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Article

Keywords: Economic–statistical design, Multivariate categorical processes, Log-linear model, Generalized Linear Test, EWMA

Posted Date: July 14th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-10155919/v1

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Additional Declarations: No competing interests reported.

Monitoring Multivariate Categorical Processes in Phase II: An Economic–Statistical Design of the GLT Control Chart

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Abstract

In contemporary manufacturing systems, product quality is frequently characterized by multiple interrelated categorical variables that can be represented through contingency tables. Although several control charts have been developed for monitoring such processes, existing studies have predominantly emphasized statistical performance, while economic aspects have received comparatively less attention. To address this limitation, this study develops an integrated economic–statistical framework for monitoring multivariate categorical processes based on contingency tables. A Generalized Linear Test (GLT) statistic derived from log-linear models is combined with an Exponentially Weighted Moving Average (EWMA) mechanism to enhance sensitivity to small and gradual process shifts. The design variables, including sample size, sampling interval, control limits, and EWMA parameters, are jointly determined through an optimization procedure that minimizes the Expected Total Cost (ETC) while satisfying predefined statistical performance requirements. Simulation results demonstrate that the proposed framework provides reliable shift detection capability and achieves lower operational costs than a purely statistical design. Therefore, the proposed EWMA–GLT approach offers an effective and practical strategy for monitoring multivariate categorical processes while simultaneously balancing economic and statistical objectives.

Keywords: Economic–statistical design; Multivariate categorical processes;Log-linear model; Generalized Linear Test; EWMA;

1. Introduction

The increasing complexity of modern manufacturing systems and the growing demand for high-quality products have intensified the need for effective process monitoring strategies. Statistical Process Control (SPC) has long been recognized as one of the most effective methodologies for maintaining process stability, reducing variability, and supporting continuous quality improvement in industrial environments. Early studies established the importance of incorporating economic

considerations into process monitoring, while subsequent developments extended conventional control charts to more sophisticated industrial systems [1,2].Nevertheless, most traditional control charts were originally developed for univariate continuous variables and may not be sufficiently effective for modern manufacturing environments involving multiple interacting quality characteristics.

In many practical applications, product quality is assessed using several categorical characteristics rather than continuous measurements. Examples include defect classifications, inspection outcomes, customer evaluations, product grades, and medical diagnoses. Since these characteristics are often correlated, monitoring them independently may result in substantial information loss regarding their dependency structure and reduce the ability to identify abnormal process behavior. Consequently, monitoring multivariate categorical processes has become an important research topic in quality engineering. Contingency tables provide an effective framework for representing the relationships among categorical variables, while contingency-table-based monitoring approaches have demonstrated promising capabilities in detecting structural changes in such processes [3,4] .In addition, generalized linear model-based methodologies have shown considerable potential for improving monitoring performance in complex industrial environments [5] .

Model-based monitoring approaches have considerably advanced the analysis of multivariate categorical processes by preserving the dependency structure among quality characteristics. Reliable online monitoring also requires the establishment of an appropriate reference model prior to implementation. Therefore, Phase I methodologies were developed to identify structural changes in historical data and establish stable in-control conditions[6]. Building upon these developments, GLT-based control charts were introduced in Phase II monitoring to detect process shifts and diagnose out-of-control conditions[7]. These studies represented important advances in monitoring multivariate categorical processes and established a foundation for developing more sophisticated monitoring frameworks.

As industrial systems become increasingly complex, researchers have attempted to address more challenging scenarios. Recent studies have investigated ordinal multivariate categorical processes by exploiting the ordering information embedded within categories, whereas others have focused on sparse contingency tables, where low-frequency observations may substantially deteriorate monitoring performance as the number of categories increases [8,9]. These findings highlight the need for robust monitoring methodologies capable of maintaining reliable performance under diverse practical conditions.

In parallel with these developments, memory-type control charts have attracted considerable attention because of their superior sensitivity to small and gradual process shifts. Among these methods, the Exponentially Weighted Moving Average (EWMA) chart has become one of the most widely adopted approaches due to its ability to incorporate both current and historical information into the monitoring procedure[2]. Recent studies have further enhanced EWMA-based

methodologies through hybrid charting schemes [10], fuzzy environments [11],and adaptive sampling strategies [12]. Although these approaches have demonstrated improved statistical performance, economic considerations have generally remained secondary.

From an industrial perspective, an effective monitoring system should not only detect process shifts rapidly but also minimize the overall operational cost. In practice, manufacturers must simultaneously consider sampling costs, inspection costs, false alarm costs, search and repair costs, and production losses incurred while the process remains out of control. Consequently, monitoring systems designed solely according to statistical criteria may not necessarily be optimal for real production environments. This limitation motivated the development of economic–statistical design methodologies, which aim to balance statistical effectiveness and economic efficiency [3]. In recent years, this concept has been extended to various monitoring structures, including attribute control charts [14], fully adaptive multivariate schemes [15], Bayesian monitoring frameworks [16], and variable sampling interval procedures [17], Although these approaches have demonstrated substantial improvements in monitoring performance and cost reduction, most of them were developed either for continuous multivariate processes under normality assumptions or for conventional univariate attribute charts.

Despite these advances, several important research gaps remain unresolved. Existing studies can generally be categorized into three main streams: contingency-table-based monitoring approaches, EWMA-based enhancement techniques, and economic–statistical optimization frameworks. However, these research streams have largely evolved independently. Contingency-table-based approaches have shown promising capabilities for detecting structural changes but rarely incorporate economic considerations. Similarly, EWMA-based approaches primarily focus on improving statistical sensitivity without explicitly optimizing economic performance. Moreover, most economic–statistical studies have been developed for continuous variables under normality assumptions or conventional attribute charts. In addition, recent studies have shown that the performance of economic–statistical control charts may be sensitive to parameter estimation, emphasizing the importance of developing robust frameworks that maintain satisfactory performance under practical implementation conditions [18].Consequently, an important gap still exists because no single framework has simultaneously integrated contingency-table-based multivariate categorical data, log-linear modeling, GLT-based monitoring, EWMA memory structures, and economic–statistical optimization.

To address these limitations, this study proposes an integrated economic–statistical framework for monitoring multivariate categorical processes based on contingency tables. First, log-linear models are employed to characterize the dependency structure among categorical variables, and a Generalized Linear Test (GLT) statistic is derived to monitor structural changes. Subsequently, the GLT statistic is integrated into an Exponentially Weighted Moving Average (EWMA) structure to enhance sensitivity to small and gradual process shifts. Finally, an economic–statistical optimization model is developed to simultaneously determine the optimal design parameters,

including sample size, sampling interval, control limits, and EWMA parameters, while minimizing the Expected Total Cost (ETC) under predefined statistical performance requirements.

Compared with previous studies, the proposed framework offers four major contributions. First, it extends economic–statistical design to multivariate categorical processes represented by contingency tables, an area that has received limited attention in the literature. Second, it establishes a unified framework that integrates log-linear models, GLT, and EWMA rather than treating these components independently. Third, it simultaneously considers economic and statistical objectives during optimization to obtain efficient chart parameters. Finally, it provides a practical monitoring tool for industrial environments characterized by multiple correlated categorical quality characteristics.

The remainder of this paper is organized as follows. Section 2 presents the methodology, detailing the log-linear modeling of contingency tables, the development of the EWMA–GLT control chart, the formulation of the Expected Total Cost (ETC) function, and the economic–statistical optimization model. Section 3 presents the numerical results, including the evaluation of the chart's statistical performance, the determination of the optimal design parameters, and comprehensive sensitivity analyses to demonstrate the effectiveness of the proposed framework. Finally, Section 4 concludes the paper and outlines directions for future research.

2. Methodology

This study presents an integrated economic–statistical framework for designing a multivariate control chart for categorical data represented by contingency tables. The proposed methodology is composed of two complementary components, namely a statistical component for process monitoring and an economic component for optimizing chart performance.

Within the statistical component, the dependence structure among categorical variables is characterized using log-linear models. Subsequently, the Generalized Linear Test (GLT) statistic is employed to detect structural changes in contingency tables. To improve the sensitivity of the monitoring procedure to small and gradual process shifts, the GLT statistic is further integrated into an Exponentially Weighted Moving Average (EWMA) framework, resulting in the development of the EWMA–GLT monitoring statistic.

The economic component aims to determine the optimal design configuration by minimizing the Expected Total Cost (ETC). To achieve this objective, several design parameters, including the sample size (n), sampling interval (h), upper control limit (UCL), EWMA smoothing parameter (λ), and EWMA control limit coefficient (L), are simultaneously optimized while maintaining satisfactory statistical performance according to criteria such as the Average Run Length (ARL) and Average Time to Signal (ATS). Monte Carlo simulation is then employed to evaluate the performance of the proposed framework from both statistical and economic perspectives and to identify the optimal combination of design parameters.

The remainder of this section is organized as follows. First, the contingency-table structure and the associated log-linear model are introduced. Next, the GLT statistic and the EWMA–GLT control chart are presented, followed by the statistical performance measures adopted in this study, including ARL and ATS. Subsequently, the economic–statistical optimization model and the ETC function are formulated, and the optimization procedure used to determine the optimal design parameters is described. Finally, the effectiveness of the proposed framework is assessed through Monte Carlo simulations implemented in MATLAB.

2.1. Log-Linear model for contingency tables

In many industrial processes, product or service quality is evaluated through several categorical variables. When these variables are examined simultaneously, the data are usually represented in the form of contingency tables. Contingency tables show the frequencies of occurrence of different combinations of the levels of categorical variables and provide the possibility of examining the dependency structure among them. Since the main objective of this study is to simultaneously monitor several dependent categorical variables, the use of contingency tables provides an appropriate framework for modeling process behavior [3].

A log-linear model is used to model the expected cell frequencies. If $N$is the sample size and $\pi_{ij}$is the probability of occurrence of cell (i, j), the expected value of each cell is obtained from

$$\boldsymbol{\mu_{ij} = N\pi_{ij}} \tag{1}$$

Then, the relationship between the expected cell frequencies and the effects of the variables is expressed through the log-linear model as

$$\ln(\mu_{ij}) = \lambda_0 + \lambda_i^X + \lambda_j^Y + \lambda_{ij}^{XY} \tag{2}$$

Here, λ_0 represents the overall effect of the model, $\lambda_i^X$and λ_j^Yrepresent the main effects of variables $X$and Y, respectively, and λ_{ij}^{XY}represents the interaction effect between the two variables. The presence of the λ_{ij}^{XY}component indicates dependency between the categorical variables, whereas the omission of this component corresponds to the assumption of independence between the variables. In this paper, the log-linear model serves as the basis for calculating the expected frequencies and developing the Generalized Linear Test (GLT) statistic for process monitoring [3].

2.2. Development of the GLT statistic for monitoring multivariate categorical processes

After establishing the underlying probabilistic structure through the log-linear model, a monitoring statistic is required to quantify the extent to which the current process departs from its in-control condition. In this study, the Generalized Linear Test (GLT) statistic is adopted as the primary monitoring measure. The GLT statistic evaluates the agreement between the observed cell frequencies and the frequencies predicted by the fitted log-linear model and, therefore, provides a numerical indication of structural changes in contingency tables.[7]

Initially, the sum of squared errors associated with the reduced model is computed as

$$SSE(R) = \sum_{i=1}^{i} \sum_{j=1}^{j} \frac{\left(n_{ij} - \mu_{ij}\right)^2}{\mu_{ij}} \tag{3}$$

where n_{ij}denotes the observed frequency in cell (i, j), and μ_{ij} represents the corresponding expected frequency estimated from the log-linear model.

Subsequently, the monitoring statistic used in this study is obtained as

$$F^* = \frac{SSE(R)}{(I-1)(J-1)} \tag{4}$$

where $(I-1)(J-1)$corresponds to the degrees of freedom associated with the interaction component of the log-linear model and reflects the dependency structure among the categorical variables.

For each generated contingency table, the value of $F^*$is calculated and employed as the monitoring statistic. Larger values of F^*indicate a greater departure from the in-control state, suggesting that the association pattern among categorical variables has changed. Consequently, an increase in F^*may be interpreted as evidence of a process shift and a potential loss of process stability.[7]

2.3. EWMA-GLT control chart

Although the GLT statistic has an appropriate ability to identify structural changes in the process, similar to many control charts based on the most recent sample, it shows limited sensitivity in detecting small and gradual shifts. To increase the detection capability, the GLT statistic is combined with the Exponentially Weighted Moving Average (EWMA) structure in this study. This approach increases the ability to detect small shifts by simultaneously using current and past information [7].

The EWMA statistic is calculated as:

$$Z_t = LF_t^* + (1-L)Z_{t-1} \tag{5}$$

where $F_t^*$is the value of the GLT statistic at time t, $Z_t$is the EWMA statistic, and $L$is the smoothing parameter, which takes a value between zero and one. Smaller values of Lassign greater weight to past information, whereas larger values are more sensitive to new changes.

To avoid notation conflicts with the parameters of the log-linear model, the EWMA smoothing parameter is denoted by L throughout this study instead of the conventional symbol λ. The initial value of the EWMA statistic is considered equal to the mean of the GLT statistic under in-control conditions:

$$Z_0 = \mu_{F^*} \tag{6}$$

The upper control limit of the chart is calculated as:

$$UCL = \mu_{F^*} + K\sigma_{F^*}\sqrt{\frac{L}{2-L}} \tag{7}$$

where $K$is the control limit coefficient, and $\mu_{F^*}$and $\sigma_{F^*}$are the mean and standard deviation of the GLT statistic under in-control conditions, respectively. The value of $L$is determined in such a way that the desired $ARL_0$is achieved [7].

Since the F^*statistic always takes positive values, the decision-making process in this study is based solely on the upper control limit. Therefore, whenever

$$Z_t > UCL \tag{8}$$

the process is considered out of control and a signal is issued.

The use of the EWMA structure allows information from previous samples to be incorporated into the decision-making process and, consequently, increases the ability to detect small and gradual shifts compared with the conventional GLT chart. Therefore, the combination of GLT and EWMA is considered an appropriate option for monitoring multivariate processes based on contingency tables [7].

2.4. Expected total cost (ETC) function

Although statistical indicators such as Average Run Length and Average Time to Signal are appropriate criteria for evaluating the performance of control charts, these indicators alone are not capable of evaluating the economic costs associated with implementing a quality control system. In practice, selecting design parameters solely based on statistical criteria may lead to a design that is statistically desirable but not economically optimal. For this reason, the economic–statistical design approach was developed to establish a balance between statistical performance and process costs [13] .In this study, the economic–statistical model proposed by Haridy et al. [14] is used as the basis for the economic evaluation of the control chart.

In this approach, the expected process cost for each shift level is calculated by combining sampling, inspection, false alarms, searching for and removing assignable causes, and the cost associated with continuing production under out-of-control conditions. The expected process cost under shift $\delta$is obtained as follows:

$$Cost(\delta) = \frac{\left(\frac{C_0 + C_1 n}{h} + \varepsilon C_d\, Delay(\delta) + \frac{\alpha C_f}{h} + \varepsilon C_s\right)}{1 + \varepsilon Delay(\delta)} \tag{9}$$

where $C_0$is the fixed sampling cost, $C_1$is the inspection cost per unit, $C_f$is the false alarm cost, $C_s$is the cost of searching for and removing the assignable cause, and $C_d$is the cost associated with continuing production under out-of-control conditions. In addition, $n$is the sample size, $h$is the sampling interval, $\varepsilon$is the assignable cause occurrence rate, and $\alpha$is the false alarm probability.

In this study, the false alarm probability is calculated based on the Average Run Length under in-control conditions as follows:

$$\alpha = \frac{1}{ARL_0} \tag{10}$$

Moreover, the expected delay time for detecting and removing a process shift is obtained as follows:

$$Delay(\delta) = ATS(\delta) - \frac{h}{2} + \frac{\varepsilon h^2}{12} + en + D \tag{11}$$

where $ATS(\delta)$is the Average Time to Signal corresponding to shift δ, $e$is the inspection time per unit, and $D$is the time required to remove the assignable cause.

The value of $ATS(\delta)$is also calculated as follows:

$$ATS(\delta) = ARL_1(\delta) \times h \tag{12}$$

Since different shifts with different magnitudes may occur in practice, the expected total process cost is calculated by combining the costs corresponding to different shift levels. Therefore, the Expected Total Cost is defined as follows:

$$ETC = \sum_{\delta} w_\delta \ \delta \, Cost(\delta) \tag{13}$$

where $w_\delta$is the weight corresponding to each shift level and $\delta$is the shift magnitude. In this study, ETC is used as the main economic–statistical evaluation criterion and serves as the basis for selecting the optimal design. Accordingly, a design is considered desirable if it maintains satisfactory statistical performance while producing the minimum expected total cost [14].

In this study, the set of out-of-control shift levels was considered as

$\delta = \{0.05, 0.10, 0.20, 0.30, 0.50, 0.75, 1.00\}$. The value $\delta = 0$represents the in-control state and is used only for calculating ARL_0; therefore, it is not included in the calculation of the Expected Total Cost. Since no actual information regarding the probability of occurrence of each shift magnitude was available, all out-of-control shift levels were assigned equal weights. Consequently, for the seven shift levels under consideration, the weight of each shift was assumed to be $w_\delta = 1/7$. Therefore, the ETC value in this study represents the weighted average of the costs corresponding to different shift levels, where equal weights were assigned and the shift magnitudes were considered according to the aforementioned values.

2.5. Economic–Statistical Optimization Framework

After formulating the Expected Total Cost (ETC) function, an economic–statistical optimization framework was developed to identify the optimal control chart design configuration. The optimization procedure aims to simultaneously achieve two objectives: minimizing the overall expected process cost and preserving an acceptable level of statistical performance.

In this framework, the decision variables consist of the sample size (n), sampling interval (h), upper control limit (UCL), EWMA smoothing parameter (λ), and EWMA control limit coefficient (L). These parameters are jointly optimized to obtain a balanced trade-off between economic efficiency and process monitoring effectiveness.

Accordingly, the optimization problem can be expressed as follows:

$$\min \text{ETC}(n,h,UCL,L,K)$$

subject to

$$ATS_0 \geq \tau \quad (14)$$

$$r = n/h \quad (15)$$

Independent decision variables : n ,

Dependent decision variables : h and UCL

where $\tau$is the minimum acceptable value of the Average Time to Signal under in-control conditions and $r$is the constant process sampling rate.

In this study, the sample size (n) is considered an independent decision variable, and the sampling interval (h) is determined based on the value of $n$and the fixed sampling rate. Finally, all possible combinations of the design parameters are evaluated, and among the designs that satisfy the statistical constraint, the design that yields the minimum Expected Total Cost is selected as the optimal economic–statistical design [14].

2.6. Simulation procedure and experimental design

The proposed EWMA–GLT framework was evaluated through Monte Carlo simulations conducted in the MATLAB R2023b software.In each simulation replication, a contingency table was first generated based on the probabilities obtained from the log-linear model. Then, the F^*statistic was calculated for each sample, and its value was incorporated into the EWMA structure. Subsequently, the process status was evaluated by comparing the EWMA statistic with the upper control limit.

In this study, the process structure was defined based on a 4×4 contingency table, and the total sample size at each sampling stage was set to $N = 500$. In addition, a constant process sampling rate of $r = 50$ was assumed.

During the optimization procedure, the sample size was selected from the set $n \in \{50,75,100,150,200,250,300,400,500\}$, and the sampling interval was calculated as $h = \frac{n}{r}$.

For the design of the EWMA-GLT chart, different values of the smoothing parameter were investigated within the range $L \in [0.01,0.30]$. In addition, to determine an appropriate control limit, a broad search was first conducted within the range $UCL \in [1.05,1.60]$, and then a finer search was performed around the promising values to achieve the target ARL_0, after which the final upper control limit was determined.

The Average Time to Signal was also calculated as $ATS = ARL \times h$. Finally, the statistical performance of the chart was evaluated using ARL_0, ARL_1, ATS_0, and ATS_1. To control the false alarm rate, the target Average Run Length under in-control conditions was set to $ARL_0 \approx 200$.

For each combination of the design parameters, the values of ARL_0, ARL_1, ATS_0, ATS_1, and ETCwere calculated. Subsequently, the designs that did not satisfy the statistical constraint were eliminated, and among the remaining designs, the combination that produced the minimum Expected Total Cost was selected as the optimal economic–statistical design.

To investigate the stability and reliability of the proposed model, sensitivity analysis was also performed with respect to variations in cost parameters, shift magnitude, sample size, control limits, and EWMA chart parameters. The results of these analyses provide a comprehensive evaluation of the model performance from both economic and statistical perspectives.

It should be noted that, in this study, the process inspection rate $(r)$was assumed to be fixed at 50 units per unit time, and the relationship $h = \frac{n}{r}$was used to determine the sampling interval. The purpose of this assumption was to maintain inspection capacity across all design scenarios and to prevent the results from being affected by changes in inspection resources. In many industrial environments, sampling capacity is determined by operational constraints such as the number of operators, measurement equipment, implementation costs, and the time available for quality control, and the inspection rate cannot be changed freely.

Therefore, the inspection rate was considered a fixed operational constraint so that the focus of the study remained on evaluating the economic–statistical performance of the proposed EWMA-GLT chart and investigating the effects of the design parameters on performance and cost indicators. It is evident that if $n$and $h$were treated as unconstrained variables, the optimization search space would become larger and different optimal values might be obtained. Therefore, extending the model by simultaneously optimizing $n$and $h$without imposing an inspection rate constraint can be considered a potential direction for future research.

Algorithm 1 illustrates the implementation steps of the proposed EWMA-GLT-based economic–statistical model. In this process, the log-linear model parameters and economic parameters are first defined, and the in-control process probability matrix is calculated. Next, contingency tables are generated using Monte Carlo simulation, and the GLT statistic is calculated. Then, the EWMA scheme is applied to increase sensitivity to small shifts, and the upper control limit (UCL) is determined so that the target $ARL_0$is achieved. Subsequently, the performance indicators, including ARL_1, ATS, and the Expected Total Cost (ETC), are calculated. Finally, the design that yields the minimum ETC is selected as the optimal economic–statistical design. In addition, sensitivity analysis is performed to evaluate the stability of the results with respect to changes in the model parameters.

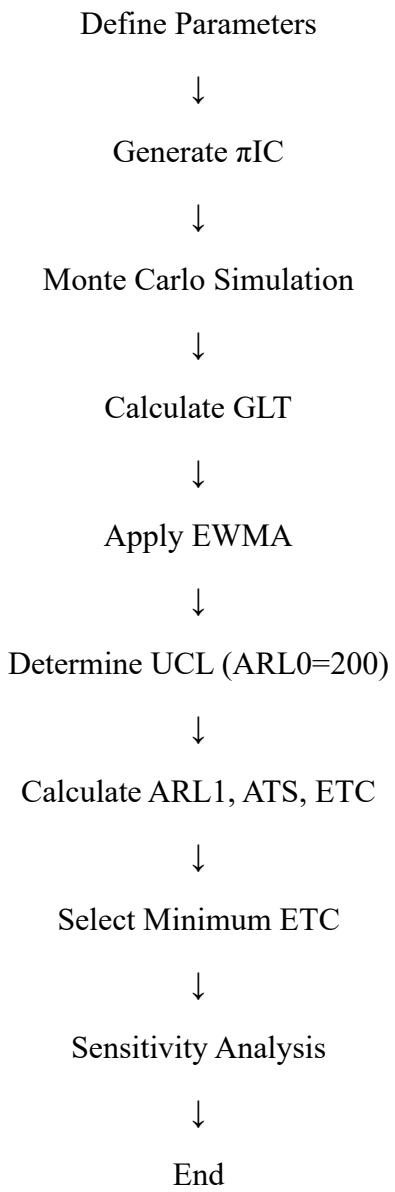


Figure.1 Flowchart of the proposed EWMA-GLT economic–statistical model

3. Results

This section presents the results obtained from the proposed EWMA–GLT economic–statistical model. First, the parameters of the log-linear model and the baseline contingency table under in-control conditions are introduced. Then, the statistical performance of the proposed chart is evaluated in terms of ARL and sensitivity to different shift magnitudes. Finally, the economic–statistical design results, sensitivity analyses, and comparisons with alternative design approaches are presented to identify the optimal monitoring scheme.

3.1. Log-Linear model parameters

For the implementation of the Phase II monitoring scheme, the values of the log-linear model parameters were assumed as follows.

Table 1. Values of the log-linear model parameters

Parameter	λ_0	λ_1^X	λ_2^X	λ_3^X	λ_4^X	λ_1^Y	λ_2^Y	λ_3^Y	λ_4^Y	λ_{11}^{XY}	λ_{22}^{XY}	λ_{33}^{XY}
Value	3.0000	-0.3000	-0.1000	0.1000	0.3000	-0.2000	-0.0667	0.0667	0.2000	0.1000	-0.0500	0.0500

Note that, only three interaction parameters including $\lambda_{11}^{XY} = 0.10$, $\lambda_{22}^{XY} = -0.05$, and $\lambda_{33}^{XY} = 0.05$, were assumed to be nonzero.

3.2. Baseline contingency table under in-control conditions

Based on the specified log-linear model parameters, the baseline contingency table representing the in-control state of the process was generated. This table describes the joint distribution of the two categorical variables and serves as the reference structure for subsequent monitoring analyses. The total sample size in each sampling stage was fixed at (N=500), and the row and column totals indicate the marginal distributions of the process under normal operating conditions.

Table 2. Baseline contingency table under in-control conditions

X / Y	**Y_1**	**Y_2**	**Y_3**	**Y_4**	**Row Total n_{+i}**
X_1	32	15	19	28	94
X_2	17	26	24	33	100
X_3	25	27	45	36	133
X_4	33	35	44	61	173
Column Total n_{+j}	107	103	132	158	500

3.3. Determination of the initial parameters of the GLT statistic

To determine the characteristics of the GLT statistic under in-control conditions, 300 Monte Carlo simulation replications were performed, and the value of the F^*statistic was calculated for each contingency table. The results showed that the value of $SSE(R)$was equal to 17.3845, and the corresponding value of the F^*statistic was 1.9316. In addition, the mean and standard deviation of the F^*statistic under in-control conditions were obtained as 1.0582 and 0.5105, respectively.

Since the objective of this study was to achieve an in-control Average Run Length of $ARL_0 = 200$, the upper control limit was determined through an iterative search procedure. In this process, different values of the control limit were examined, and the in-control Average Run Length was calculated for each value. The results showed that $UCL = 1.2775$ resulted in $ARL_0 = 201.42$, which was the closest value to the target level. Therefore, 1.2775 was selected as the final control limit of the EWMA-GLT chart and was used throughout the remainder of the study.

3.4. ARL_1 values under different shift magnitudes in the log-linear model parameters

To investigate the sensitivity of the proposed control chart to changes in the log-linear model parameters, shifts with different magnitudes were applied to the overall effect, main effects, and interaction effects. Then, for each case, the Average Run Length (ARL) was calculated using Monte Carlo simulation. Since a reduction in ARL indicates an increase in the speed of detecting process changes, smaller ARL values indicate greater sensitivity of the chart to the corresponding parameter. In addition, to quantitatively compare the influence of each parameter, the Effect Index was calculated based on the ARL values at a shift magnitude of 1.

The Effect Index is defined as

$$\text{Effect Index} = \frac{ARL_0}{ARL_\delta} \tag{16}$$

where:

- ARL_0: Average Run Length under in-control conditions
- ARL_δ: Average Run Length after applying a shift of magnitude δ

The Effect Index was calculated to evaluate the sensitivity of the chart to changes in the model parameters. A larger value of this index corresponds to a smaller ARL_δ, indicating that the chart detects process changes more rapidly. Therefore, larger Effect Index values indicate greater sensitivity of the chart to the parameter under investigation.

Table 3. ARL values under different Shift magnitudes for selected log-linear model parameters

δ	λ_0	λ_1^x	λ_2^x	λ_3^x	λ_1^y	λ_2^y	λ_{11}^{xy}	λ_{33}^{xy}	λ_{42}^{xy}	λ_{44}^{xy}

0.00	201.42	201.42	201.42	201.42	201.42	201.42	201.42	201.42	201.42	201.42
0.05	201.88	189.47	194.12	192.84	193.28	195.71	129.15	135.84	131.67	128.92
0.10	222.49	161.05	182.36	178.94	185.66	187.42	86.36	79.55	75.23	72.81
0.20	231.77	174.53	168.42	170.38	172.84	176.51	37.05	32.94	29.61	27.85
0.30	185.52	168.22	165.77	163.54	164.71	167.92	15.77	13.66	12.55	11.93
0.50	197.91	166.44	164.92	162.33	159.83	162.77	5.40	4.31	3.98	3.75
0.75	204.72	163.37	161.58	160.94	157.92	159.33	2.41	1.88	1.54	1.41
1.00	178.61	161.05	160.22	159.86	155.99	157.44	1.36	1.24	1.22	1.17
Effect Index	**1.13**	**1.25**	**1.26**	**1.26**	**1.29**	**1.28**	**148.10**	**162.44**	**165.10**	**172.15**
Sensitivity Level	**Low**	**Moderate**	**Moderate**	**Moderate**	**Moderate**	**Moderate**	**Very High**	**Very High**	**Very High**	**Very High**

The results presented in Table 3 indicate that the proposed EWMA–GLT chart exhibits different levels of sensitivity across the parameters of the log-linear model. While the overall effect (λ_0) and the main effects (λ^X and λ^Y) had only a minor influence on chart performance, the interaction effects (λ^{XY}) played a dominant role in shift detection. Increasing the shift magnitude led to a pronounced reduction in ARL values and a substantial increase in the corresponding Effect Index values. Among all examined parameters, λ_{44}^{XY}produced the highest Effect Index (172.15), indicating the strongest sensitivity. Overall, these findings demonstrate that the proposed EWMA–GLT chart is particularly suited for detecting changes in the dependency structure of categorical variables and is considerably more responsive to interaction effects than to overall or main effects.To visually investigate the sensitivity of the chart to different interaction parameters of the log-linear model, the Effect Index values for all λ^{XY}parameters were presented in the form of a heatmap.

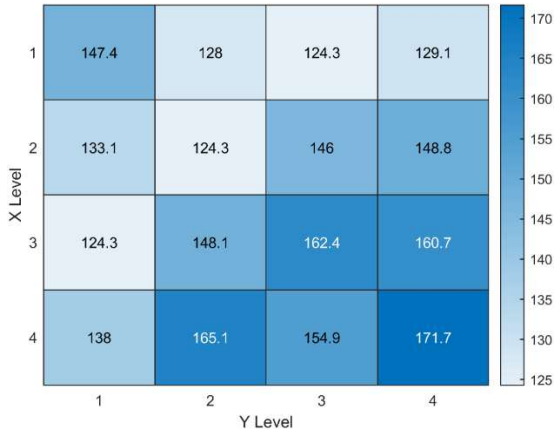


Figure 2.Heatmap of Effect Index values for λ_{ij}^{XY} shifts at δ=1

As shown in Figure 2, the highest Effect Index values were observed in the regions corresponding to λ_{44}^{XY}, λ_{42}^{XY}, and λ_{33}^{XY}, indicating the high sensitivity of the proposed chart to changes in the dependency structure among categorical variables.

3.5. Economic–Statistical design results for different sample sizes

Since the upper control limit $(UCL)$and the ARL_0index remained constant for all investigated scenarios and were equal to 1.2775 and 201.42, respectively, these values were considered identical for all designs. Therefore, the comparison was performed based on the sample size (n), sampling interval (h), ATS_0, and the Expected Total Cost $(ETC)$to more clearly investigate the effects of changes in the design parameters on the economic–statistical performance of the control chart.

Table 4. Economic–Statistical design results for different sample sizes

n	h	ATS_0	ETC	n	h	ATS_0	ETC
50	1	201.42	79.603	250	5	1007.10	89.404
75	1.5	302.12	80.938	275	5.5	1107.80	90.156
100	2	402.83	82.536	300	6	1208.50	90.830
125	2.5	503.54	83.921	350	7	1409.90	91.964
150	3	604.25	85.383	400	8	1611.30	92.973
175	3.5	704.96	86.542	450	9	1812.80	93.812
200	4	805.67	87.630	500	10	2014.20	94.503
225	4.5	906.38	88.547				

In this study, the inspection rate $(r = 50)$was considered a predefined parameter, and the sampling interval was calculated using the relationship $h = \frac{n}{r}$. Accordingly, a corresponding sampling interval was determined for each sample size, and the economic–statistical performance of the EWMA-GLT chart was evaluated. In addition, as described in the previous section, $UCL = 1.2775$ was selected such that the in-control Average Run Length was maintained at approximately $ARL_0 = 201.42$.

The results in Table 4 show that as the sample size increased from $n = 50$to $n = 500$, the value of $ARL_0 = 201.42$remained unchanged, whereas ATS_0increased from 201.42 to 2014.20. Furthermore, the Expected Total Cost (ETC)exhibited an increasing trend, rising from 79.603 for $n = 50$and $h = 1$to 94.503 for $n = 500$and $h = 10$. These results indicate that increasing the sample size and sampling interval under the investigated conditions increased the operational costs

of the control system without providing a substantial improvement in the statistical characteristics of the chart. Therefore, the combination of $n = 50$, $h = 1$, and $UCL = 1.2775$, with the minimum value of $ETC = 79.603$, was selected as the optimal economic–statistical design for the EWMA-GLT chart.

3.6. Results of the optimal economic–statistical design

Table 5. Optimal economic–statistical design results

Optimal (n)	(h)	UCL	(ARL_0)	(ATS_0)	Minimum ETC
50	1.00	1.2775	201.42	201.42	79.603

The results in Table 5 show that the optimal economic–statistical design was obtained with (n=50), (h=1), and (UCL=1.2775). Under this design, the in-control Average Run Length was maintained at the target level (ARL_0=201.42), and the value of (ATS_0) was also equal to 201.42. Furthermore, the minimum Expected Total Cost was calculated as (ETC=79.603). Therefore, this combination was selected as the optimal economic–statistical design for the proposed EWMA-GLT control chart.

3.7. Comparative Analysis of Statistical, Economic, and Economic–Statistical Designs

To investigate the effectiveness of the proposed framework from different perspectives, three design strategies, namely statistical, economic, and economic–statistical approaches, were evaluated and compared. Each design was developed according to a distinct optimization objective. The statistical design prioritized monitoring performance by satisfying predefined statistical criteria, such as $ARL_0$and ATS_0, without explicitly considering economic factors. Conversely, the economic design aimed exclusively at reducing the Expected Total Cost (ETC) and did not impose any statistical performance constraints. The economic–statistical design combined both perspectives by simultaneously minimizing the process cost and ensuring that the required statistical performance criteria were achieved.

Table 6. Comparison of statistical, economic, and economic–statistical designs

Design Type	n	h	UCL	ARL_0	ATS_0	ETC	ATS_0 Constraint Satisfied
Statistical	100	2	1.2775	201.42	402.83	82.536	Yes
Economic	50	1	1.1750	45.95	45.95	78.563	No
Economic–Statistical	50	1	1.2775	201.42	201.42	79.603	Yes

The results presented in Table 6 indicate that the economic design achieved the smallest expected total cost ($ETC = 78.563$). However, its corresponding values of $ARL_0 = 45.95$and $ATS_0 = 45.95$were substantially lower than the desired performance levels, indicating that the required statistical constraints were not satisfied. Consequently, although this design appears economically

attractive, it cannot provide reliable in-control monitoring performance and may generate an excessive number of false alarms.

By contrast, the statistical design successfully satisfied the predefined statistical requirements but resulted in a higher operating cost. The economic–statistical design provided a more balanced solution by maintaining acceptable statistical performance while keeping the overall process cost close to the minimum attainable value. These findings demonstrate that integrating economic and statistical considerations leads to a more practical and efficient monitoring strategy than relying exclusively on either economic or statistical criteria.

To compare the performance of different charts in detecting process changes, the ATS values of the GLT, classical EWMA, EWMA-GLT, and the economic–statistical model were calculated at different shift levels of the λ_{11}^{XY} parameter. The results of this comparison are presented in Figure 2.

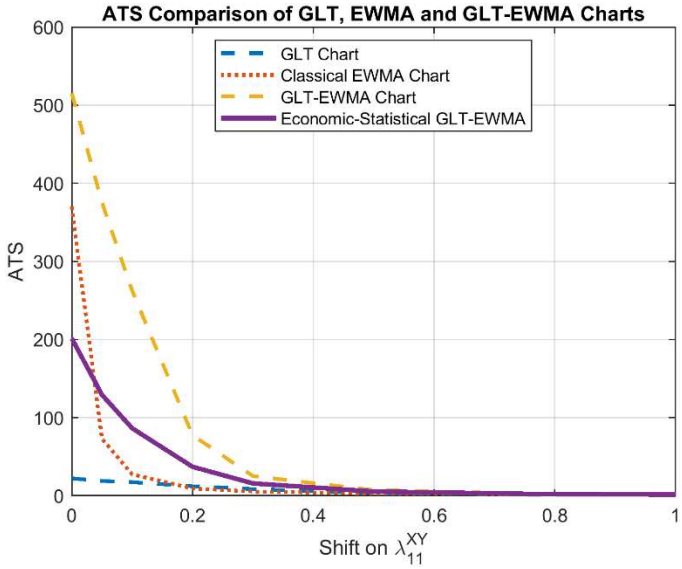


Figure 3.ATS comparison of the GLT, EWMA, GLT–EWMA, and economic–statistical GLT–EWMA charts

As shown in **Figure 3**, the ATS values decrease for all charts as the shift magnitude increases, indicating an improvement in the speed of detecting process changes. It can also be observed that the differences in chart performance are mainly noticeable for small and moderate shifts. In this region, the EWMA-GLT chart and the economic–statistical model exhibit different behavior compared with the GLT and classical EWMA charts. As the shift magnitude increases, the ATS values of all charts converge, and the performance differences gradually diminish. Overall, the results indicate that the proposed method is capable of providing satisfactory process change detection performance while maintaining economic considerations.

To assess the robustness of the proposed EWMA–GLT economic–statistical framework, a sensitivity analysis was conducted by varying the cost-related parameters, including the fixed sampling cost (C_0), variable sampling cost (C_1), false alarm cost (C_f), assignable cause removal cost (C_s), and out-of-control production cost (C_d). For each analysis scenario, the value of a single

cost parameter was adjusted from 0.25 to 2 times its baseline value, and the corresponding optimal design parameters together with the Expected Total Cost (ETC) were recalculated. The outcomes of this analysis are summarized in Table 7.

The findings indicate that the proposed model is particularly sensitive to changes in $C_d$and C_1. Specifically, the ETC varied from 33.93 to 112.89 when $C_d$was modified and from 53.25 to 105.90 when $C_1$was adjusted. In both cases, noticeable variations were also observed in the optimal values of $n$and h. Conversely, changes in $C_0$and $C_s$produced only moderate effects and led to relatively minor modifications in the optimal design under specific cost conditions.

The smallest influence was associated with the false alarm cost parameter (C_f). Although varying this parameter slightly changed the ETC value, ranging only from 79.45 to 79.81, the optimal values of n, h, and UCLremained unchanged. Overall, these results demonstrate that costs related to sampling activities and out-of-control production have the strongest impact on the economic–statistical configuration of the proposed EWMA–GLT chart.

Table 7. Sensitivity analysis of cost parameters

Parameter	Change Factor	n	h	UCL	ARL_0	ATS_0	Minimum ETC
C_0	0.25	50	1			201.42	76.44
C_0	0.50	50	1			201.42	77.49
C_0	0.75	50	1			201.42	78.55
C_0	1.00	50	1			201.42	79.60
C_0	1.25	50	1			201.42	80.66
C_0	1.50	50	1			201.42	81.71
C_0	2.00	75	1.5			302.13	83.44
C_1	0.25	50	1			201.42	53.25
C_1	0.50	50	1			201.42	62.03
C_1	0.75	50	1			201.42	70.82
C_1	1.00	50	1			201.42	79.60
C_1	1.25	50	1			201.42	88.39
C_1	1.50	75	1.5			302.13	96.57
C_1	2.00	500	10			2014.17	105.90
C_f	0.25	50	1			201.42	79.45
C_f	0.50	50	1			201.42	79.50
C_f	0.75	50	1			201.42	79.55
C_f	1.00	50	1	1.2775	201.42	201.42	79.60
C_f	1.25	50	1			201.42	79.66
C_f	1.50	50	1			201.42	79.71
C_f	2.00	50	1			201.42	79.81
C_s	0.25	50	1			201.42	74.54
C_s	0.50	50	1			201.42	76.23
C_s	0.75	50	1			201.42	77.92
C_s	1.00	50	1			201.42	79.60
C_s	1.25	50	1			201.42	81.29
C_s	1.50	50	1			201.42	82.98
C_s	2.00	50	1			201.42	86.35
C_d	0.25	500	10			2014.17	33.93
C_d	0.50	500	10			2014.17	54.12
C_d	0.75	75	1.5			302.13	70.68
C_d	1.00	50	1			201.42	79.60
C_d	1.25	50	1			201.42	87.93
C_d	1.50	50	1			201.42	96.25
C_d	2.00	50	1			201.42	112.89

3.8. Sensitivity analysis results for the EWMA parameters

Since $ARL_0 = 201.42$ and $ATS_0 = 201.42$ remained constant for all investigated combinations, these two indicators were not reported in the final table. This is because, during the sensitivity analysis, the control limits for each combination of $L$and Kwere determined in such a way that ARL_0remained fixed at its reference level. Therefore, the performance of the different combinations was evaluated solely based on $ARL_1$and ETC.

Table 8. Sensitivity Analysis Results for the EWMA Parameters

L	K	UCL	ARL₁	ETC	L	K	UCL	ARL₁	ETC
0.01	1.50	1.1125	153.18	81.907	0.15	2.00	1.3489	34.68	79.478
	1.75	1.1215	182.68	82.476		2.75	1.4579	113.23	81.055
	2.00	1.1306	223.33	83.056	0.20	1.50	1.3134	14.94	77.987
	2.75	1.1577	294.18	84.415		1.75	1.3560	20.27	78.542
0.05	1.50	1.1808	47.08	79.687		2.00	1.3985	28.93	79.184
	1.75	1.2012	61.06	80.183		2.75	1.5261	86.71	80.731
	2.00	1.2217	96.13	80.718	0.25	1.50	1.3476	13.90	77.968
	2.75	1.2830	210.31	82.132		1.75	1.3958	17.78	78.352
0.10	1.50	1.2339	25.62	78.829		2.00	1.4441	23.52	78.914
	1.75	1.2631	35.17	79.374		2.75	1.5888	73.98	80.518
	2.00	1.2924	51.19	79.896	0.30	1.50	1.3798	12.26	77.786
	2.75	1.3802	151.77	81.292		1.75	1.4335	15.80	78.213
0.15	1.50	1.2762	17.41	78.381		2.00	1.4871	21.61	78.646
	1.75	1.3126	24.95	78.862		2.75	1.6479	67.43	80.405

The results in Table 8 show that increasing the smoothing parameter Lconsiderably reduces both $ARL_1$and ETC. For example, at $K = 1.50$, the value of ARL_1decreased from 153.18 for $L = 0.01$to 12.26 for $L = 0.30$, while ETCdecreased from 81.907 to 77.786. This finding indicates that assigning a greater weight to recent observations in the EWMA statistic increases the sensitivity of the chart and improves the speed of detecting process changes.

It can also be observed that, for a fixed value of L, increasing Kleads to higher values of $ARL_1$and ETC. For example, at $L = 0.30$, ARL_1increased from 12.26 at $K = 1.50$to 67.43 at $K = 2.75$,

while ETCincreased from 77.786 to 80.405. This behavior is due to the widening of the control limits and the consequent reduction in the probability of issuing an alarm under out-of-control conditions.

A comparison of the results shows that Lhas a greater impact on chart performance than K. Increasing $L$from 0.01 to 0.30 reduced $ARL_1$by more than 92%, whereas changes in Kmainly produced gradual variations in chart performance.

Among all the investigated combinations, the best economic–statistical performance was obtained for $L = 0.30$and $K = 1.50$, which yielded the lowest values of $ARL_1 = 12.26$and $ETC = 77.786$. Therefore, this combination was selected as the optimal set of parameters for the EWMA-GLT chart.

To simultaneously investigate the effects of the EWMA parameters on the economic performance of the chart, changes in the Expected Total Cost were evaluated for different values of $L$and K.

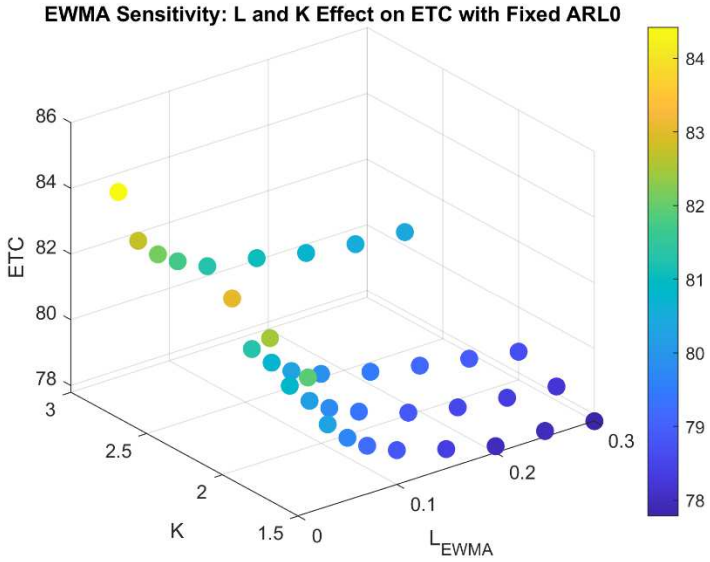


Figure 4.Effects of Land Kon the Expected Total Cost (ETC)

The results in Figure 4 show that increasing Lreduces $ETC$and improves the economic performance of the chart, whereas increasing Kleads to an increase in the Expected Total Cost. The minimum value of $ETC$was observed in the region corresponding to $L = 0.30$ and $K = 1.50$.

3.9. Sensitivity analysis of the smoothing parameter *L* in the EWMA-GLT chart

To investigate the effect of the smoothing parameter Lon the performance of the EWMA-GLT chart, the value of $K$was fixed at its optimal level of 1.50, and different values of Lwere evaluated.

In all cases, $ARL_0$and ATS_0were maintained at 201.42 so that the comparison could be performed solely based on variations in the parameter L. The obtained results are presented in **Table 9**.

Table 9. Sensitivity analysis of the smoothing parameter *L*

L	K	UCL	ARL₁	ETC
0.01	1.50	1.1125	153.18	81.907
0.03		1.1527	64.76	80.288
0.05		1.1808	47.08	79.687
0.07		1.2040	33.46	79.251
0.10		1.2339	25.62	78.829
0.15		1.2762	17.41	78.381
0.20		1.3134	14.94	77.987
0.25		1.3476	13.90	77.968
0.30		1.3798	12.26	77.786

The results in **Table 9** show that increasing Lleads to a decreasing trend in both Mean $ARL_1$and ETC. For example, Mean ARL_1decreased from 153.18 at $L = 0.01$to 12.26 at $L = 0.30$. Similarly, ETCdecreased from 81.907 to 77.786. These results indicate that assigning greater weight to recent observations in the EWMA statistic improves the speed of process change detection and reduces the Expected Total Cost.

In addition, increasing Lcaused UCL to increase from 1.1125 to 1.3798. Despite this increase, the detection performance of the chart improved, and both Mean $ARL_1$and ETCdecreased. Among all the investigated values, $L = 0.30$provided the best performance and yielded the lowest values of Mean $ARL_1$and ETC. Therefore, $L = 0.30$was selected as the optimal value of the smoothing parameter for the EWMA-GLT chart.

3.10. Optimal EWMA-GLT parameters

Table 10. Optimal EWMA-GLT parameters

Optimal L	Optimal K	UCL	ARL_0	ATS_0	Optimal n	Optimal h	Mean ARL_1	ETC
0.30	1.50	1.3798	201.42	201.42	50	1	12.26	77.786

The results in **Table 10** show that the best economic–statistical performance of the EWMA-GLT chart was obtained for the combination $L = 0.30$and $K = 1.50$. Under this setting, $UCL = 1.3798$was obtained, while $ARL_0 = 201.42$and $ATS_0 = 201.42$were maintained at their target levels. In addition, the optimal design parameters were $n = 50$and $h = 1$, indicating that small sample sizes and short sampling intervals are preferable under the investigated conditions.

From the perspective of detection performance, this combination yielded the lowest value of Mean $ARL_1 = 12.26$, indicating that the chart can detect process changes more rapidly than the other investigated combinations. Furthermore, the minimum value of $ETC = 77.786$was also obtained for this setting, demonstrating satisfactory economic performance.

Therefore, the combination $L = 0.30$and $K = 1.50$was selected as the optimal set of parameters for the EWMA-GLT chart because it simultaneously provides desirable statistical properties, the minimum Expected Total Cost, and the fastest detection capability.

It should be noted that $UCL = 1.2775$ corresponds to the baseline GLT chart, whereas the UCL values reported for the EWMA-GLT chart were calculated using the EWMA-specific formulation based on the values of $L$and K. Therefore, the difference between these UCL values is due to the different structures of the control statistics and does not indicate any inconsistency in the results.

3.11. Comparison of GLT and EWMA-GLT statistics with their corresponding control limits

To investigate the behavior of the proposed EWMA-GLT statistic and compare it with the GLT statistic, the values of both statistics were calculated and plotted for 100 consecutive samples under identical conditions. In addition, the corresponding control limits for each chart were presented to evaluate the process monitoring performance, as shown in the figure below.

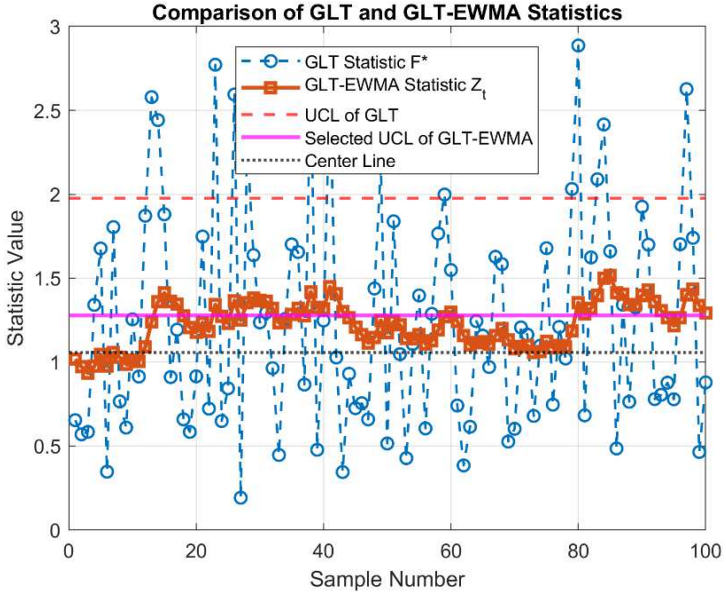


Figure 5.Comparison of GLT and EWMA-GLT Statistics with Their Corresponding Control Limits

As shown in **Figure 5**, the GLT statistic exhibits relatively large fluctuations, and its values experience considerable variations throughout the sampling process. In contrast, the EWMA-GLT statistic demonstrates a more stable and smoother behavior due to the exponential smoothing mechanism. It can also be observed that the control limit of the EWMA-GLT chart is lower than that of the GLT chart, which increases the sensitivity of the chart to small and gradual process changes. The results indicate that combining the GLT statistic with the EWMA structure not only reduces random fluctuations but also improves the capability of rapidly detecting process changes.

4. Conclusions and Future Research Directions

This study proposed an integrated economic–statistical EWMA–GLT framework for monitoring multivariate categorical processes represented by contingency tables. The dependency structure among categorical variables was modeled using a log-linear model, and the Generalized Linear Test (GLT) statistic was integrated into an Exponentially Weighted Moving Average (EWMA) structure to improve the detection of small and gradual process shifts.

The results demonstrated that the proposed framework effectively detects structural changes in contingency tables. As the shift magnitude increased, the ARL values decreased considerably, indicating improved detection performance. In addition, the sensitivity analysis showed that the interaction parameters of the log-linear model have a substantially greater influence on chart performance than the overall and main effects, highlighting the capability of the proposed chart to identify changes in the dependency structure among categorical variables.

From an economic–statistical perspective, the optimal design was obtained for (n=50), (h=1), and (UCL=1.2775), yielding (ARL_0=201.42) and (ETC=79.603). Furthermore, combining economic and statistical criteria provided a more balanced and practically applicable solution than relying solely on either economic or statistical design approaches.

Overall, the proposed EWMA–GLT framework can serve as an efficient decision-support tool for industrial environments involving multiple correlated categorical quality characteristics. Future studies may extend the framework to higher-dimensional contingency tables, compare its performance with other memory-type control charts, incorporate uncertainty through fuzzy or robust approaches, and investigate metaheuristic optimization methods using real industrial datasets.

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Funding

This research received no external funding.

Author contributions

K.A. conceived the study, developed the methodology, performed the simulations, analyzed the results, prepared the figures and tables, and wrote the original draft of the manuscript. R.K. supervised the research, contributed to the development of the methodology, critically revised the manuscript, and approved the final version of the manuscript. All authors reviewed and approved the final manuscript.

Data availability

The MATLAB codes and the simulated datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.

Competing interests

The authors declare no competing interests.