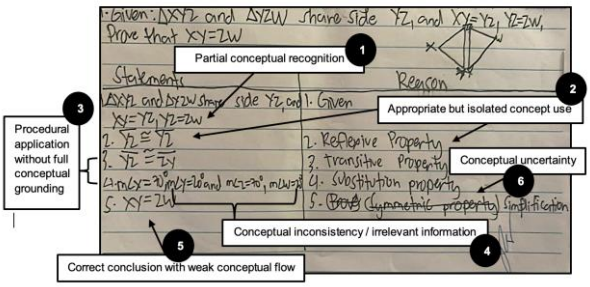


Plate 4. Annotated Student Responses Demonstrating Procedural Reasoning and Weak Conceptual Integration



These findings support the observations of Miyazaki et al. (2016) and Stylianides (2018), who emphasized that conceptual understanding involves meaningful interpretation and application of mathematical ideas rather than simple recall of definitions and properties. Similarly, previous studies have shown that fragmented conceptual understanding limits students' ability to construct coherent mathematical explanations and deductive arguments (Baglama, 2017; Perez & Ramirez, 2020; Martinez, 2021). Overall, the findings highlight that weak conceptual understanding constrained students' ability to establish meaningful geometric relationships, sustain deductive progression, and organize coherent proof structures.

Learning Difficulties in Logical Reasoning in Geometry Proof-Writing

This section examines the learning difficulties underlying students' logical reasoning in geometry proof-writing as reflected in their written proof solutions. Logical reasoning plays a critical role in

proof-writing because it enables students to initiate deductive arguments, organize logical sequences, justify mathematical claims appropriately, and construct logically valid conclusions. However, the analysis of students' written proofs revealed recurring difficulties in deductive initiation, logical sequencing, justificatory reasoning, and deductive validity, which prevented students from constructing coherent and logically supported deductive arguments. Table 5 presents the logical reasoning learning difficulties identified in students' written proof solutions.

Table 5. Logical Reasoning Learning Difficulties in Geometry Proof-Writing

Logical Reasoning Indicator	Observed Patterns in Students' Proof Solutions	Identified Learning Difficulty	Plate and Annotation References
Deductive Initiation	No initiation of deductive reasoning, completely empty statement–reason table, and restatement without logical progression	Lack of Initiation of Deductive Reasoning	Plate 14–1; Plate 14–2
Deductive Validity	Non-logical procedural action; and, Procedural reasoning replacing deductive reasoning; Uncertain or inappropriate reasoning, and conclusion with weak logical support	Surface-Level Procedural Reasoning / Emerging but Unstable Logical Reasoning	Plate 14–3; Plate 15–1; Plate 16–6; Plate 15–4; Plate 16–4
Justificatory Reasoning	Misaligned justification, misuse of justification, misapplied definition of congruent segments, unjustified transition between steps, and invalid justification	Weak Understanding of Justification in Proofs	Plate 14–4; Plate 15–3; Plate 15–6; Plate 16–2; Plate 16–5
Logical Sequencing	Absence of logical continuity, circular or non-progressive reasoning, broken logical chain, and incomplete logical sequencing	Absence of Logical Sequencing Skills	Plate 14–5; Plate 15–2; Plate 15–5; Plate 16–1
Logical Sequencing / Deductive Validity	Insertion of unrelated reasoning elements	Fragmented Deductive Reasoning	Plate 16–3

The annotated responses in Plates 5–7 revealed that several students failed to initiate the deductive process, while others merely restated given information without extending it into meaningful deductive progression. Some responses reflected procedural actions, unsupported transitions, repetitive statements, and misaligned justifications that were weakly connected to the proof structure.

In many cases, students produced isolated statements with missing intermediate deductive relationships, resulting in fragmented and incomplete logical progression toward the conclusion. Students also demonstrated difficulties in sustaining coherent deductive reasoning and validating logical relationships throughout the proof process. Several responses reflected procedural manipulation, misuse of geometric properties and definitions, fragmented deductive continuity, and weak deductive validity. Although some students demonstrated partial deductive progression, the overall reasoning process frequently remained inconsistent and logically unstable.

Furthermore, several responses demonstrated fragmented deductive sequencing, insertion of unrelated reasoning elements, procedural computation replacing deductive argumentation, and misunderstanding regarding mathematical justifications. In some cases, students incorrectly used terms such as "Prove" as justification within the proof structure. Although some responses

contained partially correct conclusions, the supporting deductive relationships frequently remained incomplete and logically insufficient.

Plate 5. Annotated Student Responses Demonstrating Difficulties in Deductive Initiation and Logical Continuity

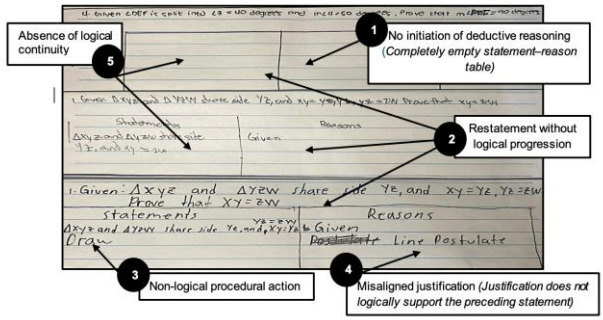


Plate 6. Annotated Student Responses Demonstrating Weak Justificatory Reasoning and Fragmented Deductive Progression

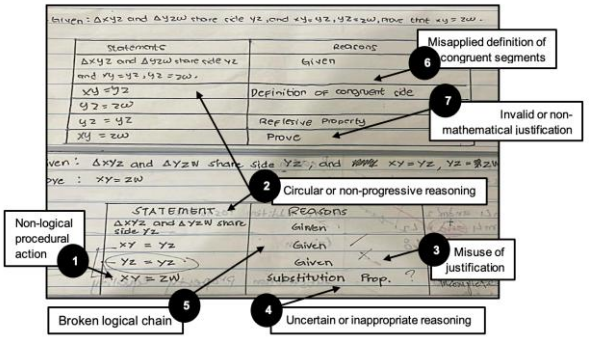
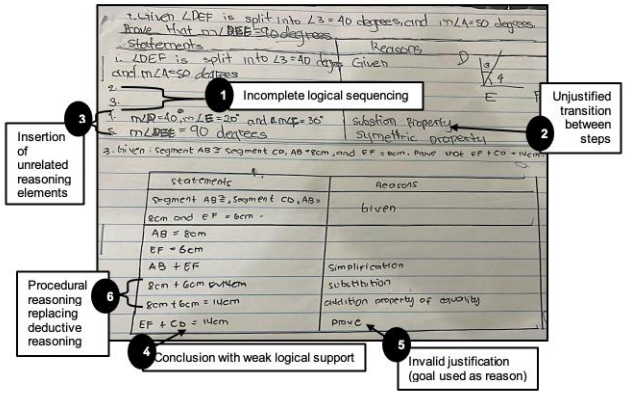


Plate 7. Annotated Student Responses Demonstrating Fragmented Deductive Sequencing and Weak Deductive Validity



Collectively, the findings indicate that students' logical reasoning remained emerging but deductively unstable. Rather than constructing logically connected arguments, students frequently relied on memorized procedures, isolated reasoning patterns, and unsupported deductive

transitions, resulting in fragmented proofs. The findings further suggest that difficulties in logical reasoning were closely associated with weaknesses in conceptual understanding.

These findings support previous studies showing that students often struggle to transition from intuitive or procedural reasoning toward formal deductive reasoning (Stylianides, 2007; Stylianides & Stylianides, 2009; Miyazaki et al., 2016). Overall, the findings highlight the need for instructional support focused on deductive sequencing, mathematical justification, and logical continuity in geometry proof-writing.

Learning Difficulties in the Structure of Deductive Proof in Geometry Proof-Writing

This section examines the learning difficulties underlying students' ability to organize and construct the formal structure of deductive proofs as reflected in their written proof solutions. The structure of deductive proof is essential in geometry proof-writing because it enables students to organize statements, reasons, and deductive relationships into coherent mathematical arguments.

Beyond conceptual understanding and logical reasoning, students must also recognize how proof components should be arranged, aligned, and completed within a formal deductive structure. However, the analysis of students' written proofs revealed recurring difficulties in recognizing proof structure, organizing deductive sequences coherently, aligning statements with appropriate justifications, and sustaining complete deductive continuity throughout the proof process.

Table 6. Structure of Deductive Proof Learning Difficulties in Geometry Proof-Writing

Structure of Deductive Proof Indicator	Observed Patterns in Students' Proof Solutions	Identified Learning Difficulty	Plate and Annotation References
Structural Awareness	Absence of proof structure and no transition from given to process; Presence of format without structural control	Lack of Structural Awareness in Proof Construction	Plate 17–1; Plate 17–2; Plate 18–1
Sequential Organization	Non-structural step in proof and non-structural element in proof sequence	Weak Organization of Sequential Proof Structure	Plate 17–3; Plate 18–3
Component Alignment	Mismatch between statement and justification, and misalignment of proof components	Difficulty in Integrating and Aligning Proof Components	Plate 17–4; Plate 18–4
Sequential Organization / Structural Completeness	Fragmented structural development, disconnected structural components	Fragmented Structural Integration	Plate 17–5; Plate 18–5
Structural Completeness	Incomplete proof structure; and, Premature closure of proof	Fragmented Structural Integration / Incomplete Deductive Structure	Plate 17–6; Plate 18–2

Table 6 presents the structural difficulties observed in students' written proof solutions. The findings revealed recurring weaknesses in structural awareness, sequential organization, component alignment, and structural completeness, which collectively hindered students from constructing coherent and formally organized deductive proofs.

The annotated responses in Plates 8 and 9 revealed that several students produced responses lacking recognizable statement–reason organization, coherent deductive progression, and complete proof development. In many cases, students merely copied given information, inserted unrelated procedural actions, or presented isolated proof components without sustaining organized deductive progression toward the conclusion. Some responses also demonstrated weak alignment between statements and corresponding justifications, resulting in structurally inconsistent and incomplete proofs.

Several students demonstrated absence of proof structure and weak transition from the given information toward the deductive process. Although some responses reflected partial awareness of proof format through statement–reason arrangements, the overall proof structures frequently remained weakly organized, inconsistently connected, and deductively incomplete. Several responses also showed premature conclusions, fragmented proof components, and disconnected deductive continuity.

Plate 8. Annotated Student Responses Demonstrating Difficulties in Structural Awareness and Fragmented Proof Organization

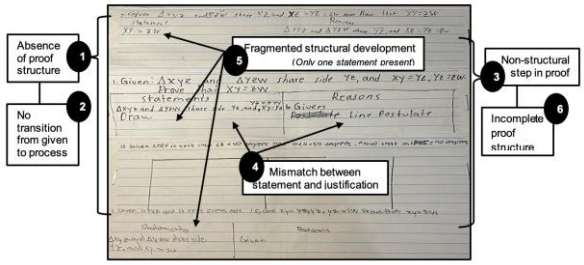
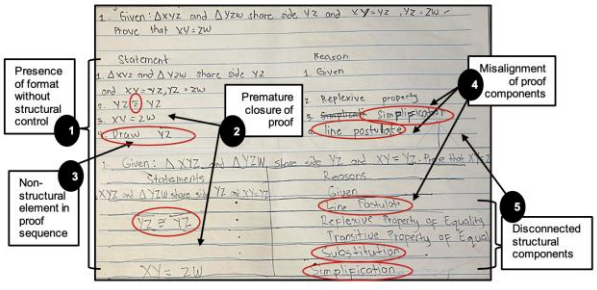


Plate 9. Annotated Student Responses Demonstrating Weak Structural Control and Fragmented Deductive Integration



The findings suggest that students possessed emerging but unstable structural understanding. While some learners demonstrated partial awareness of proof organization and attempted deductive integration, they struggled to consistently organize, align, and complete proof components within coherent deductive structures. Rather than constructing formally organized

mathematical arguments, students frequently produced fragmented and structurally inconsistent proof attempts.

These findings support the observations of Del Mundo (2018), Stylianides (2018), and Anwar and Goedhart (2019), who emphasized that students often experience difficulty organizing proof steps, aligning justifications appropriately, and sustaining complete deductive structures during proof-writing. Similarly, local studies have shown that weak structural organization and incomplete sequencing hinder students' ability to communicate deductive arguments coherently in geometry proofs (Martinez, 2021; Tolentino & Balagtas, 2023).

Furthermore, the findings reinforce the interconnected relationship among conceptual understanding, logical reasoning, and proof structure. When students fail to establish meaningful conceptual relationships and coherent deductive progression, they likewise experience difficulty organizing proof components into formally connected structures. Consequently, fragmented conceptual understanding and unstable logical reasoning contribute directly to fragmented and incomplete proof organization.

Overall, the findings indicate that students' structural difficulties were not merely formatting problems but manifestations of deeper conceptual and deductive weaknesses. The results emphasize the need for instructional approaches that explicitly strengthen proof organization, component alignment, structural continuity, and complete deductive integration in geometry proof-writing.

Conclusion

The findings revealed that Grade 9 students demonstrated weak conceptual understanding, logical reasoning, and structural organization in geometry proof-writing, with most learners classified under the Beginning level across all domains. Students' written proofs commonly reflected fragmented conceptual understanding, unsupported justifications, weak logical continuity, and incomplete deductive structures, indicating reliance on procedural rather than deductive reasoning. The findings further showed that weaknesses in conceptual understanding contributed to fragmented logical reasoning and incomplete proof organization. Overall, the study highlights the need for instructional approaches that explicitly develop conceptual understanding, deductive reasoning, and proof organization in geometry instruction.

Recommendation

Mathematics teachers are encouraged to implement scaffolded instructional approaches that explicitly develop conceptual understanding, logical reasoning, and proof organization during geometry instruction. Guided proof-writing activities, statement–reason scaffolds, reasoning prompts, collaborative discussions, and error-analysis tasks may help students strengthen deductive reasoning and proof-writing competence. Furthermore, the findings may serve as basis for developing instructional materials and interventions addressing students' recurring difficulties in geometry proof-writing.

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