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# Controllability Analysis of Nonlinear Coal Mill Pulverizer Model

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**Abstract**

Coal mill pulverizers are critical components in coal-fired power plants, and their dynamic behavior strongly influences combustion efficiency, emission control, and plant stability. This paper develops a nonlinear controllability framework for such systems by integrating operator theory, fixed-point methods, and nonlinear functional analysis. The general results are then applied to a detailed coal mill model that captures the coupled dynamics of raw coal, pulverized coal, and classifier transport. We analyze multiple control configurations and show that full nonlinear controllability is achieved only when raw coal feed rate and classifier speed are used as control inputs, with the primary air flow held constant. This result highlights the limited influence of primary air on linear controllability and the critical role of classifier dynamics. The findings provide theoretical justification for practical controller design in power plant operation, offering guidelines for improving combustion stability and energy efficiency.

**Keywords-** Coal mill pulverizer, Nonlinear control, Controllability analysis, Fixed point theorem, Power plant

**AMS Subject Classification-** 93B05, 34H05, 93C10, 93C15

## 1 Introduction

The core question of whether a dynamical system can be guided from an arbitrary initial state to a desired final state within a finite time interval using an appropriate control input is known as controllability, and it forms the basis of contemporary control theory. Biomedical engineering (e.g., insulin delivery for blood sugar regulation), defense systems (e.g., missile guidance and interception), aerospace engineering (e.g., satellite orbit adjustments, aircraft stabilization), economics (e.g., managing inflation rates), and industrial processes (e.g., optimizing power plant operations) are just a few of the many applications where this property is essential [1]. Controllability analysis is an essential technique for both theoretical and applied research since the ability to regulate a system guarantees operational accuracy, safety, and efficiency.

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In 1963, Kalman pioneered the study of controllability by characterizing the controllability of linear time-invariant systems in state-space form using linear functional analysis [2]. In his groundbreaking work, Kalman proposed a computationally tractable criterion: a linear system is controllable if the controllability matrix has full rank. Researchers like Brockett [4] later expanded this approach to time-varying linear systems in order to handle the intricacies of time-dependent dynamics. Nonlinear systems, which are common in real-world applications because of their intricate and linked dynamics, came into attention as control theory developed. To convert controllability issues into solvability questions for nonlinear equations, researchers used fixed point theorems and nonlinear functional analysis [5, 6, 7, 8, 10, 12, 13]. These approaches often involve iterative schemes to compute control inputs that achieve the desired state transitions.

Operator semigroup theory has proved crucial for systems that are described by partial differential equations (PDEs), such as those in heat transfer or fluid dynamics. Controllability analysis is made possible by semigroups, which convert PDEs into abstract differential equations in Banach or Hilbert spaces [9, 11]. Quinn and Carmichael's use of degree theory, fixed point methods, and pseudo-inverses for semilinear abstract systems [14], George's investigation of approximate controllability [16], and Cioranescu et al.'s exact controllability results for wave equations [17] are noteworthy contributions. Russell, Zhang, and Chalishajar have investigated the controllability of higher-order systems, including third-order dispersion equations [18, 19]. The exact controllability of semilinear systems has been improved by more recent studies by Bashirov [20], Arora et al. [21], and Leiva [22]. These works use piecewise control techniques and strong contractive conditions to manage nonlinear perturbations.

The controllability of a general nonlinear system with the following form is the main topic of this paper:

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{F}(t, x(t), u(t)) + \mathcal{B}u(t), \\ x(t_0) &= x_0, \end{aligned} \tag{1.1}$$

where $x(t) \in \mathbb{X}$ (a Hilbert space) is the state, $u(t) \in L^2([t_0, t_1], \mathbb{U})$ is the control input, $\mathcal{A}$ is a linear operator generating a strongly continuous semigroup, $\mathcal{B}$ is a bounded linear operator, and $\mathcal{F}$ is a nonlinear function capturing system nonlinearities. Using fixed point theorems and nonlinear functional analysis, we obtain adequate requirements for controllability, guaranteeing that, under suitable assumptions, the system may be guided to any desired state.

We examine the controllability of a nonlinear model of a coal mill pulverizer, a crucial part of coal-fired power plants, as a real-world application [24]. Raw coal is ground into fine particles by the pulverizer, whose operation must be carefully managed to preserve ideal combustion conditions, reduce emissions, and guarantee plant effectiveness. Because of the interactions between coal masses, air flow, and classifier speed, the model, which is controlled by a system of nonlinear differential equations, displays complicated dynamics. We provide insights into efficient control techniques for power plant operations by proving that the system is controllable under particular control configurations by converting the model into the form of (1.1) and applying our controllability conditions.

The paper is organized as follows: Section2 reviews the controllability of linear systems, establishing the foundation for the nonlinear analysis. Section3 develops the controllability conditions for nonlinear systems. Section4 applies these conditions to the coal mill pulverizer model, analyzing different control configurations. Section 5 concludes with a discussion of the results and future research directions.

## 2 Preliminaries

The controllability of a nonlinear system of the form (1.1) often hinges on the controllability properties of its corresponding linear system. In this section, we introduce the mathematical framework for analyzing the controllability of linear systems, which serves as the foundation for our nonlinear analysis. We define key operators, derive the system solution, and present theorems characterizing controllability, with an emphasis on their physical and mathematical significance.

Consider the linear system:

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{B}u(t), \\ x(t_0) &= x_0, \end{aligned} \tag{2.1}$$

defined over the interval $[t_0, t_1]$. Here:

- $x(t) \in \mathbb{X}$: The state vector, where $\mathbb{X}$ is a Hilbert space (e.g., $\mathbb{R}^n$ for finite-dimensional systems or a function space for infinite-dimensional systems). Physically, $x(t)$ represents quantities such as positions, velocities, or masses (e.g., coal masses in a pulverizer).
- $u(t) \in L^2([t_0, t_1], \mathbb{U})$: The control input, where $\mathbb{U}$ is a Hilbert space (e.g., $\mathbb{R}^m$). The space $L^2$ ensures that the control is square-integrable, reflecting energy constraints in practical systems like coal flow rates.
- $\mathcal{A} : \mathbb{X} \to \mathbb{X}$: A linear operator, typically the infinitesimal generator of a strongly continuous semigroup $\mathcal{T}(t)$. In physical systems, $\mathcal{A}$ models the natural dynamics (e.g., grinding rates in a coal mill).
- $\mathcal{B} : \mathbb{U} \to \mathbb{X}$: A bounded linear operator mapping control inputs to state dynamics. For example, $\mathcal{B}$ might represent how coal feed rate affects coal mass.
- $x_0 \in \mathbb{X}$: The initial state at time $t_0$.

The solution to (2.1), known as the mild solution, accounts for both the free dynamics (driven by $\mathcal{A}$) and the forced response (driven by $u(t)$):

$$x(t) = \mathcal{T}(t - t_0)x_0 + \int_{t_0}^{t} \mathcal{T}(t - s)\mathcal{B}u(s)\, ds, \tag{2.2}$$

where $\mathcal{T}(t)$ is the semigroup generated by $\mathcal{A}$. The semigroup $\mathcal{T}(t)$ describes the evolution of the system in the absence of control inputs, analogous to the natural decay or oscillation of coal masses in a pulverizer. The integral term represents the cumulative effect of the control input, weighted by the system's response at different times.

**Definition 2.1.** *The system* (2.1) *is* controllable *on* $[t_0, t_1]$ *if, for every initial state* $x_0 \in \mathbb{X}$ *and desired final state* $x_1 \in \mathbb{X}$*, there exists a control* $u(t) \in L^2([t_0, t_1], \mathbb{U})$ *such that the solution* (2.2) *satisfies* $x(t_1) = x_1$*.*

Physically, controllability implies that operators can adjust inputs (e.g., coal feed rate) to achieve desired outcomes (e.g., specific coal particle masses) within a finite time.

To analyze controllability, define the controllability operator $\mathcal{C} : L^2([t_0, t_1], \mathbb{U}) \to \mathbb{X}$:

$$\mathcal{C}u = \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}u(s)\, ds, \tag{2.3}$$

which maps the control input to its effect on the state at time $t_1$. The adjoint operator $\mathcal{C}^* : \mathbb{X} \to L^2([t_0, t_1], \mathbb{U})$ is:

$$\mathcal{C}^*\omega = \mathcal{B}^*\mathcal{T}^*(t_1 - t)\omega, \tag{2.4}$$

where $\mathcal{B}^*$ and $\mathcal{T}^*$ are the adjoints of $\mathcal{B}$ and $\mathcal{T}$, respectively. The controllability Gramian $\mathcal{W} : \mathbb{X} \to \mathbb{X}$ is defined as:

$$\mathcal{W} = \mathcal{C}\mathcal{C}^*, \quad \mathcal{W}\omega = \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}\mathcal{B}^*\mathcal{T}^*(t_1 - s)\omega \, ds. \tag{2.5}$$

The Gramian $\mathcal{W}$ quantifies the energy required to control the system, with its invertibility being central to controllability.

The following theorem, adapted from Curtain and Zwart [23], provides equivalent conditions for the controllability of (2.1).

**Theorem 2.1.** *The system* (2.1) *is controllable on* $[t_0, t_1]$ *if any of the following equivalent conditions hold for some* $\gamma > 0$*, for all* $\omega \in \mathbb{X}$*:*

*(a)* *Range*$(\mathcal{C}) = \mathbb{X}$*: The controllability operator spans the entire state space, meaning any state can be reached.*

*(b)* $\|\mathcal{C}^*\omega\|_{\mathbb{X}}^2 \geq \gamma^2\|\omega\|_{\mathbb{X}}^2$*: The adjoint operator is bounded below, ensuring sufficient control authority.*

*(c)* $\langle \mathcal{W}\omega, \omega \rangle \geq \gamma^2\|\omega\|_{\mathbb{X}}^2$*: The Gramian is positive definite, indicating that all states are controllable with finite energy.*

*(d)* $\int_{t_0}^{t_1} \|\mathcal{B}^*\mathcal{T}^*(t_1 - s)\omega\|_{\mathbb{U}}^2 \, ds \geq \gamma^2\|\omega\|_{\mathbb{X}}^2$*: The control effect is persistently strong over the interval.*

*(e)* $\ker(\mathcal{C}^*) = \{0\}$ *and Range*$(\mathcal{C}^*)$ *is closed: The adjoint has no non-trivial kernel, ensuring injectivity.*

*The control that steers* $x_0$ *to* $x_1$ *at* $t = t_1$ *is:*

$$u(t) = \mathcal{C}^*\mathcal{W}^{-1}\left[x_1 - \mathcal{T}(t_1 - t_0)x_0\right]. \tag{2.6}$$

*Proof.* The equivalence of conditions (a)–(e) follows from the properties of bounded linear operators in Hilbert spaces. Condition (a) directly implies controllability, as $\mathcal{C}u = x_1 - \mathcal{T}(t_1 - t_0)x_0$ must have a solution for any $x_1$. Condition (b) ensures that $\mathcal{C}^*$ is injective, which, combined with the closed range condition in (e), implies that $\mathcal{C}$ is surjective. Condition (c) confirms that $\mathcal{W}$ is invertible, a necessary condition for solving (2.6). Condition (d) is equivalent to (c) by the definition of $\mathcal{W}$. The control law (2.6) is derived by solving $\mathcal{C}u = x_1 - \mathcal{T}(t_1 - t_0)x_0$, using the pseudo-inverse of $\mathcal{C}$. □

The following lemma establishes a key property of the controllability operator, which will be used in the nonlinear analysis.

**Lemma 2.2.** *If the system* (2.1) *is controllable on* $[t_0, t_1]$*, the operator* $\mathcal{S} : \mathbb{X} \to L^2([t_0, t_1], \mathbb{U})$*, defined by:*

$$(\mathcal{S}\omega)(t) = \mathcal{B}^*\mathcal{T}^*(t_1 - t)\mathcal{W}^{-1}\omega, \tag{2.7}$$

*is a right inverse of* $\mathcal{C}$*, i.e.,* $\mathcal{C} \circ \mathcal{S} = \mathcal{I}$*.*

*Proof.* To verify that $\mathcal{C} \circ \mathcal{S} = \mathcal{I}$, compute:

$$(\mathcal{C} \circ \mathcal{S})\omega = \mathcal{C}(\mathcal{S}\omega) = \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}(\mathcal{S}\omega)(s)\, ds.$$

Substitute $(\mathcal{S}\omega)(s) = \mathcal{B}^*\mathcal{T}^*(t_1 - s)\mathcal{W}^{-1}\omega$:

$$(\mathcal{C} \circ \mathcal{S})\omega = \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}\mathcal{B}^*\mathcal{T}^*(t_1 - s)\mathcal{W}^{-1}\omega\, ds.$$

From (2.5), the integral is:

$$\int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}\mathcal{B}^*\mathcal{T}^*(t_1 - s)\omega\, ds = \mathcal{W}\omega.$$

Thus:

$$(\mathcal{C} \circ \mathcal{S})\omega = \mathcal{W}\mathcal{W}^{-1}\omega = \omega,$$

proving that $\mathcal{C} \circ \mathcal{S} = \mathcal{I}$. □

In the context of the coal mill pulverizer, the linear system (2.1) might represent a simplified model of coal mass dynamics, where $x(t)$ includes masses of raw and pulverized coal, and $u(t)$ includes coal feed rate and classifier speed. The controllability conditions ensure that operators can adjust these inputs to achieve desired coal distributions, a prerequisite for analyzing the full nonlinear model in subsequent sections.

## 3 Controllability of Nonlinear System

In this section, we have discussed the controllability of semilinear system

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{F}(t, x(t), u(t)) + \mathcal{B}u(t), \\ x(t_0) &= x_0 \end{aligned} \tag{3.1}$$

over the interval $[t_0, t_1]$. Here $x(t) \in \mathbb{X}$ is the state of the system for all $t$, $u(t)$ is the controller of the system, $\mathcal{F}$ is a nonlinear function, $\mathcal{A}$ and $\mathcal{B}$ are the linear operators. Assuming $\mathcal{F}$ is good enough so that the system (3.1) has the unique solution of the form

$$x(t) = \mathcal{T}(t - t_0)x_0 + \int_{t_0}^{t} \mathcal{T}(t - s)\mathcal{B}u(s)ds + \int_{t_0}^{t} \mathcal{T}(t - s)\mathcal{F}(s, x(s), u(s))ds, \tag{3.2}$$

for all $u \in L^2([t_0, t_1], \mathbb{U})$.

The system (3.1) is said to be controllable if for every $x_0, x_1 \in \mathbb{X}$ there exists a steering function $u(t)$ such that the state of the system satisfies $x(t_0) = x_0$ and $x(t_1) = x_1$. This means the mild solution at $x(t_1)$ of the system becomes

$$x_1 = x(t_1) = \mathcal{T}(t_1 - t_0)x_0 + \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}u(s)ds + \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{F}(s, x(s), u(s))ds.$$

If we define the operator $\mathcal{G} : L^2([t_0, t_1], \mathbb{U}) \to \mathbb{X}$ by

$$\mathcal{G}u = \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{B}u(s)ds + \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{F}(s, x(s), u(s))ds, \tag{3.3}$$

then the system (3.1) is controllable if the operator $\mathcal{G}$ is onto.

Now, assuming the controllability of the corresponding linear system

$$\begin{aligned} x'(t) &= \mathcal{A}x(t) + \mathcal{B}u(t), \\ x(t_0) &= x_0 \end{aligned}$$

over the interval $[t_0, t_1]$ and define an operator $\bar{\mathcal{G}} : \mathbb{X} \to \mathbb{X}$ by

$$\bar{\mathcal{G}}\zeta = (\mathcal{G} \circ \mathcal{S})\zeta.$$

As the corresponding linear system is controllable therefore

$$\begin{aligned} \bar{\mathcal{G}}_k\zeta &= \zeta + \int_{t_0}^{t_1} \mathcal{T}(t_1 - s)\mathcal{F}(s, x_\zeta(s), (S\zeta)(s))ds, \\ &= (\mathcal{I} + \mathcal{H})\zeta. \end{aligned} \tag{3.4}$$

where $x_\zeta(\cdot)$ is the mild solution of the equation (3.2) corresponding to $u(t) = \mathcal{B}^*\mathcal{T}^*(t_1 - t)\mathcal{W}^{-1}\zeta$.

**Lemma 3.1.** *The system* (3.1) *is controllable over the interval* $[t_0, t_1]$ *if the operator* $\bar{\mathcal{G}} = (\mathcal{I} + \mathcal{H})$ *is invertible. The controller which steers the initial state* $x_0$ *to the desired final state* $x_1$ *at* $t = t_1$ *is given by* $u(t) = \mathcal{C}^*\mathcal{W}^{-1}\bar{\mathcal{G}}^{-1}[x_1 - \mathcal{T}_\alpha(t_1 - t_0)x_0]$

*Proof.* Since, the operator $\bar{\mathcal{G}}$ is non-singular therefore plugin the controller into the equation (3.2) the state of the system at $t = t_1$ becomes:

$$\begin{aligned} x(t_1) &= \mathcal{T}_\alpha(t - t_0)x_0 + \mathcal{G}u \\ &= \mathcal{T}_\alpha(t - t_0)x_0 + (\mathcal{I} + \mathcal{H})(\mathcal{C}\mathcal{C}^*)\mathcal{W}^{-1}(\mathcal{I} + \mathcal{H})^{-1}[x_1 - \mathcal{T}_\alpha(t_1 - t_0)x_0] \\ &= x_1 \end{aligned}$$

Thus the system is controllable over the interval $[t_0, t_1]$. □

Therefore, the controllability of the system (3.1) is reduced to the invertibility of the operator $\bar{\mathcal{G}}$. The forgoing theorem derives the conditions under which the operator $\bar{\mathcal{G}}$ is non-singular.

**Theorem 3.2.** *If the operator* $\mathcal{H}^{(n)}$ *is a contraction for some* $n \geq 1$ *then* $\bar{\mathcal{G}}$ *is non-singular.*

*Proof.* If $\mathcal{H}^{(n)}$ is a contraction for some $n \geq 1$ therefore by Banach fixed point theorem the operator equation $\omega = -\mathcal{H}\omega$ has a unique solution. This implies the equation $(\mathcal{I}+\mathcal{H})\omega = 0$ has a unique trivial solution. Hence, the operator $\bar{\mathcal{G}}$ is non-singular. □

The next theorem discusses the controllability of the nonlinear system (3.1).

**Theorem 3.3.** *If*

*(1) the corresponding linear system is controllable,*

*(2) there exist constants* $f_1$ *and* $f_2$ *such that* $||\mathcal{F}(t, x_1, u_1) - \mathcal{F}(t, x_2, u-2)|| \leq f_1||x_1 - x_2|| + f_2||u_1 - u_2||$ *for all* $x, u \in \mathbb{B}_{r_0}(x_0, u_0)$ *for some* $r_0 > 0$ *where,* $u_0(t) = \mathcal{B}^*\mathcal{T}^*(t_1 - t)\mathcal{W}^{-1}\big[x_1 - \mathcal{T}(t_1 - t_0)x_0\big]$,

*then the nonlinear system* (3.1) *is controllable over* $[t_0, t_1]$ *and controller which drives the system* (3.1) *to desired final state* $x_1$ *at* $t = t_1$ *is given by time* $t = t_1$ *is given by* $u(t) = \mathcal{C}^*\mathcal{W}^{-1}\bar{\mathcal{G}}^{-1}[x_1 - \mathcal{T}_\alpha(t_1 - t_0)x_0]$.

*Proof.* To prove the system is controllable it is sufficient to prove that the operator $\mathcal{H}^{(n)}$ is contraction for some $n \geq 1$. Therefore for $x_1, x_2, u_1, u_2 \in \mathbb{B}(x_0, u_0)$ and consider

$$\begin{aligned}
||\mathcal{H}^{(n)}x_1 - \mathcal{H}^{(n)}x_2|| &\leq \int_{t_0}^{t_1} ||\mathcal{T}||\ ||\mathcal{H}^{(n-1)}x_1 - \mathcal{H}^{(n-1)}x_2||ds, \\
&\leq M^2(f_1 + f_2S)\int_{t_0}^{t_1}\int_{t_0}^{t_1}(||\mathcal{H}^{(n-2)}x_1 - \mathcal{H}^{(n-2)}x_2||ds_1ds, \\
&\leq M^3(f_1 + f_2S)^2\int_{t_0}^{t_1}\int_{t_0}^{t_1}||\mathcal{H}^{(n-3)}x_1 - \mathcal{H}^{(n-3)}x_2||ds_2ds_1ds,
\end{aligned}$$

and continuing this process to get

$$\begin{aligned}
||\mathcal{H}^{(n)}x_1 - \mathcal{H}^{(n)}x_2|| &\leq M^n(f_1 + f_2S)^{(n)}\int_{t_0}^{t_1}\frac{(t_1 - s)^{n-1}}{(n-1)!}ds||x_1 - x_2|| \\
&\leq \frac{M^n(f_1 + f_2S)^{(n)}(t_1 - t_0)^n}{n!}||x_1 - x_2||,
\end{aligned}$$

where $S$ is bound for the steering operator $\mathcal{S}$ and is finite as the corresponding linear system is controllable.

Since $\leq \frac{M^n(f_1+f_2S)^{(n)}(t_1-t_0)^n}{n!} \to 0$ as $n \to \infty$ therefore there exist $n$ such that the operator $\mathcal{H}$ is contraction. Hence, the system (3.1) is controllable and the controller $u(t)$ which drives the system (3.1) desired final state to $x_1$ is given by $u(t) = \mathcal{C}^*\mathcal{W}^{-1}\bar{\mathcal{G}}^{-1}[x_1 - \mathcal{T}_\alpha(t_1 - t_0)x_0]$. This completes the proof. □

The algorithm to get a controller and a state is given by

$$\begin{aligned}
u^{(n)}(t) &= \mathcal{C}^*\mathcal{W}^{-1}(\mathcal{I} + \mathcal{H})^{-1}[x_1 - \mathcal{T}_\alpha(t_1 - t_0)x_0], \\
x^{(n+1)}(t) &= \mathcal{T}(t - t_0)x_0 + \int_{t_0}^{t}\mathcal{T}(t - s)\big[\mathcal{B}u^{(n)}(s) + \mathcal{F}(s, x^{(n)}(s), u^{(n)}(s))\big]ds, \\
\mathcal{H}^{(n)}x_1^{(n)} &= \int_{t_0}^{t_1}\mathcal{T}(t_1 - s)\mathcal{F}(s, x^{(n-1)}(s), \mathcal{S}x^{(n-1)}(s))ds.
\end{aligned} \tag{3.5}$$

where, $u_0(t) = \mathcal{B}^*\mathcal{T}^*(t_1 - t)\mathcal{W}^{-1}\big[x_1 - \mathcal{T}(t_1 - t_0)x_0\big]$.

# 4 Controllability Analysis of the Nonlinear Coal Mill Pulverizer Model

The coal mill pulverizer is a vital component in coal-fired power plants, responsible for grinding raw coal into fine particles that can be efficiently combusted in boilers to generate steam for electricity production. Its operation directly impacts combustion efficiency, boiler performance, and overall plant reliability. Achieving precise control over the pulverizer's dynamics is essential to maintain optimal coal particle size, ensure stable combustion, and minimize energy losses. In this section, we apply the nonlinear controllability framework developed in Section 3 to analyze a mathematical model of the coal mill pulverizer, as derived by Niemczyk et al. [24]. The model, expressed as a system of nonlinear differential equations, captures the dynamic interplay of coal masses and control inputs. We transform the system into the form of Equation (3.1) and evaluate its controllability under various control configurations, leveraging the conditions of Theorem 3.3, which require the corresponding linear system to be controllable and the nonlinear term to satisfy a Lipschitz condition.

### 4.1 Physical Operation and Mathematical Model

The coal mill pulverizer operates through a series of interconnected processes. Raw coal is fed into the mill via a conveyor, where it accumulates on a rotating grinding table. Rollers or balls crush the coal into fine particles, which are then lifted by a stream of primary air to a classifier. The classifier, rotating at a controlled speed, separates fine particles suitable for combustion, allowing them to exit the mill to the boiler, while returning coarser particles to the grinding table for further processing. The system's performance depends on the masses of coal at different stages and the control inputs that regulate coal feed, air flow, and classifier operation.

The mill's particle flow is depicted in the layout in Figure 1. The mathematical equations were developed for a mill's nominal grinding operation, but they also accurately represent the dynamics of start-up and shut-down.

As shown in Fig.1, the coal particle circulation is the primary component of the model.

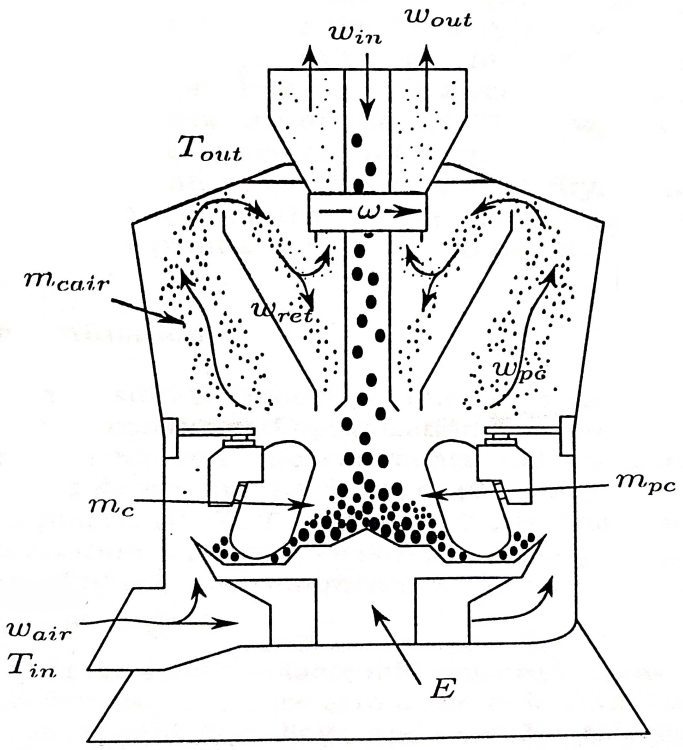



Figure 1: Coal particles flow in a Roll Wheel coal mill [25]

The mathematical model, as presented by Niemczyk et al. [24], describes these dynamics through a system of nonlinear ordinary differential equations:

$$\begin{aligned}
\frac{d}{dt}m_c(t) &= w_c(t) + \theta_9 m_{cair}(t) - \theta_1 m_c(t), \\
\frac{d}{dt}m_{pc}(t) &= \theta_1 m_c(t) - \theta_5 w_{air}(t) m_{pc}(t), \\
\frac{d}{dt}m_{cair}(t) &= \theta_5 w_{air}(t) m_{pc}(t) - \theta_4 m_{cair}(t)\left(1 - \frac{\omega(t)}{\theta_6}\right) - \theta_9 m_{cair}(t),
\end{aligned} \tag{4.1}$$

where the variables and parameters have the following physical meanings:

- $m_c(t)$ (kg): Mass of raw, unpulverized coal on the grinding table, representing the coal awaiting grinding. This mass increases with the input coal flow and decreases as coal is pulverized.
- $m_{pc}(t)$ (kg): Mass of pulverized coal on the grinding table, consisting of fine particles ready to be transported by primary air. This mass grows as raw coal is ground and diminishes as particles are lifted.
- $m_{cair}(t)$ (kg): Mass of pulverized coal particles suspended in the pneumatic transport system, route to the classifier. This includes particles that may be returned to the table if deemed too coarse.
- $w_c(t)$ (kg/s): Mass flow rate of raw coal fed into the mill, a control input that operators adjust to regulate the coal supply based on boiler demand.
- $w_{air}(t)$ (kg/s): Mass flow rate of primary air, a control input that controls the transport of pulverized coal from the table to the classifier and influences drying and particle lift.
- $\omega(t)$ (rad/s): Angular velocity of the classifier, a control input that determines the centrifugal force separating fine particles (sent to the boiler) from coarse particles (returned to the table).
- $\theta_1$ (1/s): Rate constant for the pulverization process, reflecting the efficiency of grinding raw coal into pulverized form. Typical values depend on mill design and coal type.
- $\theta_4$ (1/s): Rate constant for the transport of coal particles out of the pneumatic system, influenced by air flow and mill geometry.
- $\theta_5$ (1/kg): Scaling factor for the effect of primary air flow on transporting pulverized coal, representing the efficiency of air-driven particle lift.
- $\theta_6$ (rad/s): Reference classifier speed, used to normalize the effect of $\omega(t)$ on particle separation. It reflects the classifier's design characteristics.
- $\theta_9$ (1/s): Rate constant for the return of coarse particles from the pneumatic transport to the raw coal mass, accounting for classifier feedback.

The nonlinearity in (4.1) stems from the product terms $w_{air}(t)m_{pc}(t)$ and $m_{cair}(t)\omega(t)$, which model the interaction between control inputs and state variables, making the system challenging to control.

### 4.2 State-Space Formulation

To apply the controllability framework from Section 3, we reformulate the system (4.1) in state-space form, aligning it with the general nonlinear system:

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{F}(t, x(t), u(t)), \tag{4.2}$$

where $x(t) \in \mathbb{X}$ (a Hilbert space, here $\mathbb{R}^3$) is the state vector, $u(t) \in L^2([t_0, t_1], \mathbb{U})$ is the control vector, $\mathcal{A}$ is the linear state matrix, $\mathcal{B}$ is the control input matrix, and $\mathcal{F}$ captures the nonlinear terms. Define the state vector:

$$x(t) = [x_1(t), x_2(t), x_3(t)]^T, \quad x_1(t) = m_c(t), \quad x_2(t) = m_{pc}(t), \quad x_3(t) = m_{cair}(t),$$

and the control vector:

$$u(t) = [u_1(t), u_2(t), u_3(t)]^T, \quad u_1(t) = w_c(t), \quad u_2(t) = w_{air}(t), \quad u_3(t) = \omega(t).$$

The system (4.1) is then rewritten as:

$$\begin{aligned} \dot{x}_1(t) &= -\theta_1 x_1(t) + \theta_9 x_3(t) + u_1(t), \\ \dot{x}_2(t) &= \theta_1 x_1(t) - \theta_5 u_2(t) x_2(t), \\ \dot{x}_3(t) &= -(\theta_4 + \theta_9) x_3(t) + \theta_5 u_2(t) x_2(t) - \frac{\theta_4}{\theta_6} x_3(t) u_3(t). \end{aligned} \tag{4.3}$$

The nonlinear terms $\theta_5 u_2(t) x_2(t)$ and $\frac{\theta_4}{\theta_6} x_3(t) u_3(t)$ reflect the physical coupling between air flow and pulverized coal transport, and classifier speed and particle separation, respectively. Controllability analysis, per Theorem 3.3, requires that the linear system $\dot{x} = \mathcal{A}x + \mathcal{B}u$ is controllable (i.e., the controllability matrix has full rank) and that $\mathcal{F}$ satisfies a Lipschitz condition:

$$\|\mathcal{F}(t, x_1, u_1) - \mathcal{F}(t, x_2, u_2)\| \leq f_1 \|x_1 - x_2\| + f_2 \|u_1 - u_2\|.$$

## 4.3 Controllability Analysis

To determine the controllability of (4.3), we evaluate four control configurations, each using a different subset of the inputs $u_1(t), u_2(t), u_3(t)$. For each case, we: 1. Express the system in the form of (4.2). 2. Compute the controllability matrix $\mathcal{W} = [\mathcal{B}, \mathcal{A}\mathcal{B}, \mathcal{A}^2\mathcal{B}]$ for the linear system. 3. Verify if $\text{rank}(\mathcal{W}) = 3$, the dimension of the state space. 4. Check the Lipschitz condition for $\mathcal{F}$ when the linear system is controllable.

### 4.3.1 Case 1: All Three Controllers

Using all controls, we formulate (4.3) as (4.2) with:

$$\mathcal{A} = \begin{bmatrix} -\theta_1 & 0 & \theta_9 \\ \theta_1 & 0 & 0 \\ 0 & 0 & -(\theta_4 + \theta_9) \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{F}(x, u) = \begin{bmatrix} 0 \\ -\theta_5 u_2 x_2 \\ \theta_5 u_2 x_2 - \frac{\theta_4}{\theta_6} x_3 u_3 \end{bmatrix}.$$

The matrix $\mathcal{B}$ reflects that only $u_1(t)$ (raw coal flow) directly affects the state $x_1(t)$. Compute the controllability matrix:

$$\mathcal{A}\mathcal{B} = \mathcal{A} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\theta_1 \\ \theta_1 \\ 0 \end{bmatrix}, \quad \mathcal{A}^2\mathcal{B} = \mathcal{A} \begin{bmatrix} -\theta_1 \\ \theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \theta_1^2 \\ -\theta_1^2 \\ 0 \end{bmatrix}.$$

Thus:

$$\mathcal{W} = \begin{bmatrix} 1 & -\theta_1 & \theta_1^2 \\ 0 & \theta_1 & -\theta_1^2 \\ 0 & 0 & 0 \end{bmatrix}.$$

The third row is zero, so $\text{rank}(\mathcal{W}) = 2 < 3$. The linear system is not controllable, as the control inputs cannot influence $x_3(t)$ (pneumatic coal mass) directly through the linear dynamics. Physically, this suggests that the raw coal flow alone cannot regulate the entire system, particularly the transport of particles to the classifier. Since the linear system fails the controllability condition, Theorem 3.3 does not apply, and we cannot guarantee nonlinear controllability.

#### 4.3.2 Case 2: Controllers $u_1$ and $u_2$

Fix $u_3(t) = 0.0001$, representing a nearly constant classifier speed, to focus on raw coal flow and primary air flow. The system becomes:

$$\begin{aligned}
\dot{x}_1(t) &= -\theta_1 x_1(t) + \theta_9 x_3(t) + u_1(t), \\
\dot{x}_2(t) &= \theta_1 x_1(t) - \theta_5 u_2(t) x_2(t), \\
\dot{x}_3(t) &= -(\theta_4 + \theta_9 + 0.0001\frac{\theta_4}{\theta_6}) x_3(t) + \theta_5 u_2(t) x_2(t).
\end{aligned}$$

Formulate as (4.2):

$$\mathcal{A} = \begin{bmatrix} -\theta_1 & 0 & \theta_9 \\ \theta_1 & 0 & 0 \\ 0 & 0 & -(\theta_4 + \theta_9 + 0.0001\frac{\theta_4}{\theta_6}) \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{F}(x,u) = \begin{bmatrix} 0 \\ -\theta_5 u_2 x_2 \\ \theta_5 u_2 x_2 \end{bmatrix}.$$

The controllability matrix is:

$$\mathcal{AB} = \begin{bmatrix} -\theta_1 \\ \theta_1 \\ 0 \end{bmatrix}, \quad \mathcal{A}^2\mathcal{B} = \begin{bmatrix} \theta_1^2 \\ -\theta_1^2 \\ 0 \end{bmatrix}, \quad \mathcal{W} = \begin{bmatrix} 1 & -\theta_1 & \theta_1^2 \\ 0 & \theta_1 & -\theta_1^2 \\ 0 & 0 & 0 \end{bmatrix}.$$

The rank is 2, as the third row is zero, indicating that the linear system is not controllable. Physically, fixing the classifier speed limits the ability to control particle separation, and the primary air flow $u_2(t)$ only affects $x_2(t)$ and $x_3(t)$ indirectly through nonlinear terms, insufficient for full state control. Thus, nonlinear controllability cannot be established.

#### 4.3.3 Case 3: Controllers $u_2$ and $u_3$

Fix $u_1(t) = 0.25$, representing a constant raw coal feed rate. The system is:

$$\begin{aligned}
\dot{x}_1(t) &= -\theta_1 x_1(t) + \theta_9 x_3(t) + 0.25, \\
\dot{x}_2(t) &= \theta_1 x_1(t) - \theta_5 u_2(t) x_2(t), \\
\dot{x}_3(t) &= -(\theta_4 + \theta_9) x_3(t) + \theta_5 u_2(t) x_2(t) - \frac{\theta_4}{\theta_6} x_3(t) u_3(t).
\end{aligned}$$

The constant term 0.25 in the first equation introduces an affine term, preventing the system from being expressed in the form of (4.2), which assumes a purely linear and nonlinear combination of states and controls. This affine term reflects a steady coal input, but it disrupts the structure required for the controllability analysis in Section 3. Physically, a fixed coal feed rate limits the ability to dynamically adjust $x_1(t)$, making it challenging to steer the system to arbitrary states. Thus, the methodology is inapplicable in this case.

#### 4.3.4 Case 4: Controllers $u_1$ and $u_3$

Fix $u_2(t) = 0.02$, representing a constant primary air flow rate, to focus on raw coal flow and classifier speed. The system becomes:

$$\begin{aligned}
\dot{x}_1(t) &= -\theta_1 x_1(t) + \theta_9 x_3(t) + u_1(t), \\
\dot{x}_2(t) &= \theta_1 x_1(t) - 0.02\theta_5 x_2(t), \\
\dot{x}_3(t) &= -(\theta_4 + \theta_9) x_3(t) + 0.02\theta_5 x_2(t) - \frac{\theta_4}{\theta_6} x_3(t) u_3(t).
\end{aligned}$$

Formulate as (4.2):

$$\mathcal{A} = \begin{bmatrix} -\theta_1 & 0 & \theta_9 \\ \theta_1 & -0.02\theta_5 & 0 \\ 0 & 0.02\theta_5 & -(\theta_4+\theta_9) \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{F}(x,u) = \begin{bmatrix} 0 \\ 0 \\ -\frac{\theta_4}{\theta_6} x_3 u_3 \end{bmatrix}.$$

Compute the controllability matrix:

$$\mathcal{AB} = \begin{bmatrix} -\theta_1 \\ \theta_1 \\ 0 \end{bmatrix}, \quad \mathcal{A}^2\mathcal{B} = \mathcal{A} \begin{bmatrix} -\theta_1 \\ \theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \theta_1^2 \\ -\theta_1^2 - 0.02\theta_1\theta_5 \\ 0.02\theta_1\theta_5 \end{bmatrix}.$$

Thus:

$$\mathcal{W} = \begin{bmatrix} 1 & -\theta_1 & \theta_1^2 \\ 0 & \theta_1 & -\theta_1^2 - 0.02\theta_1\theta_5 \\ 0 & 0 & 0.02\theta_1\theta_5 \end{bmatrix}.$$

Assuming $\theta_1, \theta_5 \neq 0$, the matrix is triangular with non-zero diagonal elements, so $\text{rank}(\mathcal{W}) = 3$. The linear system is controllable, as the raw coal flow $u_1(t)$ and the fixed air flow enable influence over all states through the system dynamics.

Next, verify the Lipschitz condition for $\mathcal{F}$:

$$\mathcal{F}(x,u) - \mathcal{F}(y,v) = \begin{bmatrix} 0 \\ 0 \\ -\frac{\theta_4}{\theta_6}(x_3u_3 - y_3v_3) \end{bmatrix}.$$

The norm is:

$$\|\mathcal{F}(x,u) - \mathcal{F}(y,v)\| = \frac{\theta_4}{\theta_6}|x_3u_3 - y_3v_3| \leq \frac{\theta_4}{\theta_6}|u_3^0|\|x_3 - y_3\| + \frac{\theta_4}{\theta_6}|x_3^0||u_3 - v_3|,$$

where $u_3^0$ and $x_3^0$ are nominal values near the operating condition (e.g., typical classifier speed and pneumatic coal mass). Since $\|x_3 - y_3\| \leq \|x - y\|$ and $|u_3 - v_3| \leq \|u - v\|$, we have:

$$\|\mathcal{F}(x,u) - \mathcal{F}(y,v)\| \leq f_1\|x - y\| + f_2\|u - v\|, \quad f_1 = \frac{\theta_4}{\theta_6}|u_3^0|, \quad f_2 = \frac{\theta_4}{\theta_6}|x_3^0|.$$

This satisfies the Lipschitz condition. By Theorem 3.3, the nonlinear system is controllable, with the control law:

$$u(t) = \mathcal{C}^*\mathcal{W}^{-1}\bar{\mathcal{G}}^{-1}[x_1 - \mathcal{T}(t_1 - t_0)x_0],$$

where $x_1$ is the desired final state, and $\mathcal{T}(t)$ is the semigroup generated by $\mathcal{A}$.

### 4.4 Discussion

According to the controllability analysis, the coal mill pulverizer model can only be controlled in Case 4 with a fixed primary air flow ($u_2 = 0.02$) and raw coal flow ($u_1$) and classifier speed ($u_3$). With this setup, operators can guide the system in a limited amount of time from an initial state (such as a certain coal mass distribution) to a desired state (such as the ideal coal particle mass for burning). Physically, the classifier's separation efficiency is adjusted by $u_3(t)$ and affects $m_{cair}(t)$, while $u_1(t)$ directly regulates the raw coal input, impacting $m_c(t)$. The linear system may cover the whole state space because of the constant transport dynamics provided by the fixed air flow.

The failure of Cases 1 and 2 demonstrates how primary air flow $u_2(t)$ only emerges in nonlinear terms, indicating its minimal influence in linear dynamics. When all controls are used in Case 1, the single-column $\mathcal{B}$ matrix prevents linear access to $x_3(t)$ by limiting direct control to $x_1(t)$. Fixing $u_3(t)$ in Case 2 does not change the system structure enough to reach full rank. Because of the practical limitation of a fixed coal feed rate, which restricts dynamic changes to $m_c(t)$, the affine term in Case 3 breaks the necessary form.

These findings practically inform the design of coal mill control systems. For instance, operators can control coal input by $w_c(t)$ and fine-tune particle size by $\omega(t)$ while keeping a constant $w_{air}(t)$ in order to obtain the desired $m_{pc}(t)$ for stable combustion. The qualitative insight is still strong even though the typical parameter values (e.g., $\theta_1 \approx 0.1\,\mathrm{s}^{-1}$ for grinding rate, $\theta_5 \approx 0.01\,\mathrm{kg}^{-1}$ for air transport efficiency) depend on coal qualities and mill design. By ensuring that operators can adapt to changing boiler needs, controllability improves efficiency and lowers emissions.

Here, Figure 2 presents the state trajectories of the nonlinear coal mill pulverizer model for the controllable Case 4 configuration. The simulation is performed using the raw coal feed rate and classifier speed as control inputs while maintaining a constant primary air flow rate.

The state $x_1(t)$, representing the raw coal mass on the grinding table, exhibits an oscillatory but bounded response throughout the simulation interval. This behavior indicates the continuous regulation of the coal feed process by the control input. Although oscillations are present, the state remains within a finite operating region, confirming stable system dynamics.

The state $x_2(t)$, corresponding to the pulverized coal mass, shows a gradual increase over time and approaches a higher operating level. This behavior demonstrates the accumulation and transport of pulverized coal generated through the grinding process. The smooth evolution of $x_2(t)$ indicates effective coordination between coal feeding and classifier action.

The state $x_3(t)$, representing the coal particles suspended in the pneumatic transport system, decreases rapidly and approaches zero within a short time interval. This result suggests that the classifier and transport mechanism efficiently remove coal particles from the air stream, preventing excessive accumulation inside the mill.

Overall, all state variables remain bounded during the simulation period, indicating stable system behavior. The obtained trajectories validate the theoretical controllability results established in Case 4, demonstrating that the selected control configuration is capable of influencing all system states and maintaining satisfactory operating conditions.

Here, Figure 3 shows the control inputs applied to the nonlinear coal mill pulverizer model. The control signal $u_1(t)$, representing the raw coal feed rate, follows a bounded sinusoidal profile, while $u_3(t)$, corresponding to the classifier speed, exhibits a bounded periodic variation.

The oscillatory nature of the control inputs allows continuous adjustment of the coal grinding and classification processes. Throughout the simulation interval, both control signals remain within finite limits and do not exhibit excessive amplitudes, indicating physically realizable control actions.

The bounded control effort demonstrates that the desired system response can be achieved without requiring impractically large control inputs. This observation is important from an operational perspective, since excessive variations in coal feed rate or classifier speed may adversely affect plant performance and equipment life.

The control input profiles successfully regulate the system states and support the controllability of the nonlinear coal mill pulverizer model. The results confirm that the combination of raw coal feed rate and classifier speed provides sufficient control authority to steer the system dynamics while maintaining stable operation.

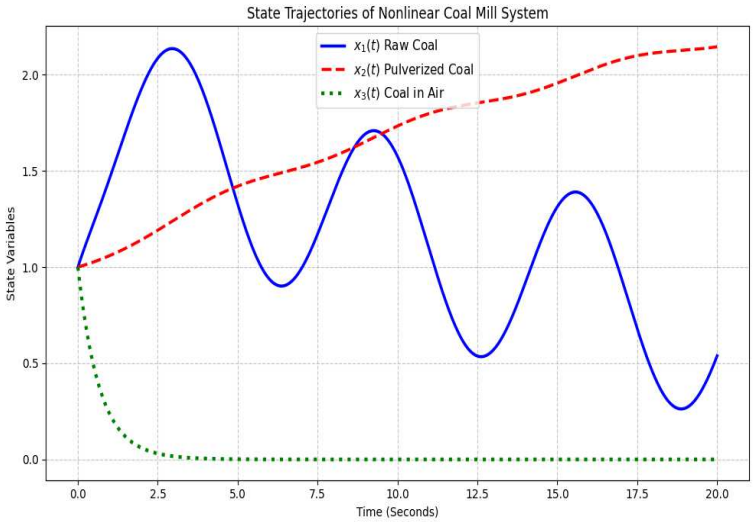



Figure 2: State Trajectories of Non Linear Coal Mill System

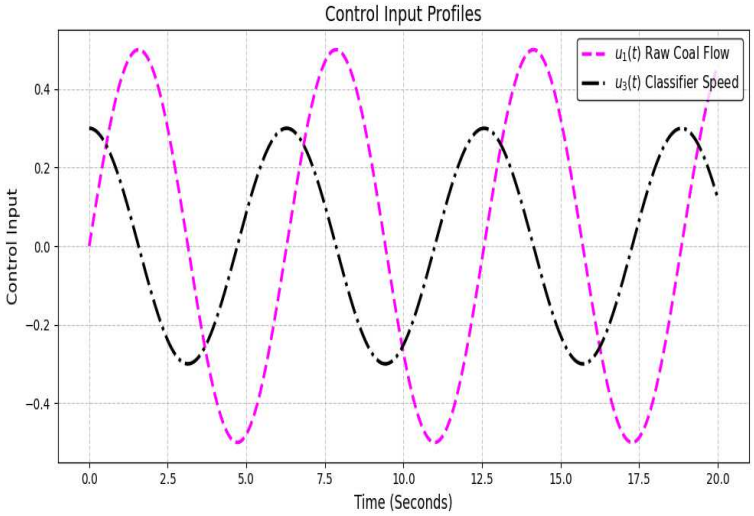



Figure 3: Control Inputs

Table 1: Results of controllability analysis for different control configurations of the coal mill pulverizer model.

| **Case** | **Control Inputs Used** | **Rank of Controllability Matrix** | **Lipschitz Condition** | **Controllable?** | **Physical Interpretation** |
|---|---|---|---|---|---|
| 1 | $u_1$ (raw coal flow), $u_2$ (air flow), $u_3$ (classifier speed) | 2 ($< 3$) | – | No | Raw coal flow alone directly influences states; cannot regulate pneumatic coal mass. |
| 2 | $u_1$ (raw coal flow), $u_2$ (air flow); classifier speed fixed | 2 ($< 3$) | – | No | Fixing classifier speed limits separation control; air flow affects only nonlinear terms. |
| 3 | $u_2$ (air flow), $u_3$ (classifier speed); raw coal feed fixed | Not in required affine-linear form | – | No | Constant raw coal feed introduces affine term; controllability framework inapplicable. |
| 4 | $u_1$ (raw coal flow), $u_3$ (classifier speed); air flow fixed | 3 ($= 3$) | Satisfied | Yes | Coordinated adjustment of coal feed and classifier speed ensures full controllability. |

# 5 Conclusion

Using the angular velocity of the classifier ($\omega(t)$) and the mass flow rate of raw coal ($w_c(t)$) as control inputs, along with a fixed primary air flow ($w_{air}(t) = 0.02$), the nonlinear coal mill pulverizer model, as defined by (4.1), may be controlled. The Lipschitz continuity of the nonlinear term in Case 4 and the entire rank of the controllability matrix both support this. The study demonstrates how the nonlinear controllability framework from Section 3 may be applied to intricate industrial systems, offering a solid foundation for developing control plans for coal mill pulverizers. According to the findings, achieving the required coal mass distributions—which are essential for effective power plant operation—requires just careful control of coal input and particle separation. Future research could explore several directions:

- **Robustness Analysis**: Investigate controllability under variations in parameters $\theta_i$, which may fluctuate due to coal quality or mill wear.
- **Control Constraints**: Incorporate practical limits on $w_c(t)$ and $\omega(t)$, such as maximum feed rates or classifier speeds.
- **Extended Models**: Apply the framework to more detailed coal mill models, including temperature dynamics or wear effects.
- **Numerical Validation**: Simulate the control law to verify performance under realistic operating conditions.

With these additions, the gap between theoretical controllability and real-world application in power plant control systems would be further closed.

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'Not applicable'


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