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Behrouz Yaghoubi

byaghoubi@gmail.com

Islamic Azad University Branch of Kermanshah <https://orcid.org/0000-0002-4575-9200>

Armin Fatahi

Islamic Azad University Branch of Kermanshah

Fariborz Yosefvand

Islamic Azad University Branch of Kermanshah

Mohammad Ali Izadbakhsh

Islamic Azad University Branch of Kermanshah

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Estimating scour depth downstream of ski-jump spillways using the electric eel foraging optimization algorithm

Armin Fatahi¹, Behrouz Yaghoubi^{1*}, Fariborz Yosefvand¹, Mohammad Ali Izadbakhsh¹

1- Department of Water Engineering, Ker. C., Islamic Azad University, Kermanshah, Iran

*Correspondence Author: Behrouz Yaghoubi

Email: byaghubi@gmail.com

Abstract

Ski-jump spillways are critical components of dams, playing a key role in flow control and energy dissipation. The impact of the jet on the downstream riverbed can cause significant scour, which threatens the stability of both the hydraulic structure and the river regime. Therefore, accurately estimating the resulting scour depth is essential for the safe and efficient design of these structures. This study presents an optimized model for predicting scour depth downstream of ski-jump spillways using the Electric Eel Foraging Optimization (EEFO) algorithm. A comprehensive model incorporating key parameters influencing scour depth was developed. Utilizing a zoning technique, the flow conditions were categorized into three distinct regions based on the Froude number, with each region defined by a specific set of parameters. This model was formulated as an optimization problem, with its decision variables calibrated against a dataset of laboratory measurements for ski-jump spillways using the EEFO algorithm. The objective function was set to minimize the sum of squared errors between predicted and observed values. The model's performance was evaluated using the statistical indices RSR, R^2 , NSE, and PBIAS, which yielded values of 0.167, 0.972, 0.972, and 0.33, respectively. These results indicate a high degree of accuracy and reliability. Furthermore, a comparative analysis with established empirical methods demonstrated the superior predictive capability of the proposed model. The findings confirm that the EEFO-based optimization method provides a highly accurate tool for scour depth prediction. It offers a practical solution for mitigating scour-related risks and optimizing the design of hydraulic structures, thereby enhancing their long-term safety and stability.

Keywords: *Scour depth, Ski-jump spillway, Optimization algorithm, Electric eel foraging*

Introduction

Ski-jump spillways are critical hydraulic structures in high dams, designed to control flow and safely dissipate flood energy. By projecting the water jet into the air, these spillways reduce the direct erosive force on the downstream riverbed, thereby protecting the dam and its associated infrastructure (Veronese, 1937b; Martins, 1975; Leopardo et al., 2002). Nevertheless, the subsequent impact of the jet with the river surface induces significant local scour. This scouring can alter bed morphology and poses a substantial threat to the stability of the hydraulic structures (Farhoudi and Smith, 1985; Chanson, 2004). Consequently, accurately estimating the scour depth downstream of ski-jump spillways is a paramount concern for engineering design and safety. The scour mechanism is highly complex, influenced by a multitude of interacting hydraulic, hydrological, and geotechnical factors (Sun et al., 2021). Early pioneering work on scour estimation due to jet impact was conducted by researchers such as Schoklitsch (1937) and Veronese (1937b), with subsequent contributions from Mason and Arumugam (1985) and Yildiz and Üzücek (1994). Over time, numerous empirical relationships have been proposed to predict this scour depth in the downstream of the ski-jump spillways; a selection of these formulas is summarized in Table (1).

Given the nonlinear and stochastic nature of the scour process, data-driven modeling techniques have emerged as powerful tools for prediction, often outperforming traditional empirical formulas (Homaei and Najafzadeh, 2022, Pandey et al., 2020, Shafagh Loron et al., 2023, Sun et al., 2021, Najafzadeh et al., 2014). For instance, Noori and Hooshyaripor (2014) demonstrated the superior accuracy of an Artificial Neural Network (ANN) over empirical methods for predicting scour in the downstream of the ski-jump spillways dimensions. Similarly, Goyal and Ojha (2011) found that Support Vector Machine (SVM) and M5P models outperformed ANN for this task. Recent studies have explored a wider array of advanced techniques as: Fuladipanah et al. (2023) compared data-driven models including SVM, Gene Expression Programming (GEP), Multilayer Perceptron (MLP), and Multivariate Adaptive Regression Splines (MARS), finding the MARS model most effective. Abd El-Hady Rady (2020) reported that a Genetic Programming (GP) model surpassed an Adaptive Neuro-Fuzzy Inference System (ANFIS). Karkheiran et al. (2021) integrated ANN with optimization algorithms like Particle Swarm Optimization (PSO) and Genetic Algorithms (GA). Wang et al. (2023) developed a hybrid SVR-AIG model, which showed better performance than a standard SVR model. These studies collectively underscore the potential of machine learning and optimization-based approaches to provide more reliable scour depth estimates, forming a critical foundation for the methodology employed in the present research.

The aim of this research is to develop an optimization model for estimating scour depth downstream of ski-jump spillways, thereby contributing to improved dam design and reduced scour-related risks. Conventional estimation methods, whether based on regression or black-box models, present significant limitations; regression models often lack sufficient accuracy, while black-box models lack a transparent mathematical formulation for reliable extrapolation to other conditions. To address these shortcomings, this study introduces a novel predictive relationship derived using the Electric Eel Foraging Optimization (EEFO) algorithm—a recent meta-heuristic approach. The resulting model is designed to offer both high accuracy and a clear mathematical expression, enhancing its utility and applicability across diverse scenarios. Given the critical importance of accurate scour prediction in hydraulic engineering, the findings of this research are intended to support both future scientific studies and practical engineering applications.

Table 1. Certain equations proposed by earlier researchers are used to determine the maximum scour depth.

Equation	Reference or approach
----------	-----------------------

$d_s = 1.9q^{0.54}H^{0.225}$	(Veronese, 1937a)
$d_s = 3.27 \frac{q^{0.6}H^{0.05}d_w^{0.15}}{g^{0.3}d^{0.1}}$	(Mason and Arumugam, 1985)
$d_s = 1.18 \frac{H^{0.6} \times q^{0.51}}{d^{0.1}}$	(Wu, 1973)
$d_s = 1.5H^{0.6} \times q^{0.6}$	(Martins, 1975)
$d_s = 0.633H^{0.25} \times F^{0.67}$	(Taraimovich, 1978)
$d_s = 2.3H^{0.1} \times q^{0.6}$	(Shafagh Loron et al., 2023)
$d_s = 1.42H^{0.3} \times q^{0.44}$	(Azmathullah et al., 2006)
$d_s = 1.16H^{0.25} \times q^{0.7}$	(Kumar and Sreeja, 2012)
$d_s = 0.526 \frac{q^{0.645}H^{0.246}}{d^{0.405}}$	(Noori and Hooshyaripor, 2014)
$d_s = 0.12d_w \times \left(\frac{H}{d_w}\right)^{1.12} \times F^{0.8}$	(Khalifehei et al., 2020)
$d_s = 6.914d_w \times F^{0.694} \times \left(\frac{H}{d_w}\right)^{0.0815} \left(\frac{R}{d_w}\right)^{-0.233} \left(\frac{d_{50}}{d_w}\right)^{0.196} \Phi^{0.196}$	(Azmathullah et al., 2006)
$d_s = 3.959d_w \times F^{0.771} \times \left(\frac{H}{d_w}\right)^{0.044} \left(\frac{R}{d_w}\right)^{0.02} \left(\frac{d_{50}}{d_w}\right)^{0.01} \Phi^{-0.01}$	(Sun et al., 2021)
$d_s = 0.15H^{0.75} \times q^{0.38}$	(Suppasri, 2007)
$d_s = 14.68H \times \left(\frac{q}{gH^3}\right)^{1.236} \times \left(\frac{d_w}{H}\right)^{-0.269} \left(\frac{R}{H}\right)^{-0.212} \left(\frac{d_{50}}{H}\right)^{0.0658} \Phi^{0.091}$	(Muzzammil and Siddiqui, 2009)
$d_s = 7.483 \frac{q^{1.4652}H^{0.246}}{g^{0.7326}d_w^{1.1978}}$	(Heng et al., 2012)

67 Materials and Methods

68 Dataset

69 The scour depth downstream of a ski-jump spillway was modeled using two primary data sources. The core
70 dataset was derived from experimental measurements published by Azmathullah et al. (2005). This was supplemented
71 with measured data from physical models of the Darian Reservoir Dam flood discharge system, as documented by the
72 Iranian Water Research Center (2011). The combined dataset comprises 104 distinct data points, each recording the
73 scour depth (d_s) alongside the corresponding hydraulic, hydrological, and geotechnical characteristics that influence
74 it.

75 The key parameters considered are: the scour depth (d_s), the unit discharge (q), The distance from the upstream
76 water level to the downstream water level (H), the bucket radius (R), the ski-jump angle (Φ), and the bed material
77 characteristics, including grain size. These variables are defined based on established literature (Azmathullah et al.,
78 2008; Ramarao and Bhajantri, 2022) and are illustrated schematically in Figure 1. The statistical characteristics of the
79 dataset are summarized in Table 2.

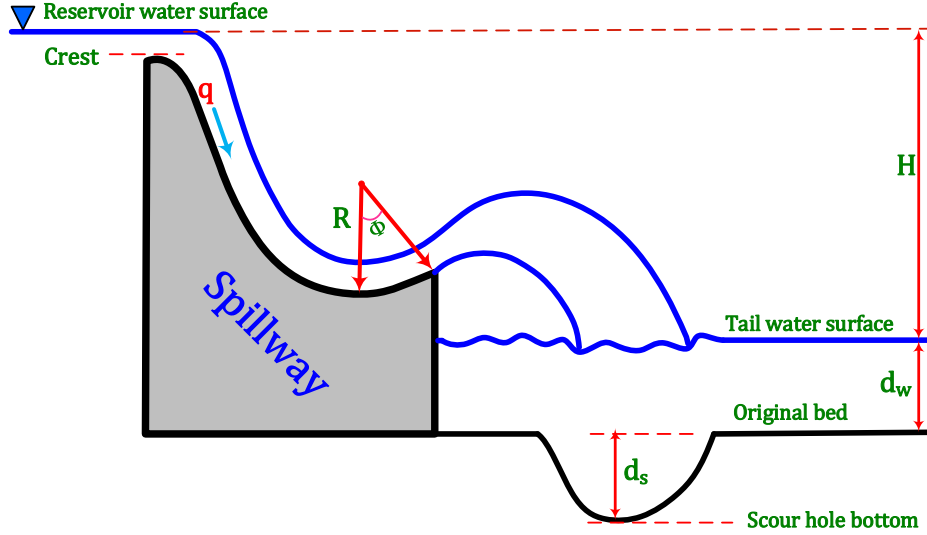


Figure 1. Schematic diagram of parameters affecting scour downstream of spillway ski- jump bucket.

Table 2. Statistical summary of the dataset

Parameters	Mean	Min.	Max.	Standard deviation
$q(\text{m}^3/\text{s}/\text{m})$	0.059	0.0089	0.236	0.051
$H(\text{m})$	0.845	0.2791	2.944	0.794
$R(\text{m})$	0.271	0.1	0.61	0.159
$d_{50}(\text{m})$	0.006	0.002	0.008	0.003
$\Phi(\text{rad})$	0.559	0.126	0.78	0.088
$d_w(\text{m})$	0.100	0.024	0.312	0.071
$d_s(\text{m})$	0.223	0.0512	0.55	0.110
$l_s(\text{m})$	1.388	0.42	30	2.974
$w_s(\text{m})$	0.963	0.6	4	0.609

As illustrated in Figure 1, the scour depth downstream of a ski-jump spillway (d_s) is governed by several key parameters. These include: the unit discharge (q), the total drop height (H), the bucket radius (R), the bucket lip angle (Φ), the tailwater depth (d_w), the median bed particle size (d_{50}), the gravitational acceleration (g), and the densities of the sediment and water (ρ_s) and (ρ_w). Dimensional analysis provides a method for deriving a mathematical relationship describing this phenomenon by systematically examining its influencing parameters. Consequently, the primary relationship for scour depth (d_s) can be expressed by the following functional form:

$$d_s = \text{fun}_1(q, H, R, \Phi, d_w, d_{50}, g, \rho_w, \rho_s) \quad (1)$$

Dimensional analysis, employing the Buckingham π theorem, was used to derive a non-dimensional relationship for the scour depth d_s . This relationship is expressed as Equation (2):

$$\frac{d_s}{d_w} = \text{fun}_2\left(\frac{q}{\sqrt{g}d_w^3}, \frac{H}{d_w}, \frac{R}{d_w}, \frac{d_{50}}{d_w}, \frac{\rho_s}{\rho_w}, \Phi\right) \quad (2)$$

The ratio of sediment to water density ($\frac{\rho_s}{\rho_w}$) is effectively constant and can therefore be omitted from Equation (2). Consequently, $\frac{d_s}{d_w}$ can be expressed as a function of the remaining dimensionless parameters, as shown in Equation

(3). This relationship is further rearranged to solve explicitly for scour depth in Equation (4). The influence of these effective dimensionless parameters on scour depth is illustrated in Figure 2.

$$\frac{d_s}{d_w} = \text{fun}_2 \left(\frac{q}{\sqrt{g d_w^3}}, \frac{H}{d_w}, \frac{R}{d_w}, \frac{d_{50}}{d_w}, \Phi \right) \Rightarrow \quad (3)$$

$$\frac{d_s}{d_w} = a \left(\frac{q}{\sqrt{g d_w^3}} \right)^b \left(\frac{H}{d_w} \right)^c \left(\frac{R}{d_w} \right)^d \left(\frac{d_{50}}{d_w} \right)^e (\Phi)^f \quad (4)$$

In this relationship, the dimensionless term $\frac{d_s}{d_w}$ represents the scour depth normalized by the flow depth and is calculated using Equation (4). This equation takes the form of a power-law expression containing a coefficient a and exponents b, c, d, e , and f . These parameters are unknowns in the model and must be determined through an optimization-based calibration process.

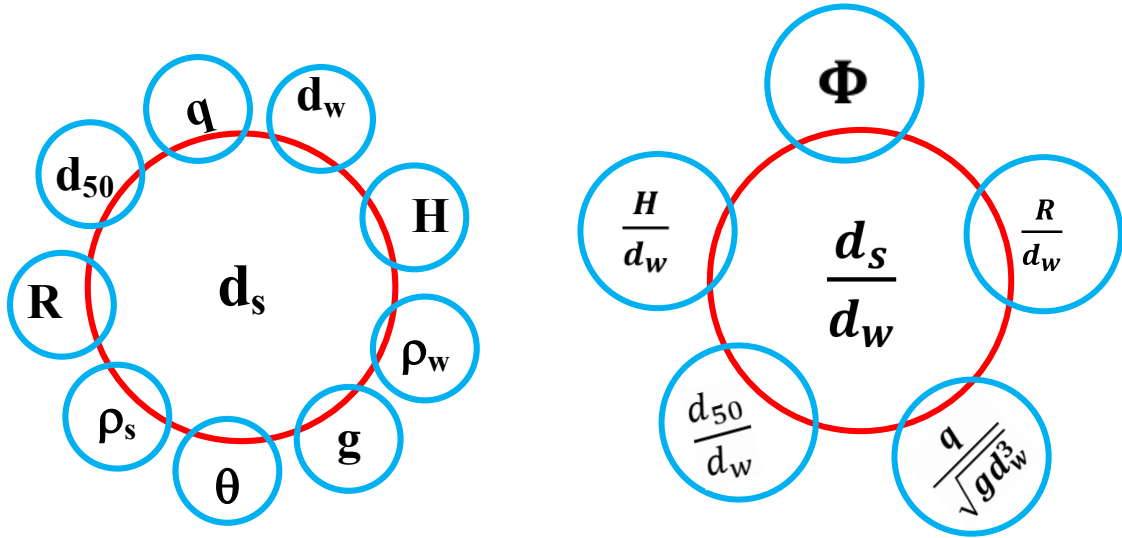


Figure 2. Schematic of effective and dimensionless parameters influencing scour depth downstream of a ski-jump spillway.

Model Development

This section presents the model formulation for determining scour depth d_s . It outlines the relevant governing equations and introduces the Electric Eel Foraging Optimization (EEFO) method and error evaluation criteria used for calibration.

Froude-Based Zoning Approach for Scour Prediction

The proposed model uses a zoning technique based on the Froude number $\left(F = \frac{q}{\sqrt{g d_w^3}} \right)$. The Froude domain, between its minimum (F_{min}) and maximum (F_{max}) observed values, is divided into a specified number of subregions (n). Each subregion i is assigned a unique set of calibrated parameters within the scour equation. For this study, the domain is divided into three subregions ($i=1,2,3$). Accordingly, the general scour equation (Equation 4) is reformulated for each zone as:

$$d_{sw} = \frac{d_s}{d_w} = a_i (F)^{b_i} \left(\frac{H}{d_w} \right)^{c_i} \left(\frac{R}{d_w} \right)^{d_i} \left(\frac{d_{50}}{d_w} \right)^{e_i} (\Phi)^{f_i} \quad | \quad L_i < F < U_i \quad (5)$$

Where L_i and U_i are the lower and upper Froude number bounds defining subregion ii . The exponents $a_i, b_i, c_i, d_i, e_i, f_i$ are the model parameters specific to each zone.

Determination of Zone Boundaries

The zone boundaries L_i and U_i are not predefined but are themselves determined through optimization. This is achieved by introducing a set of dimensionless partitioning variables k_i . $\hat{k}$ The total sum of these variables is:

$$\hat{k} = \sum_i^3 k_i \quad (6)$$

The width of each subregion Δ_i is calculated as a proportion of the total Froude range:

$$\Delta_i = \frac{k_i}{\hat{k}} (F_{max} - F_{min}) \quad (7)$$

The cumulative upper bound $\hat{\Delta}_i$ and the explicit zone bounds are then determined sequentially:

$$\hat{\Delta}_i = \begin{cases} F_{min} + \Delta_1 & \text{if } i = 1 \\ \hat{\Delta}_{i-1} + \Delta_i & \text{if } i = 2, 3 \end{cases} \quad (8)$$

$$L_i = \begin{cases} F_{min} & \text{if } i = 1 \\ \hat{\Delta}_{i-1} & \text{if } i = 2, 3 \end{cases} \quad (9)$$

$$U_i = \hat{\Delta}_i \quad (10)$$

Here, $\hat{\Delta}_i$ represents the cumulative upper limit up to zone i . Thus, the lower bound L_i for zone i (except the first zone) is the upper bound of the previous zone ($\hat{\Delta}_{i-1}$). This partitioning scheme is illustrated in Figure 3 for three subregions.

Decision Variables for Optimization

The complete set of decision variables for the optimization model comprises the scour equation parameters for each zone ($a_i, b_i, c_i, d_i, e_i, f_i$ for $i=1,2,3$) and the three partitioning variables k_i . These variables are calibrated simultaneously using the EEFO algorithm to minimize the prediction error against the observed data.

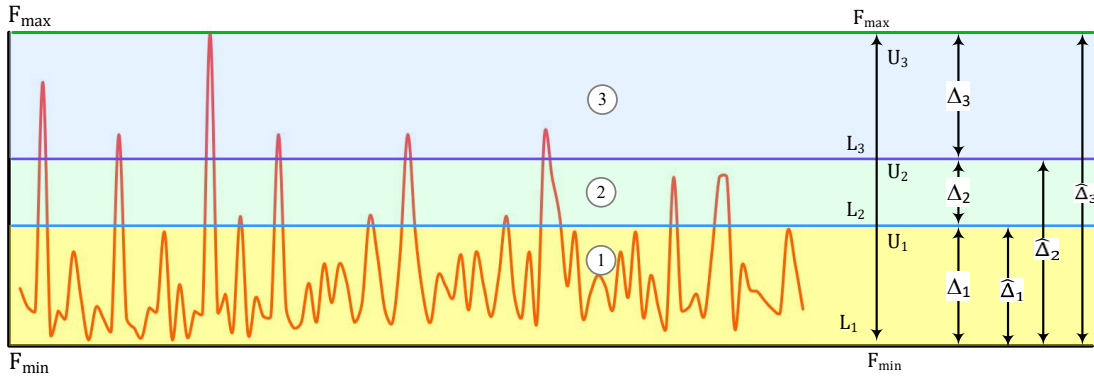


Figure 3. Partitioning of the Froude number domain into three distinct modeling regions.

Model Optimization

The calibration of the parameters in the scour depth equations (Equations 5-10) is formulated as an optimization problem. The objective is to minimize the sum of squared errors (SSd_s) between the measured scour depth ($\hat{d}_{sw}$) and the model-predicted values (d_{sw}). This objective function is defined as:

Objective function:

$$\text{Min (SSd}_s) = \text{Min} \sum_{i=1}^n (\hat{d}_{sw} - d_{sw})^2 \quad (11)$$

Subject to:

$$d_s \neq \text{Negative Number}$$

$$lb \leq X \leq ub, \quad X = [a_i, b_i, c_i, d_i, e_i, f_i, k_i], i = 1:3$$

The objective function SSd_s is defined by Equation (10), where n is the number of observed data points, X is the matrix of decision variables, and lb and ub are the lower and upper bounds for these variables, respectively.

The steps for calculating d_s within the optimization framework are as follows:

1. *Initialization*: Define the number of decision variables, the population size, and the maximum number of iterations.
2. *Parameter Bounds*: Specify the lower (lb) and upper (ub) bounds for each decision variable.
3. *Scour Depth Calculation*: For each data point, calculate the scour depth d_{sw} using Equation (5) based on the current candidate solution.
4. *Feasibility Check*: If the calculated d_{sw} is not a positive real number, correct it using the penalty function:
$$d_{sw} = \lambda |d_{sw}| \quad \text{if } d_{sw} < 0 \quad (12)$$

Here, λ denotes the penalty constant associated with d_s .
5. *Iteration*: Repeat steps 3–4 for all n data points in the experimental dataset.
6. *Objective Evaluation*: Compute the sum of squared errors SSd_s from Equation (11).
7. *Convergence Loop*: Repeat steps 2–6 until the predefined stopping criterion is met (e.g., maximum iterations reached).
8. *Solution Extraction*: Identify the solution that yields the minimum value of SSd_s .

The pseudocode for this optimization procedure is provided below.

Pseudocode of d_s :

Inputs: Number of decision variable, Input $q, \hat{d}_s, H, d_w, d_{50}, \Phi, R$, Lower Bound of decision variable (lb), Upper Bound of decision variable (**ub**), Number of Iteration, Population size

Outputs: SSd_s, d_s and decision variable

for $i:1:I$

 Calculate d_s using Equations (5)

 if $d_{sw} < 0$ then

$$d_{sw} = \lambda |d_{sw}|$$

 end if

 Calculate SSd_s using Equation (11)

end for

Electric Eel Foraging Optimization (EEFO) Algorithm

The Electric Eel Foraging Optimization (EEFO) algorithm is a metaheuristic method inspired by the collective hunting behavior of electric eels (Zhao et al., 2024). It frames the optimization process as a foraging task, where candidate solutions ("eels") explore the search space and exploit promising regions. The algorithm's two primary phases—exploration and exploitation—are modeled on four key behaviors observed in electric eel social hunting: interaction, resting, migration, and hunting.

The mathematical model for these foraging behaviors is outlined below, starting with the *interaction* phase.

Interaction

This phase corresponds to the exploration of the search space. EEFO models the interactions among individuals in the population to share information and broadly investigate potential solution areas, thereby preventing premature convergence. The interaction behavior is mathematically defined as follows:

$$C = n_1 \times [B_1, B_2, \dots B_K, \dots B_D]; \quad n_1 \sim N(0,1) \quad (13)$$

$$B_k = \begin{cases} 1 & \text{if } k == G\{L \\ 0 & \text{else} \end{cases} \quad \text{where } G = rand_{perm}(D) \quad (14)$$

$$L = 1, \dots, \left\lceil \left(\frac{T-t}{t} \times r_1 \times (D-2) + 2 \right) \right\rceil \quad (15)$$

Here, T represents the maximum number of iterations and r_1 is a random number uniformly distributed in the interval $[0,1]$. The interactive behavior is then defined as follows:

$$\begin{cases} \begin{cases} v_i(t+1) = x_j(t) + C \times (\bar{x}(t) - x_i(t)) & P_1 > 0.5 \\ v_i(t+1) = x_j(t) + C \times (x_r(t) - x_i(t)) & P_1 \leq 0.5 \end{cases} & \text{fit}(x_j(t)) < \text{fit}(x_i(t)) \\ \begin{cases} v_i(t+1) = x_i(t) + C \times (\bar{x}(t) - x_j(t)) & P_2 > 0.5 \\ v_i(t+1) = x_i(t) + C \times (x_r(t) - x_j(t)) & P_2 \leq 0.5 \end{cases} & \text{fit}(x_j(t)) \geq \text{fit}(x_i(t)) \end{cases} \quad (19)$$

$$\bar{x}(t) = \frac{1}{n} \sum_{i=1}^n x_i(t) \quad (20)$$

$$x_r = lb + r \times (up - lb) \quad (21)$$

In these relations, P_1 and P_2 are random numbers in the interval $[0,1]$, $\text{fit}(x(i))$ represents the fitness value of the i th electric eel at its candidate location, and x_j denotes the location of a randomly selected eel from the current population, where $j \neq i$. Here, n is the population size, r is a random vector in $[0,1]$, and lb and ub are the lower and upper bounds of the decision variables, respectively. According to Equation (19), this interactive behavior enables the electric eels to move diversely within the search space, thereby enhancing the exploration phase of the algorithm.

Rest Phase

Mirroring the resting behavior of electric eels between foraging periods, the EEFO algorithm incorporates a rest phase. During this stage, the algorithm evaluates and refines the solutions identified in the preceding interaction phase. The rest region is mathematically described as follows:

$$\{X | |X - Z(t)| \leq \alpha_0 \times |Z(t) - x_{prey}(t)|\} \quad (22)$$

$$\alpha_0 = 2 \left(e - e^{\frac{t}{T}} \right) \quad (23)$$

$$Z(t) = lb + Z(t) \times (up - lb) \quad (24)$$

$$z \left\{ t = \frac{x_{rand\{D\}}^{rand\{D\}} - lb^{rand\{D\}}}{up^{rand\{D\}} - lb^{rand\{D\}}} \right. \quad (25)$$

Where x_{prey} is the position vector of the best solution found so far, α_0 is the initial scale of the resting area, and the term $\alpha_0 \times |Z(t) - x_{prey}(t)|$ defines the extent of this area. Here, $x_{rand\{D\}}^{rand\{D\}}$ denotes a random position in one dimension of an individual selected from the current population, and Z is a normalized number. The resting state of an eel before entering its resting area is given by:

$$R_i(t+1) = Z(t) + \alpha \times |Z(t) - x_{prey}(t)| \quad (26)$$

$$\alpha = \alpha_0 \times \sin(2\pi r_2) \quad (27)$$

Where α is the dynamic scale of the resting area and r_2 is a random number in $[0,1]$. As the number of iterations increases, α reduces the size of the resting area, thereby promoting exploration. After determining the resting area, the eels move towards it and update their positions as follows:

$$v_i(t+1) = R_i(t+1) + n_2 \times (R_i(t+1) - \text{round}(\text{rand}) \times x_i(t)) \quad (28)$$

$$n_2 \sim N(0,1) \quad (29)$$

213 Hunting Phase

214 EEFO mimics the hunting behavior of electric eels to intensify the search around promising solutions, focusing
215 on exploitation to improve solution quality. The hunting range is defined as:

$$216 \{X | |X - x_{prey}(t)| \leq \beta_0 \times |\bar{x}(t) - x_{prey}(t)|\} \quad (30)$$

$$217 \beta_0 = 2 \left(e - e^{\frac{t}{T}} \right) \quad (31)$$

218 Here, β_0 is the initial scale of the hunting range. The eel focuses on its prey x_{prey} , with the hunting radius given
219 by $\beta_0 |\bar{x}(t) - x_{prey}(t)|$. A new position within the prey area is generated as follows:

$$220 H_{prey}(t+1) = x_{prey} + \beta \times |\bar{x}(t) - x_{prey}(t)| \quad (32)$$

$$221 \beta = \beta_0 \times \sin(2\pi r_3) \quad (33)$$

222 Here, the prey area scale β is influenced by a random number r_3 in $[0,1]$ and decreases over time to refine exploitation.

223 The eel detects and circles its prey, updating its position according to:

$$224 v_i(t+1) = H_{prey}(t+1) + \eta \times (H_{prey}(t+1) - \text{round}(\text{rand}) \times x_i(t)) \quad (34)$$

$$225 \eta = e^{\frac{r_4(1-t)}{T}} \times \cos(2\pi r_4) \quad (35)$$

226 Here, η denotes the rounding factor and the scale of the prey area, with r_4 being a random number in $[0,1]$.

227 Migration Phase

228 In EEFO, migration involves periodically searching different regions of the search space, similar to electric
229 eels moving to new feeding grounds. This process balances exploitation and exploration. The migration behavior is
230 modeled as:

$$231 v_i(t+1) = -r_5 \times R_i(t+1) + r_6 \times H_r(t+1) - l \times (H_r(t+1) - x_i(t)) \quad (36)$$

$$232 H_r(t+1) = x_{prey}(t) + \beta \times |\bar{x}(t) - x_{prey}(t)| \quad (37)$$

233 In Equation (36), H_r represents a random position within the prey area, and r_5 and r_6 are random numbers in $[0,1]$.

234 The term $H_r(t+1) - x_i(t)$ directs the eels toward the hunting area. The parameter l is the Lévy flight function,
235 introduced to avoid local optima during the exploitation phase, calculated as:

$$236 l = 0.01 \times \left| \frac{u \times \sigma}{|v|^b} \right| \quad (38)$$

$$237 \sigma = \left[\frac{\Gamma(1+b) \times \sin(\frac{\pi b}{2})}{\Gamma(\frac{1+b}{2}) \times b \times 2^{\frac{b-1}{2}}} \right]^{\frac{1}{b}} \quad (39)$$

238 where Γ is the standard gamma function, u and v are normally distributed random numbers, and b is a constant set to
239 1.5.

240 The final position of the eel is updated according to:

$$241 x_i(t+1) = \begin{cases} x_i(t) & \text{fit}(x_j(t)) \leq \text{fit}(v_i(t+1)) \\ v_i(t+1) & \text{fit}(x_j(t)) > \text{fit}(v_i(t+1)) \end{cases} \quad (40)$$

242 Energy Coefficient

243 A dynamic energy coefficient $E(t)$ governs the transition between exploration and exploitation:

$$244 E(t) = 4 \times \sin\left(1 - \frac{t}{T}\right) \times \ln \frac{1}{r_7} \quad (41)$$

Where r is a random number in $[0,1]$. This coefficient $E(t)$ decreases over iterations. When $E(t) > 1$, eels perform a global search (exploration) via mutual interaction. When $E(t) \leq 1$, each eel performs exploitation through resting, migration, or hunting behaviors with equal probability. This design ensures a higher probability of exploration in the first half of iterations and a higher probability of exploitation in the second half. The flowchart of the EEFO algorithm is shown in Figure 4.

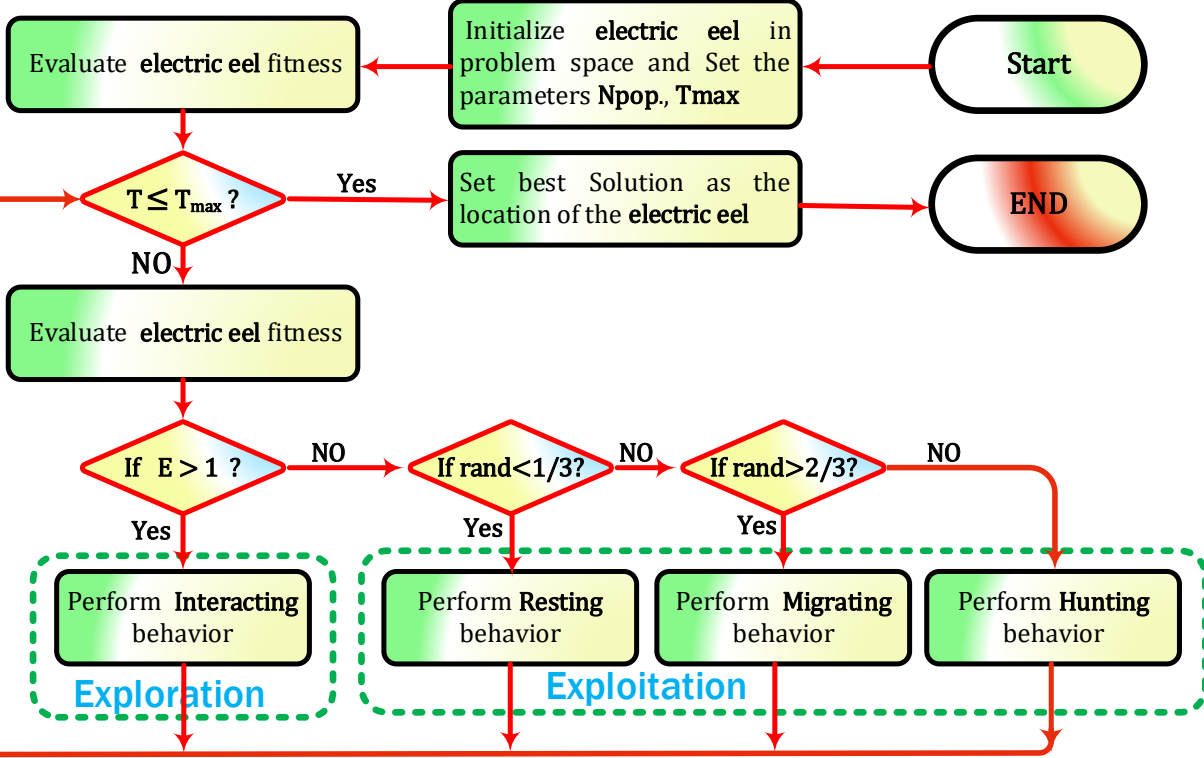


Figure 4. Flowchart of the Electric Eel Foraging Optimization (EEFO) algorithm.

Evaluation Criteria

The evaluation of predictive models is essential to validate their scientific accuracy and practical utility. To assess the performance of the developed scour depth equations for the ski-jump spillway, the following statistical indices were employed: the ratio of the root mean square error to the standard deviation of observations (RSR), the correlation coefficient (R^2), the Nash–Sutcliffe efficiency (NSE), and the percent bias (PBIAS). The formulas for calculating these indices, along with their corresponding performance rating scales, are provided in Table 3.

Table 3. Statistical criteria and rating scales for model evaluation.

Goodness-of-Fit	Equation	Performance Rating	Reference
RSR	$RSR = \frac{\sqrt{\sum_{i=1}^n (\hat{d}_{sw}^i - d_{sw}^i)^2}}{\sqrt{\sum_{i=1}^n (\hat{d}_{sw}^i - \hat{d}_{sw}^m)^2}}$	Very Good: $0 < RSR \leq 0.5$ Good: $0.5 < RSR \leq 0.6$ Satisfactory: $0.6 < RSR \leq 0.7$	(Haiati et al., 2025)
R^2	$R^2 = \left\{ \frac{\sum_{i=1}^n [(d_{sw}^i - d_{sw}^m)(\hat{d}_{sw}^i - \hat{d}_{sw}^m)]}{\sqrt{\sum_{i=1}^n (d_{sw}^i - d_{sw}^m)^2} \sqrt{\sum_{i=1}^n (\hat{d}_{sw}^i - \hat{d}_{sw}^m)^2}} \right\}^2$	Very Good: $0.8 < R^2 \leq 1$ Good: $0.7 < R^2 \leq 0.8$ Satisfactory: $0.5 < R^2 \leq 0.7$	(Afaridegan et al., 2025)

NSE	$NSE = 1 - \frac{\sum_{i=1}^n (\hat{d}_{sw}^i - d_{sw}^i)^2}{\sum_{i=1}^n (\hat{d}_{sw}^i - \hat{d}_{sw}^m)^2}$	Very Good: $0.75 < NSE \leq 1$ Good: $0.65 < NSE \leq 0.75$ Satisfactory: $0.5 < NSE \leq 0.65$	(Lang et al., 2025)
PBIAS	$PBIAS = \frac{\sum_{i=1}^n (\hat{d}_{sw}^i - d_{sw}^i)}{\sum_{i=1}^n \hat{d}_{sw}^i} \times 100$	Very Good: $PBIAS \leq \pm 10$ Good: $\pm 10 < PBIAS \leq \pm 15$ Satisfactory: $\pm 15 < PBIAS \leq \pm 25$	(Apdohan et al., 2025)

In these equations, $\hat{d}_{sw}^i$ and d_{sw}^i represent the measured and model-predicted relative scour depth values for the i -th data point, respectively. The total number of data points is n , while $\hat{d}_{sw}^m$ and d_{sw}^m denote the respective averages of the measured and predicted relative scour depth datasets.

Results and Discussion

Following the methodology outlined in the model optimization section, the mathematical model for scour depth downstream of the ski-jump spillway (Equation 11) was solved using the Electric Eel Foraging Optimization (EEFO) algorithm. The model input consists of two data groups. The first group includes the observed scour depth data, hydraulic flow parameters, spillway geometry, and downstream bed material characteristics, as described in the Dataset section. The second group comprises the optimization hyperparameters: the number of decision variables (21), their lower and upper bounds (-10, 10), the population size (100), and the maximum number of iterations (2000). Using Equations (5) through (11), the optimal parameters were identified by minimizing the objective function SSd_s across multiple runs of the EEFO algorithm. This calibration yielded the final scour depth prediction equation for the ski-jump spillway (Equation 42). The model's performance was then evaluated by comparing its predictions against the measured scour depths using standard error metrics.

The mathematical optimization model depends on the parameters F , H/d_w , R/d_w , d_{50}/d_w , and ϕ . The decision variables are a_i , b_i , c_i , d_i , e_i , f_i , and k_i . The values of a_i , b_i , c_i , d_i , e_i , and f_i are given in equations (5) and (42). The constants k_1 through k_3 are 0.6844, 1.1191, and 0.8919, respectively, and are used to determine L_i and U_i via equations (6)–(9).

The model yields the following error metrics: $RSR = 0.167$, $R^2 = 0.972$, $NSE = 0.972$, $RMSE = 0.448$, $PBIAS = 0.336$ and objective function (SSd_s) = 20.89. The ratios $\hat{d}_s/d_w$ and d_s/d_w are presented in Figures 5a and 5b. The close agreement between measured and calculated values—reflected in these error indices (Figure 6), the objective function, and the plots in Figures 5a and 5b—demonstrates that the mathematical model yields very good results for scour in jet cup spillways.

$$d_{sw} = \frac{d_s}{d_w} = \begin{cases} 7.3086(F)^{0.8182} \left(\frac{H}{d_w}\right)^{0.1101} \left(\frac{R}{d_w}\right)^{-0.2262} \left(\frac{d_{50}}{d_w}\right)^{0.176} (\Phi)^{0.2887} & | \quad F \leq 1.171 \\ 5.686(F)^{1.236} \left(\frac{H}{d_w}\right)^{0.0163} \left(\frac{R}{d_w}\right)^{-0.3363} \left(\frac{d_{50}}{d_w}\right)^{-0.2879} (\Phi)^{0.7928} & | \quad 1.171 < F \leq 2.774 \\ 7.9766(F)^{0.5378} \left(\frac{H}{d_w}\right)^{0.6715} \left(\frac{R}{d_w}\right)^{-0.1826} \left(\frac{d_{50}}{d_w}\right)^{1.2538} (\Phi)^{0.1019} & | \quad F > 2.774 \end{cases} \quad (42)$$

The optimized model is a function of the dimensionless parameters $0 \leq RSR \leq 0.5$, $0.75 < NSE \leq 1$, $0.8 < R^2 \leq 1$ and $PBIAS \leq \pm 10$. Its decision variables are the coefficients a_i , b_i , c_i , d_i , e_i , f_i and the zoning parameters k_i . The calibrated coefficients are presented in Equations (5) and (42). The zoning parameters k_1 , k_2 , and k_3 were found to be 0.6844, 1.1191, and 0.8919, respectively, which define the Froude number subregion boundaries L_i and U_i via Equations (6) to (9).

The model achieved excellent performance, with error indices as follows: $RSR = 0.167$, $R^2 = 0.972$, $NSE = 0.972$, $RMSE = 0.448$, $PBIAS = 0.336$, and $SSd_s = 20.89$. A comparison between measured ($\hat{d}_s/d_w$) and predicted (d_s/d_w) values

is shown in Figures 5a and 5b. The close alignment of data points along the 1:1 line and the low error indices confirm the model's high accuracy. According to the rating scales in Table 2, where values of $0 \leq \text{RSR} \leq 0.5$, $0.75 < \text{NSE} \leq 1$, $0.8 < R^2 \leq 1$, and $|\text{PBIAS}| < 10$ indicate "very good" performance, the model's results are classified as such. The very low value of the objective function SSd_s further corroborates the model's favorable performance.

To further validate the proposed model, its predictions were compared against those from established empirical formulas listed in Table 1. The scour depths calculated by these methods were evaluated using the same error indices, with the results summarized in Table 4. Among the empirical methods, the relationship proposed by Azmathullah et al. (2006) performed best. A direct comparison between the model from this study and the Azmathullah et al. (2006) method is presented in Figure 7. This comparison demonstrates the clear superiority of the EEFO-based optimization model in predicting scour depth with greater accuracy.

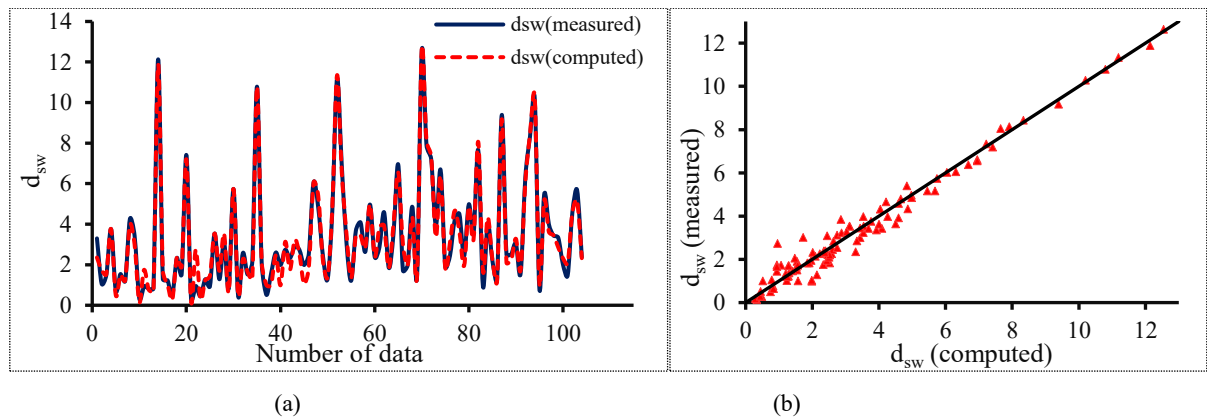


Figure 5. Comparison of measured and calculated d_s

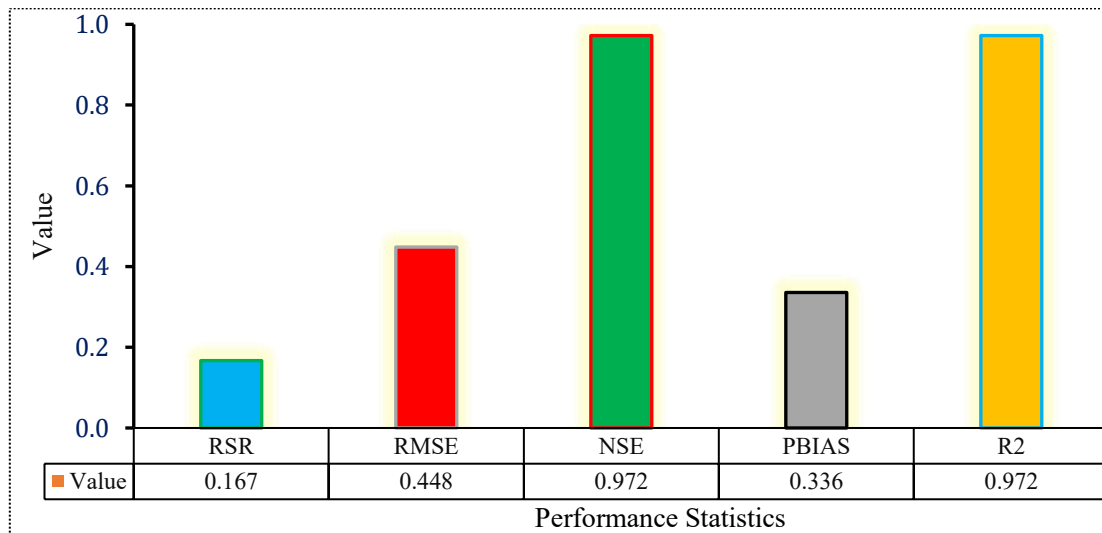


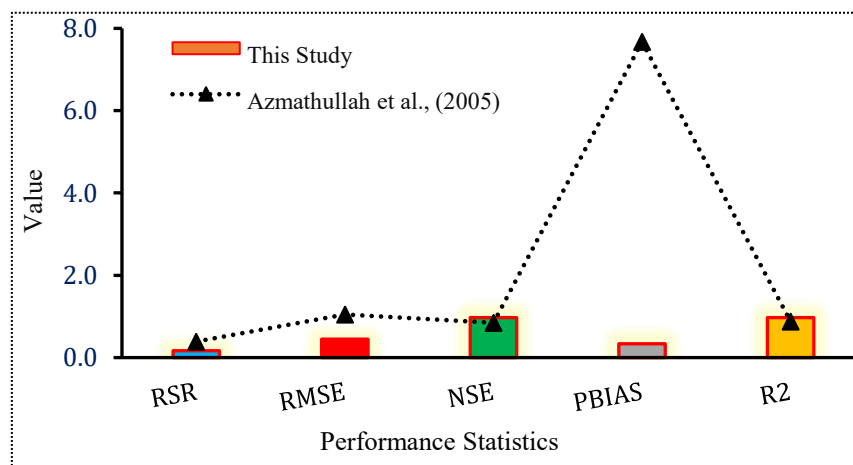
Figure 6. Error measurement indicators

Table 4. Error measurement criteria and computational results of downstream scour depth for ski-jump spillways.

Researcher (s) OR Approach	Goodness-of-Fit				
	RSR	R ²	NSE	PBIAS	SSd _s
Veronese, (1937)	0.8685	0.5801	0.2458	-39.0834	565.7
Mason and Arumugam, (1985)	0.7132	0.5643	0.4913	-21.2052	381.5

Wu, (1973)	1.4355	0.1595	-1.0608	-40.1759	1545.5
Martins, (1975)	1.0058	0.1794	-0.0117	19.6732	758.8
Taraimovich, (1978)	4.6341	0.5848	-20.4753	-201.8042	16106.1
Sofrelec (1980)	0.8981	0.6735	0.1934	-49.2729	604.9
Azmathullah et al., (2006)	0.8971	0.5288	0.1952	-36.6427	603.6
Kumar and Sreeja, (2012)	0.9931	0.4738	0.0138	48.4292	739.6
Noori and Hooshyaripor, (2014)	1.8998	0.0990	-2.6091	32.1477	2706.8
Khalifehei et al., (2020)	2.6684	0.1516	-6.1206	-138.0481	5340.3
Azmathullah et al., (2005)	0.3900	0.8731	0.8479	7.6752	114.1
Sun et al., (2021)	0.6582	0.6837	0.5668	-17.1132	324.9
Suppasri, (2007)	1.4381	0.1289	-1.0681	84.9474	1551.0
Muzzammil and Siddiqui, (2009)	1.0409	0.7648	-0.0834	-34.1535	812.6
Heng et al., (2012)	1.3899	0.3806	-0.9317	-3.7574	1448.8

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Figure 7. Comparison of error indices between the best-performing empirical model (Azmathullah et al., 2006) and the proposed EEFO-based model.

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Conclusion

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Ski-jump spillways are among the most effective hydraulic structures for safely discharging reservoir outflows. They play a critical role in energy dissipation and mitigating severe erosion of the downstream riverbed. However, the impact of the high-velocity jet inevitably creates a localized scour hole, the dimensions of which are governed by complex hydraulic and geotechnical parameters.

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This study introduced a novel predictive model for estimating scour depth downstream of a ski-jump spillway. A new Froude-number zoning approach was implemented, allowing for distinct calibration of parameters within separate flow regimes. The model was formulated by defining a power-law relationship through Buckingham π theorem and dimensional analysis. The coefficients of this equation, alongside the optimal zone boundaries, were

calibrated by minimizing the sum of squared errors (SSE) between measured and predicted scour depths using the Electric Eel Foraging Optimization (EEFO) algorithm.

The performance of the calibrated model was evaluated against established empirical methods. The proposed model demonstrated superior predictive accuracy, as evidenced by the following error indices: RMSE = 0.45 m, RSR = 0.17, NSE = 0.97, PBIAS = 0.34, and $R^2 = 0.97$. These metrics indicate a very good fit to the experimental data and greater reliability compared to traditional empirical formulas. Key advantages of this research include the use of an extensive dataset for calibration and the novel application of the EEFO metaheuristic algorithm to this hydraulic engineering problem.

The main findings of this study can be summarized as follows:

1. The developed equation outperforms existing empirical formulas for predicting ski-jump spillway scour depth, based on a comprehensive set of error metrics.
2. The primary dimensionless parameters governing scour depth are the Froude number (F), the relative drop height (H/d_w), the relative bucket radius (R/d_w), the relative grain size (d_{50}/d_w), and the bucket lip angle (Φ).
3. Within the calibrated model, scour depth exhibits a direct relationship with the Froude number, relative drop height, and bucket angle, and an inverse relationship with the relative bucket radius.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Data availability:

Data will be made available on request.

Author Contributions:

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