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Ravi Chandra Madasani

ravichandramadasani@gmail.com

Northern Arizona University

Research Article

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Posted Date: June 26th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-10094789/v1>

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Additional Declarations: No competing interests reported.

Functionally graded auxetic sub-surfaces for misalignment-tolerant polymer plain bearings

Ravi Chandra Madasani

Department of Mechanical Engineering, Northern Arizona University, Yuma, AZ, USA

Corresponding author: ravichandramadasani@gmail.com

Abstract

Rigid polymer plain bearings (machined HDPE or UHMWPE) are widely used in slow, heavily loaded, dust-exposed pivots such as solar-tracker torque tubes, where their dominant failure is *edge loading*: when the shaft misaligns, a rigid bore cannot conform, the contact patch collapses onto one axial edge, and the local pressure spikes far above nominal, driving localized wear. We propose a single-piece bearing that keeps the sliding surface smooth but replaces the solid sub-surface with a *functionally graded re-entrant (auxetic) lattice*, engineered so the bore deforms elastically to follow a misaligned shaft. The concept is evaluated with a reproducible simulation pipeline: numerical homogenization of the lattice, a two-dimensional plane-stress contact model, pressure-dependent Archard wear, viscoelastic creep over a 25-year horizon, and NSGA-II multi-objective optimization with Sobol sensitivity analysis. An optimized graded design reduces the peak contact pressure at 2° misalignment by $6.6 \times$ (213 to 32 MPa) and raises retained contact area from 1% to 30%; the solid baseline reaches wear-skin breakthrough within ~ 250 oscillation cycles whereas the graded design does not. A matched-stiffness experiment isolates a $1.7 \times$ peak-pressure benefit from negative-Poisson in-drawing, and global sensitivity shows that graded radial compliance, set by lattice density, is the dominant lever. A three-dimensional finite-element model, validated against the Lamé thick-cylinder solution, confirms the mechanism ($5.3 \times$ reduction) and shows the plane-stress model to be conservative. Graded compliance does most of the work, with the auxetic grading a secondary gain that matters when stiffness must be held against creep.

Keywords: polymer plain bearing; auxetic metamaterial; conformal contact; edge loading; Archard wear; functional grading.

Nomenclature

θ	re-entrant cell angle (deg); $\theta < 0$ auxetic
$t/l, h/l$	cell wall slenderness and aspect ratio
E_1, E_2	homogenized axial (z) and radial (r) Young's moduli
ν_{12}, ν_{21}	homogenized Poisson ratios
G_{12}	homogenized in-plane shear modulus
E_s, ν_s	solid cell-wall (polymer) modulus and Poisson ratio
β	shaft misalignment angle (deg)
z, r	axial and radial coordinates of the bearing section
L, t_w	bearing axial length and wall (core) thickness
W, p	applied radial load and contact pressure
$s(z), \delta_0$	rigid interference profile and rigid-body penetration

$\mathbf{S}_c$	condensed contact stiffness matrix
k_s, p_c	specific wear rate and critical pressure
h_w	wear depth
$c(z, t)$	viscoelastic modulus-relaxation factor
S_i, S_{Ti}	first-order and total Sobol sensitivity indices

Introduction

Polymer plain bearings and their failure mode

Self-lubricating polymer plain bearings are attractive wherever lubrication is impractical and contamination is unavoidable. In single-axis solar trackers, for example, long torque tubes rotate slowly (of order one oscillation per day) under sustained gravitational and wind loading, in dusty outdoor environments, for service lives measured in decades; comparable duty arises in agricultural linkages and slow structural pivots. Machined HDPE and ultra-high-molecular-weight polyethylene (UHMWPE) bushings run dry, tolerate grit, damp vibration, and are inexpensive, which makes them a natural choice for these applications.

The limiting failure in such service is rarely uniform sliding wear. It is *edge loading*. Real shafts deflect elastically under load and real mounts are never perfectly collinear, so the journal axis tilts relative to the bore by a small misalignment angle β . A rigid bore cannot follow this tilt: contact is lost over most of the bearing length and the entire radial load is forced onto a narrow band at one axial edge. Because the local contact pressure governs the local wear rate, this band wears rapidly, the bearing skin is breached, debris is generated, and failure cascades. Field experience in tracker bushings and the warranty cost it represents motivate designs that tolerate several degrees of misalignment without such concentration.

Existing remedies and their limitations

The classical mechanical remedy is the spherical (self-aligning) plain bearing, in which an inner race swivels within a mating concave seat. It works, but it introduces an additional sliding interface vulnerable to grit, adds cost and bulk, and complicates sealing, the very problems a polymer bushing is chosen to avoid. Softer bulk materials or elastomeric liners spread the contact but sacrifice load capacity and creep resistance, and a uniformly compliant liner that is soft enough to conform at the edges is generally too soft to hold position under sustained load. Compliant-cavity bearing concepts, in which a relief region inside a one-piece part permits local deflection, are known in the patent literature (for example, the compliant relief region of US 5,052,824 A [14]); however, a generic internal cavity provides no control over *where* and *how much* the bore yields, and offers no principled route to spatially tailor that response to a target misalignment envelope.

Auxetic metamaterials as a structural remedy

We pursue a structural rather than a mechanical or material-substitution remedy: keep the sliding surface perfectly smooth and untextured, but replace the solid sub-surface with an engineered cellular architecture whose effective compliance is *spatially graded* so that the bore elastically conforms to a misaligned shaft. Re-entrant (inwardly folded) honeycomb lattices are a natural building block because they are *auxetic*: they possess a negative Poisson’s ratio, contracting laterally under longitudinal compression instead of bulging [1, 2, 3, 4]. Under the localized radial compression imposed by a misaligned shaft, an auxetic sub-surface draws material inward toward the loaded region rather than expelling it, a kinematic effect that can recruit neighbouring material into the contact and broaden the patch. Auxetic metamaterials have been studied extensively for indentation resistance, synclastic curvature, and energy absorption [5, 6], and functional grading of cellular solids is a mature idea for tailoring stiffness fields, including graded re-entrant lattices [7]. Their combination as a graded sub-surface for *contact-pressure control in a tribological component*, with the wear and creep consequences explicitly modelled, has not to our knowledge been quantified.

Contributions

This paper evaluates the concept computationally, with every step reproducible from the released code. Specifically, we (i) homogenize the re-entrant lattice to admissible orthotropic constants and identify a degeneracy of the classical bending-only relations that must be corrected before they can be used in a plane-stress continuum; (ii) demonstrate edge-loading suppression in a two-dimensional contact model that resolves the anisotropic lattice response, rather than a one-dimensional foundation that cannot; (iii) design a controlled numerical experiment that isolates how much of the benefit is due to negative Poisson *per se* as opposed to graded compliance shared with a conventional lattice; (iv) couple the mechanism to a UHMWPE-calibrated wear law and a 25-year creep model; and (v) optimize the graded architecture and quantify, by global sensitivity analysis, the relative importance of each design variable. Throughout, we keep the claims within what a reduced-order simulation can support.

Methods

The pipeline comprises five coupled modules. All computations were performed in Python; the complete source that reproduces every figure and number is released with the paper. Representative geometric and material parameters are listed in Table 1.

Baseline geometric, material, and operating parameters.

Quantity	Symbol	Value
Bore diameter	$2R$	50 mm
Axial length	L	40 mm
Wall (core) thickness	t_w	5–12 mm (design)

Quantity	Symbol	Value
Effective contact width	b_c	10 mm
Nominal projected pressure	p_{nom}	8 MPa
Radial load	$W = p_{\text{nom}} b_c L$	3.2 kN
Wall polymer modulus	E_s	1.0 GPa
Wall Poisson ratio	ν_s	0.45
Yield / critical pressure	p_c	~22 MPa
Cell aspect ratio	h/l	2.0
Cell slenderness	t/l	0.20–0.42 (design)
Misalignment sweep	β	0–3°
Specific wear rate (UHMWPE/steel, dry)	k_s	$1 \times 10^{-5} \text{ mm}^3/(\text{N m})$
Friction coefficient	μ	0.15
Oscillation amplitude / count	φ, N	$\pm 60^\circ, 10^4$ cycles
Wear-skin thickness	t_{skin}	1.0 mm
Creep horizon	—	25 yr

Lattice homogenization (Module 1)

The re-entrant cell is parameterized by the inclined-strut angle θ ($\theta < 0$ re-entrant), the wall slenderness t/l , and the aspect ratio h/l . The classical bending-dominated relations for the in-plane orthotropic constants [3, 4] are

$$\frac{E_2}{E_s} = \left(\frac{t}{l}\right)^3 \frac{(h/l + \sin\theta)}{\cos^3\theta}, \quad \nu_{21} = \frac{(h/l + \sin\theta)\sin\theta}{\cos^2\theta}, \quad \nu_{12} = \frac{\cos^2\theta}{(h/l + \sin\theta)\sin\theta}, \quad (1)$$

with direction 2 the radial (r) and direction 1 the axial (z) axis. These reproduce the auxetic sign transition ($\nu_{21} < 0$ for $\theta < 0$) but are *degenerate* for a plane-stress continuum: Eq. (1) gives $\nu_{12}\nu_{21} = 1$ identically, so the plane-stress denominator $1 - \nu_{12}\nu_{21} \rightarrow 0$ and the stiffness is singular. Physically, the degeneracy is an artifact of neglecting axial (stretching) deformation of the cell walls.

We therefore homogenize the cell numerically with an Euler–Bernoulli beam network that carries both bending and axial strut deformation. Three unit load cases (uniaxial stress in z , uniaxial stress in r , and pure shear) applied to a representative cell with free lateral boundaries yield the full set $\{E_1, E_2, \nu_{12}, \nu_{21}, G_{12}\}$ by measuring the macroscopic strains. The inclusion of axial compliance lifts the degeneracy, giving $\nu_{12}\nu_{21} < 1$ throughout while preserving the auxetic transition (Table 2, Fig. 1). The constants are tabulated over $(\theta, t/l)$ and interpolated during finite-element assembly. The resulting plane-stress stiffness is

$$\mathbf{Q} = \frac{1}{1 - \nu_{12}\nu_{21}} \begin{bmatrix} E_1 & \nu_{12}E_2 & 0 \\ \nu_{12}E_2 & E_2 & 0 \\ 0 & 0 & (1 - \nu_{12}\nu_{21})G_{12} \end{bmatrix}. \quad (2)$$

Representative homogenized constants ($t/l = 0.30$, $h/l = 2$, $E_s = 1$ GPa). The product $\nu_{12}\nu_{21} < 1$ is admissible throughout.

θ (deg)	E_1 (MPa)	E_2 (MPa)	ν_{21}	G_{12} (MPa)	$\nu_{12}\nu_{21}$
-30	43.8	4.9	-0.305	5.1	0.838
-15	100.0	5.3	-0.191	23.3	0.693
0	300.0	6.5	0.000	300.0	0.000
+15	58.9	9.0	+0.325	39.6	0.693
+30	14.6	14.6	+0.916	15.2	0.838

Conformal contact model (Module 2)

The loaded region of the bearing is modelled as a two-dimensional plane-stress orthotropic section in the axial-radial (z, r) plane, discretized with bilinear quadrilateral elements (80×14 for production runs). The outer radius is bonded to a rigid housing ($\mathbf{u} = \mathbf{0}$); the bore surface contacts a rigid shaft tilted by β , imposing the linear interference profile

$$s(z) = \delta_0 + (z - z_c)\tan\beta - h_w(z), \quad (3)$$

where δ_0 is the rigid-body penetration, z_c the mid-length, and h_w the accumulated wear (Module 3). The lattice is graded along z through a symmetric field

$$\theta(z) = \theta_{\text{mid}} + (\theta_{\text{edge}} - \theta_{\text{mid}})(2z/L - 1)^2, \quad (4)$$

so that the cell angle (hence the local compliance and Poisson sign) varies from the axial centre to the edges. Element-wise constants from Eq. (2) enter the orthotropic elasticity weak form $a(\mathbf{u}, \mathbf{v}) = \int_{\Omega} \boldsymbol{\varepsilon}(\mathbf{v})^\top \mathbf{Q}(z) \boldsymbol{\varepsilon}(\mathbf{u}) d\Omega$, assembled into the global stiffness $\mathbf{K}$.

The frictionless unilateral contact between the deformable bore and the rigid shaft is reduced by static condensation to the bore-surface normal degrees of freedom. Partitioning into contact (c) and remaining free (i) sets and eliminating the latter gives the condensed contact stiffness

$$\mathbf{S}_c = \mathbf{K}_{cc} - \mathbf{K}_{ci}\mathbf{K}_{ii}^{-1}\mathbf{K}_{ic}, \quad (5)$$

which is symmetric positive-definite and inherits the full anisotropic coupling of the section. The Signorini contact problem is then the linear complementarity problem

$$\mathbf{u} \geq \mathbf{s}, \quad \mathbf{p} = \mathbf{S}_c \mathbf{u} \geq \mathbf{0}, \quad (\mathbf{u} - \mathbf{s})^\top \mathbf{p} = 0, \quad (6)$$

solved exactly by a primal active-set iteration. The rigid-body penetration δ_0 is found by bisection so that the integrated normal force equals the applied load, $\int_0^L p(z) b_c dz = W$ (satisfied to 10^{-5} relative). This two-dimensional treatment resolves the ν_{zr} coupling that drives auxetic in-drawing, which a one-dimensional elastic-foundation reduction cannot represent.

Because a rigid, tilted flat on a stiff section produces a near-singular, mesh-sensitive edge pressure, we report a mesh-robust peak defined as the pressure averaged over a fixed physical patch,

$$\bar{p}(z) = \frac{1}{w} \int_{|\zeta-z| \leq w/2} p(\zeta) d\zeta, \quad p_{\text{peak}} = \max_z \bar{p}(z), \quad (7)$$

with $w = 1.5$ mm; comparisons between designs are made at fixed discretization.

Wear (Module 3)

Surface recession follows a spatially resolved Archard law [9] with a pressure-dependent coefficient that accelerates above the critical pressure to represent the adhesive/plastic edge regime:

$$\frac{dh_w}{ds} = k_s \kappa(p) p, \quad \kappa(p) = 1 + \alpha [p/p_c - 1]_+, \quad (8)$$

with specific wear rate $k_s = 1 \times 10^{-5} \text{ mm}^3/(\text{N m})$, the order reported for UHMWPE sliding dry against steel [11], with acceleration $\alpha = 2$, and $p_c = 20$ MPa. The sliding distance per oscillation cycle is $s_{\text{cyc}} = 4\phi R$. Wear is integrated in cycle blocks and fed back through h_w in Eq. (3), so the surface self-conforms; following the identity $\int p dA = W$, the total wear volume is load-set and only its *distribution* differs between designs. We therefore report the peak wear depth and the cycles-to-breakthrough (when $\max_z h_w$ reaches t_{skin}), rather than total volume. This staggered scheme follows established finite-element wear practice [10].

Creep (Module 4)

Sustained-load viscoelasticity is represented as a stress-dependent relaxation of the lattice modulus,

$$c(z, t) = [1 + A (p/\sigma_{\text{ref}})^m \ln(1 + t/\tau)]^{-1}, \quad \mathbf{Q}(z, t) = c(z, t) \mathbf{Q}(z), \quad (9)$$

with parameters calibrated so that a region at the reference stress loses of order 30% of its stiffness over a 25-year horizon, consistent with UHMWPE creep behaviour; the form is monotone and self-limiting under load control. The contact problem is re-solved at logarithmically spaced times, and we report the peak-pressure drift and the radial sag $\max_z u(z)$.

Optimization and sensitivity (Module 5)

The design vector $\mathbf{x} = (\theta_{\text{edge}}, \theta_{\text{mid}}, t/l, t_w)$ is optimized with the NSGA-II elitist genetic algorithm [12] for three competing objectives evaluated at $\beta = 2^\circ$,

$$\min_{\mathbf{x}} (p_{\text{peak}}, \text{creep sag}, \text{radial compliance}), \quad \text{s.t.} \quad \varepsilon_{\text{core}} < 12\%, \quad (10)$$

where radial compliance is the bore deflection at $\beta = 0$ and the constraint keeps the softest designs within the small-strain regime. First-order and total Sobol indices [13], $S_i = \text{Var}_{x_i}$

$[\mathbb{E}(p_{\text{peak}} \mid x_i)]/\text{Var}(p_{\text{peak}})$, quantify the relative importance of each variable for the peak-pressure objective.

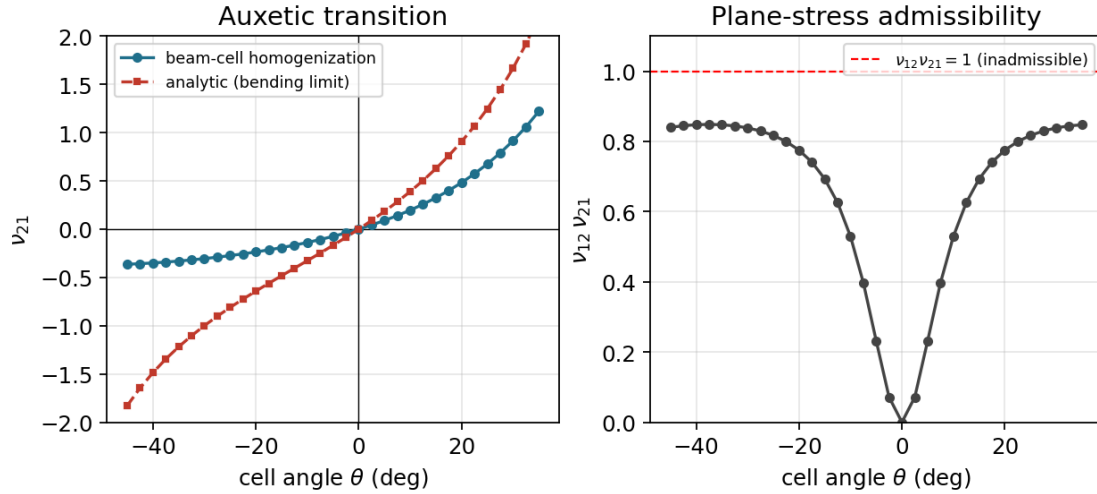
Three-dimensional confirmation model (Module 6)

To test whether the reduced-order conclusions survive in full geometry, the loaded section is replaced by a three-dimensional finite-element model of the complete cylindrical bearing wall. The annulus is discretized with trilinear hexahedral elements; the homogenized lattice is mapped onto the cylindrical axes, with the re-entrant cell occupying the radial-axial (r, z) plane and graded along z by the field $\theta(z)$ of Eq. (4), while the extruded circumferential direction ϑ is assigned a relative-density-scaled modulus $E_\vartheta = \bar{\rho} E_s$ and is decoupled in Poisson from the in-plane response. Each element uses the rotated orthotropic tensor $\mathbf{D} = \mathbf{T}^{-1} \mathbf{D}_{\text{loc}} \mathbf{T}^{-\top}$, with $\mathbf{T}$ the Voigt rotation from local cylindrical to global axes. The outer radius is bonded ($\mathbf{u} = \mathbf{0}$); a rigid shaft of radius R , tilted by β and displaced by δ_0 , contacts the bore. Interior degrees of freedom are removed by static condensation to the bore surface; the normal compliance $\mathbf{F} = \Phi^\top \mathbf{S}_c^{-1} \Phi$ is formed (Φ collecting the outward nodal normals), and the frictionless contact is solved as the complementarity problem $\mathbf{g} = \mathbf{g}_0 + \mathbf{F}\mathbf{f} \geq \mathbf{0}, \mathbf{f} \geq \mathbf{0}, \mathbf{g}^\top \mathbf{f} = 0$ by an active-set method, with δ_0 set by bisection to balance the load W . The solver was verified against the Lamé thick-cylinder solution for a bonded outer boundary, reproducing the analytic radial-displacement field to within 0.1%, and against exact rotation invariance of the isotropic stiffness.

Results

Homogenization

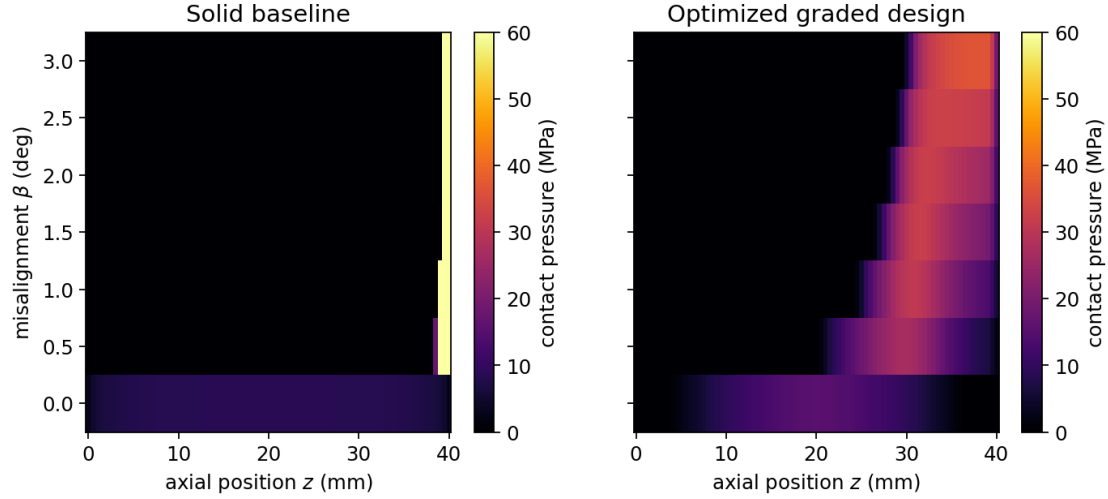
Figure 1 and Table 2 confirm the homogenization. The beam-network Poisson ratio crosses zero at $\theta = 0$ and becomes increasingly negative for re-entrant cells, matching the analytic trend in sign; the radial modulus E_2 falls as the cell becomes re-entrant, providing the soft edges the grading exploits. Critically, the product $\nu_{12}\nu_{21}$ remains below unity across the whole angle range, so the plane-stress stiffness of Eq. (2) is admissible, unlike the singular pure-bending limit.



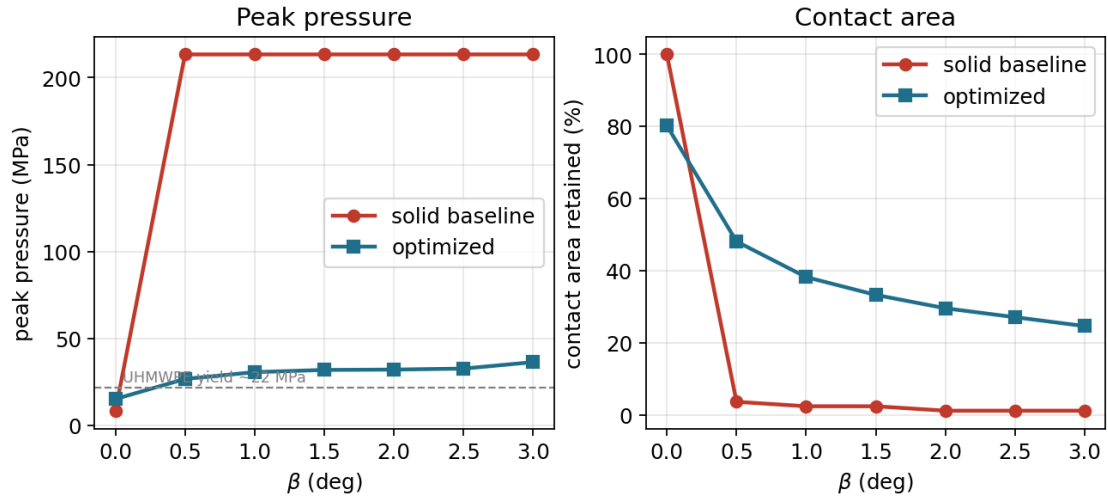
Module-1 homogenization. Left: the beam-network Poisson ratio crosses zero at $\theta = 0$ and becomes negative for re-entrant cells, matching the analytic trend in sign. Right: the homogenized constants satisfy $\nu_{12}\nu_{21} < 1$ across the range, unlike the singular pure-bending limit.

Edge-loading suppression

Figure 2 contrasts the contact-pressure field of a solid baseline and the optimized graded design over the misalignment sweep. The solid bearing collapses onto a single axial edge with lift-off elsewhere as β grows; the graded design distributes pressure across the smooth face. Quantitatively (Fig. 3), the baseline peak pressure rises to 213 MPa at $\beta = 2^\circ$. Beyond about 0.5° it saturates, since essentially all load is then carried within a sub-millimetre patch (the load-into-one-patch limit of Eq. (7)), and the retained contact area falls to 1%. The optimized design holds 32 MPa at 30% contact, a $6.6 \times$ reduction in peak pressure and a more than twenty-fold recovery of contact area. That keeps the peak near the polymer's yield pressure instead of an order of magnitude above it.



Contact-pressure field versus axial position and misalignment. The solid baseline (left) concentrates load on one edge with lift-off elsewhere; the optimized graded design (right) remains evenly loaded across the smooth face.

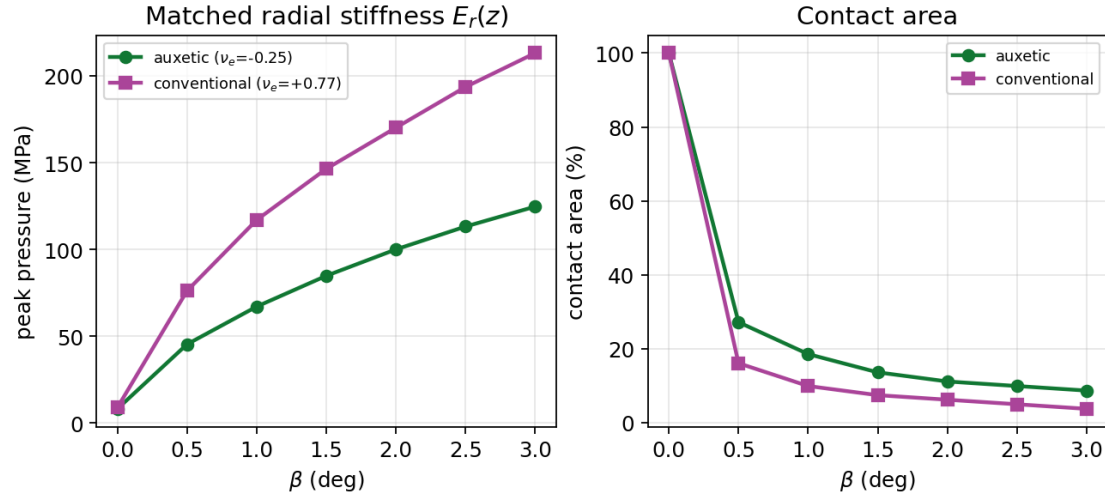


Peak contact pressure (left) and retained contact area (right) versus misalignment for the solid baseline and the optimized graded design.

Isolating the auxetic contribution

How much of the benefit comes from negative Poisson itself, and how much from compliance that any soft graded lattice would supply? To separate the two we built two graded designs with an *identical* radial-stiffness profile $E_r(z)$, matched by adjusting t/l pointwise, that differ only in Poisson sign: one auxetic ($\nu_{\text{edge}} = -0.25$), one conventional ($\nu_{\text{edge}} = +0.77$). At matched compliance (Fig. 4) the auxetic design carries a markedly lower peak pressure (100 vs 170 MPa at 2° , a factor of 1.7) and retains more contact area across the entire sweep. Because the two designs share the same radial stiffness, this difference is direct evidence that negative-Poisson in-drawing does mechanical work of its

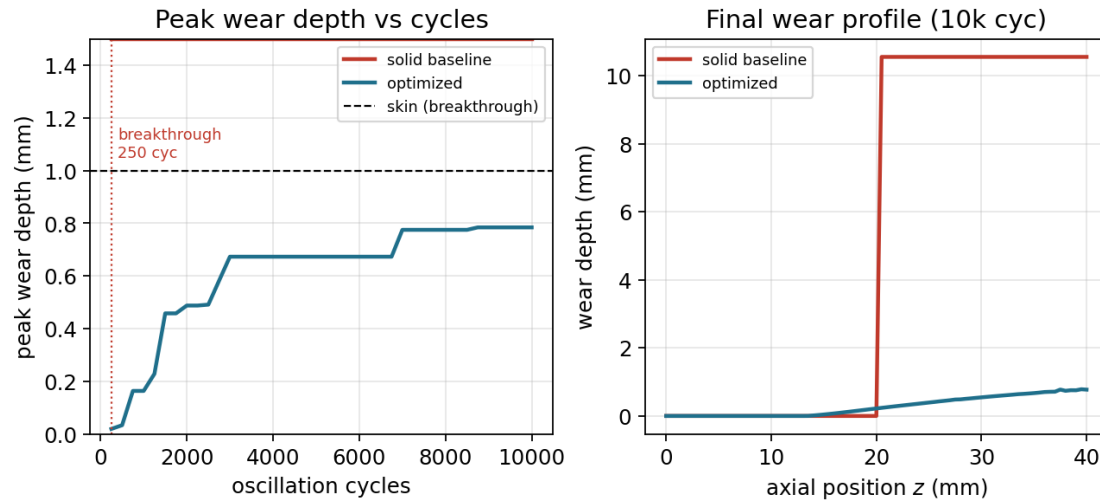
own, pulling material laterally toward the loaded edge, beyond the compliance it shares with a conventional graded lattice.



Controlled comparison at a matched radial-stiffness profile $E_r(z)$, so that only the Poisson sign differs. The auxetic design (green) sustains lower peak pressure and greater contact area than the conventional design (purple).

Wear

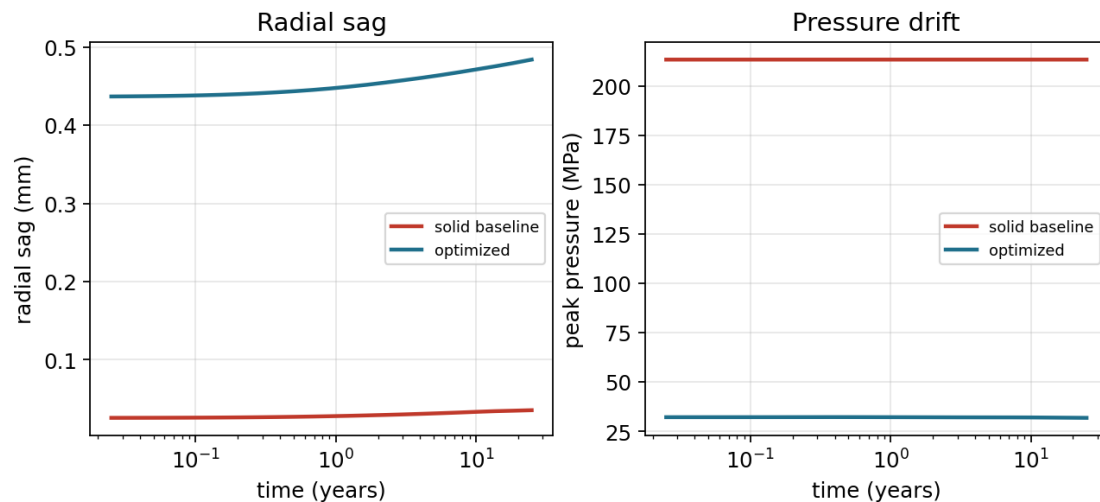
With the UHMWPE-calibrated wear rate (Fig. 5), the solid baseline's edge notch reaches the 1 mm skin thickness within ~ 250 oscillation cycles, after which material loss runs away; the optimized design's peak wear depth stays below the skin over the full 10^4 -cycle window. The final wear profiles make the mechanism explicit: the baseline concentrates removal into a deep, narrow edge notch, whereas the graded design spreads a shallow, nearly uniform layer. Consistent with the load-set total volume, the designs differ chiefly in *where* material is lost; the graded design's flat pressure field converts the same volumetric budget into a far smaller peak depth and a correspondingly longer life to breakthrough.



Wear evolution. Left: peak wear depth versus cycles; the baseline breaches the skin near 250 cycles (dotted line), the graded design does not. Right: final wear profiles: a deep edge notch (baseline) versus a shallow, uniform layer (optimized).

Creep

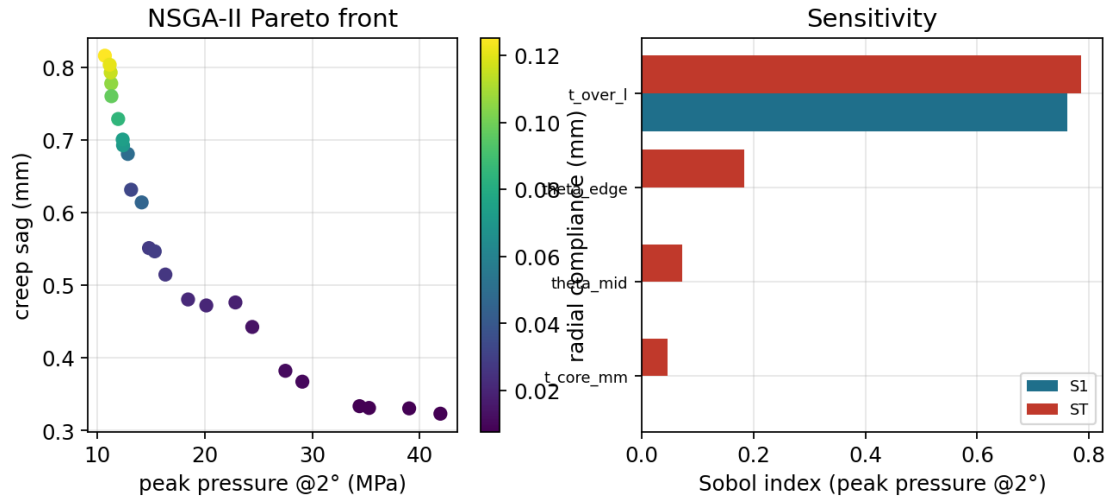
Under 25-year creep (Fig. 6) the compliant graded design accumulates more radial sag than the solid baseline, as expected of a soft structure, but it keeps its low-pressure distribution; the baseline holds a damaging stress concentration throughout its life. This extra sag is the price of conformity, and it sets a practical bound on how soft the edges can be made. The optimization treats that bound as an explicit objective.



Twenty-five-year viscoelastic creep: radial sag (left) and peak-pressure drift (right). The graded design sags more but holds its low-pressure benefit.

Optimization and sensitivity

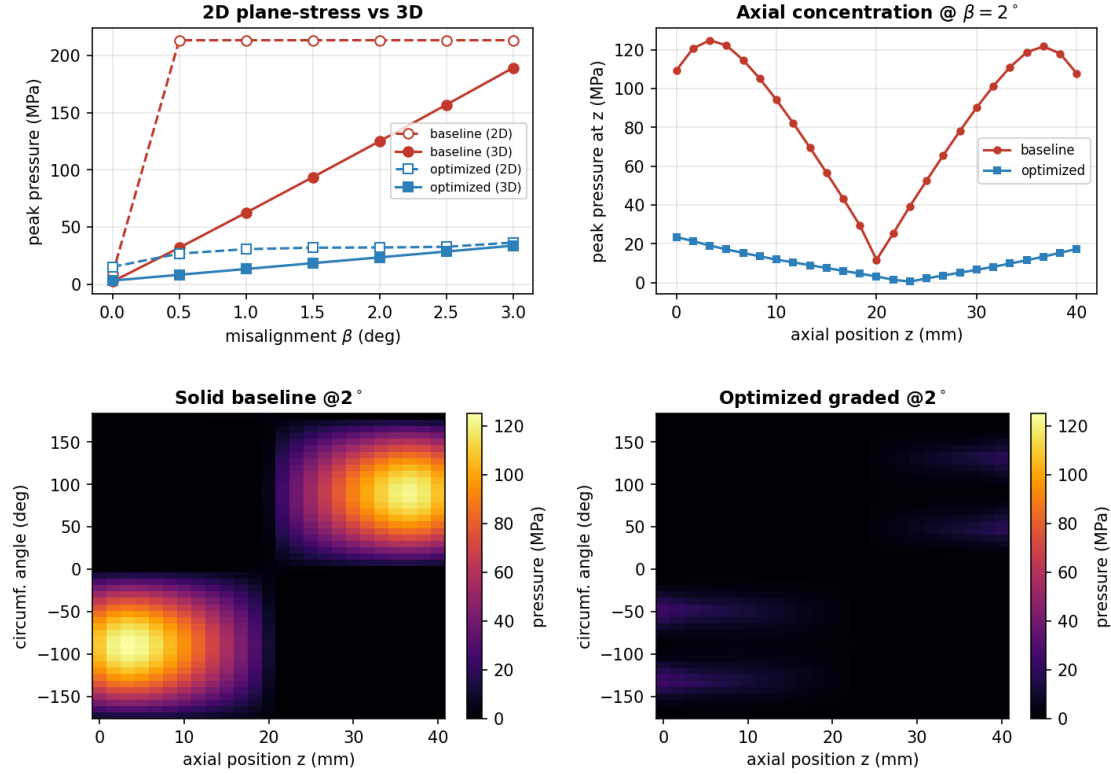
The NSGA-II Pareto front (Fig. 7) shows the basic trade-off: lower peak pressure demands more compliance, which costs stiffness and creep margin. First-order Sobol analysis of the peak-pressure objective ranks lattice density (t/l , $S_1 \approx 0.76$) far above the grading angles ($S_1 \approx 0$). Taken with the isolation experiment, this points to a consistent reading. When lattice density is free to vary, overall radial compliance dominates edge-load relief and the Poisson sign is a minor lever. The auxetic benefit matters only once stiffness is held fixed, which is exactly the regime that creep and positioning-accuracy limits impose, where the whole lattice cannot simply be softened.



Optimization and sensitivity. Left: NSGA-II Pareto front (peak pressure versus creep sag, coloured by radial compliance). Right: first-order and total Sobol indices for the peak-pressure objective; lattice density dominates.

Three-dimensional confirmation

Figure 8 reports the full three-dimensional contact solution, and the two-dimensional conclusions survive. At $\beta = 2^\circ$ the solid baseline peaks at 125 MPa and the optimized graded design at 24 MPa, a $5.3 \times$ reduction (versus $6.6 \times$ in the plane-stress section), with smooth, monotonic peak-pressure curves over the sweep. Two features emerge that a single two-dimensional section cannot show. First, the misaligned rigid shaft jams *diagonally*, touching the bore at two opposite axial edges (bottom at one end, top at the other), so the baseline pressure peaks at both ends of the bore (~ 120 MPa) while the mid-span sits nearly unloaded (~ 12 MPa); the graded design brings down both concentrations at once. Second, the plane-stress section is *conservative*. By ignoring circumferential load sharing it over-predicts the baseline spike (213 against 125 MPa), so the two-dimensional model is a safe design tool and the three-dimensional run confirms the conclusion rather than overturning it. The optimized curves of the two models nearly coincide.



Three-dimensional cylindrical-bearing contact (homogenized lattice; finite-element model validated against the Lamé solution to 0.1%). Top left: peak pressure versus misalignment, two-dimensional plane-stress versus three-dimensional, both designs. Top right: axial profile of peak pressure at $\beta = 2^\circ$, showing the baseline's two-edge concentration. Bottom: unrolled bore-pressure maps at $\beta = 2^\circ$; the baseline lights up two diagonal patches while the optimized bore stays lightly loaded.

Discussion

Mechanism: graded compliance with an auxetic refinement

The simulations support a specific, bounded claim. A smooth-bore polymer bearing with a *graded compliant sub-surface* suppresses the edge-loading pressure spike that destroys rigid polymer bearings under misalignment. It cuts peak pressure several-fold and prevents the wear breakthrough that the solid baseline reaches within a few hundred cycles. The re-entrant (auxetic) grading then adds a further, separable gain of about $1.7 \times$ at matched stiffness, through lateral in-drawing. These two effects work together. Graded compliance does the bulk of the job, and the auxetic grading lets the designer recover edge-load performance *without* giving up the stiffness needed to limit creep sag and hold positioning accuracy. The sensitivity result fits this reading: the Poisson sign counts for most in the stiffness-constrained corner of the design space that the application actually occupies. The three-dimensional model (Fig. 8) confirms the picture in full geometry, with

the benefit holding at $5.3 \times$, and shows the plane-stress section to be conservative, so the reduced-order optimization can be trusted for design.

Design envelope and operating implications

The optimizer's knee design (soft, moderately graded, with a thin compliant core) keeps the peak contact pressure near the polymer's yield pressure across the misalignment sweep, which is the condition for avoiding the plastic edge regime that triggers accelerated wear. The accompanying sag growth under creep (Fig. 6) is the principal cost and should be budgeted against the host machine's allowable radial play; for a tracker bushing of the size modelled, the predicted sag remains a small fraction of the wall thickness. The pressure-dependent wear law of Eq. (8) is what turns the flat pressure field into a life benefit: the runaway notch is driven by the near-yield edge of the solid baseline, not by its mean pressure.

Manufacturing route

Although the present study is computational, the architecture maps onto an additive-manufacturing route, now an established way to realise architected polymer lattices [8]. A smooth, fully dense polymer skin can be printed over the graded lattice core so that no cell wall reaches the sliding interface. The sliding surface then stays intact while the sub-surface supplies the engineered compliance; the graded $\theta(z)$ and t/l fields translate directly into layer-wise toolpath parameters. Keeping the lattice strictly sub-surface is a hard constraint, since cell walls at the interface would abrade the shaft.

Defensible novelty

The framing also clarifies the patentable scope. Broad "compliant cavity" bearing concepts already appear in the prior art (e.g. US 5,052,824 A [14]), so a generic soft liner is not novel on its own. What these results single out is the *inverse-designed graded re-entrant architecture*: the specific $(\theta(z), t/l)$ field that holds a flat contact patch under a target misalignment envelope, with the auxetic grading included where stiffness must be preserved. The defensible claim is that field, not "auxetic" as a label or compliance as a generic property.

Limitations

The study is reduced-order by design, so that it reproduces without commercial software, and its claims are scoped to match. (i) The design sweep and optimization use a two-dimensional plane-stress section for efficiency; a three-dimensional finite-element model of the complete bearing (Fig. 8, validated against the Lamé solution to 0.1%) confirms the mechanism and shows the section to be conservative, but the production pipeline itself remains two-dimensional. (ii) The lattice is represented by homogenized constants from a representative beam cell, not a periodic-medium or explicit-cell solution; homogenization weakens when few cells span the wall. (iii) The peak contact pressure of the stiff baseline is mesh-sensitive near the contact edge, so peak magnitudes are reported as the pressure

averaged over a fixed ~ 1.5 mm representative patch (Eq. (7)), which removes single-node contact noise, and comparisons are made at fixed discretization. (iv) The wear coefficient and its pressure dependence, and the creep relaxation, are set to representative, literature-consistent values rather than fitted to a specific material lot. (v) Kinematics are linear and small-strain; the softest edge designs approach 12% core strain, where nonlinearity begins. (vi) No thermal or pressure-velocity-limit coupling is included. Having confirmed the mechanism with a three-dimensional continuum model, the remaining confirmatory step is an explicit-cell finite-element analysis resolving individual cell walls, which would test the homogenization assumption directly and verify the isolated auxetic benefit at the cell scale.

Conclusions

A graded auxetic sub-surface beneath a smooth bore converts a misalignment-prone rigid polymer bearing into one that elastically conforms. In a coupled contact, wear, and creep study the concept reduces peak contact pressure at 2° misalignment by about $6.6\times$ (from 213 to 32 MPa), recovers contact area more than twenty-fold, and prevents the wear-skin breakthrough that the solid baseline reaches within a few hundred cycles. A matched-stiffness experiment isolates a $\sim 1.7\times$ contribution from negative-Poisson in-drawing, and global sensitivity shows that graded radial compliance is the dominant mechanism, with the auxetic effect a secondary gain that counts most under the stiffness constraints the application imposes. A three-dimensional model of the complete bearing, validated against the Lamé solution, confirms the mechanism in full geometry (a $5.3\times$ reduction, with the journal jamming diagonally against two opposite edges) and shows the plane-stress design model to be conservative. The defensible design is therefore narrow: an inverse-designed graded re-entrant architecture, with explicit-cell analysis the natural next step.

Statements and Declarations

Funding. No funding was received for conducting this study.

Competing interests. The author has no relevant financial or non-financial interests to disclose.

Author contributions. R.C.M. is the sole author and performed all aspects of the study.

Data and code availability. The complete Python pipeline that generates every figure and number in this paper is provided as supplementary material.

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