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Digital Twin-Driven Multi-Objective Optimization of Milling Parameters Under Small-Sample Conditions

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Abstract

To address the high experimental cost, insufficient parameter-space coverage, and multi-objective trade-off difficulties in milling parameter optimization under small-sample conditions, this study proposes a digital twin-driven multi-objective optimization method for milling parameters. An end-milling process is considered, with spindle speed, feed rate, axial depth of cut, and radial depth of cut selected as input parameters. A unified virtual evaluation interface is constructed to rapidly evaluate machining time, cutting force, and the probability of satisfying surface quality constraints. To improve parameter-space coverage, a diffusion-inspired noise-perturbation augmentation method is used to generate supplementary samples, which are further filtered using parameter bounds and cutting-force physical consistency constraints. A Gaussian process regression model is then established as a surface roughness threshold classification model, transforming surface quality evaluation into a probabilistic constraint satisfaction problem. Based on the virtual evaluation model, Bayesian optimization and a multi-objective genetic algorithm are combined to search for Pareto non-dominated solutions. The case study shows that the proposed method can generate a relatively continuous set of trade-off solutions among machining time, cutting force, and surface quality feasibility, providing alternative parameter combinations for different machining preferences. The optimization results are further integrated into a digital twin visualization system for interactive comparison and decision support. The recommended parameter combinations still require further validation through physical machining experiments.

Keywords: Milling parameter optimization; digital twin; virtual evaluation; surface quality; cutting force; multi-objective optimization

1 Introduction

With the development of intelligent manufacturing, the Industrial Internet, and digital twin technology,

manufacturing systems are gradually shifting from traditional experience-driven modes toward data-driven, model-predictive, and optimization-oriented decision-making. As key variables in manufacturing processes, process parameters not only affect machining efficiency and resource consumption but also directly determine performance indicators such as cutting load, surface quality, and machining stability. Therefore, rapid evaluation and optimization decision-making of process parameters under complex machining conditions have become important research issues in advanced manufacturing. By integrating physical objects, virtual models, machining data, and service applications, digital twin technology provides a new technical pathway for state mapping, performance prediction, and parameter optimization in manufacturing processes [1–4]. Recent studies have further indicated that digital twin modeling can support machine tool intelligence and machining process modeling by integrating physical models, real-time data, and decision-making services [5–6].

Milling is a widely used material removal process in aerospace, die and mold manufacturing, automotive components, and high-end equipment manufacturing. Compared with turning, drilling, and other machining processes, milling is characterized by intermittent cutting, multi-parameter coupling, and nonlinear responses. Variations in process parameters simultaneously affect multiple performance indicators, including machining time, cutting force, surface roughness, tool wear, and energy consumption. Different parameter combinations often lead to conflicts among machining efficiency, cutting load, and surface quality [7–9]. Therefore, milling process parameter optimization is essentially a multi-objective, multi-constraint, and nonlinearly coupled decision-making problem.

For milling process parameter optimization, traditional studies have mainly adopted orthogonal experiments, response surface methodology, grey relational analysis, genetic algorithms, particle swarm optimization, and NSGA-II to establish the relationships between process parameters and machining performance, and to further search for optimal parameter combinations. Jia et al. [10] conducted multi-objective parameter optimization for CNC plane milling and achieved coordinated improvements in machining efficiency, energy consumption, and surface quality. Subsequent research [11] further considered the integrated optimization of roughing and finishing parameters to improve the overall sustainability of plane milling. Zhang et al. [12] established a milling force prediction model for carbon-fiber-reinforced polymer milling and optimized the machining parameters. Hu et al. [13] investigated the multi-objective optimization of surface roughness, cutting force, and material removal rate in ultrasonic-vibration-assisted

milling. Natesh et al. [14] optimized surface roughness and material removal rate in end milling of AISI 316L stainless steel. Xu et al. [15] further applied NSGA-II to multi-objective milling parameter optimization of ultrahigh-strength steel, demonstrating the applicability of evolutionary multi-objective optimization to machining parameter decision-making. These studies indicate that multi-objective optimization has become an important approach for milling parameter optimization. However, the reliability of the optimization results depends heavily on model accuracy, sample coverage, and constraint settings.

The small-sample problem is a common challenge in manufacturing process data modeling. Since real machining experiments require materials, tools, and machine time, and because data acquisition under different parameter combinations is costly, the number of available machining samples is usually limited. In small-batch, multi-variety, and customized manufacturing scenarios, conventional data-driven models are prone to overfitting due to insufficient samples, making the optimization results sensitive to the training sample distribution and weakening their generalization ability. To alleviate this problem, virtual sample generation, physics-guided modeling, and data augmentation methods have gradually attracted attention. Liu et al. [16] proposed a data modeling method for product quality prediction under small-sample manufacturing conditions. Virtual sample generation methods for surface roughness prediction have also been shown to improve sample-space coverage and enhance the robustness of data-driven models under limited-sample conditions [17–18]. In recent years, diffusion models, as a type of generative modeling method, have shown strong potential in data augmentation and sample generation. The denoising diffusion probabilistic model proposed by Ho et al. [19] provides a basic framework for the “forward noise addition–reverse denoising” generation process. Kotelnikov et al. [20] further proposed TabDDPM, extending diffusion models to tabular data modeling tasks. These studies provide useful references for small-sample machining data augmentation.

In recent years, digital twin technology has been increasingly introduced into milling process modeling and process parameter optimization. Its core idea is to integrate virtual models, machining data, mechanistic constraints, and optimization algorithms to realize machining state mapping, performance prediction, and parameter decision-making. Liu et al. [21] reviewed digital twin-driven machining and pointed out that digital twin machining is evolving from geometric simulation and state monitoring toward intelligent prediction, process optimization, and closed-loop control. Ward et al. [22] constructed a machining-oriented digital twin system and supported closed-loop machining control through real-time model-based simulation

and look-ahead control, demonstrating that digital twins can provide a model basis for online decision-making in machining processes. For cutting load control in milling, Tong et al. [23] proposed a digital twin-driven adaptive cutting force control approach for the milling process, which combines virtual model prediction with feedback regulation to improve machining stability. Furthermore, digital twin-driven machining has been applied to thin-walled part manufacturing and adaptive surface roughness prediction with process parameter optimization, which further supports the integration of virtual modeling, quality prediction, and parameter decision-making in machining processes [24–25]. Meanwhile, Fu et al. [2] pointed out that although existing machining digital twin studies have made progress in error modeling, state prediction, and process control, further research is still needed in parameter-space expansion, physical consistency constraints, and multi-objective optimization coordination under small-sample data conditions. Therefore, constructing a digital twin process parameter optimization framework that integrates data augmentation, physical constraints, and multi-objective optimization for small-sample milling scenarios still has significant research value and engineering application potential.

In summary, although existing studies on milling parameter optimization have made progress in multi-objective modeling, intelligent optimization algorithms, and surface quality prediction, three problems remain in small-sample manufacturing scenarios. First, real machining experiments are costly, and limited samples cannot fully cover the feasible parameter space, making optimization results highly sensitive to the distribution of original samples. Second, in the absence of machining mechanism constraints, purely data-driven models may generate or recommend parameter combinations that violate basic cutting laws, thereby reducing the engineering credibility of the optimization schemes. Third, some studies focus more on outputting a single optimal parameter combination, while insufficiently expressing the trade-off relationships among machining efficiency, cutting load, and surface quality, making it difficult to satisfy the demand for multi-scheme decision-making in practical manufacturing.

To address the above problems, this paper proposes a digital twin virtual evaluation-driven multi-objective optimization method for milling parameters. Taking a typical end-milling process as the research object, a unified virtual evaluation interface is constructed in the digital twin environment, in which machining time, cutting force, and surface quality constraint satisfaction probability are taken as optimization objectives. A diffusion-inspired augmentation method is adopted to expand the parameter space under small-sample conditions, and physical consistency filtering is introduced to improve the

engineering rationality of the augmented samples. Furthermore, Gaussian process regression, Bayesian optimization, and a multi-objective genetic algorithm are combined to realize Pareto optimization search for milling parameters. Different from methods that rely only on a single prediction model or a single optimization algorithm, the proposed method emphasizes the connecting role of the virtual evaluation interface among parameter input, performance mapping, constraint filtering, and optimization decision-making, thereby providing an interpretable manufacturing decision support framework for small-sample milling parameter optimization.

The main contributions of this paper are as follows:

(1) A digital twin virtual evaluation interface for milling parameter optimization is constructed. Machining time, cutting force, and surface quality constraint satisfaction probability are unified as optimization objectives, enabling rapid evaluation of machining performance under different process parameter combinations.

(2) A small-sample data expansion method combining diffusion-inspired noise-perturbation augmentation and physical consistency filtering is proposed. This method expands the coverage of the parameter space while reducing the influence of abnormal samples that obviously violate the variation laws of cutting force on the optimization results.

(3) A multi-objective optimization workflow based on GPR probabilistic classification and BO–MOGA collaborative search is established. A Pareto trade-off solution set among machining efficiency, cutting load, and surface quality is obtained, and the optimization results are visually presented through a digital twin system.

2 Digital Twin Virtual Evaluation Framework

Existing studies have systematically summarized industrial digital twins in terms of their constituent elements, operating mechanisms, and manufacturing application scenarios [26–27]. Through the collaboration among physical objects, virtual models, data connections, and service applications, digital twins enable state mapping, performance prediction, and optimization decision-making in manufacturing processes. To support rapid evaluation and optimization decision-making of milling process parameters under small-sample conditions, this paper constructs a digital twin virtual evaluation framework for the end-milling process. This framework is not intended merely for three-dimensional visualization of the machining process. Instead, it integrates process parameters, machining performance models, constraint

conditions, and optimization algorithms into a unified virtual evaluation environment, thereby realizing a continuous mapping from parameter input to performance evaluation and then to scheme recommendation.

As shown in Fig. 1, the constructed digital twin framework mainly consists of the application object layer, data layer, model layer, and service layer. The application object layer describes the milling task, process parameter space, and performance objectives. The process parameters include spindle speed, feed rate, axial depth of cut, and radial depth of cut, while the performance objectives include machining time, cutting force, and the ability to satisfy surface quality constraints. The data layer organizes the original experimental data, augmented samples, parameter bounds, and simulated evaluation data, providing the data basis for subsequent modeling and optimization. The model layer is the core of the virtual evaluation framework. It mainly consists of the machining time model, cutting force evaluation model, and surface quality probabilistic model, which are used to map input parameters into performance indicators available for optimization. The service layer focuses on parameter optimization and scheme decision-making, including Pareto solution set generation, representative scheme selection, and interactive visualization.

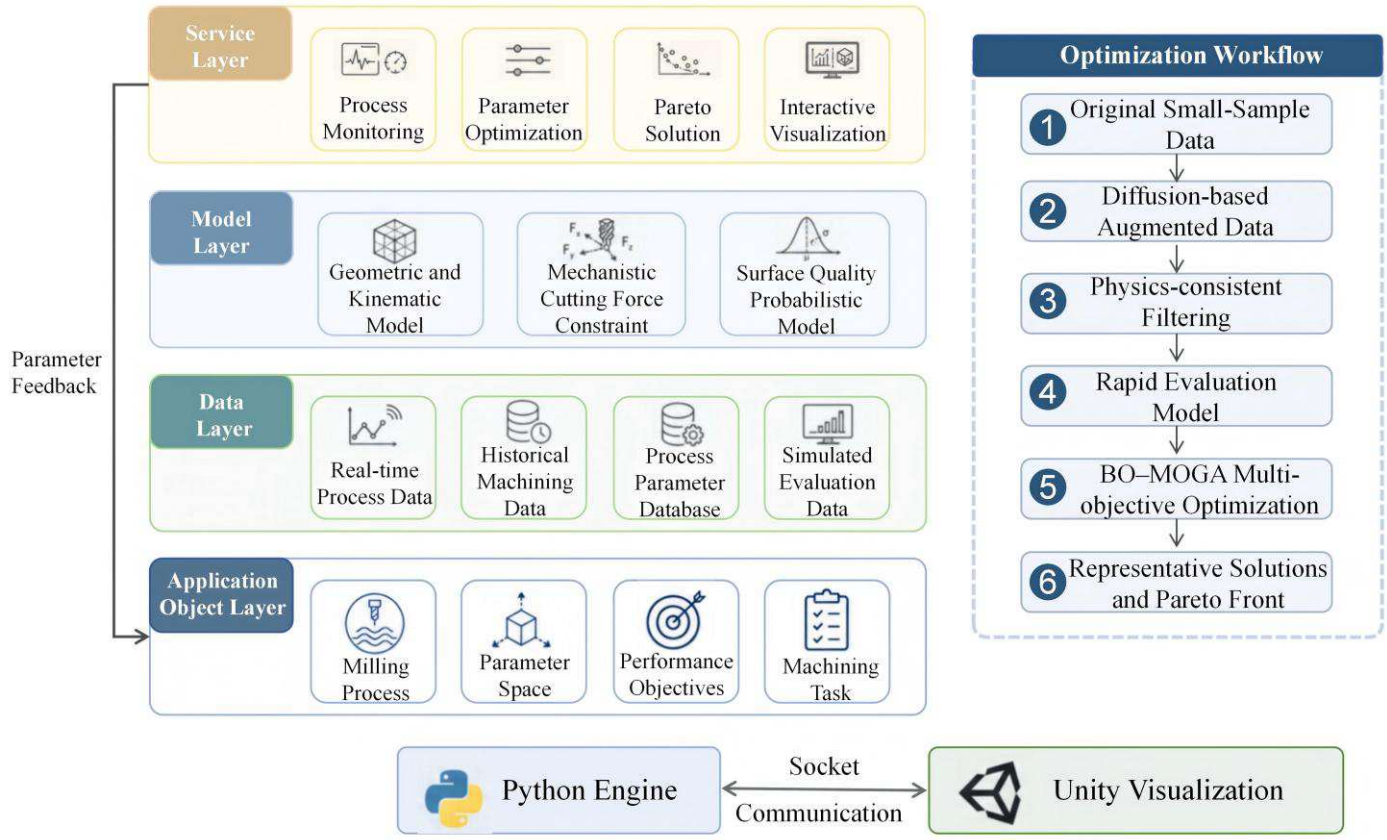


Fig. 1 Overall architecture of the digital twin framework

In this framework, the core role of the digital twin lies in the construction of the virtual evaluation

interface. For any given process parameter combination, the system can rapidly output machining time, cutting force, and surface quality constraint satisfaction probability in the virtual space, thereby replacing high-cost one-by-one machining trial evaluations during the optimization process. Compared with a single data-driven prediction model, this interface simultaneously introduces kinematic relationships, cutting-force physical constraints, and surface quality probabilistic discrimination results. As a result, the optimization process can achieve relatively high computational efficiency while maintaining a certain degree of engineering interpretability.

Therefore, the digital twin framework constructed in this paper can be understood as a virtual evaluation platform for milling parameter optimization. Its main function is not to simply reproduce the machining scene, but to provide a unified model-calling environment for parameter screening, multi-objective optimization, and manufacturing decision-making under small-sample conditions.

3 Multi-objective Optimization Framework Based on Diffusion-inspired Augmentation

In the milling process, parameters such as spindle speed, feed rate, axial depth of cut, and radial depth of cut simultaneously affect machining efficiency, cutting load, and surface quality. Since conflicts usually exist among different performance indicators, milling parameter optimization should seek engineering-feasible trade-off solutions among multiple objectives. Meanwhile, the number of real machining experimental samples is limited. If modeling and search are directly performed based on the original samples, insufficient parameter-space coverage and weak optimization stability may occur.

To address these issues, this paper proposes a BO–MOGA multi-objective optimization method based on diffusion-inspired augmentation and embeds it into the digital twin virtual evaluation framework constructed in Section 2. The proposed method first constructs an optimization-oriented virtual evaluation interface, which maps process parameters to machining time, cutting force, and surface quality constraint satisfaction probability. Then, a diffusion-inspired noise-perturbation augmentation method is adopted to expand the small-sample dataset, and the augmented samples are filtered through parameter bounds and cutting-force physical consistency constraints. Finally, Bayesian optimization and a multi-objective genetic algorithm are combined to obtain the Pareto non-dominated solution set. The overall workflow includes six stages: parameter input, virtual evaluation, sample augmentation, physical filtering, multi-objective optimization, and scheme recommendation.

3.1 Problem Definition and Virtual Evaluation Interface

Taking a typical end-milling process as the research object, this paper selects spindle speed, feed rate, axial depth of cut, and radial depth of cut as the process parameters to be optimized. The parameter vector is defined as

$$x=(n,f,a_p,a_e) \tag{1}$$

where n denotes spindle speed, f denotes feed rate, a_p denotes axial depth of cut, and a_e denotes radial depth of cut. Considering that practical process decision-making focuses not only on machining efficiency and cutting load but also on whether candidate schemes satisfy surface quality requirements, this paper transforms surface roughness evaluation into a threshold-constrained classification problem. Let the roughness threshold be

$$Ra_{\lim}=1.0\mu m. \tag{2}$$

Then, the surface quality label is defined as

$$z(\mathbf{x})=\begin{cases}1, Ra(\mathbf{x})\le Ra_{\lim},\\ 0, Ra(\mathbf{x})>Ra_{\lim}.\end{cases} \tag{3}$$

where $z(\mathrm{x})=1$ indicates that the process parameter combination satisfies the surface quality requirement, whereas $z(\mathrm{x})=0$ indicates that it does not. Furthermore, the surface quality constraint satisfaction probability $Q(\mathrm{x})$ is introduced to characterize the probability that a candidate parameter combination satisfies the roughness threshold requirement. Compared with directly outputting a single roughness prediction value, the probabilistic form can reflect the prediction uncertainty of the model under small-sample conditions and is more suitable for feasibility screening of parameter combinations.

Accordingly, the digital twin virtual evaluation interface can be uniformly expressed as

$$\mathbf{y}(\mathbf{x})=(T(\mathbf{x}),F(\mathbf{x}),Q(\mathbf{x})) \tag{4}$$

where $T(\mathrm{x})$, $F(\mathrm{x})$, and $Q(\mathrm{x})$ denote machining time, cutting force, and the probability of satisfying the roughness constraint, respectively. Based on this interface, the multi-objective optimization problem for milling parameters can be expressed as

$$\min_{\mathbf{x}\in\Omega}[T(\mathbf{x}),F(\mathbf{x}),1-Q(\mathbf{x})] \tag{5}$$

where Ω denotes the search space jointly determined by parameter bounds, equipment capability, and machining feasibility. In this optimization model, $T(\mathrm{x})$ characterizes machining efficiency, $F(\mathrm{x})$ characterizes the cutting load level, and $1\text{-}Q(\mathrm{x})$ characterizes the risk of failing to satisfy the surface quality constraint. Through the above definitions, milling process parameters, machining performance evaluation, and the multi-objective optimization problem are unified into the digital twin virtual evaluation interface, as

shown in Fig. 2.

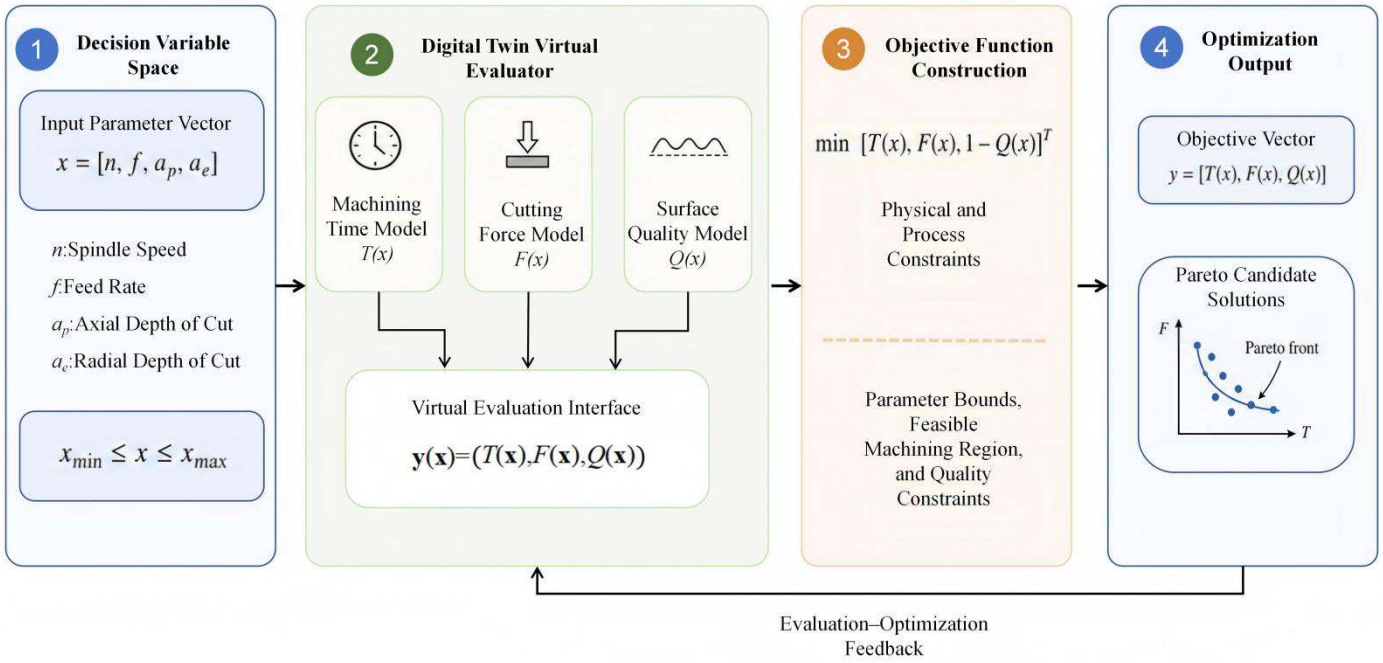


Fig. 2 Problem definition and virtual evaluation interface

3.2 Virtual Evaluation Model and Rapid Evaluation Modeling

To achieve rapid solution of the multi-objective optimization problem, a unified virtual evaluation model is constructed in the digital twin environment. This model mainly includes a machining time model, a cutting force evaluation model, and a surface quality probabilistic model, which are used to characterize machining efficiency, cutting load, and the ability to satisfy surface quality constraints, respectively.

For machining time T, the calculation is based on the basic kinematic relationship in numerical control machining. Let the total machining path length be L_{total} and the feed rate be f. The machining time can be approximately expressed as

$$T(x)=\frac{L_{total}}{f}+T_{aux} \tag{6}$$

where T_{aux} denotes the auxiliary time, including tool approach, tool retraction, idle travel, and other auxiliary operations. This model can reflect the relationship between machining time and feed motion, making it suitable as a rapid time evaluation model in the virtual evaluation interface.

For cutting force F, this paper adopts a collaborative evaluation strategy combining mechanistic constraints and data-driven prediction. First, a simplified mechanistic cutting force model is introduced as the physical prior [28]:

$$F_{mech}(x)=K_c a_p a_e \tag{7}$$

where K_c denotes the equivalent specific cutting force coefficient of the material. This model is mainly used to construct physical consistency constraints rather than to solve cutting force with high accuracy. During optimization, the cutting force is ultimately provided by the rapid evaluation model, while the mechanistic model is used to determine whether the prediction result obviously deviates from basic cutting laws.

For surface quality, Gaussian process regression (GPR) [29] is used to construct a surface roughness threshold classification model. The GPR model first outputs the predicted mean and standard deviation of surface roughness. Then, the continuous roughness prediction result is converted into the probability of satisfying the threshold constraint:

$$Q(\mathbf{x})=P(Ra(\mathbf{x})\leq Ra_{\lim}|\mathbf{x})=\Phi\left(\frac{Ra_{\lim}-\mu_{Ra}(\mathbf{x})}{\sigma_{Ra}(\mathbf{x})}\right) \tag{8}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. In this way, surface quality evaluation is transformed from a single roughness prediction into constraint satisfaction probability discrimination, which is more suitable for integration into the multi-objective optimization process.The probabilistic conversion process from the GPR-predicted roughness distribution to the surface-quality constraint satisfaction probability is illustrated in Fig. 3.

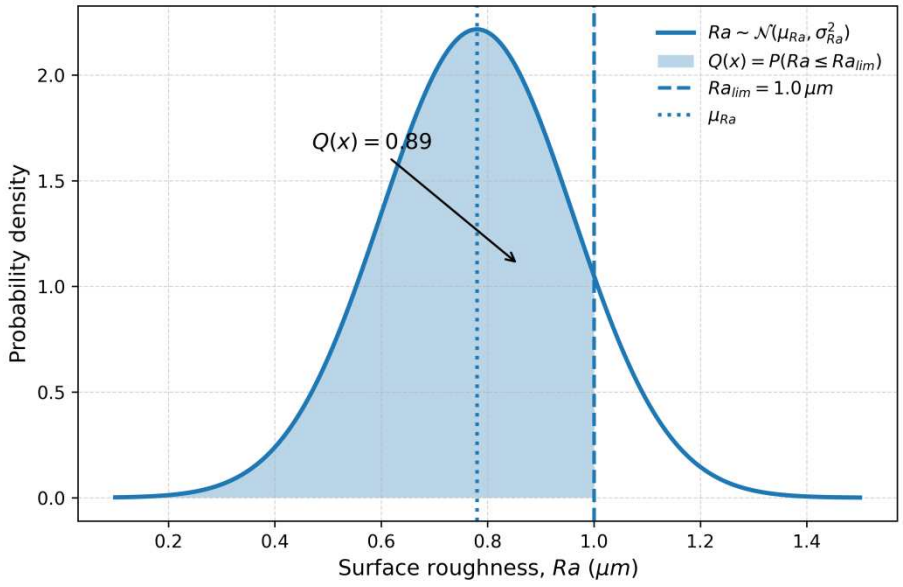


Fig. 3 Probabilistic conversion from surface roughness prediction to surface-quality constraint satisfaction probability.

Previous studies have shown that Gaussian process regression can effectively model surface roughness in machining processes and is suitable for limited-sample prediction tasks [30–32]. In this study, the RBF

kernel, the Matern 5/2 kernel, and different noise regularization parameter combinations are compared under the same input variables and repeated five-fold cross-validation strategy. Since surface roughness evaluation is transformed into a threshold-constrained classification problem in this paper, model performance is evaluated using Accuracy, Balanced Accuracy, Precision, Recall, F1-score, and AUC, rather than conventional continuous regression error metrics.

Table 1 Candidate parameter combinations and performance comparison of the GPR-based surface quality threshold classification model

Combination Number	GPR-1	GPR-2	GPR-3	GPR-4	GPR-5	GPR-6
Kernel function	RBF	RBF	RBF	Matern 5/2	Matern 5/2	Matern 5/2
α	10^{-6}	10^{-4}	10^{-3}	10^{-6}	10^{-4}	10^{-3}
Accuracy	0.833	0.833	0.833	0.837	0.837	0.837
Balanced Accuracy	0.837	0.837	0.837	0.842	0.842	0.842
Precision	0.850	0.850	0.850	0.850	0.850	0.850
Recall	0.850	0.843	0.843	0.857	0.857	0.857
F1-score	0.829	0.825	0.825	0.833	0.833	0.833
AUC	0.859	0.883	0.859	0.864	0.880	0.866

As shown in Table 1, among all candidate parameter combinations, the GPR model using the Matern 5/2 kernel function with $\alpha=10^{-4}$ achieves relatively better overall performance in terms of Balanced Accuracy, F1-score, and AUC. Therefore, this combination is finally adopted as the parameter setting of the surface quality threshold classification model in this paper.

After obtaining the training set D_{train}, a rapid evaluation model is further constructed to approximate the digital twin virtual evaluator at a relatively low computational cost. Taking process parameters as inputs, this model outputs machining time, cutting force, and surface quality constraint satisfaction probability, thereby providing the objective function calculation module for subsequent BO–MOGA optimization.

3.3 Data Augmentation and Optimization Solution

Considering the limited number of original machining samples, directly using them for rapid evaluation modeling and multi-objective optimization may result in insufficient parameter-space coverage.

Therefore, this paper introduces a diffusion-inspired noise-perturbation augmentation method to expand the original samples. Let the original sample set be

$$D_{\text{real}}=\{(\mathbf{x}_i,\mathbf{y}_i,z_i)\}_{i=1}^{N_0} \tag{9}$$

where x_i denotes the process parameter combination, y_i denotes the corresponding objective response, and z_i denotes the surface quality label defined according to the roughness threshold. This method introduces progressive noise perturbations around the original samples and generates supplementary samples under parameter-bound constraints. The augmented supplementary sample set is denoted as

$$D_{\text{aug}}=\{(\tilde{\mathbf{x}}_j,\tilde{\mathbf{y}}_j,\tilde{z}_j)\}_{j=1}^{N_{\text{aug}}} \tag{10}$$

Compared with using only the original samples for modeling and optimization, this method can expand the candidate parameter range while maintaining the main distribution characteristics of the original samples, thereby providing a richer sample basis for rapid evaluation model training and Pareto solution set search.

Since the generated samples do not necessarily all satisfy basic machining laws, a physical consistency filtering mechanism is further introduced. Taking cutting force as the main constraint object, the predicted value of the rapid evaluation model is compared with the mechanistic model estimate, and the relative deviation is defined as

$$\delta_F=\frac{|F_{\text{pred}}-F_{\text{mech}}|}{F_{\text{mech}}} \tag{11}$$

$$\delta_F\le\tau_F \tag{12}$$

where τ_F denotes the cutting-force consistency filtering threshold. Since the mechanistic cutting force model used in this paper is a simplified approximate model and is mainly used to exclude abnormal samples that obviously deviate from machining laws rather than to serve as a high-precision cutting force predictor, the threshold should not be set too strictly. In this paper, τ_F is set to 0.25. That is, when the relative deviation between the surrogate model prediction and the mechanistic model estimate exceeds 25%, the sample is considered to have insufficient physical consistency and is removed. Meanwhile, the filtering process should also satisfy the constraints of parameter bounds, equipment capability, and machining stability. After physical consistency filtering, the retained augmented sample set can be expressed as

$$D_{\text{phys}}\subseteq D_{\text{aug}} \tag{13}$$

Finally, the original samples and the physically consistent augmented samples jointly constitute the training set:

$$D_{\text{train}}=D_{\text{real}}\cup D_{\text{phys}} \tag{14}$$

Through the combined strategy of diffusion-inspired augmentation and physical filtering, the proposed method expands the sample size while improving the engineering credibility of the data.

After completing augmented sample filtering and rapid evaluation model training, this paper further adopts the BO–MOGA collaborative optimization strategy to solve the multi-objective milling parameter optimization problem. Bayesian optimization and efficient global optimization methods are suitable for expensive black-box functions and limited-evaluation scenarios [33–34], while surrogate-assisted multi-objective optimization methods can improve search efficiency when objective evaluations are costly [35]. NSGA-II provides the evolutionary multi-objective search mechanism for maintaining convergence and diversity in the Pareto solution set [36]. Based on these ideas, the proposed strategy first uses Bayesian optimization to identify potential promising regions under a limited number of evaluations. Then, the candidate solutions in the promising regions are used as initialization information for the multi-objective genetic algorithm, thereby improving the search efficiency and engineering usability of the Pareto solution set. The BO–MOGA multi-objective optimization workflow is shown in Fig. 4.

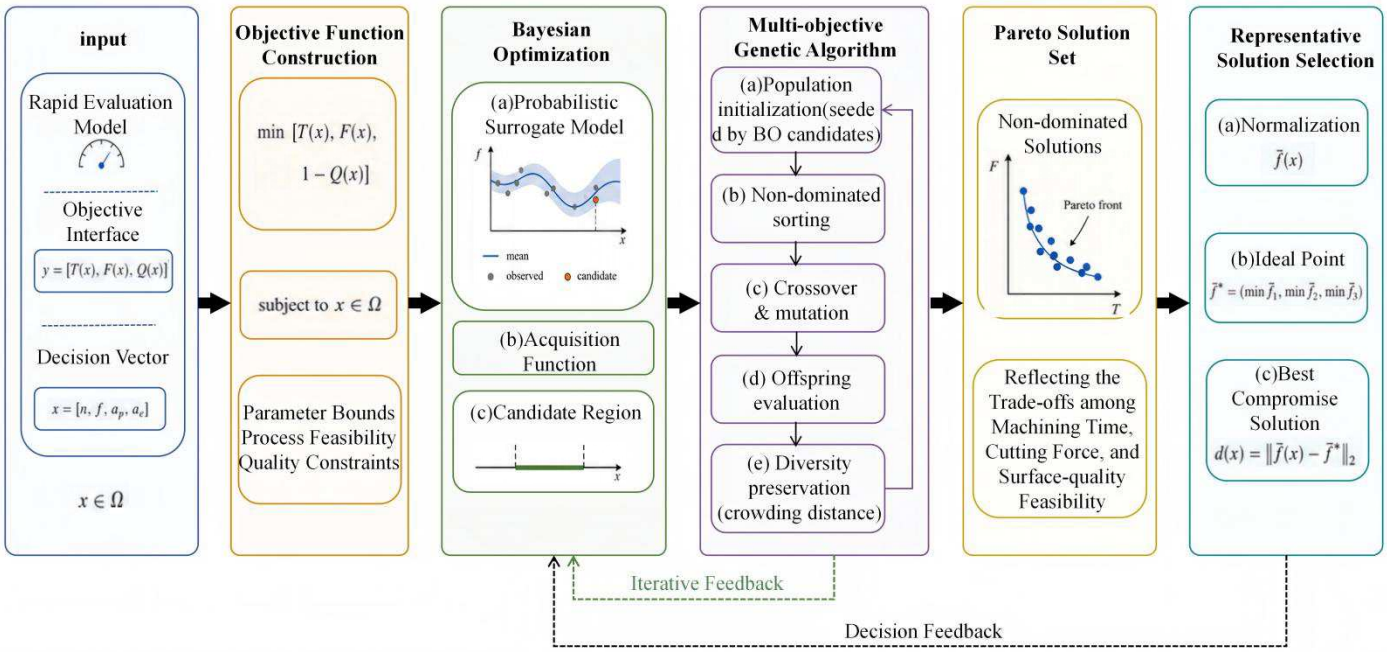


Fig. 4 BO–MOGA-based multi-objective optimization workflow

The Pareto solution set obtained through multi-objective optimization reflects the trade-off relationships among machining time, cutting force, and surface quality feasibility. To select a recommended scheme with better comprehensive performance from the solution set, this paper adopts a method combining normalization and ideal-point distance. First, each objective is normalized as follows:

$$\bar{f}_k(\mathbf{x})=\frac{f_k(\mathbf{x})-f_k^{\min}}{f_k^{\max}-f_k^{\min}} \tag{15}$$

Since the third objective is defined as 1-Q(x), the ideal point can be expressed as

$$\mathbf{z}^*=(0,0,0) \tag{16}$$

Finally, the solution that minimizes d(x) is selected as the best compromise solution:

$$\mathbf{x}^*=\arg\min d(\mathbf{x}) \tag{17}$$

This scheme can achieve a relatively balanced trade-off among machining efficiency, cutting load, and the probability of satisfying surface quality constraints. It is then used as the representative parameter configuration recommended by the digital twin system for subsequent case validation and visualization.

4 Case Study and System Implementation

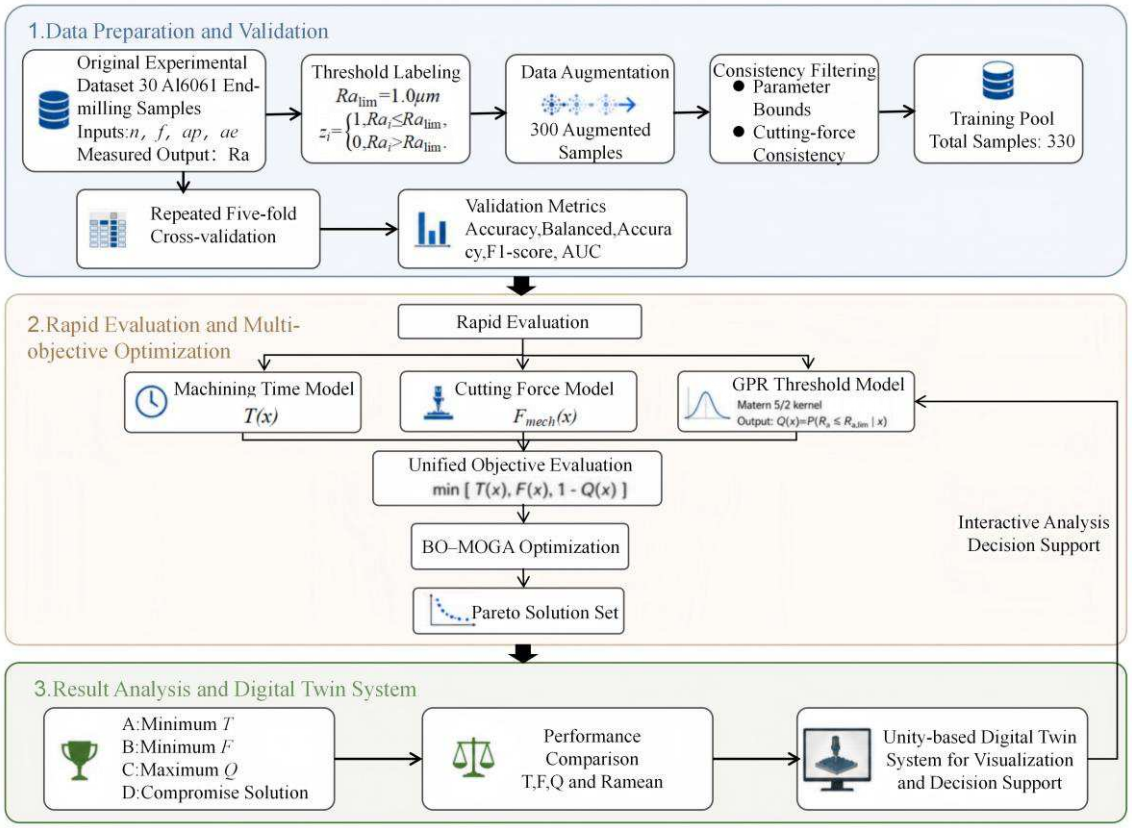


Fig. 5 Validation workflow

To verify the feasibility of the proposed digital twin-driven diffusion-augmented BO–MOGA multi-objective optimization framework under small-sample conditions, a case study was constructed based on publicly available measured end-milling data. Model validation, data augmentation analysis, multi-objective optimization comparison, and system integration demonstration were carried out in a unified

digital twin virtual evaluation environment. The validation workflow is shown in Fig. 5. It should be noted that the focus of this validation is to examine the parameter screening capability, optimization search capability, and interactive decision-support capability of the proposed framework under limited-sample conditions, rather than to conduct physical machining retests for the recommended parameter set.

4.1 Validation Object and Data Construction

In this paper, the end-milling experimental data of Al6061 aluminum alloy from the published literature were used as the original samples [37–38]. The dataset contains 30 groups of different cutting parameter combinations and their corresponding measured surface roughness Ra values. The experimental object, material type, and input variables are consistent with the end-milling parameter optimization problem considered in this study. Therefore, this dataset was used as the basic data for training and validating the surface quality threshold classification model under small-sample conditions. A unified virtual evaluation system was then constructed by integrating the machining time model, mechanistic cutting force constraint model, and surface quality probabilistic model.

Considering the current small-sample data source and the applicable range of the subsequent digital twin virtual evaluator, the parameter search space Ω was set as shown in Table 2.

Table 2 Parameter search space

Parameter	Description	Range
n	Spindle Speed	1000–3000 rpm
f	Feed Rate	100–500 mm/min
a_p	Axial Depth of Cut	10–30 mm
a_e	Radial Depth of Cut	0.5–3.5 mm

According to the surface quality threshold classification method defined in Section 3, the roughness threshold was set as $Ra_{\mathrm{lim}} = 1.0$ μm. Under this threshold, 14 of the 30 original samples satisfied the surface quality requirement, while 16 samples did not. The class distribution was therefore relatively balanced. To avoid overestimating model performance due to the distributional correlation between augmented samples and original samples, the generalization performance of the surface quality threshold classification model was evaluated only based on the original 30 experimental samples using repeated five-fold cross-validation. The augmented samples were not involved in this validation process.

Since the number of original samples was limited, a diffusion-inspired noise-perturbation augmentation

method was further used to generate 300 augmented samples. These samples were then filtered by combining parameter bounds and cutting-force consistency constraints. Finally, the original samples and the filtered augmented samples jointly formed the sample basis for the optimization stage, with a total sample size of 330. The augmented samples were mainly used to expand the candidate optimization space, assist in constructing the rapid evaluation model, and improve the parameter-space coverage of the Pareto solution set search.

4.2 Surrogate Modeling and Multi-objective Optimization Settings

After constructing the original samples and filtering the augmented samples, the rapid evaluation model for the optimization stage was built based on the unified evaluation interface defined in Section 3. Taking the process parameter combination as input, the model outputs machining time T, cutting force F, and surface quality constraint satisfaction probability Q, and serves as the objective function evaluator in the subsequent multi-objective optimization process.

Specifically, the machining time was rapidly calculated using the kinematic relationship, the cutting force was basically evaluated using the mechanistic constraint model, and the surface quality module was implemented using the GPR-based surface roughness threshold classification model. According to the model settings in Section 3, the GPR model used the original four-variable input (n, f, a_p, a_e), the Matern 5/2 kernel function, and the noise regularization parameter $\alpha = 10^{-4}$. The model first outputs the predicted mean and uncertainty of surface roughness, and then converts them into the probability Q of satisfying Ralim = 1.0 μm. The GPR model was implemented using the scikit-learn machine-learning library [39]. The performance of the surface quality classification model was evaluated using classification metrics including Accuracy, Balanced Accuracy, Precision, Recall, F1-score, and AUC.

To compare the optimization effects of different search strategies under the same virtual evaluation environment, four methods were considered: Random Search, GA, NSGA-II, and Diffusion + BO–MOGA. All methods were run under the same parameter search space, the same objective function definition, and the same rapid evaluation model. The Pareto non-dominated solution sets were extracted according to the minimization of (T, F, $1 - Q$). The settings of the four methods are shown in Table 3. This paper mainly compares the distribution characteristics of the Pareto solution sets obtained by different methods in the objective space, as well as the comprehensive performance of representative solutions in terms of machining time, cutting force, and surface quality probability.

Table 3 Comparison of optimization methods

Method	Setup Description	Function
Random Search	Generate candidate solutions randomly within the parameter space	Serves as the benchmark for unguided search
GA	Perform iterative search via selection, crossover and mutation	Used for comparison with traditional genetic optimization
NSGA-II	Adopt non-dominated sorting and crowding distance mechanisms	Used for comparison with classic multi-objective optimization
Diffusion + BO–MOGA	Initialize search guided by augmented samples and optimal regions, combined with multi-objective evolutionary optimization	Proposed method in this paper

4.3 Digital Twin System Implementation and Validation Workflow

At the system implementation level, Python and Unity were used collaboratively to deploy the digital twin platform. The Python side was responsible for sample augmentation, physical consistency filtering, virtual evaluation model training, and multi-objective optimization solving. The Unity side was responsible for three-dimensional scene construction, parameter input, Pareto solution set display, and interactive analysis of representative solutions. Socket communication was used between the two sides to realize parameter transmission and result feedback, thereby forming a closed-loop workflow of “parameter input–virtual evaluation–optimization search–result visualization”.

During the case validation process, the Python side first completed data construction, model training, and optimization solving, and then transmitted the Pareto solution set and representative solutions to the Unity side for display and interactive analysis. The current system mainly implements data augmentation, virtual evaluation, multi-objective optimization, and result display.

5 Results and Discussion

Based on the publicly available end-milling experimental data and the digital twin virtual evaluation environment, this section validates and analyzes the proposed multi-objective optimization method for milling parameters. The analysis includes the role of the virtual evaluation interface, the effect of data

augmentation, the performance of the surface quality classification model, the distribution of the Pareto solution sets, representative solutions, and the digital twin system display.

5.1 Analysis of the Digital Twin Virtual Evaluation Interface

To demonstrate the role of the digital twin system in the parameter optimization process, the virtual evaluation interface is first analyzed. Taking spindle speed, feed rate, axial depth of cut, and radial depth of cut as inputs, this interface calls the machining time model, cutting force evaluation model, and surface quality probabilistic model in the model layer, and outputs machining time, cutting force, and surface quality constraint satisfaction probability. In this way, kinematic calculation, mechanistic constraints, and data-driven prediction are integrated into an optimization-oriented virtual evaluation interface.

This interface is a key link between the digital twin system and the multi-objective optimization algorithms. On the one hand, it can rapidly evaluate the machining performance of different parameter combinations without conducting physical machining experiments one by one. On the other hand, it introduces cutting-force physical constraints and roughness probabilistic discrimination results into the evaluation process, enabling candidate schemes to simultaneously consider machining efficiency, load control, and surface quality constraints. The subsequent comparative analysis of Random Search, GA, NSGA-II, and Diffusion + BO–MOGA is conducted based on this unified interface, thereby ensuring that different methods are compared under the same evaluation criterion.

5.2 Data Augmentation and Model Validation

To verify the improvement in parameter-space coverage achieved by the diffusion-inspired noise-perturbation augmentation method, the distributions of the original and augmented samples are compared. As shown in Fig. 6, the augmented samples generally maintain the distribution trend of the original samples and form more continuous supplementary sample distributions in regions where the original samples are relatively sparse. The number of original samples is 30, and 300 supplementary samples are generated after augmentation, resulting in a total of 330 samples for the subsequent optimization stage. This result indicates that data augmentation can improve the coverage density of the parameter space without significantly deviating from the main distribution characteristics of the original samples, thereby providing a richer candidate sample basis for Pareto solution set search.

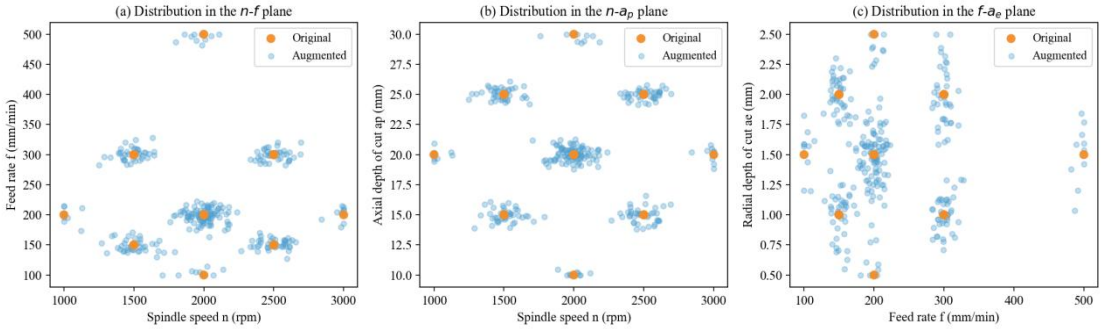


Fig. 6 Comparison of the distributions of original and augmented samples

Table 4 presents the performance metrics of the surface quality threshold classification model based on (Ra_{lim}=1.0μm). The overall classification results reflect the discrimination capability of the model on all samples, while the mean results of repeated five-fold cross-validation are used to evaluate the generalization stability under small-sample conditions. The results show that the Accuracy, Balanced Accuracy, F1-score, and AUC of the overall classification results are 0.8667, 0.8616, 0.8462, and 0.8705, respectively. For the repeated five-fold cross-validation, the mean Accuracy, Balanced Accuracy, F1-score, and AUC are 0.8333, 0.8277, 0.7944, and 0.9294, respectively. These results indicate that the GPR-based threshold classification model can relatively stably distinguish parameter combinations that satisfy the surface quality requirement from those that do not.

Table 4 Classification performance metrics of the surface quality threshold classification model

Metric	Overall Classification Result	Average Result of Repeated 5-Fold Cross Validation
Accuracy	0.8667	0.8333
Balanced Accuracy	0.8616	0.8277
Precision	0.9167	0.8487
Recall	0.7857	0.7900
F1-score	0.8462	0.7944
AUC	0.8705	0.9294

From the confusion matrix in Fig. 7, it can be observed that the model has a certain recognition capability for both classes and does not show an obvious class bias. The ROC curve in Fig. 8 further indicates that the model has good discrimination capability under different classification thresholds. Therefore, the current surface quality threshold classification model can undertake the task of rapidly

screening parameter combinations that satisfy surface quality requirements in the digital twin virtual evaluation environment, and it can serve as the surface quality constraint evaluation module in the subsequent multi-objective optimization process.

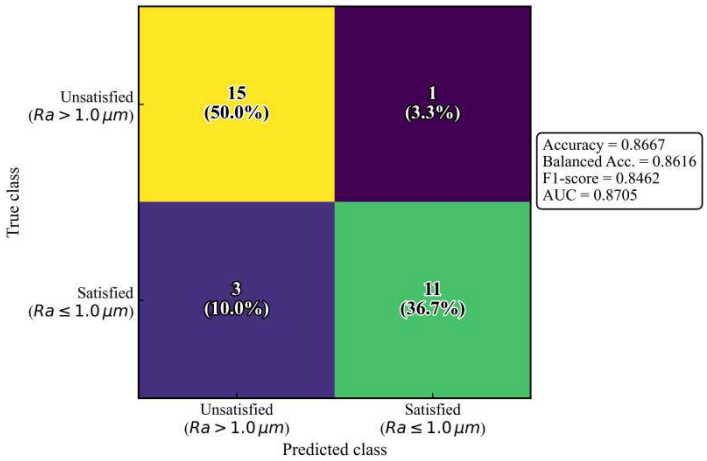


Fig. 7 Confusion matrix of the surface quality classification model

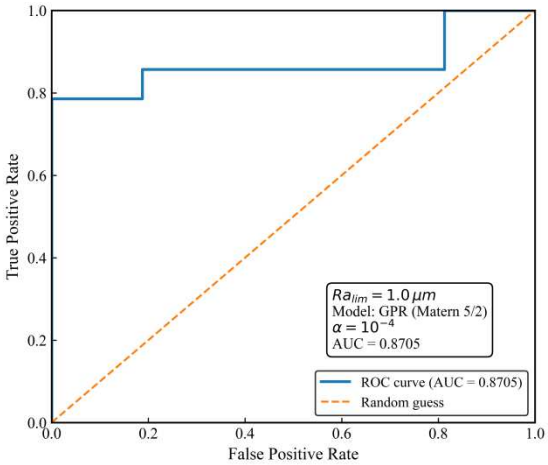


Fig. 8 ROC curve of the surface quality classification model

5.3 Pareto Solution Set and Representative Scheme Analysis

After data augmentation and rapid evaluation modeling, the performance of different optimization methods in multi-objective search is compared. Fig. 9 shows the Pareto solution set distributions obtained by Random Search, GA, NSGA-II, and the proposed Diffusion + BO–MOGA method in the objective space. Overall, the solution set obtained by Random Search is relatively scattered; the solution set obtained by GA tends to concentrate in local regions; NSGA-II improves the coverage of the solution set to some extent; and

the proposed method forms a more continuous distribution of trade-off solutions in the regions with medium-to-low cutting force and higher surface quality probability. This result indicates that the augmented samples and promising-region guidance help increase the search density in effective parameter regions under small-sample conditions, thereby improving the continuity and selectability of the Pareto solution set.

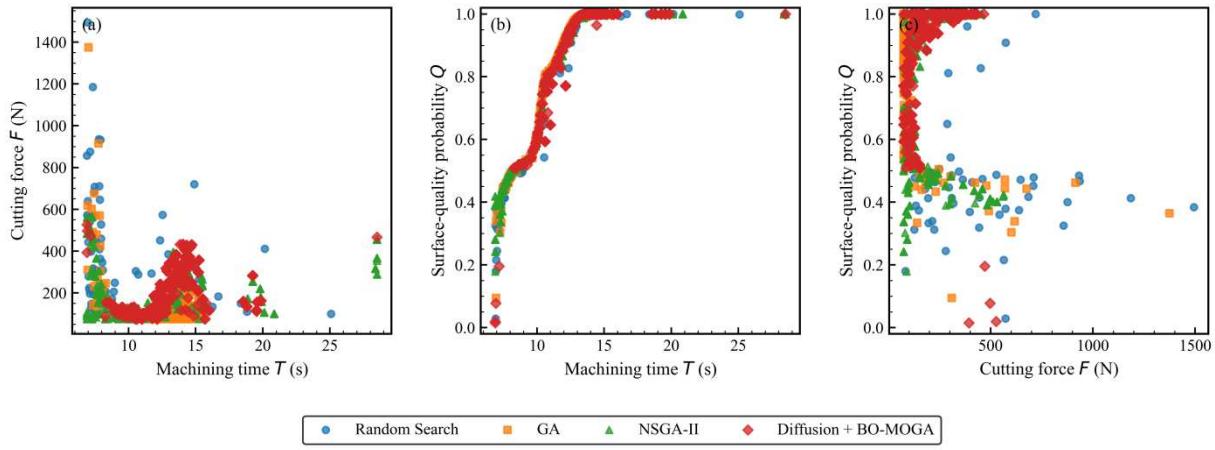


Fig. 9 Comparison of pareto solution sets obtained by different optimization methods in the objective space

The Pareto solution sets in Fig. 9 are obtained from the unified digital twin rapid evaluation model, and therefore mainly reflect the search behavior and parameter screening capability of different optimization strategies under the same evaluation environment. Compared with Random Search, GA, and NSGA-II, the proposed method can form a more balanced trade-off solution set among machining time, cutting load, and surface quality constraint satisfaction probability.

To further analyze the engineering significance of the Pareto solution set, four representative solutions are selected from the Pareto solutions obtained by the Diffusion + BO–MOGA method. These schemes correspond to minimum machining time, minimum cutting force, maximum surface quality probability, and the best compromise solution, respectively. Their parameter combinations and performance indicators are listed in Table 5, and the performance comparison is shown in Fig. 10. Although the surface quality threshold satisfaction probability Q is used as one of the optimization objectives, the roughness mean Ra_{mean} is still retained as supplementary information during scheme display to enhance the engineering interpretability of the results.

As shown in Table 5 and Fig. 10, Scheme A corresponds to the minimum machining time and is more suitable for scenarios that emphasize machining efficiency while allowing surface quality to approach the threshold boundary. Scheme B corresponds to the minimum cutting force and is more suitable for working

conditions that focus on load control and machining stability. Scheme C corresponds to the maximum surface quality probability and is more suitable for finish machining or scenarios with high requirements for surface integrity. Under the unified virtual evaluation interface, Scheme D is identified as the recommended best compromise scheme. Its machining time is 11.71 s, cutting force is 85.45 N, surface quality probability is 0.89, and Ra_{mean} is 0.78 μm. Compared with Scheme A, Scheme D significantly improves surface quality feasibility at the cost of part of machining efficiency. Compared with Scheme C, Scheme D substantially reduces machining time and cutting load while maintaining a relatively high level of surface quality. Therefore, it can be regarded as a priority candidate solution for further process verification.

Overall, the Pareto solution set analysis and representative scheme comparison show that the proposed method can form a trade-off solution set with certain continuity, selectability, and engineering interpretability among multiple objectives, thereby further supporting parameter decision-making. This analysis process, from solution set generation to recommended scheme selection, is more consistent with the practical requirements of multi-objective manufacturing optimization than simply providing a single recommended parameter combination.

Table 5 Parameter combinations and performance indicators of representative solutions in the Pareto solution set obtained by the proposed method

Item	A Minimum Machining Time	B Minimum Cutting Force	C Maximum Surface Quality Probability	D Comprehensive Optimal Compromise
n(rpm)	2730.38	2053.84	2946.07	2939.43
f(mm/min)	398.70	253.98	196.423	264.32
a_p(mm)	11.36	10.00	19.83	11.39
a_e(mm)	0.50	0.50	1.20	0.50
T (s)	8.27	12.13	15.25	11.71
F (N)	85.24	75.00	356.78	85.45
Q	0.51	0.77	1.00	0.89
Ra_{mean}(μm)	0.99	0.87	0.52	0.78
Ra_{lim}=1.0 μm?	Yes	Yes	Yes	Yes

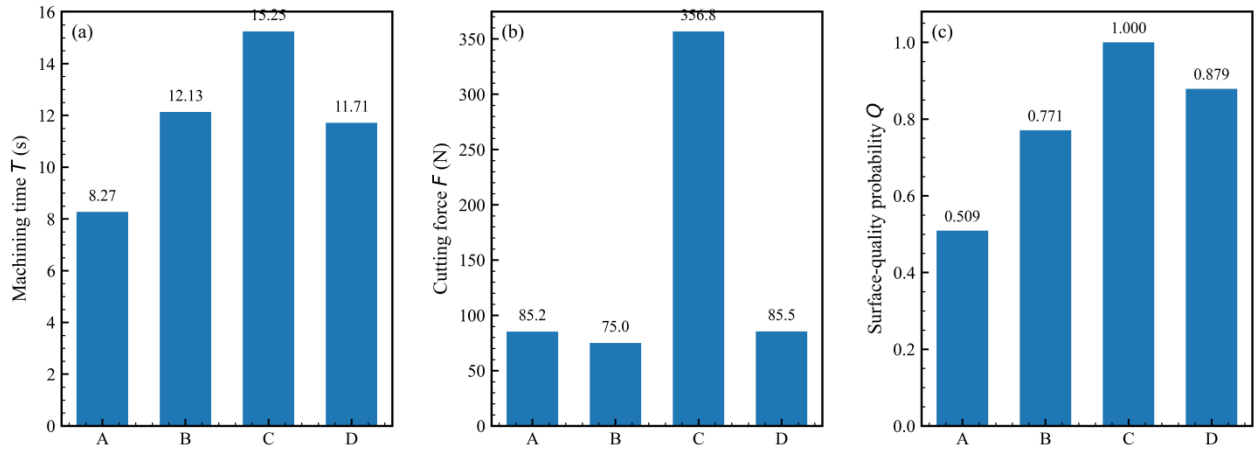


Fig. 10 Performance comparison of representative pareto solutions

5.4 Digital Twin System Validation and Interactive Display

To verify the integration and display effect of the proposed method in the digital twin system, an interactive interface for milling process parameter optimization was developed based on Unity, as shown in Fig. 11. The interface mainly includes process parameter input, three-dimensional machining scene display, Pareto front visualization, representative scheme comparison, and recommended scheme display. It can be used to realize visual expression of optimization results and interactive scheme selection.

The left side of the interface is the parameter input area, where users can input process parameters such as spindle speed, feed rate, axial depth of cut, and radial depth of cut. The middle area shows a simplified vertical machining center scene, including the worktable, fixture, workpiece, cutting tool, and machined slot. The right side is used to display the Pareto solution set distribution and representative optimization schemes. The bottom area shows the parameter combination and performance evaluation results of the currently selected scheme.

As shown in Fig. 11, after the Python side completes sample augmentation, physical consistency filtering, surface quality threshold classification modeling, and BO–MOGA optimization solving, the optimization results can be transmitted to the Unity side for display. The Unity side can visually present the Pareto solution set and typical schemes, supporting users in selecting appropriate process parameter combinations according to machining time, cutting force, and surface quality constraint satisfaction probability. The current system mainly verifies the functional closed loop of “data augmentation–virtual evaluation–multi-objective optimization–result display”. Future work will further integrate online monitoring data and machine-tool control signals to enhance the closed-loop capability of the digital twin

system; uncertainty-aware digital twin optimization provides a useful reference for this extension [40].

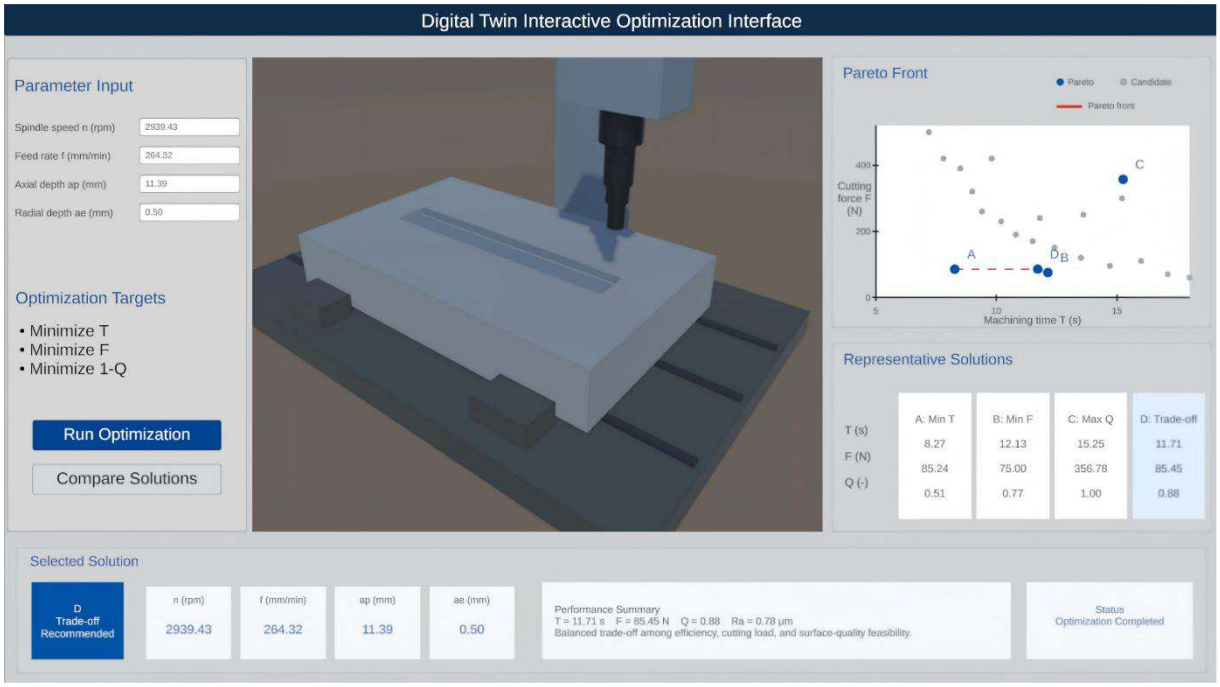


Fig. 11 Interactive interface of the digital twin system

6 Conclusions

To address the problems of insufficient parameter-space coverage, weak engineering constraints in modeling, and difficult multi-objective trade-offs in milling parameter optimization under small-sample conditions, this paper proposed a digital twin virtual evaluation-driven multi-objective optimization method for milling parameters. A unified virtual evaluation interface consisting of machining time, cutting force, and surface quality constraint satisfaction probability was constructed. This interface links process parameter input, machining performance evaluation, and optimization objective construction, thereby providing a model basis for rapid evaluation and optimization decision-making of milling parameters under small-sample conditions.

In terms of data modeling and optimization solution, a diffusion-inspired noise-perturbation augmentation method was adopted to expand the original small-sample data. Parameter bounds and cutting-force physical consistency constraints were further introduced for sample filtering, improving both the parameter coverage and engineering rationality of the augmented samples. On this basis, a GPR-based surface quality threshold classification model was established, and BO–MOGA was used to search for the Pareto non-dominated solution set. The case results show that the proposed method can form a relatively continuous trade-off solution set among machining time, cutting force, and surface quality constraint

satisfaction probability, and provide multiple optional parameter combinations for different machining preferences. By integrating the Pareto solution set and representative solutions into the digital twin visualization system, interactive display and scheme comparison of the optimization results were further realized.

This study still has some limitations. The current research was mainly conducted based on publicly available end-milling experimental data and a digital twin virtual evaluation model, and the sample size and coverage of working conditions remain limited. The obtained Pareto solution set and representative solutions have not yet been retested through physical machine-tool experiments. In addition, the digital twin system mainly realizes virtual evaluation, optimization calculation, and interactive display, but has not yet formed a closed-loop connection with real machine-tool online monitoring and control systems. Future research will further expand measured samples under different materials, tools, and working conditions, introduce online monitoring data such as cutting force, vibration, and tool wear, and conduct physical machining validation of the recommended parameter combinations, so as to improve the engineering reliability and application applicability of the proposed method.

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Statements and Declarations

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Data Availability： The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.