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Andrea D'Isidoro

University of Chieti-Pescara

Roberto Benedetti

University of Chieti-Pescara

Alessandro Marra

`alessandro.marra@unich.it`

University of Chieti-Pescara

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All proximities matter: A Bayesian combination approach to inter-firm spillovers and economic performance

Andrea D’Isidoro^a, Roberto Benedetti^b and Alessandro Marra^{c,d,e}

^a University “G. d’Annunzio” of Chieti-Pescara, Department of Economics, Pescara, Italy. ORCID: 0009-0009-5573-8102. E-mail: andrea.disidoro@unich.it.

^b University “G. d’Annunzio” of Chieti-Pescara, Department of Economics, Pescara, Italy. ORCID: 0000-0003-0763-819X. E-mail: roberto.benedetti@unich.it.

^c **Corresponding author:** University “G. d’Annunzio” of Chieti-Pescara, Department of Economics, Viale Pindaro 42, 65127 Pescara (PE), Italy. E-mail: alessandro.marra@unich.it. Tel: +39 339 5954631. ORCID: 0000-0002-6313-5047.

^d GRIF Research Centre, LUISS University, Rome, Italy.

^e BLISS Research Centre, Ca’ Foscari University, Venice, Italy.

ABSTRACT

Firm performance is shaped not only by firms’ internal capabilities but also by their proximity to other firms, which facilitates knowledge spillovers and, in turn, influences innovation and growth. Proximity, however, extends beyond mere geographic distance and encompasses multiple dimensions, including technological, cognitive, and industrial proximity, each of which may affect firms’ access to external knowledge and their ability to benefit from spillover effects. This study contributes to the literature by proposing a Bayesian methodology for convex combinations of multiple proximity networks, extending the framework from the Spatial Autoregressive to the Spatial Durbin Model. This approach enables simultaneous inference on the relative importance of each network dimension while capturing complex interdependencies within a unified dependence structure. We construct six proximity networks, one geographical and five semantic: technological, cognitive, competence-based, industrial, and a general one. Such semantic networks are derived from corporate websites and used to capture distinct dimensions of inter-firm similarity. Applied to Milan’s tech ecosystem, findings show that four proximity networks contribute significantly to spillover effects. Positive indirect impacts associated with firm size (sales) and knowledge base (number of employees) are consistent with prior findings, illuminating the channels through which inter-firm interactions propagate. The results also highlight the value of preserving the full topology of inter-firm connections through adjacency matrices rather than relying on synthetic network centrality measures. Such results contribute to advance our understanding of how multidimensional proximity networks drive firm performance and economic growth, with implications for both business strategy and place-based innovation policy.

Keywords: Convex combination, Multidimensional proximity, Knowledge spillovers, Bayesian model uncertainty, Semantic networks, Tech ecosystems.

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1 Introduction

The performance of firms, particularly in knowledge-intensive technology sectors, is profoundly shaped by their proximity to other firms within the ecosystem in which they operate. Proximity facilitates knowledge spillovers, which are critical drivers of innovation and growth (Wang and Yang 2022). Proximity is not solely geographical: it encompasses multiple dimensions, including technological, cognitive, and industrial variants, generating spillovers that influence revenue growth, innovation, and competitive dynamics (Boschma 2005; Broekel and Bednarz 2018).

Traditional approaches typically rely on geographic metrics and/or synthetic network positioning indices, such as centrality measures or clustering coefficients, that summarize each observation’s location within a purpose-built network (Broekel 2015; Cao et al. 2019; Qiu et al. 2020; Turkina et al. 2025). While informative, such measures overlook the multifaceted nature of inter-firm connections, which can be described more comprehensively through proximity matrices within a spatial frame (Autant-Bernard and LeSage 2011; Paci et al. 2014; Marra and D’Isidoro 2025). By employing proximity matrices rather than synthetic indices, we preserve disaggregated relational information, incorporating a convex combination of six proximity matrices into spatial econometric models and enabling assessment of both individual contributions and their interactions (Hazir et al. 2018). The proposed methodology builds on the convex combination approach with Bayesian estimation, developed for the Spatial Autoregressive (SAR) model by Debarsy and LeSage (2022). We extend the framework to the Spatial Durbin Model (SDM): by incorporating covariate lags, the SDM captures both local and global spillovers within a flexible specification, rendering the framework more informative and empirically relevant (Pace and LeSage 2009; Ben Amara et al. 2025; Bassibas and Kamal 2026). Moreover, Bayesian inference enables testing the significance of each proximity dimension’s contribution while addressing model uncertainty through model averaging (Shang and LeSage 2021; Debarsy and LeSage 2022). This framework offers policy insights for fostering innovation in technology ecosystems by identifying the dominant proximity channels (i.e., industrial, technological, or cognitive), through which spillovers operate (Buczkowska et al. 2019; Cao et al. 2019).

Applied to Milan’s tech ecosystem, semantic or non-geographical similarities are based on keyword vectors derived from company websites, capturing adopted technologies, knowledge bases, competencies, and industrial specializations (Marra et al. 2026), and all similarity measures are transformed from a linear to an inverted-U shape following seminal contributions (Boschma 2005; Noteboom et al. 2007). Relative to prior work, the contribution is threefold. First, the approach explicitly incorporates semantic proximity measures derived from corporate websites, overcoming the informational limitations of sources such as formal collaboration records, patent classes, or industry codes (Nathan and Rosso 2015; Autant-Bernard 2012). This choice is motivated by two complementary considerations. On the one hand, advances in text analysis and large language models have dramatically expanded the informational reach of textual data, making corporate websites a rich and updatable source of firm-level knowledge profiles. On the other hand, grounding all proximity dimensions in the same textual source yields a methodologically coherent framework: the network position variables used in the empirical model are derived directly from the semantic proximity matrices, ensuring that all relational constructs share a common measurement basis and remain mutually interpretable. Second, as anticipated above, a convex combination with Bayesian estimation is extended to the SDM and

applied at firm-level scale, allowing to capture all closer firms’ neighborhood even within cities (Lavoratori and Castellani 2021). Third, the framework is empirically validated in a dynamic urban context, demonstrating the SDM’s superiority over the SAR specification and identifying significant spillover effects with concrete policy implications.

The paper is organised as follows. Section 2 reviews the literature on multiple proximities, networks, and spillovers, discussing the integration of geographical and non-geographical dimensions. Section 3 presents the methodological framework, covering the construction of semantic similarity matrices and their transformation into proximity matrices (3.1), the Bayesian convex combination approach (3.2), and the full estimation procedure for the SDM with convex combined W (3.3), including direct, indirect, and total impacts and the model uncertainty procedure. Section 4 describes data and variables, results on proximity relevance, variable significance, and model selection. Section 5 concludes with policy implications and directions for future research.

2 Background

The literature has long emphasized the role of proximity in facilitating knowledge spillovers and enhancing firm performance. Early research focused on geographical proximity, arguing that physical closeness enables tacit knowledge exchange through face-to-face interactions and reduces transaction costs (Audretsch and Feldman 1996, 2004; Mariotti et al. 2015; Bouba-Olga et al. 2015, Qiu et al. 2020). As supply chains and digital technologies evolved, however, scholars advocated for a multidimensional understanding of proximity.

Boschma’s (2005) seminal framework identifies several proximity types. Among these, there is cognitive proximity involving shared knowledge bases that enable absorptive capacity (Cohen and Levinthal 1990), technological proximity facilitating recombinant innovation through complementary technologies (Li et al. 2021), and industrial proximity supporting related variety, where firms in similar sectors benefit from specialized spillovers (Frenken et al. 2007).

These dimensions are not mutually exclusive: they interact, sometimes substituting for and sometimes reinforcing one another (Capello and Caragliu 2018; Ben Amara et al. 2025). Researchers have increasingly explored the advantages of combining multiple proximity dimensions to understand how firms exchange knowledge (Boschma et al. 2009). Geographical proximity facilitates face-to-face interactions and informal exchanges, yet lower transportation costs mitigate its influence, but alone cannot fully capture the complexity of knowledge flows (Hazir et al. 2018; Ben Amara et al. 2025). Spatial economic systems benefit from interconnected networks, highlighting that geographical and non-geographical proximities play complementary roles in innovation networks (Reggiani et al. 2002; Wang and Yang 2022). Liu and Ma (2019) find that low technological proximity within dense R&D regions enhances recombinant innovation potential, while Li et al. (2021) show that regional co-location fosters the recombination of diverse technologies. Jespersen et al. (2018) show that technological proximity bridges geographical and market distances, enhancing collaboration potential, and Cao et al. (2019) find that some proximity dimensions can offset the absence of geographical closeness while others support scientific collaboration. Multiple integrated layers enable deeper analysis of knowledge spillovers (Lopolito et al. 2022; Panori et al. 2022), and Marra et al. (2024) propose a spatial model that simultaneously accounts for geographical and business proximity.

A key point concerns the interplay between network metrics and spatial proximities (Bassibas and Kamal 2026). Networks provide channels for knowledge flows both in rela-

tion to geographical and non-geographical dimensions (Ascani et al. 2025; Turkina et al. 2025). Turkina et al. (2019) demonstrate that network centrality enhances innovation in clusters by connecting diverse actors, with brokerage positions enabling access to novel information and closeness fostering trust through repeated interactions (Burt 2005). Physical proximity can amplify these network effects, as co-located firms tend to form denser ties (Löfsten et al. 2023). Empirical studies suggest that network proximity may substitute for geographical closeness, especially in knowledge-intensive industries (Aguilera et al. 2012). Turkina et al. (2025) find that collaborative networks have stronger associations with innovation than mere spatial proximity, with network and spatial dimensions acting as substitutes when interacted, and the relationship varying by tie type and network structure. Similarly, Löfsten et al. (2023) show that professional networks positively affect startup growth, while geographical proximity to customers and competitors yields mixed effects. Spillovers from foreign multinationals further illustrate this dynamic: Mariotti et al. (2015) find that productivity spillovers to domestic firms diminish with distance but are stronger when technological gaps are moderate, while network ties can extend spillovers beyond local boundaries (Cao et al. 2019). Also, proximity dimensions need to be considered jointly because they co-evolve: Broekel (2015) shows that cognitive and geographical proximities develop in tandem, while organizational proximity may substitute for other dimensions in mature networks, suggesting that the relative importance of each dimension shifts over time, shaped by both firm strategies and policy interventions (Bouba-Olga et al. 2015).

This interdependence makes the case for modelling proximities simultaneously rather than sequentially: a framework that treats each dimension as fixed and independent will systematically misattribute spillover effects as the underlying proximity structure changes (Buczkowska et al. 2019). Capturing this complexity requires moving beyond single-matrix approaches toward methods capable of estimating the joint contribution of multiple proximity dimensions. Higher-order spatial models incorporate multiple lags but suffer from identification issues (Paci et al. 2014; LeSage and Pace 2011; Elhorst et al. 2012), while interaction terms provide insights but ignore topological nuances (Turkina et al. 2025). Convex combinations blend matrices into a unified structure with weights reflecting relative importance (Debary and LeSage 2018), but maximum likelihood approaches are limited to few dimensions (Hazir et al. 2018; Marra and D’Isidoro 2025), whereas Bayesian methods handle many, enabling inference and model averaging (Debary and LeSage 2022). Sheng and LeSage (2021) apply convex combinations to Chinese regional knowledge production, finding that non-spatial connectivity dominates, while Hazir et al. (2018) show that collaboration ties outweigh geography in French ICT R&D networks, and Marra and D’Isidoro (2025) find greater geographical relevance in Sweden’s tech ecosystem.

Our study extends this line of research by applying the convex combination with Bayesian approach to the SDM and considering firm-level proximity, which enables controlling for more granular levels of geography results than areal units (Lavoratori and Castellani 2021). The application is in Milan’s context, combining six proximity dimensions, previously considered in isolation (Marra et al. 2026) or in tandem (Marra and D’Isidoro 2025), within a unified framework, and quantifying channel-specific spillovers while addressing whether networks substitute or complement geographical proximity.

3 Methodological framework

This section details the construction of semantic proximity matrices and the Bayesian estimation approach, detailing the impacts estimation and the adopted model selection procedure.

3.1 The Semantic Proximity Matrices

Traditional metrics such as industry codes or patent classes are static and fail to capture evolving firm activities (Nathan and Rosso 2015; Marra et al. 2024). Industry codes are typically declared at firm founding and rarely updated as strategies shift, making them particularly ill-suited for technology firms that operate across sectors and change specializations rapidly. Patent classes face analogous limitations: technological proximity estimates derived from them tend to lag behind real-world dynamics and struggle to reflect modern industrial complexity. Semantic approaches using text data from firm websites offer a flexible alternative, deriving proximities from self-described profiles (Kinne and Lenz 2021; Marra and D’Isidoro 2025). Nathan and Rosso (2015) show how text data deepens understanding of digital industries, while Kinne and Lenz (2021) demonstrate its ability to uncover collaboration networks and innovative outcomes. Topic modelling has been applied to patent texts to measure cognitive proximity (Qin et al. 2021, Faure and Ducruet 2026) and identify technology opportunities (Peng et al. 2023; Qi et al. 2022). Website text has been used to classify firms and technological trajectories (Marra and Baldassari 2022) and detect AI adoption at the firm level (Dahlke et al. 2024; Kinne and Axenbeck 2020). Marra et al. (2026) apply this approach to Milan’s tech ecosystem, constructing industrial, cognitive, and technological proximity matrices and finding significant effects on firm performance, though without capturing the interactions among all dimensions simultaneously.

We employ textual content from company websites to ensure data are self-reported and up-to-date, accurately reflecting each firm’s positioning. Keywords are extracted to identify core technologies, knowledge bases, competencies, and industrial specialisations. To standardise across firms, we apply Latent Dirichlet Allocation (LDA), a probabilistic topic modelling method that uncovers latent themes and maps firms onto a common set of topics, increasing comparability and supporting the identification of semantic proximity. Text is processed through standard NLP procedures, including cleaning, tokenisation, lemmatisation, and stop-word removal, yielding a vector of tokens for each firm. Pair-wise similarity is then computed via cosine similarity, which measures orientation rather than magnitude and is therefore robust to differences in text length. The resulting similarity matrices are converted into standard formats for subsequent analysis. Following Boschma (2005) and Nooteboom et al. (2007), cosine similarity scores are transformed into an inverted-U function to construct the semantic proximity matrix. This captures the non-linear relationship between similarity and knowledge exchange: moderate proximity maximises mutual understanding and competitive stimulation, whereas too little or too much similarity hinders learning and limits idea diversity (Kok et al. 2020; Marra et al. 2024). The spatial weight matrix W is accordingly constructed using a Gaussian kernel centred around the third quartile of the cosine similarity distribution, with diagonal elements set to zero:

$$w_{ij} = \exp \left(-\frac{(s_{ij} - q_3)^2}{2h^2} \right) \quad (1)$$

where the distance is the difference between the cosine similarity value s_{ij} and the third quartile value q_3 (which corresponds to the maximum adjacency); h is the bandwidth, settled to reach zero adjacency in correspondence of maximum value of cosine similarity (one) and for values lower than the second quartile of the distribution. Weights are also set to zero for firm pairs exhibiting very low or very high cosine similarity, as intermediate levels are most conducive to spillovers and performance gains (Marra and D’Isidoro 2025).

3.2 The combination of proximity matrices

The geographical proximity matrix is constructed from the distances between firms’ headquarters coordinates. We decide to apply the same U-inverted transformation used for the semantic matrices – as in Formula (1) – to fully align with the rationale of Boschma (2005). We consider together geographical proximity and non-geographical ones adopting the convex combination framework, which enables the integration of multiple structures, each weighted by its own parameter that captures its relative contribution (Debarsy and LeSage 2022). This approach generalizes beyond two matrices (e.g., geographical and semantic) to L matrices, allowing additional dimensions of proximity if needed (e.g., technological, cognitive or industrial). Moreover, it avoids many of the estimation and interpretation issues associated with higher-order models, while providing parameters that reflect the relative importance of each type of connectivity in the global cross-sectional dependence scheme.

Formally, let W_ℓ ($\ell = 1, \dots, L$) represent $n \times n$ connectivity matrices (with n being the number of firms), where each W_ℓ is row-normalized, has zeros on the main diagonal, and captures a specific type of proximity ($L = 6$: W_1 for geographical, W_2 for general, W_3 for technological, W_4 for cognitive, W_5 for competence-based, and W_6 for industrial). The convex combined weight matrix $W_c(\Gamma)$ is defined as:

$$W_c(\Gamma) = \sum_{\ell=1}^L \gamma_\ell W_\ell, \quad (2)$$

where $\Gamma = (\gamma_1, \dots, \gamma_L)'$, $0 \leq \gamma_\ell \leq 1$, and $\sum_{\ell=1}^L \gamma_\ell = 1$.

This ensures $W_c(\Gamma)$ is a valid weight matrix reflecting a weighted blend of proximities, with the γ_ℓ parameters indicating the relative strength of each connectivity type.

Subsequently to its construction, the $W_c(\Gamma)$ is ready to be used in spatial econometric models. The introduction of an estimation scheme for the SDM is justified by several reasons. In addition to the spatial lag of the dependent variable, considered by the SAR, in the SDM also the spatial lags of covariates are involved. This circumstance gives more flexibility to the latter model, which could detect both local and global spillovers (Ben Amara et al. 2025; Bassibas and Kamal 2026). In fact, LeSage (2014) suggests that one of the best specifications to start with is the SDM, since it may overcome the bias related to omitted variables (Pace and LeSage 2009). Moreover, the SDM nests both the SAR and the Spatial Lag of Xs (SLX), and it is recommended when spatially autocorrelated errors are present, as it provides greater robustness. Under certain constraints on the coefficients, the SDM can be reduced to a Spatial Error Model (SEM) (LeSage and Pace 2009).

3.3 Bayesian Estimation and Model Selection

To estimate the model and account for uncertainty in the convex combination, we employ a Bayesian framework using Markov Chain Monte Carlo (MCMC) methods, following Debarsy and LeSage (2022). We extend their approach from the Spatial Autoregressive (SAR) model to the SDM, which includes spatially lagged explanatory variables to capture additional spillover effects from covariates, detailing in the following subsections the necessary differences respect to their work.

A *Mathlab* code version of convex combination SDM was released by LeSage (2021) in the context of spatial panel data. That version of the SDM avoids using the convex combination term in (2) for the covariates, and is expressed as follows:

$$y = \rho W_c(\Gamma)y + X\beta + \sum_{l=1}^L W_l X \theta_l + \varepsilon, \quad (3)$$

$$\varepsilon \sim N(0, \sigma^2 I_n).$$

$$\text{where } W_c(\Gamma) = \sum_{l=1}^L \gamma_l W_l, \quad 0 \leq \gamma_l \leq 1, \quad \sum_{l=1}^L \gamma_l = 1, \quad \Gamma = (\gamma_1, \dots, \gamma_L)'.$$

This formulation effectively turns the model into a model resembling a SAR augmented by a very large set of regressors (i.e. the spatially lagged X s for each W_l).

Differently from the outlined approach, our model applies the same combined weight matrix W_c to both the spatial lag of the dependent variable and the spatial lags of the covariates:

$$y = \rho W_c(\Gamma)y + X\beta + W_c(\Gamma)X\theta + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2 I_n) \quad (4)$$

where y is the $n \times 1$ vector of the dependent variable, X is the $n \times k$ matrix of explanatory variables, β is the $k \times 1$ vector of coefficients on X , θ is the $k \times 1$ vector of coefficients on the spatially lagged X , ρ is the spatial dependence parameter, and ε is the disturbance term.

We believe our formulation (4) has several advantages. First of all, it helps the interpretability of the gamma parameters: the weights gammas reflect the relative importance of each proximity dimension for the entire spatial dependence structure, including both endogenous interaction, and exogenous spillovers. According to formula (3), instead, the gammas only weight the lag of y , while the θ_l coefficients mix the effect of the covariate with that of its specific spatial lag, making it harder to isolate the contribution of each proximity channel. Another important advantage is linked to the parsimony and scalability of our method: with 10 explanatory variables and 6 proximity matrices, as in our empirical application, the model in (3) would require estimating 60 additional parameters (10 θ vectors $\times$ 6 matrices). The model proposed in (4) requires only one β and one θ for each independent variable. This difference becomes even more relevant if the number of proximity matrices or covariates increases. Lastly, but theoretically relevant, the specification in (4) preserves the intrinsic spirit of the SDM: it allows a single spatial weight matrix to govern both the lag of y and the lags of X . Our SDM specification adheres strictly to this logic while incorporating the convex combination of multiple proximity matrices.

3.3.1 Model estimation

The model in (4) can be rewritten in a computationally convenient form by isolating the dependence parameters:

$$(I_n - \rho W_c(\Gamma))y = X\beta + W_c(\Gamma)X\theta + \varepsilon \quad (5)$$

Define $\tilde{y} = (y, W_1y, W_2y, \dots, W_Ly)$ as the $n \times (L+1)$ matrix, and $\omega = (1, -\rho\Gamma')'$ as the $(L+1) \times 1$ vector. The left-hand side becomes $\tilde{y}\omega$. For the right-hand side, precompute $Z_\ell = W_\ell X$ for each ℓ , so $W_c(\Gamma)X = \sum_{\ell=1}^L \gamma_\ell Z_\ell$. Define the augmented explanatory matrix $V = [X, W_c(\Gamma)X]$ and $\delta = (\beta', \theta')'$ as the $2k \times 1$ parameter vector. The model is then:

$$\tilde{y}\omega = V\delta + \varepsilon \quad (6)$$

The parameter space is $\Omega = \Omega_\rho \times \Omega_\Gamma \times \Omega_\sigma \times \Omega_\delta = (-1, 1) \times [0, 1]^L \times (0, \infty) \times \mathbb{R}^{2k}$, ensuring stationarity.

We assign flat priors: $p(\rho) \sim U(-1, 1)$, $p(\Gamma) \sim U(0, 1)$ subject to the simplex constraints, $p(\delta) \propto 1$, and a flat prior on $\log(\sigma^2)$ to restrict $\sigma^2 > 0$. Consequently, we have:

$$p(\delta, \sigma^2, \rho, \Gamma \mid y, X, W) \propto |I_n - \rho W_c(\Gamma)| (\sigma^2)^{-n/2} \exp \left\{ -\frac{1}{2\sigma^2} (\tilde{y}\omega - V\delta)' (\tilde{y}\omega - V\delta) \right\} \quad (7)$$

where $|I_n - \rho W_c(\Gamma)|$ is the Jacobian term.

MCMC estimation proceeds via conditional sampling:

- Conditional on ρ, Γ, σ^2 , $\delta \sim N(\hat{\delta}, \sigma^2(V'V)^{-1})$, where $\hat{\delta} = (V'V)^{-1}V'(\tilde{y}\omega)$;
- Conditional on ρ, Γ, δ , $\sigma^2 \sim [(\tilde{y}\omega - V\delta)'(\tilde{y}\omega - V\delta)]/\chi_n^2$ (scaled chi-squared);
- For ρ conditional on Γ, δ, σ^2 , use griddy-Gibbs: evaluate the conditional posterior over a grid of ρ values in $(-1, 1)$ and sample from the numerical distribution;
- For Γ conditional on ρ, δ, σ^2 , use block Metropolis-Hastings with proposals from a Dirichlet(1, ..., 1) distribution (uniform on the simplex as in Debarsy and LeSage (2022)), tuned by scaling the variance to achieve 40-60% acceptance. For each proposal Γ^* , compute $W_c(\Gamma^*)$ and $V^* = [X, W_c(\Gamma^*)X]$ approximately (feasible given large n , ideally $n > 1000$ and precomputed Z_ℓ).

As in Debarsy and LeSage (2022), the Jacobian $|I_n - \rho W_c(\Gamma)|$ is approximated using a Monte Carlo trace estimator (Pace and Barry, 1997), as $W_c(\Gamma)$ varies across draws:

$$\log |I_n - \rho W_c(\Gamma)| \approx \frac{1}{m} \sum_{j=1}^m u_j' (I_n - \rho W_c(\Gamma)) u_j \quad (8)$$

where u_j are random vectors (from $N(0, 1)$), and m is chosen for precision (e.g., $m = 100$).

3.3.2 Calculating effects estimates

The non-linear structure of the SDM requires the computation of partial derivatives of the expected value of y with respect to each explanatory variable in order to obtain economically meaningful measures of direct, indirect, and total impacts. Following the approach developed for the SAR model with convex combinations by Debarsy and LeSage (2022), we extend the derivation to the SDM specification. Starting from the reduced-form expression of the model in (4) and differentiating with respect to the k -th column of X (denoted x_k), the $n \times n$ matrix of partial derivatives is:

$$\frac{\partial y}{\partial x_k} = S_k(W_c(\Gamma)) = (I_n - \rho W_c(\Gamma))^{-1}(I_n \beta_k + W_c(\Gamma) \theta_k) \quad (9)$$

where β_k and θ_k are the parameters associated with the r -th explanatory variable in the direct and spatially lagged components, respectively. This expression simultaneously captures both the endogenous interaction effect (via ρ) and the exogenous spatial spillovers (via θ_k).

Scalar summary measures of these effects are obtained as follows. The average direct impact is the average of the diagonal elements of $S_k(W_c(\Gamma))$:

$$\bar{M}(k)_{\text{direct}} = n^{-1} \text{tr}[S_k(W_c(\Gamma))]. \quad (10)$$

The average total impact is the average row sum of $S_k(W_c(\Gamma))$:

$$\bar{M}(k)_{\text{total}} = n^{-1} \iota_n' S_k(W_c(\Gamma)) \iota_n, \quad (11)$$

where ι_n is an $n \times 1$ vector of ones. The average indirect (spillover) impact is simply the difference:

$$\bar{M}(k)_{\text{indirect}} = \bar{M}(k)_{\text{total}} - \bar{M}(k)_{\text{direct}}. \quad (12)$$

Because $W_c(\Gamma)$ is a convex combination, direct computation of the inverse and traces for every MCMC draw would be computationally prohibitive. We therefore adopt the computationally efficient trace-based approximation strategy introduced by Debarsy and LeSage (2022) and adapted here for the SDM. The matrix $S_k(W_c(\Gamma))$ is expanded as an infinite series:

$$S_k(W_c(\Gamma)) = \sum_{r=0}^{\infty} \rho^r W_c(\Gamma)^r (I_n \beta_k + W_c(\Gamma) \theta_k). \quad (13)$$

Truncating at order ψ (typically $\psi = 100$; LeSage and Pace 2009) and pre-computing the required traces of powers of the combined weights matrix ($\text{tr}(W_c(\Gamma)^r)$) for $r = 2, \dots, \psi$ (the first-order trace is zero by construction of the row-normalized matrices) yields accurate approximations of the direct impact. The total impact remains closed-form: $\bar{M}(k)_{\text{total}} = (\beta_k + \theta_k)/(1 - \rho)$ when $\rho \neq 1$, provided the series converges.

Empirical dispersion measures are constructed directly from the resulting posterior distributions of the impacts. This procedure ensures that inference properly reflects uncertainty in both the spatial dependence parameter ρ and the convex-combination weights γ_ℓ . The extension from SAR to SDM preserves the computational advantages of the trace approximation while incorporating the richer spillover structure induced by the spatially lagged covariates.

3.3.3 Model uncertainty

To address model uncertainty in selecting the optimal convex combination (which subsets of the L matrices to include), we use Bayesian model averaging (BMA). Each model M_j corresponds to a subset of connectivity matrices ($2^L - L - 1$ possible models if L is small; otherwise, restrict to plausible subsets). Posterior model probabilities are:

$$p(M_j | y) = \frac{p(y | M_j) p(M_j)}{\sum_{q=1}^{\eta} p(y | M_q) p(M_q)} \quad (14)$$

with equal priors $p(M_j) = 1/(2^L - L - 1)$. The log-marginal likelihood $p(y \mid M_j)$ is computed via Metropolis-Hastings tuned Monte Carlo integration over the parameters (Debarsy and LeSage 2022).

Practically, given the estimates of log-marginal likelihoods ($\log M_j$, for each alternative model M_j we can compute the posterior probability as in Sheng and LeSage (2021):

$$p(M_j \mid y) = \frac{\exp(\log M_j)}{\sum_{q=1}^{\eta} \exp(\log M_q)} \quad (15)$$

where η is the total number of models compared. In the present work we have $L = 6$ different matrices and we also aim to use posterior model probabilities to assess if SDM models are more suitable with respect to the SAR models. Therefore, the number of possible models is $2^6 - 6 - 1 = 64$ doubled (128) to use both model specifications. For this reason, we decide to restrict the possible models to a plausible subset, proceeding with a backward selection, starting from the complete specification with 6 connectivity matrices and then excluding the one with the least significant convex combination parameter γ_ℓ , until the log-marginal likelihood reaches its maximum. This procedure reduces the number of models to be compared, helping interpretation of results.

At the end, each model estimation is associated with a posterior probability and the model with the highest one is considered as the best specification. This Bayesian approach provides credible intervals for the γ_ℓ parameters, allowing inference on their significance (e.g., whether a proximity dimension contributes meaningfully), while handling the SDM’s additional complexity.

4 Empirical application

The dataset covers Milan’s tech ecosystem and comprises 505 firms drawn from Dealroom.co (Dealroom 2024). Dealroom.co provides comprehensive coverage of innovative firms. The sample’s representativeness is established in Marra et al. (2026) across firm size (Table 1), sectoral composition (Table 2), sales (Table 3), and geographic distribution (Table 4), thereby ensuring the robustness and generalizability of our findings.

Table 1: Distribution of Milan’s tech companies according to size (sample vs. full dataset)

Size	Subset	Dataset
Micro (<10 employees)	44.6%	47.3%
Small (10 – 50 employees)	38.0%	37.6%
Medium (50 – 250 employees)	11.3%	9.9%
Big (> 250 employees)	6.1%	5.2%

Table 2: Composition of Milan’s tech ecosystem by industries (sample vs. full dataset)

Industries	Subset	Dataset
Fintech	14.0%	12.8%
Marketing	9.9%	9.8%
Software	9.4%	9.4%
Media	7.8%	7.6%
Health	7.2%	8.6%
Food	6.2%	7.1%
Energy	4.8%	4.5%
Real estate	4.4%	4.2%
Home living	4.1%	3.7%
Transportation	3.5%	3.7%
Fashion	3.4%	4.2%
Education	3.4%	3.4%
Travel	2.7%	3.0%
Wellness	2.3%	2.2%
Sports	1.9%	1.9%
Others	15.0%	14.8%

Note: Some tech companies belong to more than one industry.

Table 3: Distribution of Milan’s tech companies according to sales (sample vs. full dataset)

Quartile	Subset	Dataset
Min	0.9	0.2
25%	5.2	5.1
50%	5.8	5.6
75%	6.4	6.3
Max	10.1	10.2
Mean	5.9	5.7

Table 4: Geographical distribution of company headquarters in Milan by zone (sample vs. full dataset)

Zones	Subset	Dataset
Zone 1	30.8%	33.1%
Zone 2	4.0%	3.1%
Zone 3	9.8%	8.2%
Zone 4	10.5%	9.5%
Zone 5	7.8%	7.7%
Zone 6	6.2%	6.5%
Zone 7	7.4%	6.0%
Zone 8	13.6%	15.0%
Zone 9	9.9%	10.9%

This section estimates the impacts of multiple proximities on firm performance by comparing four specifications: OLS without network variables, OLS with network variables, SAR, and SDM with combined matrix. This progression isolates the effects of proximity structures and their interactions.

The dependent variable is sales growth (*Sales_growth*) between 2022 and 2023, measured as the log difference in sales (Coad and Rao 2008). Controls include initial sales (*Initial_sales*) as log sales in 2022 (Civera et al. 2025; Schenkenhofer et al. 2025), average number of employees in the time interval (*Know_base*) as a proxy for knowledge base (Bloom et al., 2013; Balsmeier et al. 2014), employment growth (*Know_growth*) to capture knowledge base expansion (Lindelöf and Löfsten 2004), firm age (*Age*) (Rydehell et al. 2019; Guerrero et al., 2023), and the firm’s positioning in the different networks (*Geo_neighbours*; *Gen_centrality*; *Tech_centrality*; *Cogn_centrality*; *Comp_centrality*; *Ind_centrality*) (Mariotti et al. 2015; Rao-Nicholson et al. 2019). Semantic network covariates follow an established literature: closeness centrality is derived from each of the five semantic matrices using proximity weights as links and is highly correlated with eigenvector centrality in our application (Broekel 2015; Cao et al. 2019; Turkina et al. 2025). Summary and network statistics are reported in Tables 5 and 6.

Table 5: Summary of variables’ distribution

Variable	Min	20%	40%	60%	80%	Max	Median	Mean
<i>Sales_growth</i>	-0.95	0.00	0.01	0.18	0.35	2.18	0.03	0.09
<i>Initial_sales</i>	0.90	5.12	5.58	5.99	6.48	10.1	5.82	5.94
<i>Know_growth</i>	-0.89	-0.25	0.11	0.40	0.85	1.49	0.21	0.33
<i>Know_base</i>	1	4.86	9.84	16.02	38.86	22940	13.34	178.66
<i>Age</i>	2	5	6	8	11	17	7	10.86
<i>Geo_neighbours</i>	10	18	21	33	42	50	23	29.14

Table 6: Summary of the closeness centrality distribution for semantic networks

Variable	Min	20%	40%	60%	80%	Max	Median	Mean
<i>Gen_centrality</i>	0.045	0.055	0.057	0.060	0.064	0.072	0.058	0.060
<i>Tech_centrality</i>	0.017	0.057	0.059	0.061	0.062	0.076	0.060	0.060
<i>Cogn_centrality</i>	0.030	0.055	0.057	0.059	0.063	0.069	0.057	0.058
<i>Comp_centrality</i>	0.083	0.107	0.115	0.118	0.124	0.147	0.116	0.116
<i>Ind_centrality</i>	0.024	0.056	0.059	0.063	0.067	0.079	0.061	0.066

4.1 Single model estimates

We start presenting the results of baseline OLS model that explains firms’ sales growth through their characteristics. As shown in Table 7, the intercept is positive and significant (0.7045), indicating baseline growth. Initial sales (*Initial_sales*) have a negative coefficient (-0.1124), suggesting reduced returns for larger firms at inception, consistent with convergence effects in growth models (Sheng and LeSage 2021). The average number of employees (*Know_base*) and the growth rate in the number of employees (*Know_growth*), respectively the knowledge base and its expansion rate, are positively

associated with sales growth, reflecting human capital's role in scaling (Löfsten et al. 2023). Contra, firm age is in not statistically significant.

Adding network variables ("OLS Networks" in Table 7) incorporates aggregate network positioning metrics as regressors. Geographical network (the number of geographical neighbours (*Geo_neighbours*)) becomes positive and statistically significant (0.0042), aligning with local spillover arguments (Mariotti et al. 2015). Technological proximity exhibits a strong positive effect (11.171), underscoring technological alignment's importance (Turkina et al. 2025). Other network indices are not statistically significant, suggesting varying roles across dimensions (Broekel 2015).

Transitioning to the "SDM" estimated using the convex combined matrix and focusing only on the variables' coefficients of the covariates, we can observe that initial sales partially weaken, while knowledge growth does not change. This indicates that spatial dependence absorbs some direct effects, highlighting indirect channels (Debarsy and LeSage 2022). The full "SDM Networks" model combines both, providing the richest specification. Here, initial sales further diminish, knowledge growth persists, and *Tech_centrality* coefficient strengthen. *Ind_centrality* gains statistical significance, while general cognitive centralities become insignificant, indicating possible substitution effects (Turkina et al. 2025).

Table 7: Estimated beta coefficients

	OLS	OLS Networks	SDM	SDM Networks
Intercept	0.7045*** (0.0000)	0.1966 (0.1946)	0.1861 (0.2951)	0.1776 (0.6463)
<i>Initial_sales</i>	-0.1124*** (0.0000)	-0.1158*** (0.0000)	-0.0842** (0.0322)	-0.0635** (0.0348)
<i>Know_growth</i>	0.1530*** (0.0000)	0.1561*** (0.0000)	0.1504*** (0.0000)	0.1509*** (0.0000)
<i>Know_base</i>	0.0034* (0.0695)	0.0041** (0.0309)	0.0003 (0.2487)	0.0003 (0.5940)
<i>Age</i>	-0.0052 (0.7365)	-0.0006 (0.6791)	-0.0006 (0.7741)	-0.0010 (0.7719)
<i>Geo_neighbours</i>		0.0042** (0.0160)		0.0026* (0.0416)
<i>Gen_centrality</i>		-0.3387 (0.6438)		-1.1847 (0.8238)
<i>Tech_centrality</i>		11.171** (0.0215)		11.629** (0.0194)
<i>Cogn_centrality</i>		-2.667 (0.3055)		-8.0659 (0.5455)
<i>Comp_centrality</i>		-2.9602 (0.6208)		-3.7232 (0.6570)
<i>Ind_centrality</i>		6.0730 (0.1246)		7.8956* (0.0946)

Note: p-values in brackets; *p<0.1; **p<0.05; ***p<0.01.

SDM needs effect estimates to be properly interpreted (Ben Amara et al. 2025). Accordingly, we examine scalar summary impacts of the SDM with all six proximity matrices combined (Table 8). Direct impacts represent own-firm effects, while indirect im-

pacts capture spillovers, and total impacts represent their sum (LeSage and Pace 2011). For initial sales, direct effect is negative, but the indirect one is positive (but strongly significant). This suggests compensatory effects from the neighbours' scale (Sheng and LeSage 2021). Employment growth shows strong direct and indirect effects, with the consequent total impact indicating broad spillovers, consistent with human capital mobility in clusters (Löfsten et al. 2023). Average employment and age have insignificant direct and indirect impacts, but age's total impact is weakly negative, hinting at maturity-induced slowdown (Broekel 2015). Centrality impacts reveal that technologies' direct impact is positive (though the indirect component is insignificant), which aligns with the recombinant-innovation mechanism (Li et al. 2021). This lack of interpretation is particularly relevant, since the contribution of network indices appear to be overcome by the presence of the proximity matrix. For instance, high ρ value (0.7633) confirms strong overall dependence, reflecting Milan's dense ecosystem and the pivotal role of the autocorrelation between firms' performance according to their multifaced proximity structure (Marra et al. 2026). This complex structure could be better described looking at γ parameters (Table 9), which quantify relative importance of each channel. Each proximity parameter is associated in brackets, to account for significance, the proportion of values lower than 0.01 obtained from the draws' distribution, since inference is delicate due to the parameter bounds (0 is the lower bound, not allowing tests of $\gamma = 0$ based on the draws' distribution). We rely on this proportion for its interpretation similar to that of the p-values, since it is the proportion on the tail of the distribution (close to the lower bound, i.e. 0).

Technological (0.2508) and cognitive (0.2281) dimensions dominate, followed by industrial (0.1824) and geographical (0.1248). General (0.1181) and competence-based (0.0959) proximities are less significant, suggesting redundancy (Boschma 2005). Such evidence suggests the need of a deeper analysis to assess which proximities matter more and deserve to be included in the best specification model. This scope is pursued through the implementation of model uncertainty analysis described in section 3.3.3.

A comparison between the estimations presented according with our convex SDM specification, and the results of the specification proposed in LeSage (2021) code, could be performed looking at γ and ρ parameters (with their inference), AIC, and adjusted R-squared. These estimations for the model specification in (3) are summarized in Table 10. Impacts are not presented, since there could be more than 70 effects to interpret, that is one of the weaknesses of that model. The ρ parameter is much lower than the one in our specification, and significant only at 10% level. The adjusted R-squared is lower, while the AIC is higher, both signaling a worse adaptation, due to the huge number of variables. Focusing on the γ estimations, the first two are similar (geographical and general), while technological and cognitive decreases, competence-based and, mainly, industrial increases. However, according to the inference based on the draws, only technological and industrial proximities present a proportion of < 0.01 values lower than 5%, and three proximities (geographical, general, and cognitive) have proportions higher than 10%. The comparison with γ parameters estimated with our SDM specification, all with inference proportions lower than 10% (four under 5%), highlights the higher insight potential of our method, at least in this empirical application.

Table 8: Direct and indirect impacts of SDM with all proximities

	Direct Impacts	Indirect impacts	Total impacts
<i>Initial_sales</i>	-1.9619*** (0.0000)	2.2295*** (0.0098)	0.2676 (0.1951)
<i>Know_growth</i>	3.6992*** (0.0000)	0.3030*** (0.0002)	4.0022*** (0.0000)
<i>Know_base</i>	0.0009 (0.2464)	0.0049 (0.2498)	0.0058 (0.2487)
<i>Age</i>	-0.0348 (0.1273)	-0.7054 (0.2421)	-0.7402* (0.0905)
<i>Geo_neighbours</i>	0.0364 (0.2499)	0.1063 (0.2497)	0.1427 (0.2496)
<i>Gen_centrality</i>	-4.0435 (0.1908)	51.4832 (0.2499)	47.4397 (0.1540)
<i>Tech_centrality</i>	13.8116* (0.0759)	23.9750 (0.2235)	37.7866 (0.1724)
<i>Cogn_centrality</i>	-0.1476 (0.2281)	4.1088 (0.2472)	3.9632 (0.1689)
<i>Comp_centrality</i>	-0.2901 (0.2073)	0.3711 (0.2444)	0.0810 (0.2429)
<i>Ind_centrality</i>	2.5562 (0.2462)	-51.0860 (0.2495)	-48.5298 (0.1478)
ρ	0.7633*** (0.0000)		
adj R-squared	0.3003		
AIC	66.0831		

Note: p-values in brackets; *p<0.1; **p<0.05; ***p<0.01.

Table 9: Convex combination parameters of SDM with all proximities

	geographical	general	technological	cognitive	competence-based	industrial
γ	0.1248 (0.044)	0.1181 (0.056)	0.2508 (0.019)	0.2281 (0.017)	0.0959 (0.091)	0.1824 (0.032)

Note: proportion of draws' values lower than 0.01 in brackets.

Table 10: Estimations of the SDM using formula (3) convex combination specification

Parameters	Estimate	$P(\gamma < 0.01)$
Geographical	0.1016	0.1460
General	0.1181	0.1284
Technological	0.2074	0.0496
Cognitive	0.1482	0.1027
Competence-based	0.1451	0.0827
Industrial	0.2796	0.0191
ρ	0.4562 (0.0848)	
adj R-squared	0.2959	
AIC	68.7995	

Note: $P(\gamma < 0.01)$ reports the proportion of MCMC draws in which γ is smaller than 0.01.

4.2 Model uncertainty

As detailed in the methodological section, we do not estimate all the possible models (128) but we limit the number of models by a backward elimination scheme, until the last proximity elimination generates a worst log-marginal likelihood with respect to the previous step. We replicate this procedure with both SDM and SAR specifications. For each model, a set of 10,000 draws were used with the first 1,000 discarded for burn-in. Model comparison via posterior probabilities favours SDM over SAR, validating our methodological effort and confirming the intuition of Paci et al. (2014). Table 11 summarizes all the procedure, which involved eight different models estimations. For each model, the specification implied, the log-marginal likelihood (the higher the better), and the associated posterior probability are reported. From γ_1 to γ_6 are presented the convex combination parameters associated with each proximity, according to the following order: geographical, general, technological, cognitive, competence-based, and industrial.

The posterior probabilities of all the SAR specification are equal to 0. Consequently, the four SDM models are the ones to be considered. Among them, the model with all proximities combined reaches something more than 5% probability, while the model with three proximities, which excludes geographical interactions, is less than 3%. Only two models achieve remarkable probabilities, i.e. the one with five proximity matrices (with the exclusion of the competence-based), with 16,5% probability, and the SDM with four proximity matrices (namely geographical, technological, cognitive, and industrial), which is so far the best model specification, reaching a posterior probability of more than 75%.

It is interesting to note that the proportions of γ parameters' draws lower than 0.01 are all under 5%, when considering the best specification with four proximities, while at least one of them is always higher than 5%, in models with five or six combined structures (both in SDM and SAR cases). This circumstance, together with better fitting indices (higher R-squared and lower AIC, Table 12), confirms again SDM with four combined matrices as the best specification. Also according to the Bayesian Model Averaging, available in Table 11, we can state, as in Debarsy and LeSage (2022), that two proximities (γ_2 and γ_5) are too low to be considered higher than 0.

The structure of the best combined matrix is highlighted in bold in Table 11, where it can be appreciated that the four dimensions involved are the best known and inves-

Table 11: Model uncertainty backward

	Log-marginal	Prob(M)	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6
SDM	-420.4842	0.0561	0.1248 (0.044)	0.1181 (0.056)	0.2508 (0.019)	0.2281 (0.017)	0.0959 (0.091)	0.1824 (0.032)
SDM	-419.4059	0.1650	0.1249 (0.057)	0.1250 (0.061)	0.2871 (0.014)	0.2505 (0.016)	–	0.2124 (0.018)
SDM	-417.8896	0.7518	0.1352 (0.044)	–	0.3242 (0.009)	0.2891 (0.012)	–	0.2514 (0.017)
SDM	-421.2151	0.0270	–	–	0.4180 (0.003)	0.3037 (0.007)	–	0.2782 (0.011)
SAR	-457.4114	0.0000	0.1478 (0.026)	0.0976 (0.095)	0.3292 (0.004)	0.1816 (0.027)	0.0690 (0.134)	0.1747 (0.028)
SAR	-454.8955	0.0000	0.1599 (0.024)	0.0997 (0.078)	0.3592 (0.005)	0.1934 (0.029)	–	0.1876 (0.029)
SAR	-452.4291	0.0000	0.1719 (0.031)	–	0.4142 (0.000)	0.2112 (0.015)	–	0.2026 (0.021)
SAR	-455.8620	0.0000	0.2337 (0.002)	–	0.4927 (0.010)	0.2735 (0.014)	–	–
BMA	-418.3333	1.0000	0.1294	0.0272	0.3165	0.2797	0.0054	0.2418

Note: Highest log-marginal and posterior probability in bold. Proportion $\gamma < 0.01$ in brackets.

tigated in multiple proximity studies, since they account, respectively, for geographical, technological, cognitive, and industrial proximity. Regarding their relevance, we pinpoint technological proximity as primary channel (more than 32%), cognitive (about 29%) and industrial specializations network (25%) are both relevant in line with similar previous findings (Bassibas and Kamal 2026), and we find that the geographical one is less important than others, accounting for almost 13% of the overall structure, but still significant. Compared with Marra et al. (2026), which did not combine the above proximities and investigated them separately, we find that interactions amplifying spillovers, as underlined by ρ 's magnitude (Table 12), which shows strong autocorrelation between firm performance and underscores network topology's role over aggregates (Hazir et al. 2018). These findings link to literature: positive indirect effects for size/knowledge base mirror (Marra et al. 2026), but here could be attributable to technological/cognitive proximities (Broekel 2015). Generally, compared to the results obtained for the overall model, impacts shown in Table 12 have a similar interpretation, but the higher spatial autocorrelation parameter and the elimination of redundancy in the proximity structure clearly point that this best specification could be very helpful, with the aim of providing useful methodological tools to unbox the underlying mechanisms behind firms' interaction, both in geographical and non-geographical dimensions (Audretsch and Feldman 2004; Ascani et al. 2025).

Table 12: Direct and indirect impacts of SDM best combination of 4 proximities

	Direct Impacts	Indirect impacts	Total impacts
<i>Initial_sales</i>	-1.8769*** (0.0000)	1.9539*** (0.0098)	0.0743 (0.2338)
<i>Know_growth</i>	3.3155*** (0.0000)	0.9413*** (0.0001)	4.2578*** (0.0000)
<i>Know_base</i>	0.0010 (0.2457)	0.0111 (0.2498)	0.0121 (0.2474)
<i>Age</i>	-0.0374 (0.1086)	-0.7729 (0.2421)	-0.8103* (0.0980)
<i>Geo_neighbours</i>	0.0404 (0.2499)	0.0587 (0.2499)	0.0991 (0.2497)
<i>Gen_centrality</i>	-6.4177 (0.1542)	43.5163 (0.2347)	37.0986 (0.1573)
<i>Tech_centrality</i>	11.7540* (0.0899)	14.3578 (0.2221)	26.1118 (0.1835)
<i>Cogn_centrality</i>	-0.5185 (0.2419)	5.3489 (0.2488)	4.8304 (0.1226)
<i>Comp_centrality</i>	-0.3057 (0.2029)	-0.8248 (0.2427)	-1.1305 (0.2208)
<i>Ind_centrality</i>	2.3366 (0.2139)	-33.7673 (0.2444)	-31.4307 (0.1614)
ρ	0.7558*** (0.0000)		
adj R-squared	0.3005		
AIC	62.0704		

Note: p-values in brackets; *p<0.1; **p<0.05; ***p<0.01.

5 Conclusions

This study develops and applies a Bayesian convex combination framework for the SDM to investigate how multiple proximity dimensions shape firm performance in Milan’s tech ecosystem. By constructing six proximity matrices (geographical and five semantic: general, technological, cognitive, competence-based, and industrial) from corporate websites and estimating them jointly within a unified spatial dependence structure, the approach overcomes the limitations of single-proximity analyses and maximum-likelihood methods restricted to few dimensions (Hazır et al. 2018; Marra and D’Isidoro 2025). The results are clear: the SDM with four proximities (geographical, technological, cognitive, and industrial) receives more than 75% posterior probability. Technological and cognitive proximities carry the largest weights ($\gamma \approx 0.32$ and 0.29), while general and competence-based proximity prove redundant within the richer structure. The high spatial autoregressive parameter ($\rho \approx 0.76$) and strong positive indirect impacts on employment growth and knowledge base confirm that spillovers operate primarily through technological and cognitive channels, consistent with recombinant innovation mechanisms and human-capital mobility in dense urban clusters (Löfsten et al. 2023; Li et al. 2021). The findings contribute to the literature in two ways. First, they provide the first firm-level application of the Bayesian convex combination approach of Debarsy and LeSage (2022), extending it

to the SDM and demonstrating its superiority over SAR through formal posterior model probabilities. Second, deriving proximities from web content rather than industry or patent codes captures the dynamic nature of firm activities that traditional classifications miss (Nathan and Rosso 2015; Marra et al. 2024). The results also speak to the substitution versus complementarity debate between network and spatial effects (Turkina et al. 2025; Ascani et al. 2025): geographical proximity remains relevant (13% weight) but is clearly secondary to technological and cognitive channels, suggesting that policy should prioritise semantic alignments over physical co-location. For technological ecosystem or industrial cluster initiatives, identifying dominant proximity channels offers concrete guidance. Technology-matching platforms, university-firm knowledge-transfer programmes, and targeted R&D subsidies strengthening knowledge and technological linkages can amplify spillovers more effectively than generic co-location incentives. In less specialised regions, the methodology can diagnose which proximity dimensions are absent and tailor interventions accordingly, reducing innovation divides (Capello and Caragliu 2018; Ascani et al. 2025).

Several avenues for future research emerge. Potential selection bias from analysing a subset of the firm population calls for techniques that control for spatial sampling design effects (Benedetti et al. 2020). Missing data, particularly among early-stage startups, could be addressed by integrating Bayesian data-augmentation or multiple-imputation procedures into the MCMC sampler, with solutions suited to spatial contexts (Panzera et al. 2016). Comparative applications across diverse ecosystems would reveal how proximity weights shift with context, testing the generalisability of the findings. Finally, incorporating additional dimensions, such as institutional proximity or temporary co-location through events and trade fairs, could further enrich the framework and deepen understanding of knowledge spillovers in modern innovation systems.

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