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Ideological Tolerance, Voter Abstention, and Party Polarization: An Agent-Based Extension of the Hotelling Model

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Abstract

This paper develops an agent-based extension of the Hotelling model in which voter participation depends on ideological distance. We introduce a tolerance threshold that determines whether voters abstain when candidates are too far from their preferred positions. The model shows that this participation constraint can destabilize the classical median voter equilibrium. When tolerance is low and voter preferences are concentrated, candidates tend to diverge and compete for localized voter groups. As tolerance increases or voter preferences become more dispersed, convergence toward the center re-emerges. The results also reveal non-monotonic relationships between voter dispersion and candidate polarization, indicating shifts between median-focused and peripheral competition. These findings provide a micro-foundation linking voter abstention to political polarization.

Keywords: Hotelling model, voter abstention, political polarization, agent-based simulation

1.Introduction

The classical Hotelling spatial competition model predicts that competing political candidates converge toward the median voter. However, recent political developments in many democratic systems have shown increasing ideological polarization rather than convergence. This empirical pattern raises questions about the explanatory power of traditional spatial voting models.

One possible explanation lies in voter participation behavior. While classical models typically assume universal turnout, in reality, voters may abstain when all available candidates are perceived as too distant from their own ideological preferences.

Such abstention behavior can fundamentally reshape the strategic incentives faced by political candidates.

This paper introduces a distance-based abstention mechanism into a one-dimensional Hotelling framework. Voters are assumed to participate only when at least one candidate lies within a tolerance threshold of their ideal point. In addition, voter preferences are modeled using a truncated normal distribution rather than the standard uniform assumption.

Using agent-based simulations, this study examines how voter dispersion and ideological tolerance jointly influence candidate positioning and turnout outcomes. The results suggest that abstention mechanisms can destabilize the classical median voter equilibrium and generate conditions under which political polarization emerges. This provides a potential micro-foundation for understanding divergence in contemporary electoral competition.

This paper makes three contributions.

First, it introduces a participation constraint into the spatial competition framework, linking voter abstention to ideological distance.

Second, it demonstrates that this mechanism can destabilize the median voter equilibrium and generate divergent candidate positioning.

Third, it identifies non-monotonic and regime-dependent relationships between voter dispersion and political polarization.

2.Literature Review

The spatial theory of electoral competition provides a foundational framework for understanding strategic candidate positioning in democratic elections. In the classical model developed by Duncan Black (1948) and later formalized by Anthony

Downs (1957), voters are assumed to select the candidate whose ideological position lies closest to their own preference on a one-dimensional policy spectrum. Under conditions of majority rule and universal voter participation, this framework leads to the well-known median voter theorem, which predicts that competing candidates converge toward the ideological position of the median voter.

Subsequent research has explored the conditions under which this convergence result may fail. Extensions of the spatial voting model have introduced multidimensional policy spaces, party activists, and institutional constraints that can produce divergent equilibrium positions (John H. Aldrich 1983; Gary W. Cox 1990). More broadly, the spatial voting tradition has been further formalized and extended through analytical treatments of voter preferences and candidate strategies (James M. Enelow & Melvin J. Hinich 1984; Melvin J. Hinich & Michael C. Munger 1997).

A second line of research focuses on voter participation and turnout behavior. Classical spatial models typically assume universal participation, but empirical research suggests that turnout varies systematically across individuals and groups. Rational choice models emphasize the costs and benefits of voting and the role of mobilization in shaping participation decisions (Steven J. Rosenstone & John Mark Hansen 1993; John H. Aldrich 1993). Other theoretical work highlights strategic uncertainty in large elections (Thomas Palfrey & Howard Rosenthal 1985) and the role of ethical or expressive motivations in voter participation (John Feddersen & Alvaro Sandroni 2006).

These insights have important implications for candidate strategy. If voters who feel ideologically distant from available candidates are less likely to participate, turnout patterns may alter the incentives facing political parties. Under such conditions, candidates may find it advantageous to target specific ideological constituencies rather than converge toward the median voter.

A related literature examines the causes and consequences of political polarization in contemporary democracies. Scholars have documented increasing

ideological polarization in both elite politics and mass public opinion (Nolan McCarty, Keith T. Poole, & Howard Rosenthal 2006). Other research suggests that the ideological center of the electorate may be weakening in some political systems, contributing to more fragmented electoral competition (Alan Abramowitz 2010; Morris Fiorina 2008).

In recent years, computational approaches have increasingly been used to explore complex political dynamics that are difficult to analyze using purely analytical models. Agent-based modeling allows researchers to simulate heterogeneous populations and strategic interactions among political actors (Robert Axelrod 1997; Joshua Epstein 2006; Lars-Erik Cederman 2003; Nigel Gilbert 2008). Such models have proven particularly useful for examining emergent phenomena such as political polarization, institutional dynamics, and collective behavior.

Despite these advances, relatively little research has systematically examined how participation thresholds based on ideological distance may reshape spatial electoral competition. If voters participate only when at least one candidate lies within a tolerable ideological distance from their preferred position, the strategic incentives of candidates may shift away from the classical median voter equilibrium. Under such conditions, electoral competition may instead encourage candidates to focus on mobilizing ideologically proximate voter clusters.

The present study contributes to this literature by developing an agent-based extension of the Hotelling spatial competition model in which voter turnout depends on ideological distance. By simulating candidate adaptation across repeated elections, the model explores how voter dispersion and ideological tolerance jointly influence candidate positioning and turnout outcomes.

3.1 Model Specification

The traditional Hotelling model of spatial competition assumes that consumers or voters are evenly distributed along a continuous interval $[0, 1]$, representing either

a linear city or a one-dimensional ideological spectrum. Political candidates or economic competitors choose a single point on this line as their political stance or product location. Voters or consumers then select the candidate or firm that is closest to them according to the principle of minimum distance.

Under this framework, the equilibrium outcome of the dynamic game is that the two candidates (or firms) converge toward the midpoint of the interval. Voters or consumers located on both sides of the midpoint are then evenly divided between the two candidates/firms.

When the two competitors converge toward the same position, the marginal utility approaches zero, implying that neither party has an incentive to further adjust their position.

While the original Hotelling model provides a useful explanation for the convergence tendency observed in two-party competition, recent developments in Western democratic politics—both in two-party and multi-party systems—have shown increasing political polarization. Candidates often adopt strategies that diverge from their opponents rather than converging toward the median voter position predicted by the traditional Hotelling model. This phenomenon poses a challenge to the explanatory power of the original model.

3.2 Model Modifications

This study introduces two modifications to the traditional Hotelling framework.

The “Ideological tolerance threshold”

The proposed “ideological tolerance threshold” assumes that when the ideological distance between a candidate's political position and a voter's preference exceeds a certain threshold, the voter chooses to abstain rather than vote for the candidate. This abstention is interpreted as a form of political silence expressing voter dissatisfaction or anger.

Voter Positions

Voters are sampled from a truncated normal distribution over the interval $[0, 1]$with mean $0.5$and a variable standard deviation σ. Values falling outside the interval are truncated to the boundary values.

Voting Rule

A voter considers voting for a candidate only if the ideological distance between them is less than or equal to a threshold r.

- If both candidates fall within the threshold r, the voter chooses the closer candidate (ties are split evenly).
- If neither candidate falls within the threshold, the voter abstains.

Candidate Strategy

Candidates adopt an alternating local best-response strategy. At each step, a candidate tests a small movement to the left or right (step size ε). If moving in a particular direction increases their vote share, they move in that direction. This process repeats until convergence or until a maximum number of steps is reached.

Parameter Scan

The simulation explores the following parameter ranges:

- $\sigma \in \{0.03, 0.08, 0.15, 0.25\}$
- $r \in \{0.05, 0.12, 0.25\}$

For each parameter combination, the simulation is repeated multiple times and the results are averaged.

Other parameters include:

- Number of voters: $N = 2000$
- Maximum repetitions per configuration: $reps = 4$
- Maximum number of dynamic steps: $T = 300$

5.Simulation Results and Interpretation

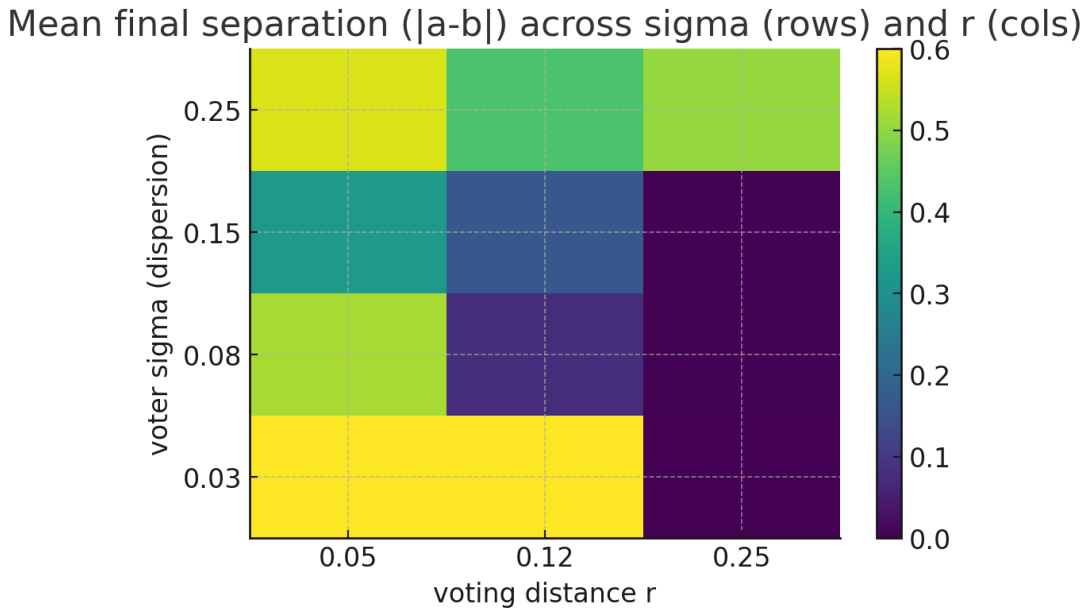


Figure 1

Figure 1 presents the averaged simulation results in a two-dimensional space where the **voting distance threshold** r is plotted on the horizontal axis and the **standard deviation of voter distribution** σ is plotted on the vertical axis.

When the voter distribution is highly concentrated (small σ) and the voting distance threshold r is small, the two parties tend to maintain a larger ideological distance from each other. In this situation, capturing nearby voters yields clear benefits, making differentiation a more advantageous strategy.

In contrast, when the voter distribution becomes more dispersed (larger σ) and the voting distance threshold r is larger, candidates are more likely to move toward the ideological center, approaching the behavior predicted by the traditional Hotelling model.

The voting distance threshold r has a particularly strong effect on the equilibrium outcome.

- When r is **small**, voters consider candidates only when they are very close to their own ideological position. This increases the likelihood of **political**

divergence between candidates and lowers the overall level of electoral participation.

- When $r$is **large**, more voters are willing to compare the two candidates, which encourages **convergence toward the center** and increases overall participation.

Interestingly, regardless of whether we vary voter distribution while holding the voting distance constant, or vary the voting distance while holding voter distribution constant, the resulting changes in the ideological distance between the two parties are neither uniform nor monotonic.

For example, when the voting distance is fixed at $r = 0.05$and the standard deviation of the voter distribution gradually increases, the ideological distance between the two parties first decreases and then increases, with the minimum occurring approximately around $(r = 0.05, \sigma = 0.15)$. This pattern may indicate that as the voter distribution becomes wider, the strategic competition between the two parties shifts between targeting median voters and targeting marginal voters. Initially, parties prioritize appealing to median voters, but as the distribution widens further, competition increasingly shifts toward voters located at the ideological margins.

Similarly, when the voter distribution is fixed at $\sigma = 0.25$, the ideological distance between the two parties as a function of the voting distance threshold $r$also exhibits a non-monotonic pattern, first decreasing and then increasing. This suggests that changes in the voting distance threshold can alter the strategic balance between competing for centrist voters and peripheral voters.

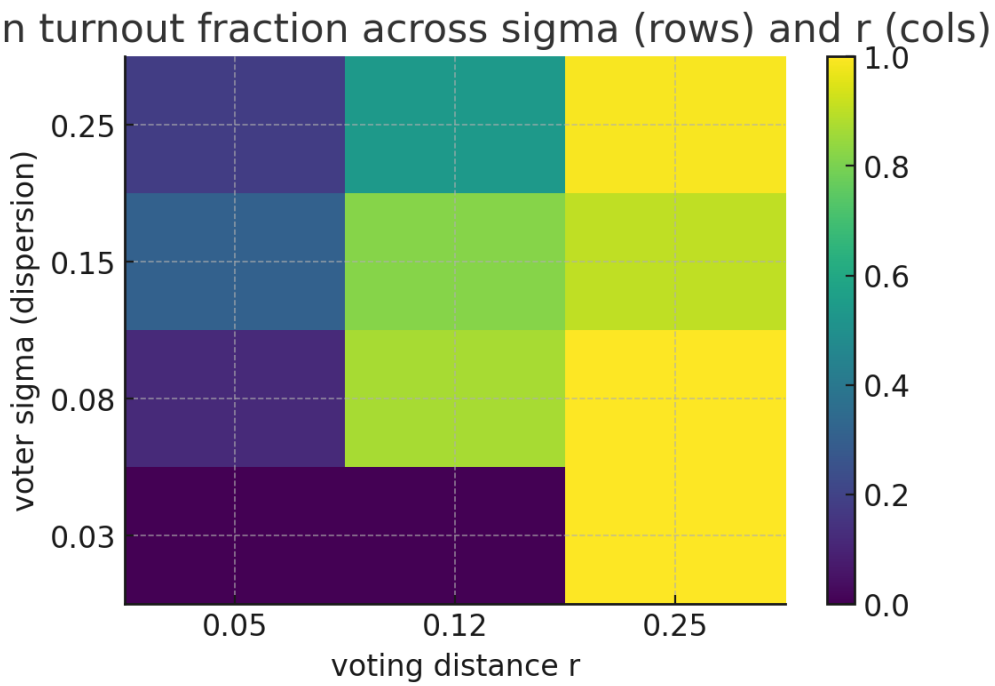


Figure 3 Average Total Turnout under Different (σ, r)Combinations

Figure 3 illustrates how the overall level of voter participation changes as both the voter distribution (σ) and the voting distance threshold (r) vary.

In general, a smaller voting distance threshold leads to lower average turnout, which is intuitively reasonable. A smaller $r$implies that voters are more selective about which candidates they consider acceptable, thereby reducing the number of voters who ultimately participate in the election.

However, a particularly interesting pattern emerges when examining turnout as a function of voter dispersion. For a given voting distance threshold r, total turnout exhibits a non-monotonic relationship with σ, reaching an interior extremum around σ = 0.15. Notably, these extrema differ across values of r: the minimum turnout observed under high voting distance can still exceed the maximum turnout observed under low voting distance. One possible interpretation is that, under both high and low voting-distance conditions, the dominant drivers of turnout alternate between marginal voters and median voters. In other words, the relative importance of these two voter groups

shifts depending on the tolerance threshold for ideological distance, and the direction of this shift differs under different voting-distance regimes.

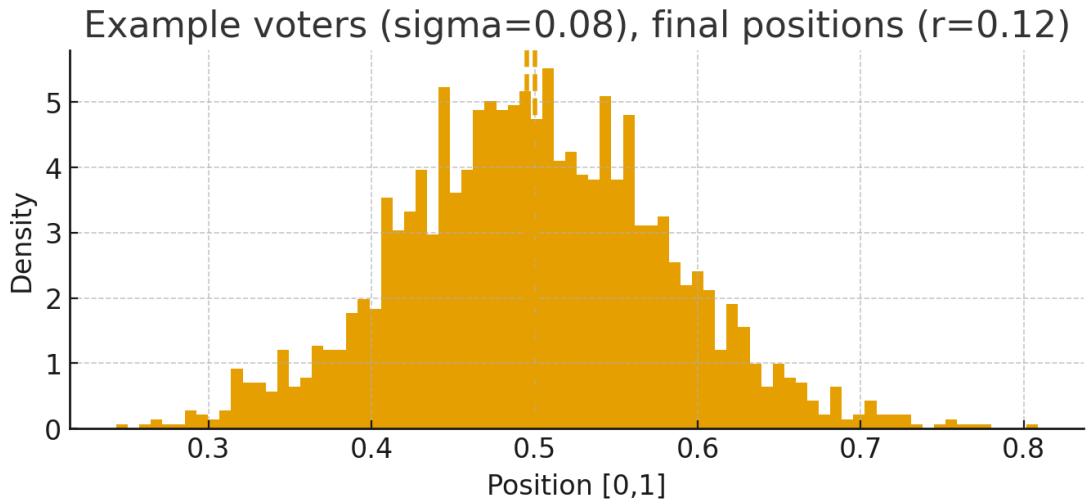


Figure 4 Example Case: Final Equilibrium Positions of Candidates ($\sigma = 0.08, r = 0.12$)

Figure 4 presents the final equilibrium positions of the two candidates in an illustrative simulation case where $\sigma = 0.08$and $r = 0.12$.

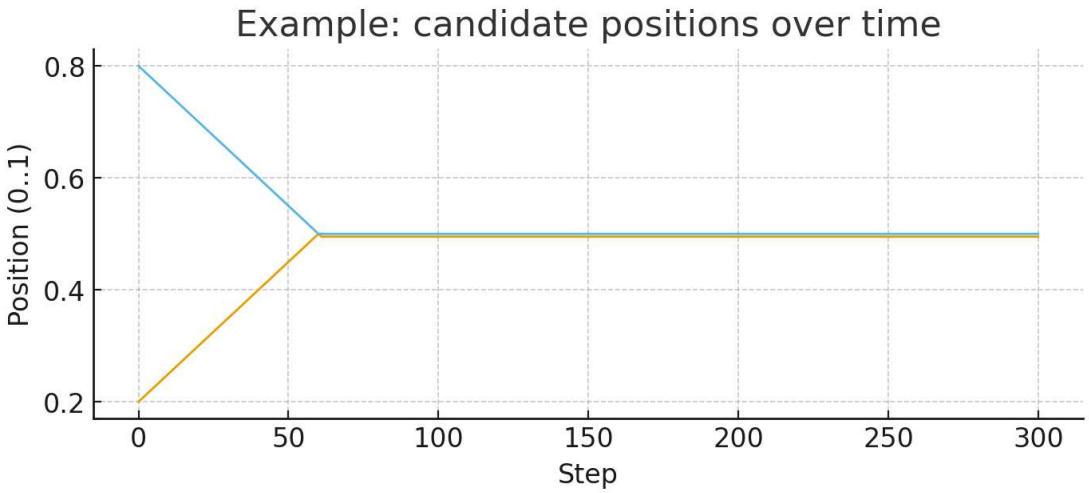


Figure 4-1 Dynamic Convergence Process

Figure 4-1 illustrates the dynamic process through which the two candidates converge to equilibrium. Since similar convergence patterns appear under other parameter configurations, these additional cases are not presented in detail.

6.Robustness Checks

We conducted a series of robustness checks to assess the sensitivity of our model to key parameters. First, we varied the dispersion of voter preferences (σ). Larger σ, representing more dispersed electorates, tended to produce more polarized equilibrium positions for the candidates. Conversely, very small σ, where voters are highly concentrated near the center, sometimes resulted in negligible turnout if candidates were initially far from the voter mass, preventing candidates from effectively adjusting their positions. Second, we examined the effect of the voting threshold (r). Smaller r values, which restrict voters' willingness to support distant candidates, reduced turnout and constrained candidate movement, whereas larger r values increased turnout and pulled candidates toward the voter center. Third, we evaluated the impact of sample size (N). Small N generated noisy, uneven voter samples, causing unstable equilibrium positions and larger candidate separation, while larger N produced smoother distributions, leading to more stable candidate positions closer to the electorate center. Finally, increasing the number of repetitions (reps) had little effect on the average outcomes, indicating that stochastic variability was not a major source of instability. Overall, our model exhibits robust behavior within a reasonable parameter range (moderate σ, r, and sufficiently large N), while extreme parameter values or small sample sizes can induce substantial fluctuations in equilibrium outcomes.

7.Conclusion

This paper develops an agent-based extension of the classical Hotelling model by incorporating a distance-based voter participation mechanism. By introducing an ideological tolerance threshold, the model relaxes the standard assumption of universal turnout and allows voter abstention to emerge endogenously from ideological mismatch.

The results demonstrate that participation constraints can fundamentally reshape the strategic landscape of electoral competition. Rather than consistently converging toward the median voter, candidate positioning exhibits regime-dependent dynamics. When ideological tolerance is low, candidates tend to diverge and compete for localized voter clusters, leading to persistent political polarization. As tolerance increases, convergence toward the center re-emerges, restoring the classical prediction of the Hotelling framework.

In addition, the relationship between voter dispersion and candidate polarization is shown to be non-monotonic. Changes in voter distribution do not produce uniform shifts in equilibrium outcomes but instead generate transitions between distinct competitive regimes, characterized by varying emphasis on median versus peripheral voters.

Robustness checks further indicate that these patterns are stable across a wide range of parameter settings, although extreme parameter values and small sample sizes may introduce instability in candidate positioning.

Overall, this study provides a micro-foundation linking voter abstention to political polarization and highlights the importance of endogenous turnout in spatial models of electoral competition. The findings suggest that incorporating participation constraints is essential for understanding divergence in contemporary democratic systems.

Future research could extend this framework by incorporating multidimensional policy spaces, heterogeneous candidate strategies, or dynamically evolving voter preferences, thereby offering a more comprehensive account of political competition in complex electoral environments.

8. Implications and Limitations

In the simulations presented in this study, both the voter distribution and the voting distance threshold are treated as exogenous variables. However, it is necessary to further consider the potential endogeneity of these variables and their implications in real-world political contexts.

In practice, the degree of similarity between the policy positions chosen by two competing parties—represented in the model by the proximity of their final equilibrium positions—may itself influence voters' willingness to support either party. When the political platforms of the two parties become excessively similar, some voters may interpret this convergence as a form of elite collusion or a staged alternation of power between the two parties. Such perceptions may intensify voter dissatisfaction or "anger."

Under these circumstances, voters may adopt several different responses. Some voters may engage in strategic punishment voting, choosing to support a party that is ideologically more distant from their own preferences in order to punish their previously preferred party. Others may express dissatisfaction through political abstention, refusing to vote and thereby weakening the electoral base of the established parties. In more extreme situations, voters may even turn to extra-institutional or anti-system political actions, thereby departing from the democratic framework altogether.

Another important factor is voters' political sensitivity, referred to in this paper as the voting distance threshold. This threshold represents how close a candidate's ideological position must be to a voter's preference for the voter to consider supporting that candidate. In reality, such sensitivity may depend strongly on voters' retrospective evaluations of previous governments.

For example, if an ideologically extreme voter believes that the party they supported in the previous election has successfully implemented policies that protect their interests, that voter's political sensitivity may remain stable or even decrease. In other words, the voter may become more willing to support relatively moderate candidates. Conversely, if the voter feels dissatisfied with the previous government's

performance, they may lose confidence in moderate parties. As a result, they may either withdraw from electoral participation or shift their support toward smaller parties representing more extreme interests, if such alternatives exist.

The concentration or dispersion of voter distribution—represented by the standard deviation σ—can also be interpreted as an indicator of the degree of political fragmentation within a society. Ideally, voters would be concentrated near the center of the ideological spectrum (i.e., a relatively small σ), indicating a society characterized by relatively moderate and shared political views. Under such conditions, political competition between parties may focus more on policy details and practical governance issues rather than fundamental ideological conflicts.

However, the degree of voter concentration or dispersion may itself be influenced by various structural factors, such as the effectiveness of governance, the degree of political communication and propaganda, the intensity of identity politics, and levels of education within society.

In extreme situations, the distribution of voters may not resemble a normal distribution at all. Instead, it may take the form of a bimodal or multimodal distribution, indicating deep social polarization. Under such conditions, party strategies would become significantly more complex, and the political agenda of competing parties may become increasingly fragmented and polarized.

Ultimately, party competition in parliamentary or electoral politics can be understood as a reflection of the underlying structure of voter preferences and political consciousness. Party strategies rarely evolve independently of the electorate. If party politics were to become completely detached from voter preferences, it would likely fall outside the scope of modern democratic politics altogether.

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