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Comparative Performance of Markov Regime Switching Models and Neural Network State Switching Models in Predicting Regimes of Nigeria Exchange Rate Returns

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Comparative Performance of Markov Regime Switching Models and Neural Network State Switching Models in Predicting Regimes of Nigeria Exchange Rate Returns

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ABSTRACT
This study evaluates whether the hybrid model that integrates neural networks with Markov State/Regime-Switching methods, proposed by Adejumo-Agbailu et al. (2025), outperforms the widely used Markov State Switching Model (SSM) and Markov Autoregressive State Switching Model (ARSSM) in detecting and forecasting regimes, specifically bull and bear states, in Nigerian exchange rate returns under small sample conditions. Comparative analyses were conducted across sample sizes of 30, 50, 70, and 100, using standard estimation and prediction performance metrics. Results from model estimation, based on the Akaike Information Criterion (AIC) and log-likelihood values, reveal that the Multilayer Perceptrons State Switching Model (MLPS^{2}M) delivers the best fit relative to classical counterparts, particularly for samples below 100 observations. Additionally, predictive assessments using RMSE and MAE confirm the MLPSSM's superior regime-classification accuracy, highlighting its strength in identifying bull and bear states in limited data settings. The findings suggest that the MLPS^{2}M provides a more effective framework for modeling and forecasting financial return regimes, and its adoption is recommended for applications involving small-sample financial data.

Keywords: Nigerian exchange rate returns, Markov process, Regime switching, Neural network, MLPS^{2}M

INTRODUCTION

Nigeria's exchange rate has been characterized by significant policy interventions, episodes of market segmentation, and recurrent devaluations. The Central Bank of Nigeria (CBN) has historically maintained multiple exchange-rate windows, leading to official and parallel market spreads. These episodes have often resulted in abrupt transitions in volatility and depreciation trends, with substantial implications for investors and policymakers. For instance, Ilu (2019) demonstrated that oil price shocks transmitted into naira volatility in a nonlinear fashion, with high-volatility regimes persisting during periods of declining crude oil revenue. Similarly, Zira & Adejumo-Agbailu (2023) identified distinct low- and high-volatility regimes in the Nigerian naira/dollar exchange rate using a Markov regime-switching autoregressive model, emphasizing the policy-driven nature of regime shifts. These findings highlight the inadequacy of linear models and motivate the use of regime-switching approaches in capturing the peculiar structure of Nigeria's exchange-rate dynamics.

The formalization of regime-switching analysis in time-series econometrics is credited to Hamilton (1989), who developed a Markov-switching autoregressive (MS-AR) model to explain U.S. business cycles. The central idea is that economic and financial time series often evolve according to unobserved regimes, each with distinct parameters such as means, variances, or autoregressive coefficients, and that these regimes transition probabilistically following a Markov chain. This framework has since been widely applied in macroeconomics and finance to model interest rates, exchange rates, and equity returns. In Nigeria, recent applications have built on Hamilton's approach. A 2025 ResearchGate study on the Nigerian stock market applied MS-AR(2) and found evidence of persistent bull and bear regimes, underscoring the model's interpretability and relevance to emerging-market financial time series. The appeal of MRS models lies in their ability to produce regime probabilities and transition matrices, offering both statistical and intuitive insights into state-dependent dynamics.

While the basic Hamilton model is valuable, financial returns often exhibit conditional heteroskedasticity, fat tails, and volatility clustering that simple MS-AR specifications cannot fully capture. To address this, researchers have developed Markov-switching GARCH (MS-GARCH) models, which allow volatility parameters to vary across regimes. For example, Krolzig (1997) generalized MS models to include autoregressive and GARCH components, enabling richer dynamics. In the Nigerian context, Zira & Adejumo-Agbailu (2023) employed an MS-AR approach, but the study noted unexplained volatility spikes during crisis periods, suggesting that MS-GARCH would capture such features better. Globally, MS-GARCH has proven effective in capturing financial crises, as reported by Haas, Mittnik, & Paolella (2004), who found that MS-GARCH significantly outperformed single-regime GARCH in explaining stock return volatility. These extensions are crucial for exchange-rate studies since volatility forecasting is central for risk management, hedging, and policy evaluation.

Advances in machine learning, particularly deep learning, have opened new frontiers for modeling nonlinear time-series dynamics. Neural network–based state-switching models attempt to learn latent regimes without strong parametric assumptions. For instance, Nikolaev (2019) developed a regime-switching recurrent neural network (RS-RNN) to forecast wind power data, demonstrating that the network could learn distinct operating states more flexibly than traditional hidden Markov models. Similarly, Xu, Peng, & Chen (2021) proposed a Deep Switching State Space Model (DS^3M) capable of combining continuous latent factors with discrete switching dynamics, showing superior performance in nonlinear forecasting tasks. In finance, Mari & Mari (2023) applied deep regime-switching models to energy prices and reported that neural approaches captured asymmetric responses to shocks that were missed by classical models. These studies underscore the potential of neural models to capture complex nonlinearity, which is highly relevant in Nigeria's exchange-rate context where structural breaks and policy shocks create non-Gaussian, nonlinear patterns.

A growing body of research suggests that combining econometric regime models with machine-learning architectures may yield the best of both worlds. Ilhan *et al.* (2020) introduced the Markovian RNN, a recurrent neural network augmented with hidden Markov components, and found that it adapted well to nonstationary financial environments. Their results showed superior forecasting accuracy compared to standalone RNNs or HMMs. In the same vein, Adejumo-Agbailu *et al.* (2025) introduced neural network state-switching models designed to enhance regime prediction accuracy for nonlinear time-series returns, particularly in small-sample contexts. The simulation comparative study reveals that while MRS models remain valuable benchmarks, the hybrid of neural networks and MRS models often achieve superior interpretability and predictive performance when small data and high nonlinearity are present. However, the empirical evidence is missing thus highly context-dependent, emphasizing the need for careful application in country-specific settings like Nigeria.

Despite the rich global literature, Nigerian studies remain limited in scope. Most research has relied on variants of the MRS framework, particularly MS-AR models, to study exchange-rate or stock market regimes (e.g., Zira and Adejumo-Agbailu, 2023; Ilu, 2019). These studies consistently find multiple regimes in Nigerian data but stop short of comparing classical approaches to modern hybrid machine-learning counterparts such as Adejumo-Agbailu *et al.* (2025) proposed methods; namely Recurrent Neural Network State Switching Model (RNNSSM), Generalized Regression Neural Network State Switching Model (GRNNSSM), Radial Basis Function Network State Switching Model (RBFSSM) and Multilayer Perceptrons State Switching Model (MLPSSM). Given the unique policy-driven shocks in Nigeria, this study pursues to examine whether the Adejumo-Agbailu *et al.* (2025) proposed hybrid of neural networks and MRS models would outperform classical MRS in detecting and predicting the regimes (i.e. bull and bear) of the Nigeria Exchange Rate Returns. Addressing this empirical gap is necessary to guide both policymakers (e.g., for FX policy and reserve management) and market participants (for hedging and trading strategies).

RESEARCH METHODS

Markov State/Regime Switching Model (MSSM)

Drawing on Hamilton (2005), the Markov Switching Model is used to overcome one of the key limitations of the Hidden Markov Model, namely its treatment of unobserved or hidden states. In specifying the Markov Switching Model, we consider a two-state system, such that the state variable S_t belongs to the set $\Omega = \{1,2\}$. This means that, for example, the log returns of a financial time series may be generated from different normal distributions depending on the prevailing latent state of the model at time t. Based on this assumption, the model can be expressed as follows:

$$\begin{aligned} Y_{t=1} &= \mu_1 + \epsilon_t; \quad where\ \ \epsilon_t \sim (N, \sigma_1^2) \quad \text{for state 1} \\ Y_{t=2} &= \mu_2 + \epsilon_t; \quad where\ \ \epsilon_t \sim (N, \sigma_2^2) \quad \text{for state 2} \end{aligned} \tag{1}$$

This implies that if the hidden Markov state at time t is 1, the conditional mean of the dependent variable is μ_1, while the variance of the error term is σ_1^2. Likewise, if the system is in state 2, the dependent variable has conditional mean μ_2 and innovation variance σ_2^2. The same interpretation applies to additional states where applicable.

Because the Markov chain is latent, the actual state occupied by the model at any point in time cannot be directly observed. Instead, the state process is inferred from the observed dynamics of Y_t. To derive the probability distribution that characterizes the observed series Y_t, it is necessary to define a probabilistic mechanism that governs movements from one state to another, that is, from $S_t = i$ to $S_t = j$. For a two-state hidden Markov model, this mechanism is represented by the transition probabilities:

$$\rho_{i,j} = P_r(S_t = j | S_{t-1} = i) \quad i, j \in \Omega = \{1,2\} \tag{2}$$

By the Markov property, the transition probability depends on the history of the process only through the immediately preceding state. This is a fundamental feature of the Markov regime-switching framework: the movement between latent states is itself governed by a stochastic process.

The parameters of the two-state Markov-switching model specified in equation (1) can be estimated using different techniques, one common approach being maximum likelihood estimation.

The Markov Switching Autoregressive Model

Generally, as described in Zira & Adejumo-Agbailu (2023) an autoregressive model of order n with first-order, N-state Markov-switching mean and variance may be written as:

$$\phi(L)\left(r_t - \mu_{S_t}\right) = e_{t,} \quad e_{t,} \sim N\left(0, \sigma_{S_t}^2\right), \tag{3}$$

$$Pr[S_t = j \mid S_{t-1} = i] = p_{ij}, \quad i, j = 1, 2, \ldots, N,$$

$$\sum_{j=1}^{M} p_{ij} = 1; \qquad \mu_{S_t} = \mu_1 S_{1t} + \mu_{2t} S_{2t} + \ldots + \mu_N S_{Nt}; \qquad \sigma_{S_t}^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t} + \ldots + \sigma_2^2 \sigma_2^2,$$

where $S_{nt} = 1,$ if $S_t = n$, and $S_{nt} = 0$ otherwise.

Let S_t denote a discrete state variable that takes values from the finite set {1, 2, . . ., N}. Assume that the probability of S_t, taking a specific value *j* is determined by the previous history of the process only through its immediately preceding state, S_{t-1}.

$$Pr[S_t = j \mid S_{t-1} = i, \quad S_{t-2} = k, \ldots]$$

$$= Pr[S_t = j \mid S_{t-1} = i] = p_{ij} \tag{4}$$

Such a process is described as an *N*-state Markov Chain with transition probabilities $\{p_{ij}\}_{i,j=1,2,3,\ldots,N}$ the transition probability p_{ij} gives the probability that the state *i* will be followed by state *j*.

The two-regime Markov Autoregressive State-Switching Model (ARSSM), originally introduced by Hamilton (1989), allows the data-generating process to move between regimes in a probabilistic manner. Mendy and Widodo (2018) describe an ARSSM with regime changes and an autoregressive process of order *p* as follows;

$$Y_t = \{a_1 + \beta_{11} Y_{t-1} + \cdots + \beta_{p1} Y_{t-p} + \varepsilon_t$$

$$S_t = 1 \; a_2 + \beta_{12} Y_{t-1} + \cdots + \beta_{p2} Y_{t-p} + \varepsilon_t$$

$$S_t = 2 \; a_3 + \beta_{13} Y_{t-1} + \cdots + \beta_{p3} Y_{t-p} + \varepsilon_t \tag{5}$$

Where the regimes in (5) are indexed by S_t. In the MS-AR model, the intercept and the parameters of the AR part depend on the regime at time *t*. These regimes are assumed to be distinct unobservable variables. Hence, in this study, regime one describes the periods of bull of the exchange rate returns, r_t, and regime two symbolises the period of bear of the returns r_t. The transitions that are between the regimes are assumed to follow an ergodic and intricate first-order Markov process. This implies impacts of all past observations for the variables and the regime is fully captured in the recent observation of the regime variable as represented below;

$$\rho_{ij} = Prob\left(S_t = \frac{j}{S_{t-1}} = i\right) \quad \forall\, i, j = 1,2$$

$$\sum_{i=1}^{2} \rho_{ij} = 1$$

Matrix P captures the probability of switching which is known as a transition matrix;

$$P = [P_{11} \quad P_{12} \; P_{21} \quad P_{22} \;] \tag{6}$$

where $P_{11} + P_{21} = 1$, and $P_{12} + P_{22} = 1$. The closer the probability ρ_{ij} is to one the longer it takes to shift to the next regime.

The Network State Switching Models

To enhance the regime-classification ability of the conventional Markov state-switching model in equation (1), particularly for nonlinear return series and small-sample applications, Adejumo-Agbailu et al. (2025) proposed a hybrid modelling framework that combines Markov two-state switching models with neural network techniques. The neural networks incorporated in their approach include the Recurrent Neural Network (RNN), Radial Basis Function Network (RBF), Generalized Regression Neural Network (GRNN), and Multilayer Perceptron (MLP).

The neural network state-switching framework developed by Adejumo-Agbailu et al. (2025) exploits the ability of neural networks to capture complex nonlinear patterns while retaining the interpretability of the Markov two-state switching structure. This makes the approach useful for identifying and predicting the two major regimes in Nigerian exchange-rate returns, namely bull and bear market states. The modelling procedure is presented in two stages.

First Stage: Generative Network based on RNN, GRNN, RBF and MLP

Given a time series dataset of Y_t, the generative network procedure is implemented as follows. The dataset is first divided into training and testing samples, where the training sample is defined up to time step t_k, while the remaining observations from t_{t-k} are used for testing. At time step t_k, one of the selected neural network architectures, namely RNN, GRNN, RBF, or MLP, is applied to the training data. The hidden representation generated from the input series can be written as: $h_t \sim f_h(y_t)$ where f_h denotes the mapping function associated with the chosen neural network architecture.

Second stage: Embedding the generative network into the Markov two-state switching model

In the next stage, the generated neural network component h_t is incorporated into the Markov two-state switching model together with the specified training data. This integration produces four hybrid regime-switching models: the Recurrent Neural Network State-Switching Model (RNNSSM), Generalized Regression Neural Network State-Switching Model (GRNNSSM), Radial Basis Function Network State-Switching Model (RBFSSM), and Multilayer Perceptron State-Switching Model (MLPSSM). Based on this formulation, the resulting model can be expressed as follows:

$$Y_{t_k \sim h_t} = g_t = c_{s_t} + \mathrm{B}_1\left(g_{t-1} - c_{s_t-1}\right) + \mathrm{B}_2\left(g_{t-2} - c_{s_t-2}\right) + \epsilon_t \quad (7)$$

where

$$c_{s_t} = c_0 S_{0t} + c_1 S_{1t} + c_2 S_{2t}$$
$$\sigma_{s_t}^2 = \sigma_1^2 S_{1t} + \sigma_2^2 S_{2t}$$
$$\epsilon_t \sim i.i.d(0, \sigma_{s_t}^2)$$

c_{s_t} denotes the regime-specific mean, while $\sigma_{s_t}^2$ represents the regime-dependent variance. The model coefficients, denoted by B_1 or B_2, may vary across different regimes or subsamples. The proposed specification treats the state variable S_t as the realization of an unobserved two-state Markov chain, with S_t assumed to be independent of the disturbance term ϵ_t for every time period t.

The evolution of S_t, is assumed to follow an ergodic and irreducible first-order Markov process. This implies that the effects of previous observations of g_t and earlier states are fully summarized by the most recent observed state information. This relationship is represented in equation (3);

$$\rho_{ij} = Prob\left(S_t = {}^{j}/_{S_{t-1}} = i\right) \quad \forall i,j = 1,2 \sum_{i=1}^{2} \rho_{ij} = 1 \quad (8)$$

Matrix P captures the probability of switching which is known as a transition matrix;

$$\rho_{ij} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \quad (9)$$

where $P_{11} + P_{21} = 1$, and $P_{12} + P_{22} = 1$

The closer the transition probability ρ_{ij} is to unity, the more persistent the current regime becomes, implying that a switch to another regime is less likely to occur quickly.

For the two-regime Markov regime-switching model specified in equation (7), estimation can be carried out using Hamilton's filter. The central idea of this filtering procedure is to compute the filtered probabilities for each regime by using the available information to infer the likelihood of the system being in each unobserved state.

Data Source

The daily data of Nigeria Exchange Rate (ExchR) i.e. USD/Naira was utilized in this study. The data were sourced from Central Bank of Nigeria (CBN) website and cover the period of 26th February 2025 to 23rd July 2025. The returns of the ExchR were determined using $r_t = \ln\left({}^{ExchR_t}/_{ExchR_{t-1}}\right)$; where r_t is the ExchR-Returns (ExchRR).

Model Estimation Procedure and Assessment of Predictive Performance

Before estimating the proposed neural network state-switching models, namely RNNSSM, GRNNSSM, RBFSSM, and MLPSSM, and comparing them with the conventional Markov State-Switching Model (SSM) and Markov-Switching Autoregressive model (MS-AR), the exchange-rate return series (Y_t) is first examined to determine its suitability for nonlinear modelling. For this purpose, the study applies the BDS nonlinearity test developed by Brock, Dechert, and Scheinkman. In addition, the return series is tested for stationarity using the Augmented Dickey-Fuller (ADF) test, while the presence of volatility clustering is assessed using the ARCH test.

For model selection, the study adopts the widely used Akaike Information Criterion (AIC), expressed as: $AIC = T\ ln(residual\ sum\ of\ squares) + 2n$, where T represents the number of usable observations and n denotes the number of estimated parameters. The log-likelihood value of each fitted model is also considered. Accordingly, the best-performing model is identified as the one with the lowest AIC value and the highest log-likelihood.

To evaluate predictive accuracy, the proposed models are compared with the Markov State-Switching Model using standard forecast-error measures, including the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). These measures are defined as:

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|(A_t - F_t)|$$

where A_t is the observed value at time *t,* and F_t is the predicted value at the same time point. The predictive performance of the models is then examined using the Nigeria Exchange Rate Returns described earlier, using four categories of small sample sizes.

RESULTS AND DISCUSSION

Data Description and Preliminary Analysis

This section reports and interprets the descriptive statistics and selected time-series characteristics of Nigerian exchange-rate returns. Figures 1 to 4 display the trend plots of the exchange-rate series and the corresponding return series for sample sizes of 30, 50, 70, and 100, respectively.

From the plots, the series across the different sample sizes appear to exhibit a degree of stationarity, as reflected in relatively stable means, variances, and autocorrelation patterns, although these features may still be influenced by external factors or underlying trends. In addition, the plots show evidence of clustering behaviour, suggesting the presence of volatility in the return series..

Figure 1

Time Series Plots of Exchange Rate (USD/Naira) and Its Returns Series for n=30

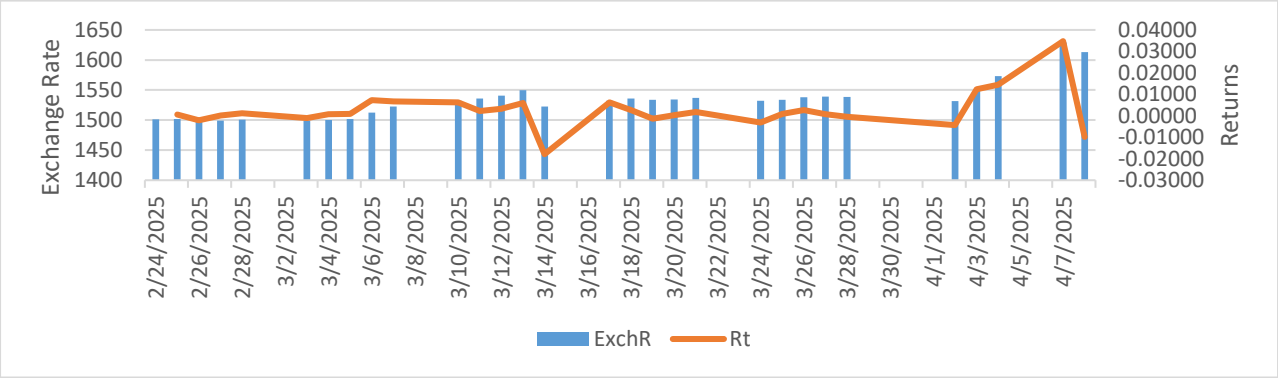


Figure 2

Time Series Plots of Exchange Rate (USD/Naira) and Its Returns Series for n=50

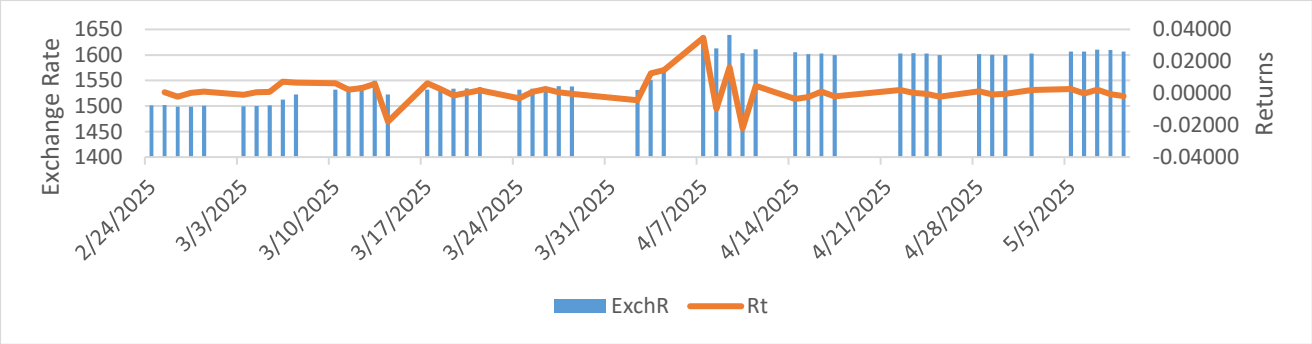


Figure 3

Time Series Plots of Exchange Rate (USD/Naira) and Its Returns Series for n=70

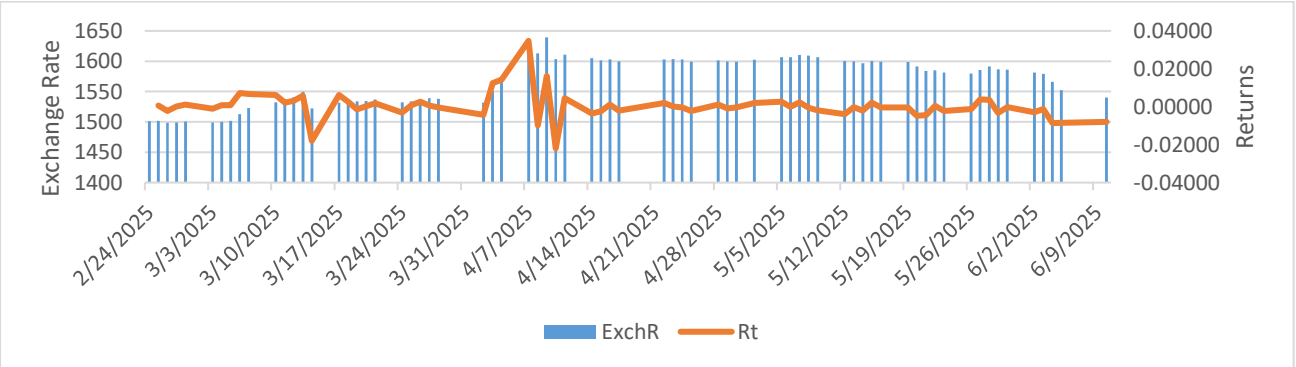


Figure 4

Time Series Plots of Exchange Rate (USD/Naira) and Its Returns Series for n=100

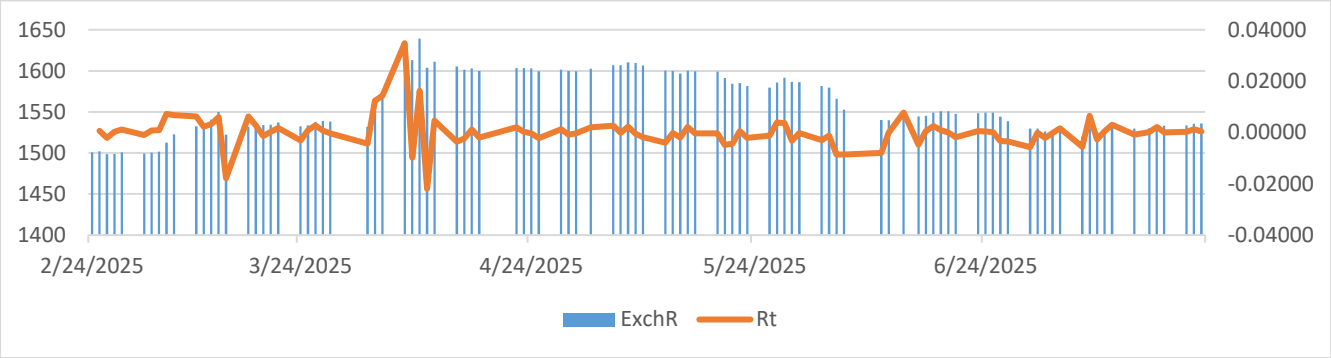


Table 1

Summary Statistics and Preliminary Tests of Real-Life Dataset

	30		50		70		100	
Summary Statistics								
	ExchR	**Rt**	**ExchR**	**Rt**	**ExchR**	**Rt**	**ExchR**	**Rt**
Mean	1532	0.0024	1562	0.0014	1568	0.0004	1578	0.0004
Median	1533	0.0009	1545	0.0007	1583	-0.0001	1583	-0.00005
Min.	1499	-0.0179	1499	-0.0219	1499	-0.0021	1499	-0.0219
Max.	1629	0.0348	1639	0.0348	1639	0.0348	1639	0.03477
Std. Dev.	30.6078	0.0085	43.6581	0.0078	39.1042	0.0070	39.1042	0.0070
Preliminary Tests of Returns								
ADF Test	*I(0)* 0.01*		*I(0)* 0.01*		*I(0)* 0.01*		*I(0)* 0.01*	
BDS Test	0.0043*		0.0039*		0.0035*		0.0035*	
ARCH Test	1.65e-05*		1.79e-06*		1.72e-10*		1.72e-10*	

Note: ** denotes significant at 0.05 level* ***Source:*** *Researchers' Compilations from R-Output*

Table 1 reports the descriptive statistics, stationarity test, nonlinearity test, and volatility test results for the Nigerian exchange rate and its return series across four small-sample categories. The results show that the exchange-rate return series recorded mean returns of 0.0024±0.0085, 0.0014±0.0078, 0.0004±0.0070, and 0.0004±0.0070 for sample sizes of 30, 50, 70, and 100, respectively. Following the patterns observed in the time-series plots, the return series were further examined for stationarity, nonlinearity, and volatility. The Augmented Dickey-Fuller (ADF) test results indicate that all the return series are stationary at level, as expected. The BDS test results show p-values that are significant at the 5% level, implying rejection of the linearity assumption for the simulated return series across all sample sizes. Similarly, the ARCH test results are significant at the 5% level, indicating rejection of the null hypothesis of no volatility effect in the exchange-rate returns for all the sample categories. These findings demonstrate that Nigerian exchange-rate returns exhibit significant nonlinear and volatility characteristics across the different sample sizes. Therefore, the application of nonlinear and volatility-based modelling techniques, such as the State-Switching Model and the proposed neural network state-switching models, is appropriate..

Model Estimation

This section presents and discusses the estimation results for the Adejumo-Agbailu et al. neural network state-switching models, the conventional Markov State-Switching Model (SSM), and the Markov Autoregressive State-Switching Model (ARSSM). Each model was estimated using the training dataset corresponding to the selected sample sizes. The goodness-of-fit diagnostics for the estimated return models show that the Q-statistic p-values are greater than 0.05. This suggests that there is no statistically significant residual dependence or autocorrelation remaining in the squared standardized residuals. Hence, the estimated models can be regarded as adequately specified.

Table 2

Summary of Models' Estimations for Real-Life Dataset at 30 Sample Size

		SSM	ARSSM	RNNS²M	RBFS²M	GRNNS²M	MLPS2M
State 1	**Intercept**	0.001	0.0009	0.0054*	4659.15*	0.0305	4673.969*
	Training_Network	-	ar(1) = 0.4805	rnn= -0.0115*	rbf=-1188.09*	grnn=-36.1202	mlp=-4705.83
	Std Error	0.0077	0.0002	0.0013	0.0103	0.0081	0.00001
State 2	**Intercept**	0.001	-0.0119	0.0434	-1922.40*	0.0007	47.64.497*
	Training_Network	-	-1.0151	rnn=-0.0888	rbf=8444.43*	grnn=1.1272	mlp=-4796.973
	Std Error	0.0017	0.0000	0.0063	0.0003	0.0029	0.00009
Transition Prob.	**State 1 State 2**	0.8228 0.0979	0.9524 0.5000	0.9276 0.1283	0.5977 0.3952	0.0108 0.0896	0.5705 0.5839
	State 2 State 1	0.1772 0.9021	0.0476 0.5000	0.0724 0.8717	0.4023 0.6048	0.9892 0.9104	0.4295 0.4160
	AIC	-199.8338	-271.8073	-205.9632	-320.1243	-115.7039	-351.4241
	logLik	100.9169	139.9036	106.9816	162.9512	61.8519	179.7121

Note: ** denotes significant at 0.05 level* ***Source:*** *Researchers' Compilations from R-Output*

Figure 5

Models' Transition Probabilities of Exchange Rate Returns at Sample Size 30

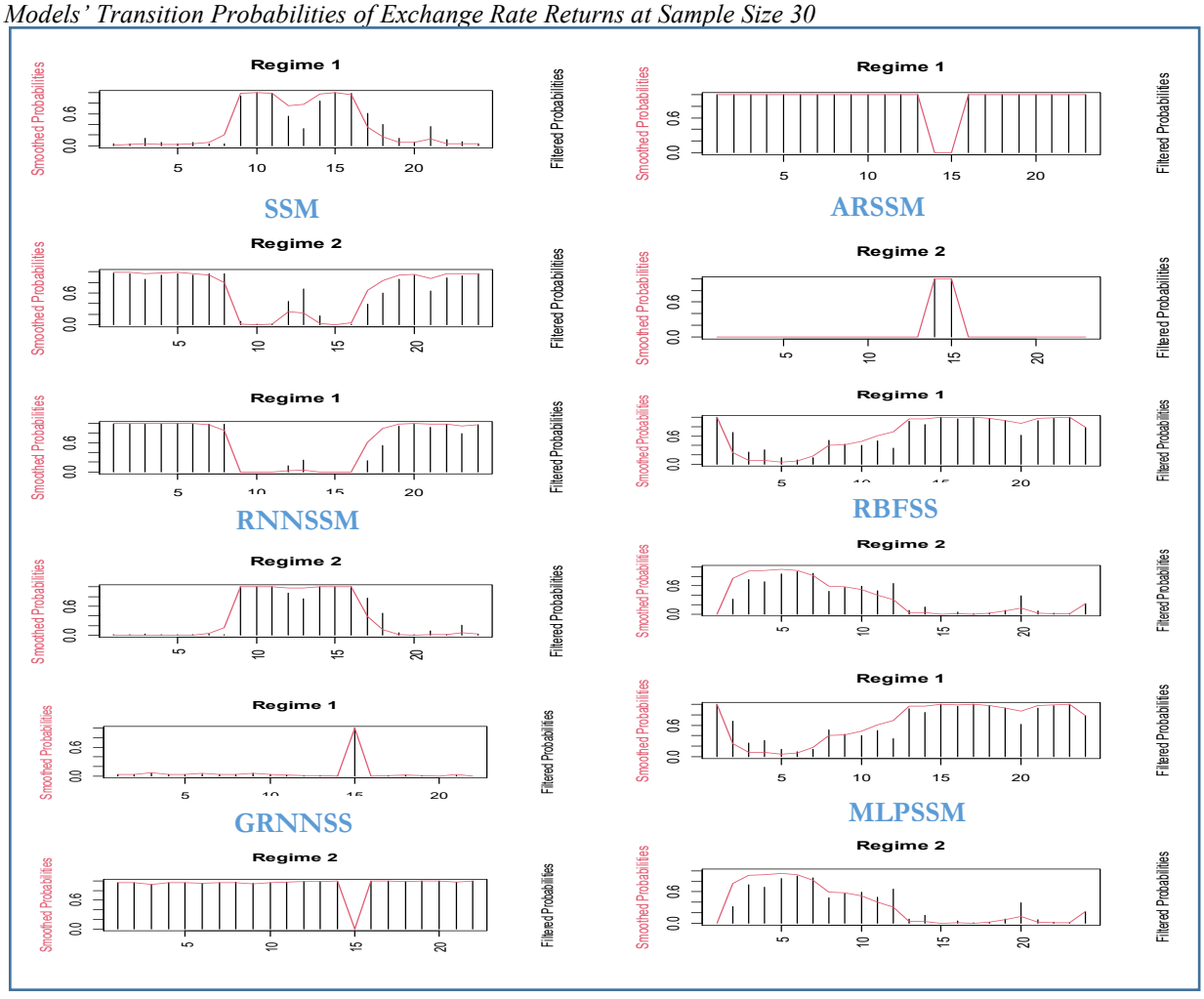


Table 3

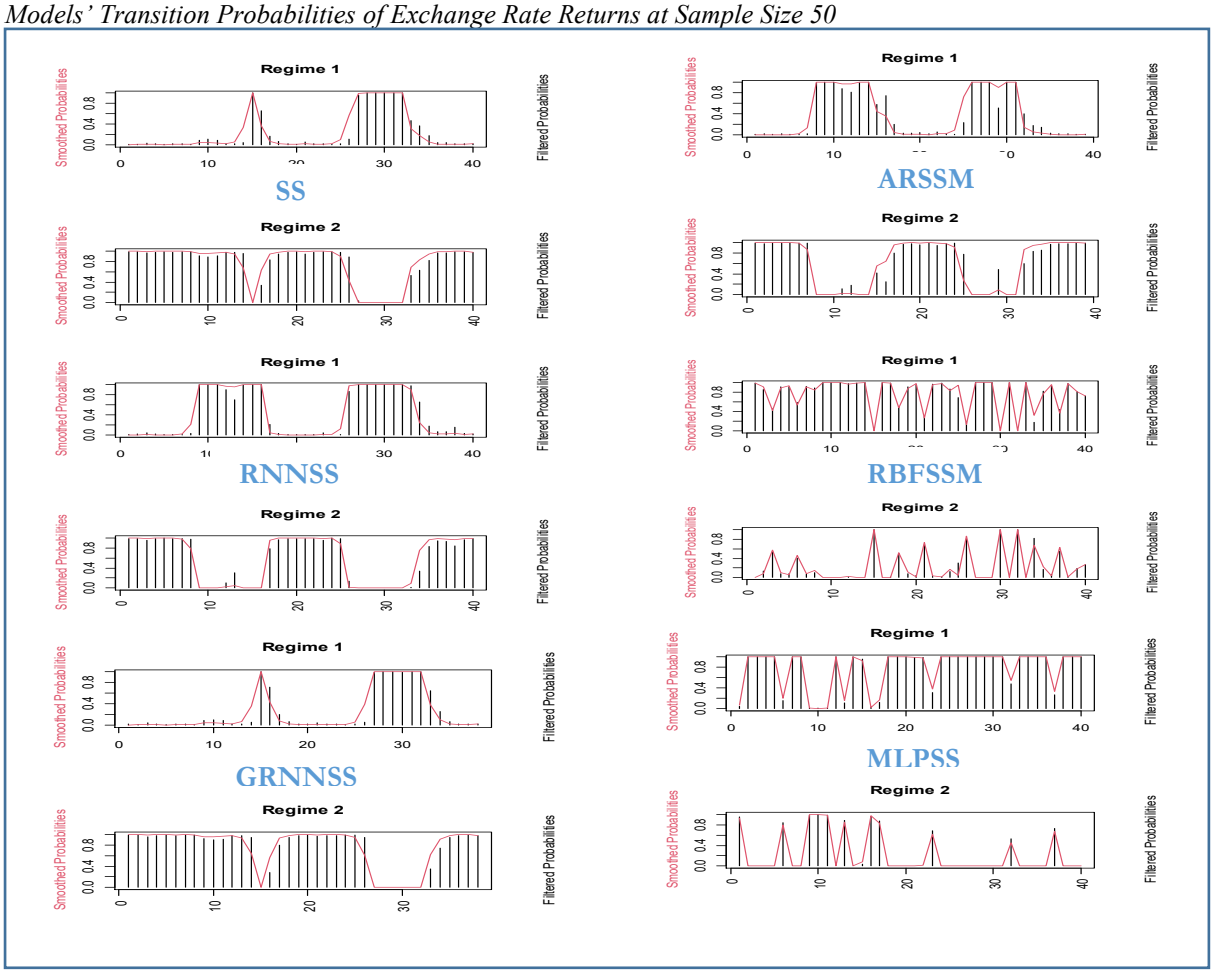
Summary of Models' Estimations for Real-Life Dataset at 50 Sample Size

		SSM	ARSSM	RNNS²M	RBFS²M	GRNNS²M	MLPS2M
State 1	Intercept	0.0016*	0.0059	-0.0762*	2.86e+03*	-0.0255	4108.5*
	Training_Network	-	ar(1) = -0.3198	rnn= 0.4163*	rbf=-5.72e+07*	grnn=-56.4775	mlp=-4118.8*
	Std Error	0.0169	0.0129	0.0093	0.0028	0.0167	0.00012
State 2	Intercept	0.0016*	0.0000	-0.0104	-5.47e+03*	0.0046	4175.5*
	Training_Network	-	ar(1)= -0.2391*	rnn=0.0548	rbf=1.09e+08*	grnn=7.8799	mlp=-4185.9*
	Std Error	0.0029	0.0017	0.0012	0.0022	0.0028	0.000031
Transition Prob.	State 1 State 2	0.7614 0.0726	0.8501 0.0934	0.8710 0.0911	0.6813 1.0000	0.7594 0.0763	0.7924 0.5851
	State 2 State 1	0.2386 0.9274	0.1499 0.9066	0.1290 0.9089	0.3187 0.0000	0.2406 0.9237	0.2075 0.4149
	AIC	-303.333	-297.8423	-325.6951	-333.5842	-281.4482	-590.6862
	logLik	152.6665	152.9211	166.8475	170.7921	144.7241	299.3431

Note: ** denotes significant at 0.05 level* ***Source:*** *Researchers' Compilations from R-Output*

Figure 6

Models' Transition Probabilities of Exchange Rate Returns at Sample Size 50



Tables 2, 3, 4, and 5 summarize the model estimation results for Nigerian exchange-rate returns using sample sizes of 30, 50, 70, and 100, respectively. For the sample size of 30, Table 2 shows that the Adejumo-Agbailu et al. MLPS^{2}M and RBFS^{2}M models produced the lowest AIC values of −351.4241 and −320.1243, respectively. These models also recorded relatively high log-likelihood values of 179.7121 and 162.9512, respectively. Based on these selection criteria, the MLPS^{2}M emerges as the most parsimonious model among the estimated state-switching models. Table 2 also reports the transition probabilities for the two identified regimes, namely the bull and bear states. For the MLPS^{2}M model, the transition probabilities suggest a moderate tendency for the exchange-rate return process to remain in its current regime, while also allowing a reasonable likelihood of switching between states. Specifically, the model indicates a 57% probability of remaining in the bull state and a 43% probability of moving from the bull state to the bear state. Conversely, when the system is in the bear state, there is a 58% probability of remaining there and a 42% probability of transitioning to the bull state. These transition probabilities imply that movements between the bull and bear regimes can occur with appreciable likelihood, suggesting that shocks or market-related events may trigger regime changes in the exchange-rate return process, as illustrated in Figure 5.

Furthermore, the models were estimated using the training dataset with a sample size of 50. Table 3 summarizes the estimation results obtained from this dataset. The results show that the MLPS^{2}M and RBFS^{2}M models produced the lowest AIC values of −590.6862 and −333.5842, respectively. They also recorded comparatively high log-likelihood values of 299.3431 and 170.7921, respectively. On the basis of these criteria, MLPS^{2}M is identified as the most parsimonious model among the competing models. Table 3 also reports the transition probabilities for the two regimes, namely the bull and bear states. For the best-performing model, MLPS^{2}M, the transition probabilities indicate a strong likelihood that the exchange-rate return process will remain in the bull regime, while the probability of remaining in the bear regime is moderate. Specifically, the model shows a 79% probability of persistence in the bull state and a 21% probability of moving from the bull state to the bear state. In contrast, when the process is in the bear state, there is a 59% probability that it will remain there and a 41% probability that it will switch to the bull state. These results suggest that a major or extreme shock may be required to move the system out of the bull regime, whereas transitions from the bear regime may occur more readily in response to market events, as shown in Figure 6. For the sample size of 70, the results presented in Table 4 indicate that MLPS^{2}M and RBFS^{2}M again performed best in terms of model selection criteria. The two models produced the lowest AIC values of −778.5124 and −606.9728, respectively, together with high log-likelihood values of 393.2562 and 307.4864, respectively. Consequently, MLPS^{2}M emerges as the most parsimonious model among the estimated alternatives.

In addition, Table 4 presents the transition probabilities for the bull and bear regimes. The MLPS^{2}M transition probabilities suggest that the exchange-rate return process has only a fair to moderate tendency to remain in the same regime, implying a relatively high possibility of switching between states. The results show a 52% probability of remaining in the bull state and a 48% probability of moving to the bear state. Similarly, when the system is in the bear state, the probability of staying in that regime is 50%, while the probability of switching to the bull state is also 50%. These findings indicate that regime changes may occur easily, meaning that market events can trigger movements between the bull and bear phases, as illustrated in Figure 7.

Table 4

Summary of Models' Estimations for Real-Life Dataset at 70 Sample Size

		SSM	ARSSM	RNNS^{2}M	RBFS^{2}M	GRNNS^{2}M	MLPS^{2}M
State 1	**Intercept**	0.0011*	-0.0001	0.0003	2.85e+03*	0.0011	8530.84*
	Training_Network	-	ar(1) = -0.2137*	rnn= -0.0017	rbf=-5.70e+07*	grnn=-5.2911	mlp=-8542.8*
	Std Error	0.0026	0.0018	0.0017	0.0101	0.0130	0.00012
State 2	**Intercept**	0.0011*	0.0060	-0.0314	5.23e+03*	0.0009	8627.41*
	Training_Network	-	ar(1)= -0.3267	rnn=0.1427	rbf=-1.05e+08*	grnn=1.4146	mlp=-8639.5*
	Std Error	0.0165	0.0129	0.0119	0.0005	0.0018	0.00013
Transition Prob.	**State 1 State 2**	0.9502 0.2291	0.9459 0.1449	0.9451 0.1281	0.7364 0.0435	0.8511 0.0621	0.5239 0.5011
	State 2 State 1	0.0498 0.7709	0.0541 0.8551	0.0549 0.8719	0.2636 0.9565	0.1489 0.9379	0.4761 0.4989
	AIC	-451.9118	-451.0783	-459.0957	-606.9728	-435.7454	-778.5124
	logLik	222.9559	229.5392	233.5478	307.4864	221.8727	393.2562

***Note:** * denotes significant at 0.05 level* ***Source:** Researchers' Compilations from R-Output*

Figure 7

Models' Transition Probabilities of Exchange Rate Returns at Sample Size 70

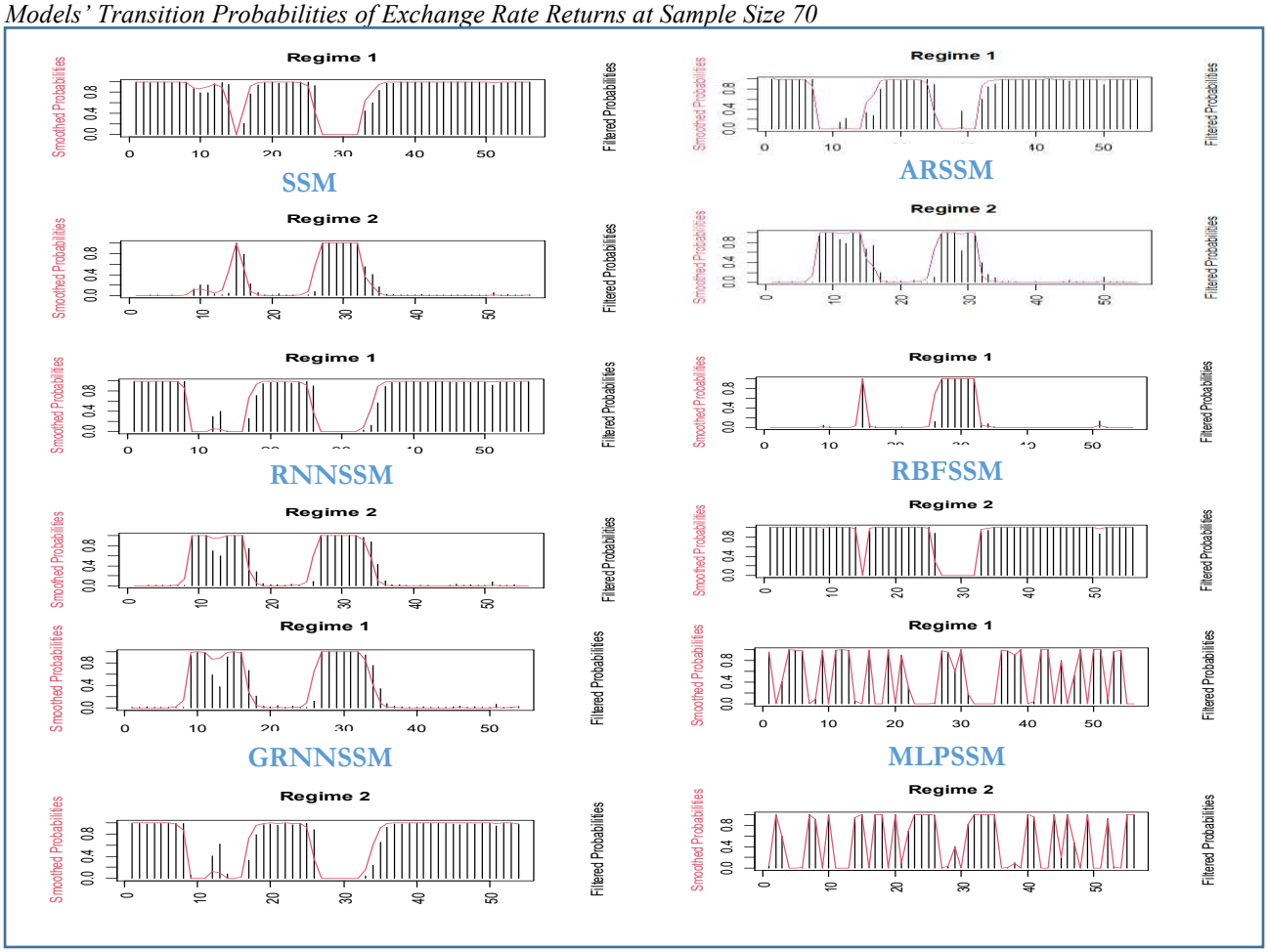


Table 5

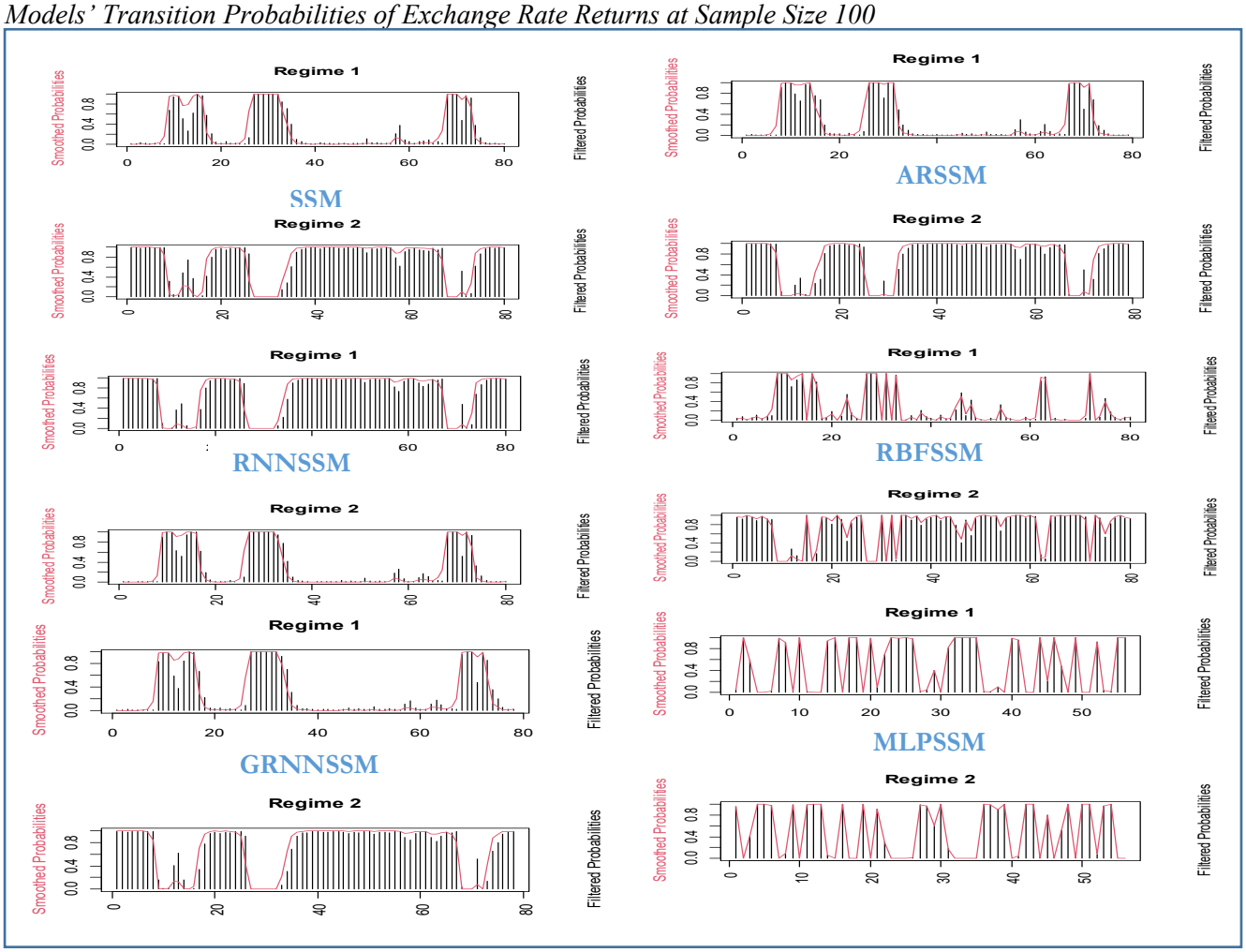
Summary of Models' Estimations for Real-Life Dataset at 100 Sample Size

		SSM	ARSSM	RNNS²M	RBFS²M	GRNNS²M	MLPS²M
State 1	**Intercept**	0.0004*	0.0023	0.0024	2.93e+03*	0.0163	8627.44*
	Training_Network	-	ar(1) = -0.0976	rnn= -0.0129	rbf=-1.17e+08*	grnn=-27.4923	mlp=-8639.5*
	Std Error	0.0119	0.0122	0.0019	0.0026	0.0115	0.00013
State 2	**Intercept**	0.0004*	-0.0003	0.0408	5.30e+03*	0.0012	8530.76*
	Training_Network	-	ar(1)= -0.1760	rnn=-0.2038	rbf=2.12e+08*	grnn=-2.6300	mlp=-8542.71*
	Std Error	0.0021	0.0021	0.0102	0.0019	0.0021	0.00012
Transition Prob.	**State 1 State 2**	0.8348 0.0649	0.8336 0.0602	0.9393 0.1511	0.4862 0.1713	0.8399 0.0644	0.4995 0.4775
	State 2 State 1	0.1652 0.9351	0.1664 0.9398	0.0607 0.8489	0.5138 0.8287	0.1600 0.9356	0.5005 0.5225
	AIC	-648.5142	-639.1177	-656.7784	-698.4455	-628.9584	-778.5184
	logLik	325.2571	323.5589	332.3892	353.2227	318.4792	393.2592

Note: ** denotes significant at 0.05 level* ***Source:*** *Researchers' Compilations from R-Output*

Figure 8

Models' Transition Probabilities of Exchange Rate Returns at Sample Size 100



Similarly, the models were estimated using the training dataset with a sample size of 100, and the summarized estimation results are reported in Table 5. The results show that MLPS^{2}M and RBFS^{2}M produced the lowest AIC values and relatively higher log-likelihood estimates compared with the other models. Specifically, Table 5 indicates that MLPS^{2}M and RBFS^{2}M recorded AIC values of −778.5184 and −698.4455, respectively, along with log-likelihood values of 393.2592 and 353.2227. Based on these criteria, MLPS^{2}M is identified as the most parsimonious model among the estimated alternatives. Table 5 also presents the transition probabilities for the two regimes, namely the bull and bear states. For the MLPS^{2}M model, the transition probabilities suggest a fair to moderate likelihood that the return process will remain in its current regime, implying a reasonable possibility of switching between states. The results show that the model has a 50% probability of remaining in the bull state and a 50% probability of moving to the bear state. Conversely, when the process is in the bear state, there is a 48% probability of remaining there and a 52% probability of switching to the bull state. These findings suggest that market events may easily trigger movements between the two regimes, that is, from state 1, the bull phase, to state 2, the bear phase, as illustrated in Figure 8.

Following this, the section examines the goodness-of-fit assessment of the estimated models in capturing Nigeria's exchange-rate return regimes. Figures 9 and 10 display the standard errors of the estimated models across the selected sample sizes for state 1, representing the bull regime, and state 2, representing the bear regime, respectively.

Figure 9
Models' Std Error Estimates across the Sample Sizes – State 1 (Bull)

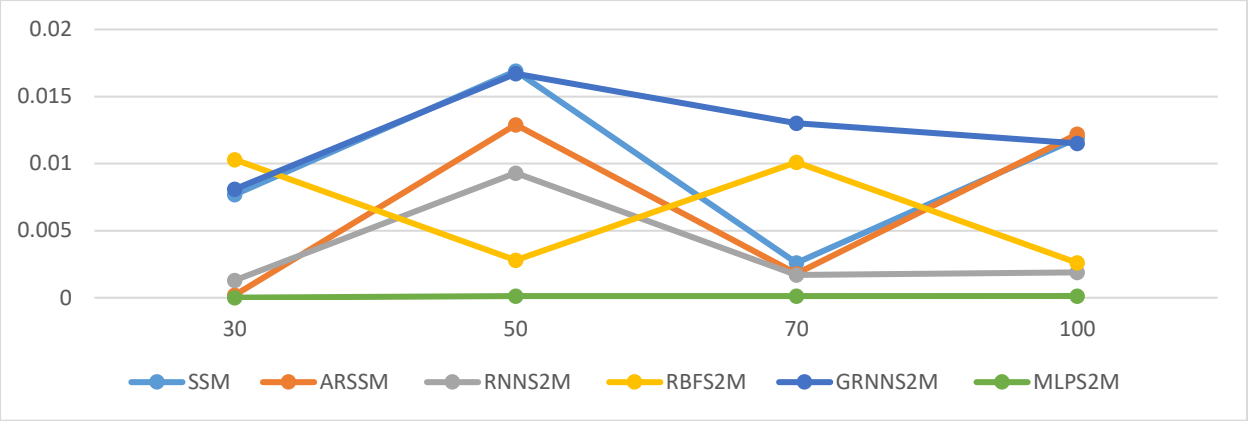


Figure 10
Models' Std Error Estimates across the Sample Sizes – State 2 (Bear)

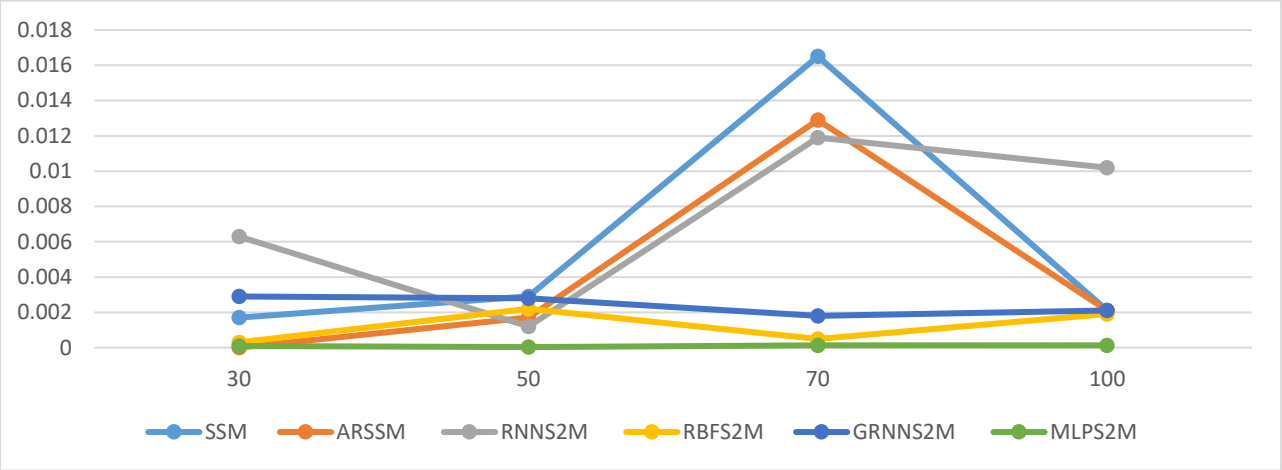


Figure 11
Models' AIC Estimates across the Sample Sizes

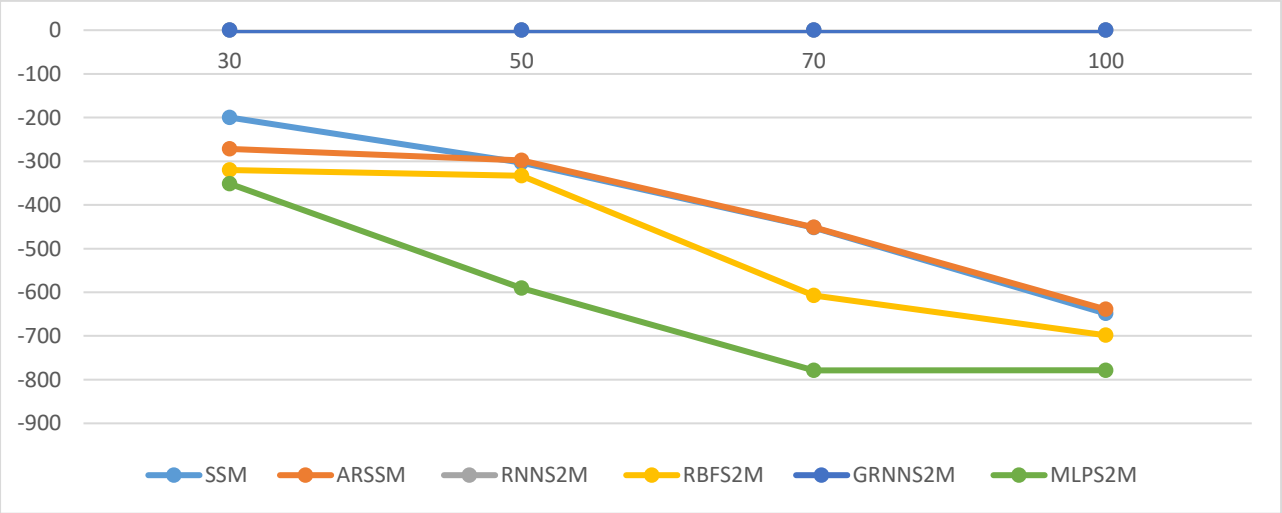
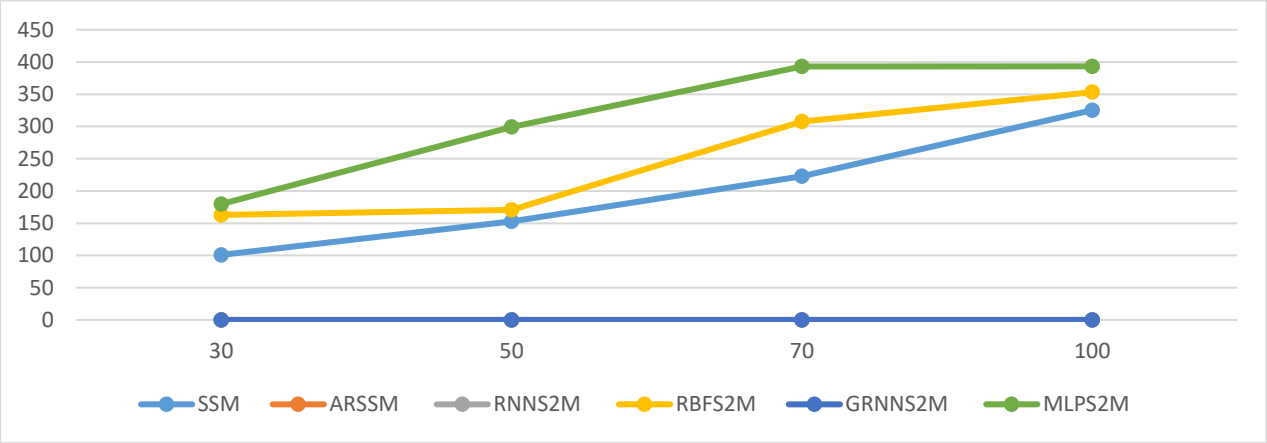


Figure 12

Models' Loglikelihood Estimates across the Sample Sizes



As shown in Figures 9 and 10, the MLPS2M model proposed by Adejumo-Agbailu et al. (2025) produced the lowest standard errors across the selected small sample sizes for both the bull and bear regimes. In addition, Figures 11 and 12 display the line plots of the AIC and log-likelihood values for the estimated models, respectively. The results indicate that two of the four models introduced by Adejumo-Agbailu et al. (2025), namely RBFS^{2}M and MLPS^{2}M, performed better than the conventional SSM and the other proposed neural network state-switching models. This superior performance is reflected in their lower AIC values and higher log-likelihood estimates across all four small-sample categories. Specifically, Figure 11 shows that MLPS^{2}M consistently recorded the lowest AIC values, while Figure 12 indicates that the same model achieved the highest log-likelihood values across all the sample sizes considered.

It is also worth noting that the AIC and log-likelihood values appear to stabilize when the sample size reaches 100. This suggests that MLPS^{2}M performs particularly well for sample sizes below 100. Based on these findings, the Multilayer Perceptron State-Switching Model developed by Adejumo-Agbailu et al. (2025) can be regarded as the best-fitting model for capturing Nigeria's exchange-rate return regimes across the small sample sizes examined.

Evaluation of Regime Prediction Performance

This section evaluates and discusses the predictive performance of the six estimated models. The assessment is based on in-sample predictions for sample sizes of 30, 50, 70, and 100. Prediction accuracy is measured using the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

Table 6 and Figure 13 present the prediction performance results for Nigerian exchange-rate returns. The results show that MLPS^{2}M consistently produced the lowest RMSE and MAE values across all the sample sizes considered. These findings indicate that the fitted MLPS2M model provides superior in-sample predictive accuracy and is therefore more effective for forecasting Nigerian exchange-rate returns under small-sample conditions.

Table 6

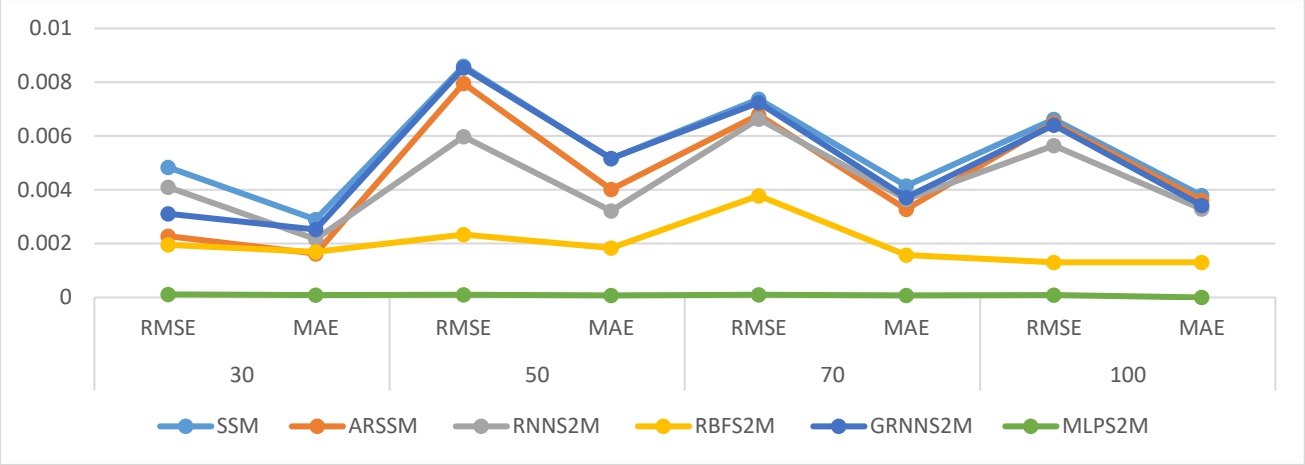
In-Sample Prediction Performances of Estimated Models

	30		50		70		100	
	RMSE	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
SSM	0.004833	0.002899	0.008597	0.005157	0.007374	0.004144	0.006620	0.003792
ARSSM	0.002277	0.001629	0.007956	0.004018	0.006785	0.003272	0.006485	0.003614
RNNS^{2}M	0.004094	0.002168	0.005976	0.003222	0.006637	0.003646	0.005645	0.003284
RBFS^{2}M	0.001956	0.001693	0.002339	0.001837	0.003783	0.001570	0.001306	0.001306
GRNNS^{2}M	0.003116	0.002524	0.008538	0.005170	0.007237	0.003712	0.006410	0.003425
MLPS2M	0.000114	0.000086	0.000101	0.000078	0.000096	0.000077	0.000090	0.000000

Source: *Researchers' Compilations from R-Output*

Figure 13

Graph of the In-Sample Prediction Performances of Estimated Models under the Real-Life Dataset Application



Furthermore, Tables 7 and 8 present the MLPS^{2}M 20% in-sample regime predictions for Nigerian exchange-rate returns across the selected small-sample sizes. The results indicate that the predicted regimes are relatively stable across the sample categories considered. For the Nigerian exchange-rate return series, Tables 7 and 8 show that the MLPS^{2}M model of Adejumo-Agbailu et al. (2025) predicted bull-regime values of +0.001039, +0.001652, +0.001128, and +0.000393 for sample sizes of 30, 50, 70, and 100, respectively. The results further indicate that, at a sample size of 30, the predicted bull value has a moderate probability of remaining in the bull regime, with a persistence probability of 57%. At a sample size of 50, however, the predicted bull value shows a stronger likelihood of remaining in the same regime, with a persistence probability of 79%.

For sample sizes of 70 and 100, the predicted bull values have approximately equal chances of remaining in the bull regime or switching to the bear regime, with persistence probabilities close to 50%. Therefore, based on the in-sample prediction results for the empirical dataset, although the proposed MLPS^{2}M predicts bull-regime values across all the small sample sizes considered, the bull regimes predicted for sample sizes 30 and 50 appear to have a relatively greater likelihood of persistence.

Table 7

Novel MLPS^{2}M 20% In-Sample Daily Market-Returns Regimes Predictions across Small Sample Sizes 30 and 50

Daily Period	30		50	
	Nigerian exchange rate returns		Nigerian exchange rate returns	
	Testing Data	Predicted $P_{11} = 0.57$, $P_{21} = 0.43$	Testing Data	Predicted $P_{11} = 0.79$, $P_{21} = 0.21$
1	-0.00042	+0.001039	-0.002242	+0.001652
2	-0.00432	+0.001039	+0.001222	+0.001652
3	+0.01240	+0.001039	-0.000836	+0.001652
4	+0.01440	+0.001039	-0.000341	+0.001652
5	+0.03477	+0.001039	+0.001995	+0.001652
6	-0.00982	+0.001039	+0.002627	+0.001652
7			-0.000097	+0.001652
8			+0.002306	+0.001652
9			-0.000550	+0.001652
10			-0.001824	+0.001652

Note: *+ and - denote Bull and Bear Regime respectively* ***Source:*** *Researchers' Compilations from R-Output*

Table 8

Novel MLPS²M 20% In-Sample Daily Market-Returns Regimes Predictions across Sample Sizes 70 and 100

Daily Period	70		100	
	Nigerian exchange rate returns		Nigerian exchange rate returns	
	Testing Data	Predicted $P_{11} = 0.52$, $P_{21} = 0.48$	Testing Data	Predicted $P_{11} = 0.50$, $P_{21} = 0.50$
1	-0.004821	+0.001128	+0.000079	+0.000393
2	-0.004404	+0.001128	-0.003410	+0.000393
3	+0.000462	+0.001128	-0.003448	+0.000393
4	-0.002145	+0.001128	-0.005791	+0.000393
5	-0.001231	+0.001128	-0.000067	+0.000393
6	+0.003863	+0.001128	-0.002174	+0.000393
7	+0.003678	+0.001128	-0.000205	+0.000393
8	-0.003216	+0.001128	+0.001631	+0.000393
9	-0.000189	+0.001128	-0.005570	+0.000393
10	-0.002974	+0.001128	+0.006489	+0.000393
11	-0.001284	+0.001128	-0.002708	+0.000393
12	-0.008588	+0.001128	+0.000552	+0.000393
13	-0.008548	+0.001128	+0.002943	+0.000393
14	-0.008007	+0.001128	-0.000882	+0.000393
15			+0.000141	+0.000393
16			+0.002183	+0.000393
17			+0.000064	+0.000393
18			+0.000210	+0.000393
19			+0.001194	+0.000393
20			+0.000370	+0.000393

Note: *+ and - denote Bull and Bear Regime, respectively* ***Source:*** *Researchers' Compilations from R-Output*

Moreover, in the empirical application to Nigerian exchange-rate returns, Table 7 reports stable predicted regime estimates of approximately 0.0010 across six forecasts for the sample size of 30. This value corresponds to the bull regime, or state 1, and shows a reasonable degree of agreement with the testing data. Similarly, for the sample size of 50, Table 7 shows repeated predicted regime estimates of about 0.0017 across ten forecasts, again indicating the bull regime and demonstrating a fairly close correspondence with the testing observations.

For the sample sizes of 70 and 100, Table 8 presents consistent predicted regime estimates of 0.0011 and 0.0004, respectively. Both values also correspond to the bull regime, or state 1. The predicted regimes show a weaker but still acceptable alignment with the testing data for the sample size of 70, while the match is stronger for the sample size of 100. Overall, the results suggest that the fitted MLPS²M, proposed model by Adejumo-Agbailu *et al.* (2025), provides largely reliable regime predictions for exchange-rate returns across all the small sample sizes considered.

CONCLUSIONS

This study examined whether the hybrid neural network and Markov regime-switching framework proposed by Adejumo-Agbailu et al. (2025) performs better than conventional Markov regime-switching models in detecting and forecasting Nigerian exchange-rate return regimes, particularly bull and bear states. The study compared the competing models using estimation results and regime-prediction outcomes based on standard evaluation measures.

The model estimation findings, especially the Akaike Information Criterion (AIC) and log-likelihood values, show that the Multilayer Perceptron State-Switching Model developed by Adejumo-Agbailu et al. (2025) provides the best fit when compared with the traditional Markov state-switching and Markov-switching autoregressive models. This performance is particularly evident under small-sample conditions involving fewer than 100 observations.

In addition, the prediction results based on RMSE and MAE further support the superior performance of the MLPSSM in forecasting exchange-rate regimes. The model demonstrates stronger accuracy in classifying bull and bear states in small-sample settings. Based on these findings, the study concludes that the MLPS²M is the most appropriate model for analyzing Nigerian exchange-rate returns when the available sample size is limited, as it combines good in-sample fit with strong regime-prediction performance.

Consequently, the study recommends the use of the Adejumo-Agbailu et al. (2025) Multilayer Perceptron State-Switching Model for modelling and forecasting financial return regimes.

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