Additional file 2
Full unified implementation algorithm and supplementary implementation material

This file accompanies the manuscript "Value of Information with Imprecise Probabilities for Decision Modeling." It states in full the unified IP-VOI implementation algorithm summarized in the Methods (subsection "Implementation") and describes the executable R/Quarto template provided as Additional file 3.

Equations referenced below. For a bounded scalar output X with support in $[\ell, u]$ and p-box bounds $\underline{F}_X, \overline{F}_X$,

$$\begin{aligned} \underline{\mathbb{E}}^{\square}(X) &= \ell + \int_{\ell}^{u} \{1 - \overline{F}_X(x)\}\, dx, \\ \overline{\mathbb{E}}^{\square}(X) &= \ell + \int_{\ell}^{u} \{1 - \underline{F}_X(x)\}\, dx, \end{aligned} \tag{1}$$

$$\underline{\mathbb{E}}^{\square}(X) \le \underline{\mathbb{E}}_{\mathcal{P}}(X) \le \overline{\mathbb{E}}_{\mathcal{P}}(X) \le \overline{\mathbb{E}}^{\square}(X). \tag{2}$$

Full unified implementation algorithm

The definitions above lead to a direct implementation. There is an outer representation of the admissible set and an inner decision-model calculation. The outer representation specifies which probability measures are allowed by the evidence; the inner calculation estimates the relevant VOI quantity under each measure or under a chosen decision rule. Suppose $\mathcal{P}$ is represented by a finite or discretized index set $\mathcal{H} = \{h_1, \ldots, h_M\}$, where h_m indexes the probability measure $P_{h_m} \in \mathcal{P}$. For each h_m, simulate N input draws $\theta_m^{(i)}$, $i = 1, \ldots, N$, and compute $B_{m,i,a} = \mathrm{NB}_a(\theta_m^{(i)})$ for every strategy $a \in \mathcal{A}$. Superscripts in parentheses denote simulation draws, not powers. If the target is EVPPI or EVSI, also store the observed information $z_m^{(i)}$, equal to $\phi(\theta_m^{(i)})$ for EVPPI or a simulated future dataset $Y_d^{(i)}$ for EVSI.

Algorithm 1. Unified IP-VOI implementation with branches for EVPI, EVPPI, EVSI, and fixed-measure envelopes

1. **Define the admissible set.** Specify the evidence constraints and the corresponding set $\mathcal{P}$. Choose a computational representation $\mathcal{H} = \{h_1, \ldots, h_M\}$, either exact for a finite set or an approximation to a continuous set. If a reference probability measure is used, identify its index m_0 such that $P_{h_{m_0}} = P_0$.

2. **Define the decision problem.** Specify $\mathcal{A}$, Θ, and the net-benefit functions $\mathrm{NB}_a(\theta)$ for all $a \in \mathcal{A}$. Specify the information target: $Z = \theta$ for EVPI, $Z = \phi(\theta)$ for EVPPI, or $Z = Y_d$ for EVSI.

3. **Choose the estimand.** Decide whether the analysis will report a decision-rule-specific IP-VOI, the fixed-measure VOI envelope, or both. For a rule-specific analysis, choose and justify the decision rule ρ before seeing the results.

4. **Generate inner simulations.** For each $h_m \in \mathcal{H}$, simulate $\theta_m^{(i)} \sim P_{h_m}$, evaluate $B_{m,i,a}$ for every strategy, and simulate $z_m^{(i)}$ when the target is EVPPI or EVSI.

5. **Compute current expected values.** For each strategy and outer setting, compute

$$\widehat{\mu}_a(h_m) = \frac{1}{N}\sum_{i=1}^{N} B_{m,i,a}.$$

Under the lower-expectation rule, estimate the current value by

$$\widehat{V}_0^{\mathrm{LE}} = \max_{a\in\mathcal{A}} \min_{1\le m\le M} \widehat{\mu}_a(h_m).$$

6. **Branch A: lower-expectation EVPI.** Estimate the perfect-information value by

$$\widehat{V}_\Theta^{\mathrm{LE}} = \min_{1\le m\le M} \frac{1}{N}\sum_{i=1}^{N} \max_{a\in\mathcal{A}} B_{m,i,a}.$$

Report $\widehat{\mathrm{EVPI}}_{\mathcal{P}}^{\mathrm{LE}} = \widehat{V}_\Theta^{\mathrm{LE}} - \widehat{V}_0^{\mathrm{LE}}$.

7. **Branch B: lower-expectation EVPPI or EVSI.** Define a candidate policy class Δ_Z. If Z is continuous, discretize Z into interpretable states or restrict attention to a parameterized policy class. For each candidate policy $\delta \in \Delta_Z$, compute

$$\widehat{v}_\delta(h_m) = \frac{1}{N}\sum_{i=1}^{N} B_{m,i,\delta(z_m^{(i)})}.$$

Estimate $\widehat{V}_Z^{\mathrm{LE}} = \max_{\delta\in\Delta_Z} \min_{1\le m\le M} \widehat{v}_\delta(h_m)$ and report $\widehat{\mathrm{VOI}}_Z^{\mathrm{LE}} = \widehat{V}_Z^{\mathrm{LE}} - \widehat{V}_0^{\mathrm{LE}}$. For EVSI, report $\widehat{\mathrm{ENBS}}_{\mathcal{P}}^{\mathrm{LE}}(d) = \widehat{\mathrm{VOI}}_Z^{\mathrm{LE}} - C_{\mathrm{study}}(d)$.

8. **Branch C: fixed-measure VOI envelope.** For each h_m, compute the conventional VOI target $\widehat{T}(h_m)$ using any validated single-measure estimator. For EVPI, for example,

$$\widehat{T}(h_m) = \frac{1}{N}\sum_{i=1}^{N} \max_{a\in\mathcal{A}} B_{m,i,a} - \max_{a\in\mathcal{A}} \frac{1}{N}\sum_{i=1}^{N} B_{m,i,a}.$$

Estimate the envelope as $[\min_{1\le m\le M} \widehat{T}(h_m), \max_{1\le m\le M} \widehat{T}(h_m)]$ and compare the reference value $\widehat{T}(h_{m_0})$ with the endpoints.

9. **Branch D: p-box implementation.** If the admissible set is represented through p-boxes rather than explicit draws from each P_{h_m}, propagate the shared uncertain inputs through the vector $(\mathrm{NB}_a(\theta))_{a\in\mathcal{A}}$ or directly through the decision quantity, such as $B(\theta) = \max_{a\in\mathcal{A}} \mathrm{NB}_a(\theta)$. Use Equation (1) to obtain expectation bounds for the full p-box band, and use Equation (2) to interpret them as outer bounds for the original admissible set unless the full band is the intended credal set.

10. **Assess stability.** For each VOI target, report the reference-measure value, endpoint values, endpoint-defining assumptions, the threshold classification (robust vs. assumption-dependent, as defined in the Methods), and Monte Carlo error at candidate endpoints.

This integrated algorithm clarifies the relation between the objects in the paper. Branches A and B estimate VOI under a stated imprecise-probability decision rule. Branch C estimates the range of conventional VOI results over admissible precise probability measures. Branch D describes how the same logic is implemented when the admissible set is propagated through p-boxes. Analysts can report more than one branch, but each branch answers a different question and should be labeled accordingly.

Supplementary implementation material (Additional file 3)

The supplementary Quarto/R file implements the branching algorithm stated above. It contains modular code blocks for defining an admissible set, computing p-box expectation bounds, comparing decision rules, estimating lower-expectation EVPI, estimating lower-expectation EVPPI or EVSI by finite policy search, computing the fixed-measure VOI envelope, and classifying research thresholds. Users replace the example sampling and net-benefit functions with their own decision model while retaining the outer imprecise-probability calculations. A companion supplementary file provides a reporting checklist that extends CHEERS-VOI (CHEERS-VOI; Kunst et al., *Value in Health* 2023 and *Medical Decision Making* 2023) with the additional items an IP-VOI analysis requires.