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Quantum Tensor Magnetometry with Bright Multi-Dimensional Entangled Lights

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Tensor measurements play a vital role in both fundamental sciences and interdisciplinary applications such as navigation and remote sensing which require sensitive perception of the gradient of vector fields. Fundamentally, these measurements rely on the estimation of multiple parameters, which is a key focus in metrology. Quantum metrology has achieved remarkable success in surpassing the standard quantum limit for single-parameter estimation, offering extraordinary sensitivity. Quantum multiparameter estimation has also been demonstrated recently with few-particle quantum systems. Here, we develop tensor quantum metrology by advancing multiparameter quantum sensing to an unprecedented level with both high scalability and multidimensional performance, fulfilling the essential demands on sensitivity for tensor measurements. We generate a bright, multi-dimensional polarization-entangled light source from an atomic ensemble, with three simultaneously squeezed Stokes operators, enabling superior performance in multiparameter quantum sensing compared to conventional polarization-entangled light. By employing such a quantum light in a tensor magnetometry, we achieve a 6 dB quantum enhancement in both direction and amplitude measurements of magnetic field gradient, with a directional gradient sensitivity of $21 \mu\text{rad}/\text{cm}/\sqrt{\text{Hz}}$ and an amplitude gradient sensitivity of $6 \text{fT}/\text{cm}/\sqrt{\text{Hz}}$. As the first realization of high-sensitivity quantum-enhanced sensing of a tensor field, this work marks a milestone in the advancement of quantum metrology.

With advancements in quantum optics, quantum sensors have enabled the development of a wide range of technologies, including atomic magnetometers [1–7], interferometers [8–11], and microwave sensors [12–14], all of which play pivotal roles in both fundamental and applied physics. The sensitivity of these sensors is fundamentally constrained by the standard quantum limit (SQL), which is determined by spin projection noise and photon shot noise (PSN) [15]. Quantum metrology has enabled investigations to overcome this limitation by leveraging techniques such as squeezing and entanglement to suppress quantum noise, enabling better sensitivity than classic methods [16–25]. Notably, the performance of state-of-the-art atomic magnetometers and atomic clocks has been significantly improved according to recent demonstrations of entangled sensors [22, 25–28], marking substantial progress toward practical quantum-enhanced atomic sensors.

Building upon these foundational advances, quantum metrology is expanding into various research fields including fundamental physics [29–35], chemical analysis [36, 37], biological magnetic field measurements [38–40], navigation [41, 42], and geological remote sensing [43]. While quantum techniques have successfully enhanced sensitivity for scalar field measurements through single-

parameter estimation, most practical scenarios in these domains require measurement of vector or tensor fields, demanding the simultaneous estimation of multiple parameters.

Multiparameter quantum sensing has witnessed substantial theoretical developments [44–46]. These advancements have been experimentally validated in proof-of-principle demonstrations, including optical systems [47–50], trapped ions [51, 52], Bose-Einstein condensates [53, 54], and superconducting circuits [55]. While these studies elucidate fundamental quantum limits and feasibility conditions, current experimental platforms typically operate with few particles, falling short by orders of magnitude compared to real-world requirements. Bridging this gap requires significant advancements in high scalability, multidimensional performance, and capability for suppression of both quantum and classical noises.

To address these challenges in scalable multiparameter quantum sensing, we propose and demonstrate a novel approach for quantum-enhanced tensor magnetometry. Unlike conventional methods that extract only the magnetic field strength via the Larmor frequency, our scheme additionally exploits the oscillation amplitude of the spin precession, which carries critical information about the magnetic field direction. By constructing a differential

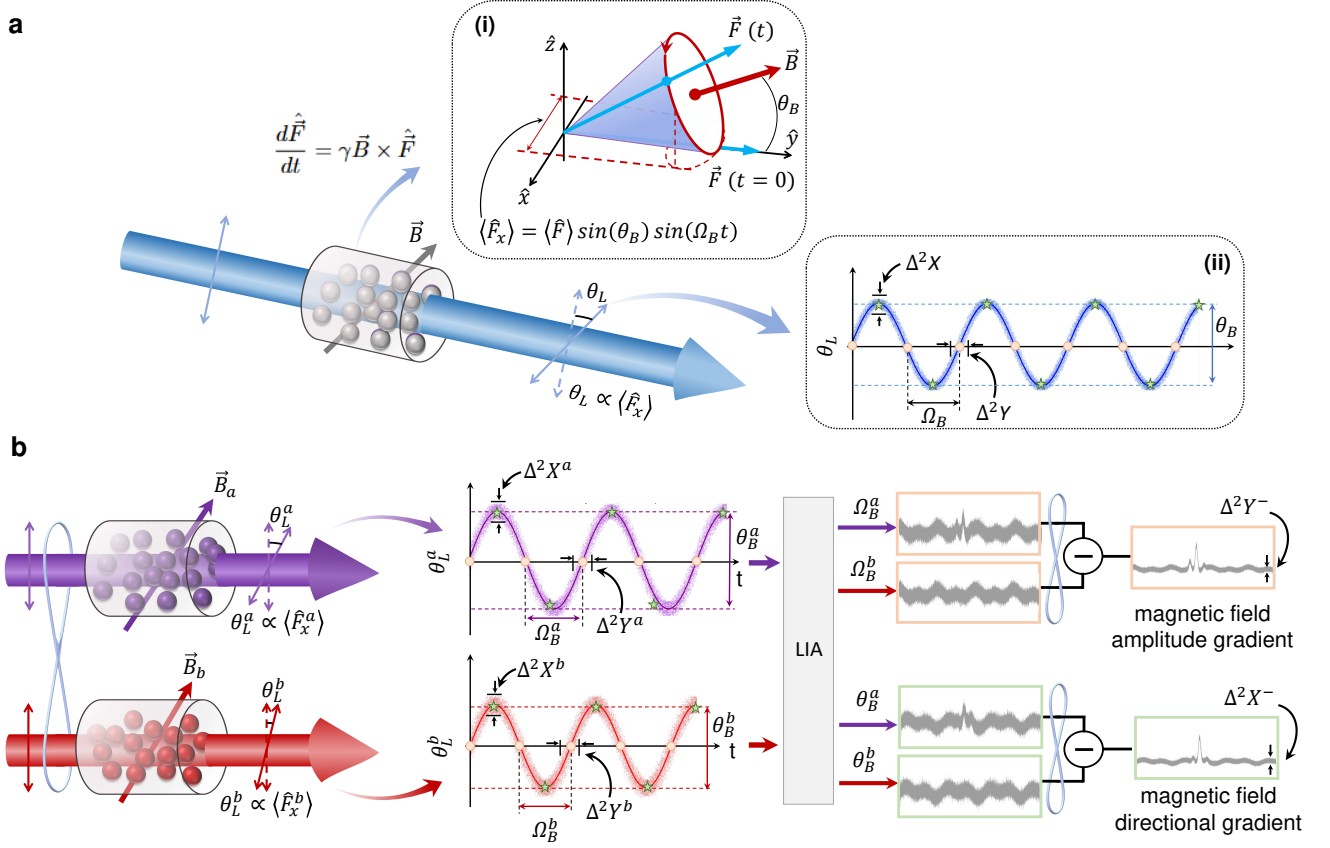


FIG. 1. **Schematic diagram of tensor magnetometry with multi-dimensional continuous-variable polarization-entangled (m-CVPE) lights.** **a**, Schematic diagram of vector magnetic field measurement. As the probe propagates with the polarized atomic spin undergoing Larmor precession, the polarization angle θ_L is modulated at the Larmor frequency Ω_B . $\langle \hat{F} \rangle$ is the total angular momentum of the polarized spins. γ is the gyromagnetic ratio. B is the leading magnetic field. Inset figure (i): Atomic spins undergo Larmor precession. θ_B is the magnetic field direction at the atomic sensor. $\langle \hat{F}_x \rangle$ is the projection of spin on $\hat{x}$ axis. Inset figure (ii): The polarization of the output light oscillates sinusoidally. Green stars: $\Omega_B^k t = (1/2 + q)\pi$, Pink circles: $\Omega_B^k t = q\pi$, ($k = a, b, q = 0, 1, 2, \dots$). Here, $\Delta^2 Y = \Delta^2 \hat{S}_x^{in}$, $\Delta^2 X = \cos(\xi) \Delta^2 \hat{S}_x^{in} + \sin(\xi) \Delta^2 \hat{S}_y^{in}$. **b**, Schematic of Quantum-enhanced tensor magnetic component measurement. Sensors a and b are utilized to form a magnetic gradiometer. Here, $\Delta^2 Y^k = \Delta^2 \hat{S}_x^k$, $\Delta^2 X^k = \cos(\xi) \Delta^2 \hat{S}_x^k + \sin(\xi) \Delta^2 \hat{S}_y^k$, $\Delta^2 Y^- = \Delta^2 \hat{S}_{x,-}^{in}$, $\Delta^2 X^- = \cos(\xi) \Delta^2 \hat{S}_{x,-}^{in} + \sin(\xi) \Delta^2 \hat{S}_{y,-}^{in}$, $\xi = 2\kappa \langle \hat{F} \rangle L \sin(\theta_B^k)$. L is the length of the atomic sensor and κ is the first order coupling constant. LIA: Lock-in Amplifier.

measurement between two atomic sensors, we extract gradient information of both the field direction and amplitude, which correspond to specific tensor components of the magnetic field. This two-parameter measurement protocol paves the way toward full-field tensor reconstruction using scalable quantum sensors.

To enable simultaneous quantum enhancement in both parameters, we utilize the specifically engineered multi-dimensional continuous-variable polarization-entangled (m-CVPE) lights as bright quantum probes. The m-CVPE lights comprise two distinct modes, a and b , and are generated via two orthogonally polarized four-wave mixing (FWM) processes [56–68] in a warm atomic ensemble. The entanglement properties of m-CVPE lights are characterized by combined Stokes operators $\hat{S}_{m,\pm}$, defined as $\hat{S}_{m,\pm} = \hat{S}_m^a \pm \hat{S}_m^b$ ($m = x, y, z$), where $\hat{S}_m^a$ and $\hat{S}_m^b$ denote the Stokes

operators corresponding to modes a and b , respectively. In contrast to conventional CVPE (c-CVPE) lights [69–84], three combined Stokes operators $\hat{S}_{x,-}$, $\hat{S}_{y,-}$ and $\hat{S}_{z,+}$ are simultaneously squeezed in our generated m-CVPE lights, thereby enabling superior performance in multiparameter quantum sensing. Applied to tensor magnetometry, the multidimensional entanglement enables simultaneous quantum-enhanced sensitivity to both magnetic field direction and amplitude gradients, achieving $21 \mu\text{rad}/\text{cm}/\sqrt{\text{Hz}}$ and $6 \text{ fT}/\text{cm}/\sqrt{\text{Hz}}$, respectively. The polarization-entangled source from an atomic vapor cell is anticipated to considerably impact practical quantum metrology and quantum network applications.

Tensor magnetometry

In this work, we demonstrate the quantum-enhanced measurement of two specific components of the magnetic

field gradient tensor, namely $\partial_z B_z$ and $\partial_z B_y$. Under an external magnetic field, atomic spins undergo Larmor precession, with the precession frequency determined by the magnetic field amplitude, and the precession amplitude dependent on the angle between the initial spin state and the magnetic field.

For an applied magnetic field at any arbitrary orientation, the direction of the vector magnetic field can be determined by measuring the $\langle \hat{F}_x \rangle$ and $\langle \hat{F}_y \rangle$ components (see detail in Methods).

To demonstrate the measurement process, we measure vector magnetic fields with Larmor frequencies Ω_B aligned in $\hat{y} - \hat{z}$ plane, with an angle of θ_B with respect to $\hat{x}$ axis as depicted in Fig. 1 (a). For an atomic ensemble initially polarized along the $\hat{y}$ axis, undergoing Larmor precession with such magnetic field, the projection of spin on $\hat{x}$ axis is as shown in Fig. 1 (a)(i):

$$\langle \hat{F}_x \rangle = \langle \hat{F} \rangle \sin(\theta_B) \sin(\Omega_B t), \quad (1)$$

where $\langle \hat{F} \rangle$ is the total angular momentum of the polarized spins. The spin state is mapped to the polarization of light through the first order atom-light interaction:

$$\hat{H}_L = \kappa \hat{S}_z \hat{F}_x, \quad (2)$$

where, κ is the first order coupling constant. After the optical field passes through the atomic sensor, its polarization angle is rotated as follows:

$$\begin{pmatrix} \hat{S}_x^{out} \\ \hat{S}_y^{out} \end{pmatrix} = \begin{pmatrix} \cos \theta_L & \sin \theta_L \\ -\sin \theta_L & \cos \theta_L \end{pmatrix} \begin{pmatrix} \hat{S}_x^{in} \\ \hat{S}_y^{in} \end{pmatrix}, \quad (3)$$

where, $\hat{S}_{x(y)}^{in}$ are the Stokes operators of the light before it interacts with the atomic sensor. After this atom-light interaction, $\hat{S}_{x(y)}^{out}$ are a mixture of $\hat{S}_x^{in}$ and $\hat{S}_y^{in}$. The polarization of the output light oscillates sinusoidally. The magnetic field information is encoded in the waveform of the polarization angle, constituting a multi-parameter measurement [85, 86] wherein the field's amplitude and direction are respectively embedded in the oscillation frequency and amplitude of the spin precession, as illustrated in Fig. 1(a)(ii). Experimentally, the output polarization is detected via a photon-detector and demodulated using a lock-in amplifier (LIA). The amplitude and direction of the magnetic field are then extracted simultaneously from the Y (out-of-phase) and X (in-phase) outputs demodulated from polarization rotation signal.

For tensor magnetometry, the Larmor frequencies Ω_B with perturbations $\Delta\Omega_B$ and the angles θ_B with perturbations $\Delta\theta_B$ are $\Omega_B^k = \Omega_B + \Delta\Omega_B^k$ and $\theta_B^k = \theta_B + \Delta\theta_B^k$ ($k = a, b$) at the positions of two atomic sensors, as illustrated in Fig. 1 (b). Here, we measure the magnetic field amplitude gradient and direction gradient, which can be further converted into gradient components $\partial_z B_z$ and $\partial_z B_y$ (see details in Methods).

The magnetic field amplitude gradient, which is extracted from the Y output of LIA, is determined in principle by measuring the time between two zero crossings of the oscillating spin components, i.e. $\Omega_B^k t =$

$q\pi$ ($q = 0, 1, 2, \dots$). The output gradient signal reads as $\langle \hat{S}_{x,-}^{out} \rangle = \langle \hat{S}_x^a \rangle - \langle \hat{S}_x^b \rangle = -\xi \langle \hat{S}_y^{in} \rangle (\Delta\Omega_B^a - \Delta\Omega_B^b)$, with the noise given by $\Delta^2 \hat{S}_{x,-}^{out} = \Delta^2 \hat{S}_{x,-}^{in}$ (see detail in Methods), where $\xi = 2\kappa \langle \hat{F} \rangle L \sin(\theta_B)$, L is the length of the atomic sensor. Here, since the measurement depends solely on $\hat{S}_{x,-}^{in}$, using c-CVPE lights [69–84] that squeezes $\hat{S}_{x,-}^{in}$ is sufficient to achieve quantum enhancement of amplitude sensitivity.

The magnetic field direction gradient, which is extracted from the X output of LIA, can be inferred from the oscillating amplitude of the polarized spins, i.e., $\Omega_B^k t = (1/2 + q)\pi$. The output signal reads as $\langle \hat{S}_{x,-}^{out} \rangle = -\xi' (\sin \xi \langle \hat{S}_x^{in} \rangle + \cos \xi \langle \hat{S}_y^{in} \rangle) (\Delta\theta_B^a - \Delta\theta_B^b)$, with the noise given by $\Delta^2 \hat{S}_{x,-}^{out} = \Delta^2 (\cos \xi \hat{S}_{x,-}^{in} - \sin \xi \hat{S}_{y,-}^{in})$, where $\xi' = \xi \cot(\theta_B)$ (see detail in Methods). The noise in the direction measurement is jointly determined by the noise of the two combined Stokes operators ($\hat{S}_{x,-}^{in}$ and $\hat{S}_{y,-}^{in}$). In this scenario, the measurement sensitivity is jointly limited by the noise of both $\hat{S}_{x,-}^{in}$ and $\hat{S}_{y,-}^{in}$. c-CVPE lights, which squeezes only one of the Stokes operators, cannot provide simultaneous quantum enhancement. To address this, we develop m-CVPE lights with simultaneous squeezing of both $\hat{S}_{x,-}^{in}$ and $\hat{S}_{y,-}^{in}$, enabling quantum-enhanced measurements of magnetic field direction gradients.

M-CVPE lights

The generation of CVPE light typically requires mixing of two squeezed vacuum states of lights with orthogonal polarizations. However, the resulting entangled lights exhibit a low photon number, limiting their applicability in high-sensitivity metrology [47–50]. Conventional protocols for photon number enhancement involve the coherent superposition of squeezed vacuum with an orthogonally polarized coherent state at a beam splitter. In such c-CVPE lights, quantum enhancement arises from squeezing in one polarization, while the polarization measurement capability originates from the coherent light in the orthogonal polarization. However, due to the introduction of coherent light, one of the Stokes operators $\Delta \hat{S}_{x,-}$ is consistently constrained to the SQL [87]. When the optical field traverses a birefringent medium, the change in its polarization causes mixture of two Stokes operators, as demonstrated in Eq. (3). This effect is particularly critical for high-sensitivity magnetometers, where the polarization rotation angle may exceed 45° [88, 89], resulting in a complete exchange of the noise properties between the two Stokes operators. In our experiment, the maximum polarization rotation angle reaches 28.5° , which still leads to significant mixing of the quantum noise of the two operators. When only one of them is initially squeezed, this mixing effectively introduces vacuum noise into the squeezed component, thereby substantially degrading the squeezing [90], which fundamentally limits the application of such quantum states in metrology.

To overcome the inherent SQL limitation in c-CVPE

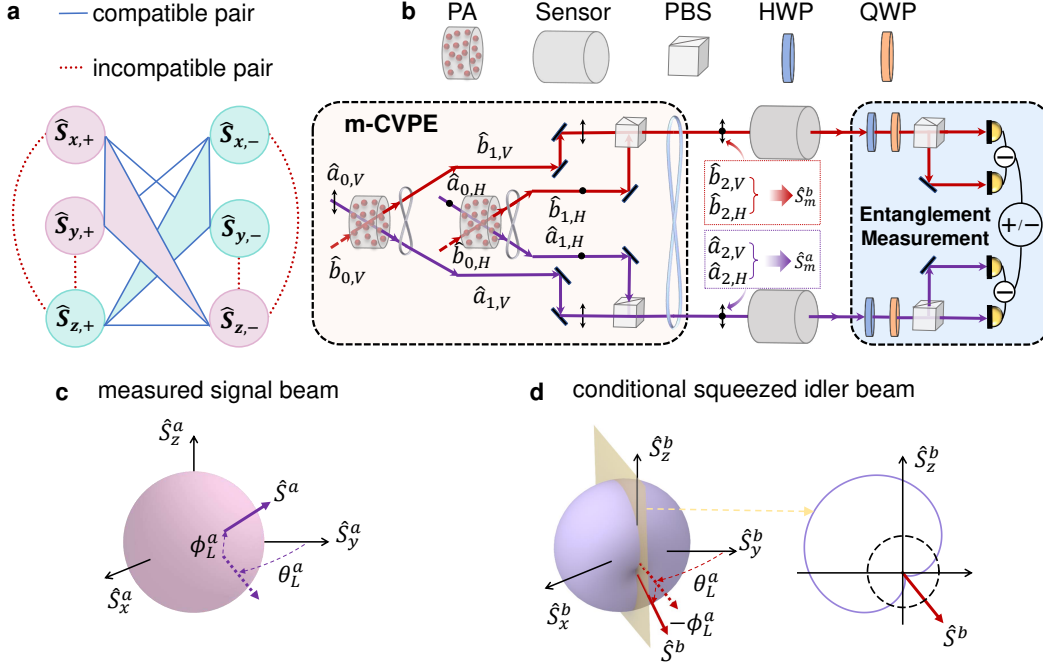


FIG. 2. **Schematic diagram of m-CVPE lights generation.** **a**, $\hat{S}_m^a$ and $\hat{S}_m^b$ are Stokes operators for two modes, and $\hat{S}_{m,\pm} = \hat{S}_m^a \pm \hat{S}_m^b$ ($m = x, y, z$). Solid lines and dashed lines represent compatible pair and incompatible pair, respectively. Here, $\hat{S}_{x,-}$, $\hat{S}_{y,-}$ and $\hat{S}_{z,+}$ can be squeezed simultaneously. Squeezing of $\hat{S}_{x,-}$ and $\hat{S}_{y,-}$ lead to the anti-squeezing of $\hat{S}_{z,-}$. Similarly, the squeezing of $\hat{S}_{z,+}$ leads to the anti-squeezing of $\hat{S}_{x,+}$ and $\hat{S}_{y,+}$. **b**, PA: parametric amplifier. PBS: polarizing beam splitter. $\hat{a}_{u,j}$ and $\hat{b}_{u,j}$ are the field operators with $u = 0, 1, 2$ and $j = H, V$. The index 0 represents the seed beams before PAs, 1 represents the quadrature entangled beams after PAs, and 2 represents the m-CVPE lights after PBSs. The index H and V represent the horizontally and vertically polarization modes. HWP: half-wave plate. QWP: quarter-wave plate. **c**, noise surface of general Stokes operators in mode $\hat{a}$ with tracing out mode $\hat{b}$. **d**, noise surface of the general Stokes operators in mode $\hat{b}$ with heralded detection of mode $\hat{a}$. The purple line corresponds to the yellow cross-section in **d**.

configurations, we develop m-CVPE lights. The potential for simultaneous squeezing is first illustrated by the compatibility relations among the combined Stokes operators for two linearly-polarized optical fields with identical amplitude and polarization, as shown in Fig. 2(a). To realize this potential, we develop m-CVPE lights by combining two bright polarization-orthogonal quadrature-entangled beam pairs, replacing the coherent-light element in traditional schemes with a second entangled source [90], thereby enabling simultaneous squeezing across three combined Stokes operators, as illustrated in Fig. 2(b). The two pairs of quadrature entangled twin beams (consisting of signal beams $\hat{a}_{1,V}$ and $\hat{a}_{1,H}$, and idler beams $\hat{b}_{1,V}$ and $\hat{b}_{1,H}$) are generated after parametric amplifiers (PAs) via FWM processes [66, 91]. The input-output relations are:

$$\begin{aligned}\hat{a}_{1,j} &= G\hat{a}_{0,j} + g\hat{b}_{0,j}^\dagger, \\ \hat{b}_{1,j}^\dagger &= G\hat{b}_{0,j}^\dagger + g\hat{a}_{0,j},\end{aligned}\quad (4)$$

where, $j = H, V$ denotes the horizontally and vertically polarized modes, $\hat{a}$ and $\hat{b}$ denote signal and idler modes, G and g are the gain factors satisfying $G^2 - g^2 = 1$. The input field of two FWM processes is $|\alpha\rangle_H|0\rangle_H^b|\alpha\rangle_V^a|0\rangle_V^b$,

where $|\alpha\rangle$ and $|0\rangle$ are coherent state and vacuum state, respectively. Each pair of twin beams after PA shares the same polarization, while the polarization of the two pairs are perpendicular to each other. We transform the entanglement into a polarization basis by combining the two signal beams and two idler beams from the upper (H-polarized) and lower (V-polarized) channels via two polarizing beam splitters (PBSs). This spatial-to-polarization conversion preserves the quantum correlations in both polarization modes, eliminating classical noise contamination inherent to hybrid quantum-classical architectures.

With our m-CVPE lights, the variances of three combined Stokes operators ($\hat{S}_{x,-}$, $\hat{S}_{y,-}$ and $\hat{S}_{z,+}$) are given by (see Supplementary materials):

$$\Delta^2 \hat{S}_{x,-} = \Delta^2 \hat{S}_{y,-} = \Delta^2 \hat{S}_{z,+} = \frac{N_t}{G^2 + g^2}, \quad (5)$$

where N_t is the total photon number of m-CVPE lights. The variances of the three combined Stokes operators ($\hat{S}_{x,-}$, $\hat{S}_{y,-}$, and $\hat{S}_{z,+}$) are simultaneously squeezed, ensuring that such a quantum light field is suitable for quantum estimation of multiple parameters.

To characterize the noise properties of the gener-

ated m-CVPE lights across all polarization components, we define general Stokes operator by rotating the coordinates $\hat{S}_y^k$ via two rotating operations $e^{i\phi_L^k \hat{S}_y^k/2} e^{i\theta_L^k \hat{S}_z^k/2}$, ($k = a, b$):

$$\hat{S}^k(\theta_L^k, \phi_L^k) = \sin \theta_L^k \cos \phi_L^k \hat{S}_x^k + \cos \theta_L^k \hat{S}_y^k + \sin \theta_L^k \sin \phi_L^k \hat{S}_z^k. \quad (6)$$

Fig. 2(c,d) illustrate the noise surfaces for the general Stokes operators. The radial distance from the origin to the surface in any direction (θ_L^k, ϕ_L^k) represents the noise in that direction. By tracing out one of the two modes, e.g., mode $\hat{b}$, the surviving mode $\hat{a}$ is described by a thermal state with anti-squeezing in all directions as shown in Fig. 2 (c). With the heralded detection of mode $\hat{a}$ along the direction θ_L^a, ϕ_L^a , the general Stokes operator of mode $\hat{b}$ along the direction $\theta_L^b = \theta_L^a, \phi_L^b = -\phi_L^a$ is then conditionally squeezed as shown in Fig. 2 (d).

To further explore the entanglement properties between modes $\hat{a}$ and $\hat{b}$, we investigate the noise characteristics of combined Stokes operators $\hat{S}^a - \hat{S}^b$, which exhibit rich angular dependencies, reflecting the interplay between polarization orientations and entanglement structure. The theoretical noise surfaces of $\hat{S}^a - \hat{S}^b$, under specific relationships between the angles $\theta_L^a, \theta_L^b, \phi_L^a$ and ϕ_L^b are illustrated in Fig. 3 (a, b), while the corresponding experimental measurements are plotted in Fig. 3 (c).

The noise power is measured at 0.6 MHz relative to the PSN, which is calibrated using coherent light with the same optical power as the entangled beams. With a gain of $G^2 = 6$ and an external loss of $\epsilon = 0.185$, the maximum squeezing reaches 7 dB. As it turns out, this entanglement extends the dimensions of entanglement between signal and idler beams. For $\theta_L^a = \theta_L^b, \phi_L^a = -\phi_L^b$, the combined operator exhibits continuous squeezing across all directions (green sphere). The black curve on this sphere corresponds to $\phi_L^a = \phi_L^b = 0, \theta_L^a = \theta_L^b$, matching the experimentally measured black curve in Fig. 3(c), where the noise remains squeezed. Experimentally, this configuration is realized by synchronously rotating the half-wave plates (HWPs) for modes $\hat{a}$ and $\hat{b}$ such that $\theta_L^a = \theta_L^b = 2\theta_h$, where θ_h is the physical rotation angle of the HWPs. When $\theta_L^b = \theta_L^a + \pi, \phi_L^b = -\phi_L^a$, the noise surface (pink sphere) shows uniform anti-squeezing. The purple curve ($\phi_L^a = \phi_L^b = 0, \theta_L^b = \theta_L^a + \pi$) aligns with the experimental purple curve in Fig. 3(c), which is implemented by rotating the HWP for mode $\hat{b}$ by an additional $\pi/2$ relative to $\hat{a}$, confirming persistent anti-squeezing. The light blue and yellow surfaces correspond to $\theta_L^b = \theta_L^a + \pi, \phi_L^b = \phi_L^a$ and $\theta_L^b = \theta_L^a, \phi_L^b = \phi_L^a$, respectively. Experimentally, we probe the tangent lines of these spheres to the $\hat{x} - o - \hat{z}$ plane by rotating the quarter-wave plates (QWPs) with angle $\phi_L^b = \phi_L^a = 2\theta_q$, showing oscillating noise properties between squeezing and anti-squeezing.

Sensitivity

To fully exploit the quantum correlations in our system, we employ the m-CVPE lights source as the probes for a

a composed stokes operators **b** composed stokes operators

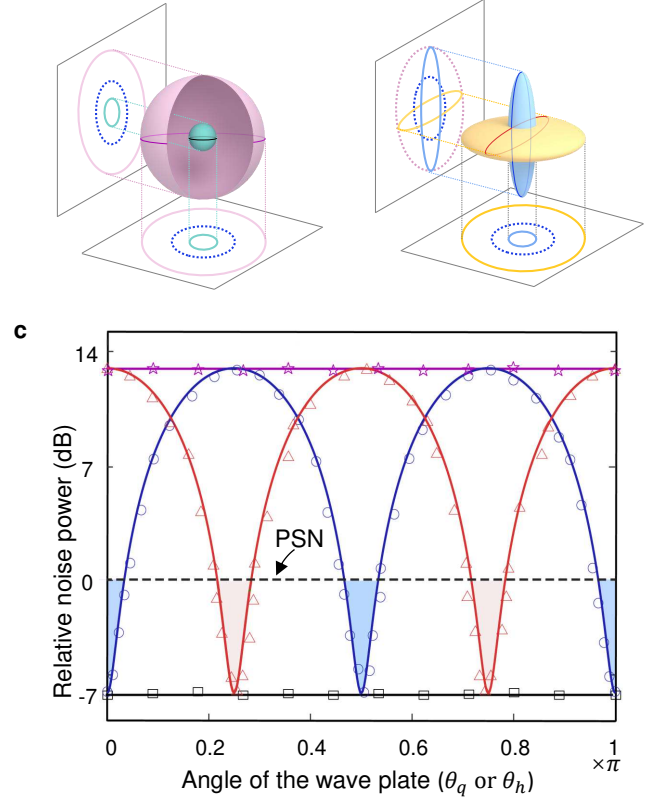


FIG. 3. The characteristics of m-CVPE lights. **a** and **b**, the green, pink, yellow, and light blue surfaces represent the noise of combined Stokes operators with $\theta_L^b = \theta_L^a, \phi_L^b = -\phi_L^a$; $\theta_L^b = \theta_L^a + \pi, \phi_L^b = -\phi_L^a$; $\theta_L^b = \theta_L^a + \pi, \phi_L^b = \phi_L^a$; $\theta_L^b = \theta_L^a, \phi_L^b = \phi_L^a$, respectively. The blue dashed line denotes the photon shot noise (PSN). **c**, Variation in the noise power relative to the PSN with the angles θ_q and θ_h , shown as curves (theoretical simulation) and scattered data (experimental data). All the data are normalized with respect to the PSN. Here, $G = \sqrt{6}$, $\alpha = 10^7$, and $\epsilon = 0.185$.

tensor magnetometer. This allows us to simultaneously reduce the noise contributions associated with both $\hat{S}_{x,-}^{in}$ and $\hat{S}_{y,-}^{in}$, thereby enabling quantum-enhanced sensitivity in both directional and amplitude gradient measurements. The sensitivity data shown below verify that this simultaneous noise suppression directly translates into enhanced performance in both measurement channels.

For magnetic measurement, the optical pumping and repumping beams are periodically modulated at a frequency Ω_m , synchronously polarizing the atomic spins in both sensors. The polarization rotation signal of the probe light is demodulated by LIA referenced to the same modulation frequency. As Ω_m is swept across the Larmor frequency, the atomic spins are resonantly driven, producing a characteristic magnetic resonance curve in the

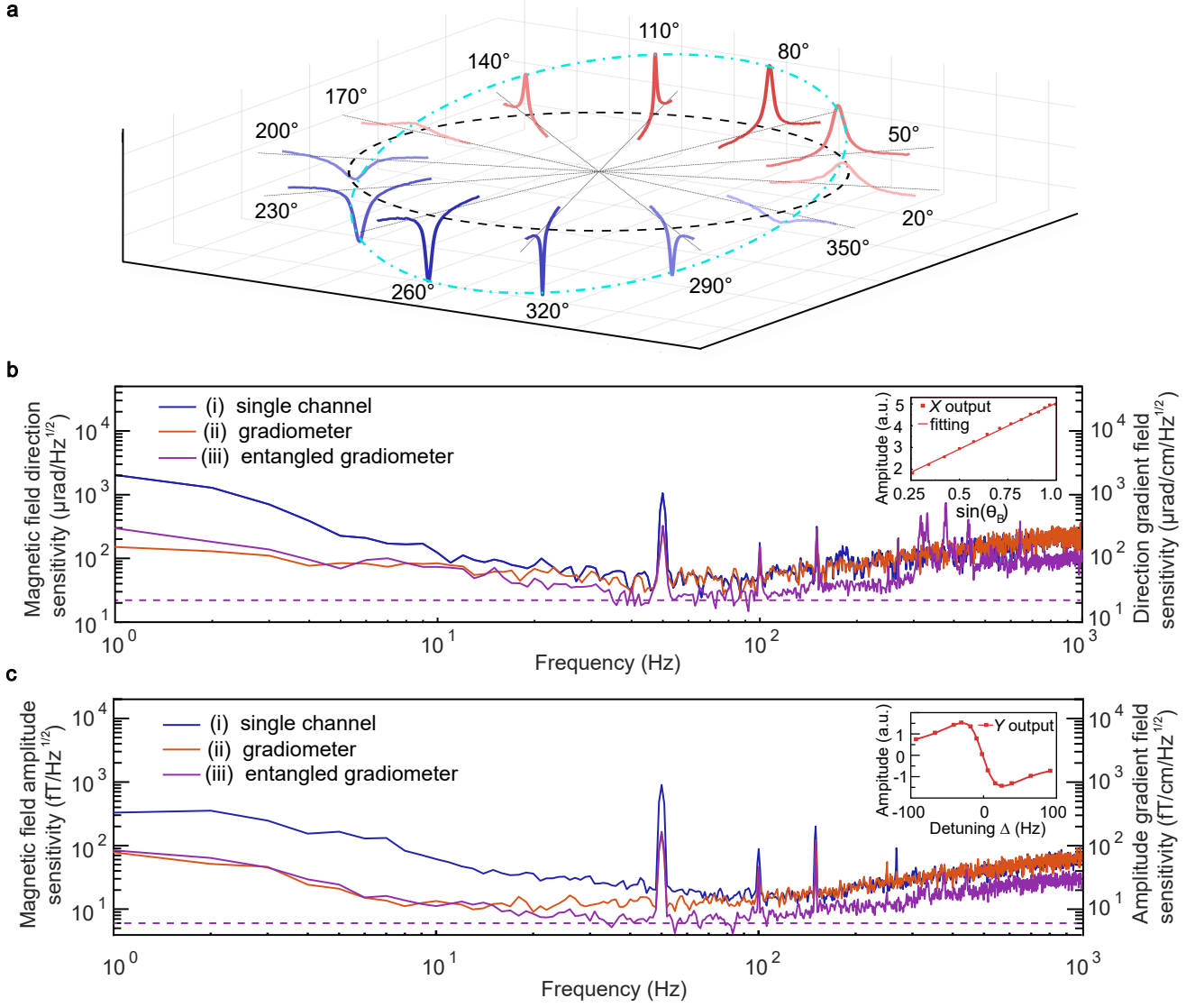


FIG. 4. **The characteristics of the magnetic field.** **a**, Experimentally measured X output of the magnetometer as a function of the modulation frequency Ω_m for different magnetic field angles θ_B . Regions with positive X output amplitudes are marked in red, while those with negative amplitudes are marked in blue, with color intensity proportional to the signal magnitude. Directional (**b**) and amplitude (**c**) sensitivities. In each panel: (i) sensitivity of the magnetometer limited by PSN. (ii) Gradient field sensitivity of the gradiometer limited by PSN. (iii) Gradient field sensitivity of the gradiometer with 6 dB polarization-entangled squeezing. Dashed lines denote the optimal sensitivity of the entangled gradiometer for direction (**b**) and amplitude (**c**), respectively. The detuning is defined as $\Delta = \Omega_m - \Omega_B$, where the Ω_m is the modulation frequency of the sensor's optical pumping.

LIA outputs. Fig. 4 (a) displays the resulting magnetic resonance curves of the X output for various field angles θ_B . Each trace corresponds to a magnetic resonance curve of the atomic ensemble, and the envelope of these curves (blue dotted line) follows $\sin(\theta_B)$ dependence.

To quantify the sensitivity of the tensor magnetometer, we fix the modulation frequency on resonance. The resulting directional and amplitude sensitivities, obtained by monitoring the X output at $\theta_B = 45^\circ$ and the Y output at $\theta_B = 90^\circ$ respectively, as shown in Fig. 4 (b) and (c). The insets in (b) and (c) show the correspond-

ing signal slopes, which are used to convert the measured noise spectra into the magnetic field direction and amplitude sensitivities, respectively. The blue line (i) gives the sensitivity of a conventional magnetometer, which is obtained by blocking the optical pumping of one sensor and is limited by both common-mode magnetic (CMM) noise and photon shot noise (PSN). The red line (ii) shows the PSN-limited sensitivity of the gradiometer, achieved by turning on the optical pumping and repumping of both sensors to suppress the CMM noise. The purple line (iii) depicts the quantum-enhanced sensitivity of the

gradiometer, utilizing 6 dB m-CVPE light to further mitigate PSN. The dashed lines indicate the optimal sensitivities of the entangled gradiometer for direction (b) and amplitude (c) measurements, respectively. In the directional sensitivity spectrum (Fig. 4b, purple curve), excess noise is observed in the 300–500 Hz range. This noise originates from the phase-locking system used to stabilize the relative phase between the horizontally and vertically polarized entangled beams during their combination on a polarizing beam splitter. The frequency response of each sensor, which acts as a low-pass filter with the transfer function $b/\sqrt{f^2 + b^2}$ [3, 94], leads to the signal decrease as the measurement frequency f increases beyond the bandwidth $b = 100$ Hz.

For the directional sensitivity shown in Fig. 4(b), the conventional magnetometer achieves a sensitivity of approximately $80 \mu\text{rad}/\sqrt{\text{Hz}}$ at 95 Hz (blue line i). The gradiometer configuration suppresses CMM noise to reach a PSN-limited directional gradient sensitivity of $50 \mu\text{rad}/\text{cm}/\sqrt{\text{Hz}}$ (red line ii). With 6 dB m-CVPE light, the quantum-enhanced gradiometer achieves an optimal directional gradient sensitivity of $21 \mu\text{rad}/\text{cm}/\sqrt{\text{Hz}}$ at 30 Hz (purple line iii). For the amplitude sensitivity shown in Fig. 4(c), this configuration achieves approximately $20 \text{ fT}/\sqrt{\text{Hz}}$ at 20 Hz for the conventional magnetometer (blue line i). Operating the setup as a gradiometer suppresses CMM noise and achieves an optimal PSN-limited amplitude gradient sensitivity of $11 \text{ fT}/\text{cm}/\sqrt{\text{Hz}}$ (red line ii). The introduction of 6 dB m-CVPE probe light further reduces PSN, yielding a quantum-enhanced sensitivity that reaches $6 \text{ fT}/\text{cm}/\sqrt{\text{Hz}}$ at 20 Hz (purple line iii).

Conclusion

In this work, we develop and demonstrate a novel approach for achieving quantum-enhanced measurements of both the direction and amplitude of magnetic field gradient by combining the m-CVPE lights with an atomic magnetic gradiometer. The m-CVPE lights are generated by integrating sets of independent entangle-

ment sources with orthogonal polarizations, yielding a noise squeezing degree of 7 dB. Compared with previous entangled magnetic gradiometer, the sensitivity in magnetic field measurements is improved by an order of magnitude since the entangled and measured quantity changes from quadrature to polarization which is consistent with results in classical atomic magnetometry [3] and atomic clock [21]. By leveraging the 6 dB noise squeezing of such m-CVPE lights at 20 Hz, we achieve a quantum-enhanced tensor magnetometry with an amplitude gradient sensitivity of $6 \text{ fT}/\text{cm}/\sqrt{\text{Hz}}$ and a directional gradient sensitivity of $21 \mu\text{rad}/\text{cm}/\sqrt{\text{Hz}}$. The deployment of this multi-dimensional polarization-entangled source, derived from a warm atomic vapor cell, holds significant promise for transformative impacts in practical quantum metrology. Furthermore, our quantum-enhanced tensor magnetometry could illuminate the overarching design principles applicable to multiparameter quantum sensing technologies.

In this paper, we measure the gradient components $\partial_z B_z$ and $\partial_z B_y$. Notably, with a quantum network of entangled magnetic sensors, this methodology can be systematically extended to characterize other components of the field to form a full tensor magnetometer without loss of generality, as the underlying measurement principles remain consistent across spatial dimensions. Further enhancements in tensor magnetometry performance can be realized by incorporating advanced quantum manipulation techniques and optimizing atom-light interactions.

Online content

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at <http://doi.org/XXX>.

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Methods

Quantum theory of tensor magnetometry

As mentioned above, we measure the Stokes operator $\hat{S}_{x,-}^{out}$ of m-CVPE lights to obtain the direction and amplitude of magnetic field gradient, $\Delta\theta_B^a - \Delta\theta_B^b$ and $\Delta\Omega_B^a - \Delta\Omega_B^b$. As mentioned in the main text, the direction and amplitude of a magnetic field gradient can be written as $\theta_B^k = \theta_B + \Delta\theta_B^k$ and $\Omega_B^k = \Omega_B + \Delta\Omega_B^k$, respectively. Therefore, the $\theta_L^k = -\kappa L \langle \hat{F} \rangle \sin \theta_B^k \sin \Omega_B^k t$ can be expanded as:

$$\begin{aligned} \theta_L^k &= -\kappa L \langle \hat{F} \rangle \sin(\theta_B^k + \Delta\theta_B^k) \sin(\Omega_B t + \Delta\Omega_B^k t) \\ &\approx -\kappa L \langle \hat{F} \rangle (\sin \theta_B^k + \Delta\theta_B^k \cos \theta_B) \\ &\quad \times (\sin(\Omega_B t) + \Delta\Omega_B^k t \cos \Omega_B t). \end{aligned} \quad (7)$$

When $\Omega_B t = q\pi$, we have $\theta_L^k \approx -\kappa L \langle \hat{F} \rangle \sin \theta_B^k \Delta\Omega_B^k t \equiv -\xi \Delta\Omega_B^k$ which is a small quantity. From the input-output formula of the Stokes operators in Eq. (3), we derive:

$$\begin{aligned} \hat{S}_{x,-}^{out} &= \cos \theta_L^a \hat{S}_x^{a,in} + \sin \theta_L^a \hat{S}_y^{a,in} \\ &\quad - (\cos \theta_L^b \hat{S}_x^{b,in} + \sin \theta_L^b \hat{S}_y^{b,in}) \\ &\approx \hat{S}_{x,-}^{in} - \xi (\Delta\Omega_B^a \hat{S}_y^{a,in} - \Delta\Omega_B^b \hat{S}_y^{b,in}). \end{aligned} \quad (8)$$

Because the mean values of $\hat{S}_{x(y)}^{a,in}$ and $\hat{S}_{x(y)}^{b,in}$ are the same, we define $\langle \hat{S}_{x(y)}^{a,in} \rangle = \langle \hat{S}_{x(y)}^{b,in} \rangle = \langle \hat{S}_{x(y)}^{in} \rangle$. The signal and noise can be written as:

$$\begin{aligned} \langle \hat{S}_{x,-}^{out} \rangle &= -\xi \langle \hat{S}_y^{in} \rangle (\Delta\Omega_B^a - \Delta\Omega_B^b), \\ \Delta^2 \hat{S}_{x,-}^{out} &\approx \Delta^2 \hat{S}_{x,-}^{in}. \end{aligned} \quad (9)$$

Here the small high-order term in noise $\Delta^2 \hat{S}_{x,-}^{out}$ has been ignored.

When $\Omega_B t = (q + 1/2)\pi$, the derivations are similar to those described above. We have $\theta_L^k \approx -\kappa L \langle \hat{F} \rangle (\sin \theta_B^k + \Delta\theta_B^k \cos \theta_B^k) \equiv -\xi + \xi' \Delta\theta_B^k$:

$$\begin{aligned} \hat{S}_{x,-}^{out} &= \cos \theta_L^a \hat{S}_x^{a,in} + \sin \theta_L^a \hat{S}_y^{a,in} \\ &\quad - (\cos \theta_L^b \hat{S}_x^{b,in} + \sin \theta_L^b \hat{S}_y^{b,in}) \\ &\approx \cos \xi \hat{S}_{x,-}^{in} - \sin \xi \hat{S}_{y,-}^{in} \\ &\quad - \xi' \sin \xi (\Delta\theta_B^a \hat{S}_x^{a,in} - \Delta\theta_B^b \hat{S}_x^{b,in}) \\ &\quad - \xi' \cos \xi (\Delta\theta_B^a \hat{S}_y^{a,in} - \Delta\theta_B^b \hat{S}_y^{b,in}). \end{aligned} \quad (10)$$

The signal and noise can be written as:

$$\begin{aligned} \langle \hat{S}_{x,-}^{out} \rangle &= -\xi' \sin \xi \langle \hat{S}_x^{in} \rangle (\Delta\theta_B^a - \Delta\theta_B^b) \\ &\quad - \xi' \cos \xi \langle \hat{S}_y^{in} \rangle (\Delta\theta_B^a - \Delta\theta_B^b), \\ \Delta^2 \hat{S}_{x,-}^{out} &\approx \Delta^2 (\cos \xi \hat{S}_{x,-}^{in} - \sin \xi \hat{S}_{y,-}^{in}). \end{aligned} \quad (11)$$

Here, the small high-order term in noise $\Delta^2 \hat{S}_{x,-}^{out}$ has been ignored.

Negligibility of atomic spin noise

The foregoing analysis proceeds under the assumption that $\hat{F}_x$ is treated as $\langle \hat{F}_x \rangle$. Here, we give a proof that under the experimental conditions considered, the contribution of atomic spin noise is negligible. When $\hat{F}_x$ is instead considered as an operator, the input-output relationship dictates that:

$$\hat{S}_y^{out} = \cos \hat{\theta}_L \cdot \hat{S}_y^{in} - \sin \hat{\theta}_L \cdot \hat{S}_x^{in}, \quad (12)$$

where

$$\hat{\theta}_L = -\kappa \hat{F}_x L \sin(\theta_B) \sin(\Omega_B t). \quad (13)$$

To evaluate the transformation of the optical Stokes operator due to interaction with a polarized atomic ensemble, we begin from the rotation formula Eq. (12), with the Faraday rotation angle operator given by Eq. (13). We expand the trigonometric functions up to second order and the field operators are decomposed into their mean values and fluctuations as follows:

$$\begin{aligned} \langle \hat{S}_y^{out} \rangle + \delta \hat{S}_y^{out} &= \left[1 - \frac{\kappa^2 L^2 (\langle \hat{F}_x \rangle + \delta \hat{F}_x)^2}{2} \right] \delta \hat{S}_y^{in} \\ &\quad + \kappa L (\langle \hat{F}_x \rangle + \delta \hat{F}_x) (\langle \hat{S}_x^{in} \rangle + \delta \hat{S}_x^{in}). \end{aligned} \quad (14)$$

Applying the mean-field approximation and keeping only first-order fluctuation terms, we obtain the fluctuation of the output field as:

$$\begin{aligned} \delta \hat{S}_y^{out} &= \left(1 - \frac{\kappa^2 L^2 \langle \hat{F}_x \rangle^2}{2} \right) \delta \hat{S}_y^{in} \\ &\quad + \kappa L \langle \hat{F}_x \rangle \delta \hat{S}_x^{in} + \kappa L \langle \hat{S}_x^{in} \rangle \delta \hat{F}_x. \end{aligned} \quad (15)$$

For measuring amplitude of magnetic field i.e. $\langle \hat{F}_x \rangle = 0$, increasing either photon number or atom number will accentuate the atomic spin noise contribution. For measuring direction of magnetic field i.e. $\langle \hat{F}_x \rangle = \langle \hat{F} \rangle$, in the regime where the atom number is much larger than photon number ($\langle \hat{F} \rangle \gg \langle \hat{S}_x^{in} \rangle$), particularly relevant for achieving large rotation angles, the output noise originates solely from optical Stokes components, with atomic spin noise rendered negligible.

Reconstruction of magnetic-field gradient tensor

The gradient tensor measurement of the magnetic field, a second-order tensor characterizing the spatial rate of change of a magnetic field, is mathematically defined as the matrix of partial derivatives of the magnetic field vector $\vec{B} = [B_x, B_y, B_z]$ with respect to the three-dimensional spatial coordinates (x, y, z) . The complete

representation is expressed as:

$$\nabla \times \vec{B} = \begin{pmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{pmatrix}. \quad (16)$$

In a source-free region (where there is no current or magnetic monopole), the tensor satisfies the Laplace condition ($\nabla \cdot \vec{B} = 0$), meaning the sum of the diagonal components is zero:

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0. \quad (17)$$

This feature can reduce the number of independent measurement components. If the magnetic field is generated by a static magnetic source (without eddy currents), the tensor satisfies the symmetry:

$$\frac{\partial B_i}{\partial j} = \frac{\partial B_j}{\partial i}, (i, j = x, y, z), \quad (18)$$

further reducing the independent components to 5. In the main text, we measure the direction and amplitude gradients of a magnetic field. Here, we convert these values into the gradients of the B_y and B_z fields:

$$\begin{aligned} \Delta B_y &= B_a \cos \theta_B^a - B_b \cos \theta_B^b \\ &\approx (B_0 + \Delta B_a)(\cos \theta_B - \sin \theta_B \Delta \theta_B^a) \\ &\quad - (B_0 + \Delta B_b)(\cos \theta_B - \sin \theta_B \Delta \theta_B^b) \\ &\approx (\Delta B_a - \Delta B_b) \cos \theta_B - B_0 \sin \theta_B (\Delta \theta_B^a - \Delta \theta_B^b), \end{aligned} \quad (19)$$

$$\begin{aligned} \Delta B_z &= B_a \sin \theta_B^a - B_b \sin \theta_B^b \\ &\approx (B_0 + \Delta B_a)(\sin \theta_B + \cos \theta_B \Delta \theta_B^a) \\ &\quad - (B_0 + \Delta B_b)(\sin \theta_B + \cos \theta_B \Delta \theta_B^b) \\ &\approx (\Delta B_a - \Delta B_b) \sin \theta_B + B_0 \cos \theta_B (\Delta \theta_B^a - \Delta \theta_B^b). \end{aligned} \quad (20)$$

Here, the difference in the y (z)-component of two vector magnetic fields $\Delta B_{y(z)} = B_{y(z)}^a - B_{y(z)}^b$ is measured by a magnetic gradiometer. a and b are the labels of these two magnetic fields. B_a and B_b are their strengths, and these fields are in the $\hat{y} - \hat{z}$ plane with angles θ_B^a and θ_B^b with respect to the y axis. Because difference of the direction and amplitude of these two magnetic fields are small, the expressions for the magnetic field gradients can be expanded by taking $\theta_B^k = \theta_B + \Delta \theta_B^k$ and $B_k = B_0 + \Delta B_k$, and we omit the small high-order terms, as above.

In this paper, the magnetic field is aligned in the $\hat{y} - \hat{z}$ plane, for a magnetic field in an arbitrary direction, the equation of motion for the spin under such a magnetic field is:

$$\frac{d\hat{F}}{dt} = \gamma \vec{B} \times \hat{F} \quad (21)$$

$$\begin{aligned} \hat{F}(t) &= \hat{F}(0) \cos(\gamma|B|t) + \vec{B} \times \hat{F}(0) \sin(\gamma|B|t) + \\ &\quad \vec{B} \cdot \hat{F}(0) [1 - \cos(\gamma|B|t)] \vec{B} \end{aligned} \quad (22)$$

For a spin initially polarized along $\hat{y}$ axis, i.e. $\langle \hat{F}_x(0) \rangle = \langle \hat{F}_z(0) \rangle = 0$,

$$\begin{cases} \langle \hat{F}_x \rangle = \frac{B_x B_y \langle \hat{F} \rangle}{|B|^2} + \frac{\langle \hat{F} \rangle}{|B|} \sqrt{B_z^2 + \frac{B_x^2 B_y^2}{|B|^2}} \sin(\Omega_B t) \\ \langle \hat{F}_y \rangle = \frac{B_y^2 \langle \hat{F} \rangle}{|B|^2} + \frac{(B_x^2 + B_z^2) \langle \hat{F} \rangle}{|B|^2} \cos(\Omega_B t) \\ \langle \hat{F}_z \rangle = \frac{B_y B_z \langle \hat{F} \rangle}{|B|^2} + \frac{\langle \hat{F} \rangle}{|B|} \sqrt{B_x^2 + \frac{B_y^2 B_z^2}{|B|^2}} \sin(\Omega_B t) \end{cases} \quad (23)$$

Here, $\Omega_B = \gamma|B|$. The component of the magnetic field along the y -direction can be determined by measuring $\langle \hat{F}_y \rangle$. Once the B_y component is known, the direction of the magnetic field in the $\hat{x} - \hat{z}$ plane can be determined by measuring $\langle \hat{F}_x \rangle$.

Energy level diagram

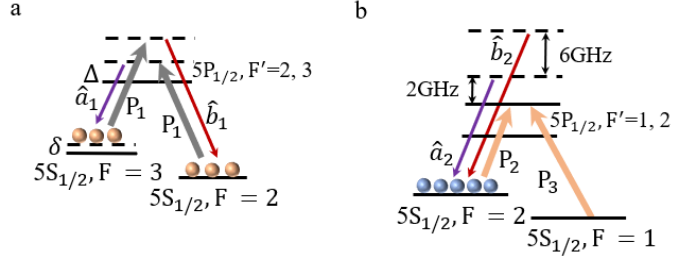


FIG. 5. **Energy level diagrams of ^{85}Rb and ^{87}Rb .** a, Energy level diagram in the D_1 line of ^{85}Rb for four-wave-mixing (FWM) process. Δ : one-photon detuning; δ : two-photon detuning. P_1 : FWM pump beam. b, Energy level diagram in the D_1 line of ^{87}Rb for atomic magnetometry. P_2 and P_3 are optical pumping and repumping beams of atomic magnetometry, respectively.

Experimental setup

The detailed experimental arrangement is shown in Fig. 6. To generate m-CVPE lights, an external cavity diode laser (ECDL) is tuned to the blue of the D_1 line of the ^{85}Rb $5S_{1/2}, F=2$ to $5P_{1/2}, F'=2$ transition, at approximately 1 GHz. The laser power is amplified by a tapered amplifier (TA) to 2 W. A PBS is used to split the beam into a strong pump beam with a power of 400 mW and a weak seed beam with a power of 30 μW . The seed beam is red shifted approximately 3.04 GHz from the pump beam by an acousto-optic modulator (AOM) in a double-pass configuration. The pump and seed beams are split by PBSs into two channels, namely, the upper and lower channels, which are separated by a distance of approximately 5 mm. For each channel, the powers of the pump and seed beams are 110 mW

and $15\mu\text{W}$, respectively. The waist diameters of the pump beam and seed beam are $500\mu\text{m}$ and $250\mu\text{m}$, respectively, and they are focused at the center of the ^{85}Rb atomic vapor cell. The pump beam and seed beam have orthogonal linear polarizations and are combined through a Glan-Laser polarizer (GL) at a crossing angle of 0.4° . The ^{85}Rb atomic vapor cell (12.5 mm long) is heated to 120°C , with a high transmission efficiency of $\sim 95\%$. After passing through the atomic vapor cell, the seed beams are amplified named signal beams and accompanied by the generation of the idler beams, which are red and blue shifted 3.04GHz from the pump beam, respectively. A Glan Thomson (GT) polarizer with an extinction ratio of $10^5:1$ is used to filter the excess pump beams. The polarizations of the signal and idler beams in the upper channel are adjusted to H-polarization via HWPs. The entangled beams with H-polarization and V-polarization are combined via a PBS. 99% of the beams are output from one side of the PBS, with a measured mode matching efficiency of 99%, generating the m-CVPE lights. 1% of the beam, output from the other side of the PBS, is used to lock the relative phase between the horizontal and vertical modes to zero.

To achieve magnetic field measurements, the entangled gradiometer utilizes two paraffin-coated and antireflection (AR)-coated cylindrical cells (1 cm in length, 1 cm in diameter) as sensors, placed side-by-side in a three-layer μ -metal magnetic shield. The generated m-CVPE lights serve as the probe lights for the entangled gradiometer. The frequencies of $\hat{a}_{1,j}$ and $\hat{b}_{1,j}$ are detuned by approximately 2GHz and 8GHz , respectively, from the $|5^2S_{1/2}, F=2\rangle \rightarrow |5^2P_{1/2}, F=2\rangle$ transition of the ^{87}Rb atoms. To facilitate the generation of a common-mode magnetic (CMM) field and a magnetic field gradient across the two sensors, we construct a set of Helmholtz coils and a set of anti-Helmholtz coils, each with a diameter of 92mm , centered around the two ^{87}Rb vapor cells.

Two DL pro lasers are employed to provide optical pumping and optical repumping for atomic magnetometry, respectively. Here, the optical pumping beam is tuned to resonate with the $5S_{1/2}, F=2$ to $5P_{1/2}, F'=2$ transition, polarizing the atoms. The optical repumping beam is tuned to the $5S_{1/2}, F=1$ to $5P_{1/2}, F'=2$ transition, ensuring that most atoms are pumped into the $F=2$ states. Both the circularly polarized optical pumping and repumping are split and sent to the two sensors along the $\hat{y}$ axis. Using an AOM, the intensities of the optical pumping and repumping are modulated with a square-wave (3% duty cycle) at a frequency of Ω_p . The atomic spins in the two sensors are synchronously oriented along the $\hat{y}$ axis and polarized to the $|5S_{1/2}, F=2, m_F=2\rangle$ state. During the “on” part of the cycle, the powers of the optical pumping and repumping beams are set to 2mW and 6mW , respectively. Here, to remove the degeneracy in the magnetic field direction, the folding mirrors control the orientation of

the m-CVPE lights direction, enabling separate measurements of the atomic spin projections $\hat{F}_x$ (without folding mirrors) and $\hat{F}_z$ (with folding mirrors). As the m-CVPE lights pass through the two sensors, their polarization angles, electric field amplitudes, overall phases, and ellipticities are modulated. In this study, we primarily examine the alterations in the polarization angle. The Larmor frequency is positioned at $\Omega_B = 80\text{kHz}$ to balance the non-linear Zeeman (NLZ) effect [92], optical noise squeezing [25], and residual magnetic resonance effects associated with the $F=1$ state [93]. Due to the absorption losses of 8% and 6% for the signal and idler beams, respectively, the available optical squeezing at $\Omega_B^a = 80\text{kHz}$ remains at 6 dB.

The polarization characteristics are described by the Stokes operators, which are detected by the measurement devices as shown in Fig. 2. After passing through an HWP and a QWP, the signal ($\hat{a}_{2,j}$) and idler ($\hat{b}_{2,j}$) beams are split via a PBS, and the outputs are detected by a balanced photo-detector (BPD, Thorlabs PDB450A) with two high quantum efficiency (96%) photodiodes (S 3883). To measure the noise properties of combined Stokes operators, we detect the combined Stokes operators $\hat{S}^a - \hat{S}^b$ by altering the angles of the HWP axis (θ_h) and QWP axis (θ_q) relative to the light polarization for both signal and idler beams with $\theta_L = 2\theta_h$, $\phi_L = 2\theta_q$. The fluctuations of combined Stokes operators are measured by a spectrum analyzer. With the Larmor precession of atomic spin, $\hat{S}^a - \hat{S}^b$ oscillates at the Larmor frequency as an optical rotation signal, which is demodulated at the frequency of Ω_m by a lock-in amplifier.

Phase locking

The pump and seed beams can be finely adjusted by using a combination of PBS and HWP, respectively. To ensure phase stability, we employ two sets of phase-locked loops, each dedicated to individually phase-locking two signal beams and two idler beams. The PBS in this setup has an extinction ratio of 99:1, with the vertical and horizontal polarizations lights being detected by the balanced photo-detector (BPD). The BPD's electrical signal is monitored via an oscilloscope, while a fraction of the signal is fed into a lock-in amplifier (Stanford Model SR830 DSP) to generate an error signal. This error signal is subsequently input into a proportional-integral-derivative (PID) controller (model SIM960), which generates an output signal that is used to regulate the driving voltage of a piezoelectric transducer (PZT). This regulation compensates for any phase fluctuations between the two channels, ultimately achieving phase locking at either the highest or lowest point of the interference fringe.

Calibration of the coherent magnetometer and coherent gradiometer

By adjusting the temperature of the ^{85}Rb vapor cell to 100°C , we optimize the gain factor in the FWM

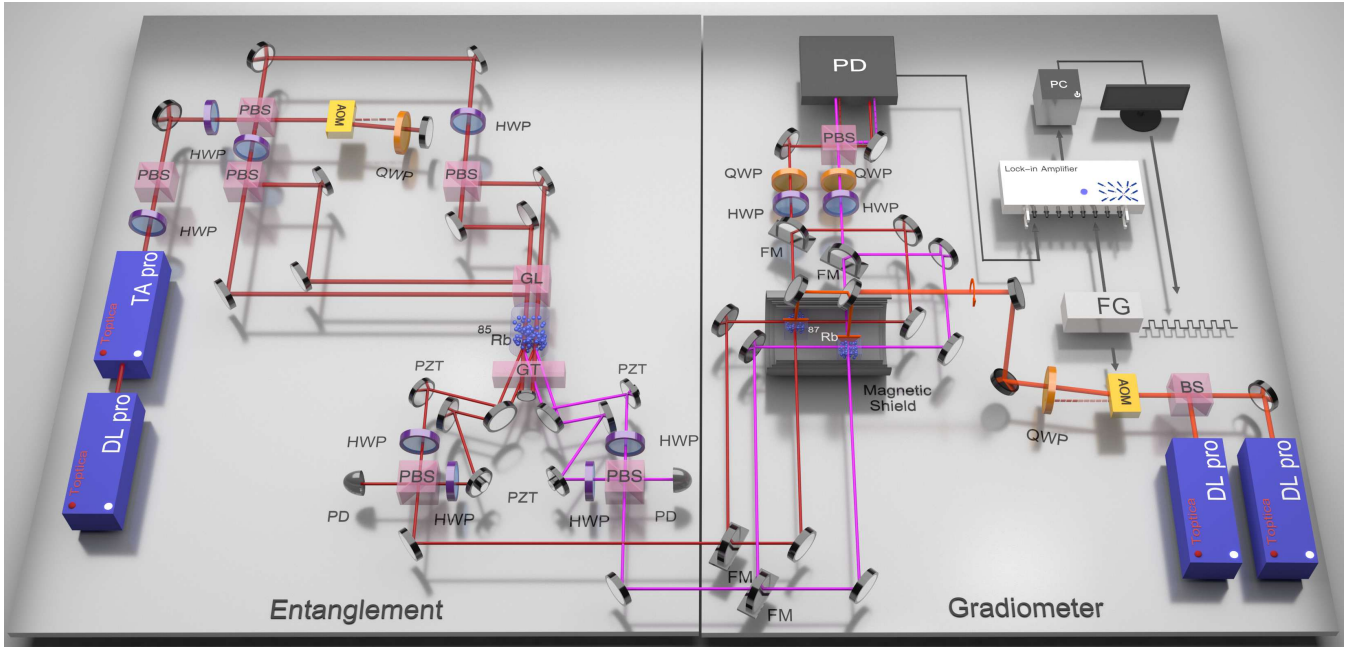


FIG. 6. **Experimental setup.** HWP: half-wave plate. PBS: polarizing beam splitter. AOM: acousto-optic modulator. QWP: quarter wave plate. GL: Glan-laser polarizer. GT: Glan-Thomson polarizer. PZT: piezo-electric transducers; FM: folding mirror; FG: function generator. BPD: balanced photo-detector. BS: 50/50 beam splitter.

process, ensuring that the noise spectrum of the intensity-difference is consistent with that of coherent light. Here, we calibrate the laser power of each channel using an oscilloscope to align with the coherent light intensity. We converted the system into a conventional magnetometer by blocking the optical pumping and repumping beams of one sensor, while “turned on”, the beams for both sensors configured it as a gradiometer.

Data availability

The data supporting the findings of this study are available in this paper and its Supplementary Information or from the corresponding authors upon request.

Code availability

The codes related to the findings of this study are available from the corresponding authors upon request.

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Author contributions W.Z. supervised the whole project. G.Z.B., and W.Z. conceived the research. G.Z.B., and W.Z. designed the experiments. S.H.W., L.J.D., D. Z., S.Y.Li., W.D., Y.Q.Y., J.C., and G.Z.B. performed the experiment. S.H.W., L.J.D., D. Z., and G.Z.B. and W.Z. contributed to the theoretical study. S.H.W., L.J.D., D. Z., and G.Z.B. analyzed the data. S.H.W., L.J.D., D. Z., and G.Z.B. draw the diagrams. S.H.W., L.J.D., D. Z., G.Z.B., and W.Z. wrote the paper. All authors contributed to the discussion and review of the manuscript.

Competing interests The authors declare that they have no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at <https://doi.org/xx>.

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Supplementary materials

PBSs are:

Stokes operators definition

The polarization state of light is described by Stokes operators, which are defined as:

$$\begin{aligned}\hat{S}_0^k &= \hat{k}_H^\dagger \hat{k}_H + \hat{k}_V^\dagger \hat{k}_V = \hat{k}_+^\dagger \hat{k}_+ + \hat{k}_-^\dagger \hat{k}_-, \\ \hat{S}_x^k &= \hat{k}_H^\dagger \hat{k}_H - \hat{k}_V^\dagger \hat{k}_V = \hat{k}_+^\dagger \hat{k}_- + \hat{k}_-^\dagger \hat{k}_+, \\ \hat{S}_y^k &= \hat{k}_H^\dagger \hat{k}_V + \hat{k}_V^\dagger \hat{k}_H = i(\hat{k}_+^\dagger \hat{k}_- - \hat{k}_-^\dagger \hat{k}_+), \\ \hat{S}_z^k &= i(\hat{k}_V^\dagger \hat{k}_H - \hat{k}_H^\dagger \hat{k}_V) = \hat{k}_+^\dagger \hat{k}_+ - \hat{k}_-^\dagger \hat{k}_-, \end{aligned} \quad (24)$$

where $\hat{k}_H$ and $\hat{k}_V$ are the annihilation operators of the H and V polarization modes, $\hat{k}_+$ and $\hat{k}_-$ are the annihilation operators of the left and right circular polarization modes, and $k = a, b$ corresponds to the signal and idler beams, respectively.

Quantum states rotation

For clarity, we provide an explicit description of the rotation operations applied to the polarization states of light. The rotation operators about the axes $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ are given by the following matrices:

$$\begin{aligned}\hat{\mathbf{R}}_x(\gamma) &= e^{\frac{i}{2}\gamma\hat{S}_x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{pmatrix}, \\ \hat{\mathbf{R}}_y(\phi) &= e^{\frac{i}{2}\phi\hat{S}_y} = \begin{pmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{pmatrix}, \\ \hat{\mathbf{R}}_z(\theta) &= e^{\frac{i}{2}\theta\hat{S}_z} = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (25)$$

These operations correspond to rotations of the polarization state around the respective axes on the Poincaré sphere. The general Stokes operators introduced in Eq.(6) of the main text are constructed based on sequential applications of such rotations, specifically $e^{i\phi_L^k \hat{S}_y^k/2} e^{i\theta_L^k \hat{S}_z^k/2}$ for each mode k .

Generation and quantum correlations of m-CVPE lights

As shown in Fig. 2(b), the output fields after the PAs are:

$$\begin{aligned}\hat{a}_{1,j} &= G\hat{a}_{0,j} + g\hat{b}_{0,j}^\dagger, \\ \hat{b}_{1,j} &= G\hat{b}_{0,j} + g\hat{a}_{0,j}^\dagger. \end{aligned} \quad (26)$$

The Jones vector of the field of one beam $\hat{E} = \hat{E}_H \vec{e}_H + \hat{E}_V \vec{e}_V$ is defined as:

$$\hat{E} = \begin{pmatrix} \hat{E}_H \\ \hat{E}_V \end{pmatrix}, \quad (27)$$

where $\hat{E}_H$ and $\hat{E}_V$ are the H and V polarized fields, respectively, and $\vec{e}_{H(V)}$ are the horizontal and vertical direction vectors. The new fields combined by the two

$$\begin{aligned}\hat{a}_{2,j} &= \begin{pmatrix} \hat{a}_{1,H} \\ \hat{a}_{1,V} \end{pmatrix}, \\ \hat{b}_{2,j} &= \begin{pmatrix} \hat{b}_{1,H} \\ \hat{b}_{1,V} \end{pmatrix}. \end{aligned} \quad (28)$$

We spatially combine the beams $\hat{a}_{1,V}(\hat{b}_{1,V})$ and $\hat{a}_{1,H}(\hat{b}_{1,H})$ using two PBSs, generating m-CVPE lights as shown in Fig. 7. After the PBSs, the resultant beams exhibit entanglement in both polarization modes, defining them as m-CVPE lights.

The Stokes operators are derived from Eq. (26) and

Eq. (28):

$$\begin{aligned}
\hat{S}_x^a &= \hat{a}_{2,H}^\dagger \hat{a}_{2,H} - \hat{a}_{2,V}^\dagger \hat{a}_{2,V} \\
&= (G\hat{a}_{0,H}^\dagger + g\hat{b}_{0,H})(G\hat{a}_{0,H} + g\hat{b}_{0,H}^\dagger) \\
&\quad - (G\hat{a}_{0,V}^\dagger + g\hat{b}_{0,V})(G\hat{a}_{0,V} + g\hat{b}_{0,V}^\dagger) \\
&= G^2(\hat{a}_{0,H}^\dagger \hat{a}_{0,H} - \hat{a}_{0,V}^\dagger \hat{a}_{0,V}) + g^2(\hat{b}_{0,H}^\dagger \hat{b}_{0,H} - \hat{b}_{0,V}^\dagger \hat{b}_{0,V}) \\
&\quad + Gg(\hat{a}_{0,H}^\dagger \hat{b}_{0,H}^\dagger + \hat{a}_{0,H} \hat{b}_{0,H} - \hat{a}_{0,V}^\dagger \hat{b}_{0,V}^\dagger - \hat{a}_{0,V} \hat{b}_{0,V}), \\
\hat{S}_x^b &= \hat{b}_{2,H}^\dagger \hat{b}_{2,H} - \hat{b}_{2,V}^\dagger \hat{b}_{2,V} \\
&= (G\hat{b}_{0,H}^\dagger + g\hat{a}_{0,H})(G\hat{b}_{0,H} + g\hat{a}_{0,H}^\dagger) \\
&\quad - (G\hat{b}_{0,V}^\dagger + g\hat{a}_{0,V})(G\hat{b}_{0,V} + g\hat{a}_{0,V}^\dagger) \\
&= G^2(\hat{b}_{0,H}^\dagger \hat{b}_{0,H} - \hat{b}_{0,V}^\dagger \hat{b}_{0,V}) + g^2(\hat{a}_{0,H}^\dagger \hat{a}_{0,H} - \hat{a}_{0,V}^\dagger \hat{a}_{0,V}) \\
&\quad + Gg(\hat{b}_{0,H}^\dagger \hat{a}_{0,H}^\dagger + \hat{b}_{0,H} \hat{a}_{0,H} - \hat{b}_{0,V}^\dagger \hat{a}_{0,V}^\dagger - \hat{b}_{0,V} \hat{a}_{0,V}), \\
\hat{S}_y^a &= \hat{a}_{2,H}^\dagger \hat{a}_{2,V} + \hat{a}_{2,V}^\dagger \hat{a}_{2,H} \\
&= (G\hat{a}_{0,H}^\dagger + g\hat{b}_{0,H})(G\hat{a}_{0,V} + g\hat{b}_{0,V}^\dagger) \\
&\quad + (G\hat{a}_{0,V}^\dagger + g\hat{b}_{0,V})(G\hat{a}_{0,H} + g\hat{b}_{0,H}^\dagger) \\
&= G^2(\hat{a}_{0,H}^\dagger \hat{a}_{0,V} + \hat{a}_{0,V}^\dagger \hat{a}_{0,H}) + g^2(\hat{b}_{0,V}^\dagger \hat{b}_{0,H} + \hat{b}_{0,H}^\dagger \hat{b}_{0,V}) \\
&\quad + Gg(\hat{a}_{0,H}^\dagger \hat{b}_{0,V}^\dagger + \hat{a}_{0,H} \hat{b}_{0,V} + \hat{a}_{0,V}^\dagger \hat{b}_{0,H}^\dagger + \hat{a}_{0,V} \hat{b}_{0,H}), \\
\hat{S}_y^b &= \hat{b}_{2,H}^\dagger \hat{b}_{2,V} + \hat{b}_{2,V}^\dagger \hat{b}_{2,H} \\
&= (G\hat{b}_{0,H}^\dagger + g\hat{a}_{0,H})(G\hat{b}_{0,V} + g\hat{a}_{0,V}^\dagger) \\
&\quad + (G\hat{b}_{0,V}^\dagger + g\hat{a}_{0,V})(G\hat{b}_{0,H} + g\hat{a}_{0,H}^\dagger) \\
&= G^2(\hat{b}_{0,H}^\dagger \hat{b}_{0,V} + \hat{b}_{0,V}^\dagger \hat{b}_{0,H}) + g^2(\hat{a}_{0,V}^\dagger \hat{a}_{0,H} + \hat{a}_{0,H}^\dagger \hat{a}_{0,V}) \\
&\quad + Gg(\hat{b}_{0,H}^\dagger \hat{a}_{0,V}^\dagger + \hat{b}_{0,H} \hat{a}_{0,V} + \hat{b}_{0,V}^\dagger \hat{a}_{0,H}^\dagger + \hat{b}_{0,V} \hat{a}_{0,H}), \\
\hat{S}_z^a &= i\hat{a}_{2,V}^\dagger \hat{a}_{2,H} - i\hat{a}_{2,H}^\dagger \hat{a}_{2,V} \\
&= i(G\hat{a}_{0,V}^\dagger + g\hat{b}_{0,V})(G\hat{a}_{0,H} + g\hat{b}_{0,H}^\dagger) \\
&\quad - i(G\hat{a}_{0,H}^\dagger + g\hat{b}_{0,H})(G\hat{a}_{0,V} + g\hat{b}_{0,V}^\dagger) \\
&= iG^2(\hat{a}_{0,V}^\dagger \hat{a}_{0,H} - \hat{a}_{0,H}^\dagger \hat{a}_{0,V}) + ig^2(\hat{b}_{0,H}^\dagger \hat{b}_{0,V} - \hat{b}_{0,V}^\dagger \hat{b}_{0,H}) \\
&\quad + iGg(\hat{a}_{0,V}^\dagger \hat{b}_{0,H}^\dagger + \hat{a}_{0,V} \hat{b}_{0,H} - \hat{a}_{0,H}^\dagger \hat{b}_{0,V}^\dagger - \hat{a}_{0,H} \hat{b}_{0,V}), \\
\hat{S}_z^b &= i\hat{b}_{2,V}^\dagger \hat{b}_{2,H} - i\hat{b}_{2,H}^\dagger \hat{b}_{2,V} \\
&= i(G\hat{b}_{0,V}^\dagger + g\hat{a}_{0,V})(G\hat{b}_{0,H} + g\hat{a}_{0,H}^\dagger) \\
&\quad - i(G\hat{b}_{0,H}^\dagger + g\hat{a}_{0,H})(G\hat{b}_{0,V} + g\hat{a}_{0,V}^\dagger) \\
&= iG^2(\hat{b}_{0,V}^\dagger \hat{b}_{0,H} - \hat{b}_{0,H}^\dagger \hat{b}_{0,V}) + ig^2(\hat{a}_{0,H}^\dagger \hat{a}_{0,V} - \hat{a}_{0,V}^\dagger \hat{a}_{0,H}) \\
&\quad + iGg(\hat{b}_{0,V}^\dagger \hat{a}_{0,H}^\dagger + \hat{b}_{0,V} \hat{a}_{0,H} - \hat{b}_{0,H}^\dagger \hat{a}_{0,V}^\dagger - \hat{b}_{0,H} \hat{a}_{0,V}). \tag{29}
\end{aligned}$$

Furthermore, six combined Stokes operators can be de-

rived as:

$$\begin{aligned}
\hat{S}_{x,-} &= \hat{S}_x^a - \hat{S}_x^b \\
&= (\hat{a}_{0,H}^\dagger \hat{a}_{0,H} - \hat{a}_{0,V}^\dagger \hat{a}_{0,V}) - (\hat{b}_{0,H}^\dagger \hat{b}_{0,H} - \hat{b}_{0,V}^\dagger \hat{b}_{0,V}), \\
\hat{S}_{x,+} &= \hat{S}_x^a + \hat{S}_x^b \\
&= (G^2 + g^2)[(\hat{b}_{0,H}^\dagger \hat{b}_{0,H} - \hat{b}_{0,V}^\dagger \hat{b}_{0,V}) \\
&\quad + (\hat{a}_{0,H}^\dagger \hat{a}_{0,H} - \hat{a}_{0,V}^\dagger \hat{a}_{0,V})] \\
&\quad + 2Gg(\hat{b}_{0,H}^\dagger \hat{a}_{0,H}^\dagger + \hat{b}_{0,H} \hat{a}_{0,H} - \hat{b}_{0,V}^\dagger \hat{a}_{0,V}^\dagger - \hat{b}_{0,V} \hat{a}_{0,V}), \\
\hat{S}_{y,-} &= \hat{S}_y^a - \hat{S}_y^b \\
&= (\hat{a}_{0,H}^\dagger \hat{a}_{0,V} + \hat{a}_{0,V}^\dagger \hat{a}_{0,H}) - (\hat{b}_{0,V}^\dagger \hat{b}_{0,H} + \hat{b}_{0,H}^\dagger \hat{b}_{0,V}), \\
\hat{S}_{y,+} &= \hat{S}_y^a + \hat{S}_y^b \\
&= (G^2 + g^2)[(\hat{a}_{0,H}^\dagger \hat{a}_{0,V} + \hat{a}_{0,V}^\dagger \hat{a}_{0,H}) \\
&\quad + (\hat{b}_{0,V}^\dagger \hat{b}_{0,H} + \hat{b}_{0,H}^\dagger \hat{b}_{0,V})] \\
&\quad + 2Gg(\hat{a}_{0,H}^\dagger \hat{b}_{0,V}^\dagger + \hat{a}_{0,H} \hat{b}_{0,V} + \hat{a}_{0,V}^\dagger \hat{b}_{0,H}^\dagger + \hat{a}_{0,V} \hat{b}_{0,H}), \\
\hat{S}_{z,-} &= \hat{S}_z^a - \hat{S}_z^b \\
&= i(G^2 + g^2)[(\hat{a}_{0,V}^\dagger \hat{a}_{0,H} - \hat{a}_{0,H}^\dagger \hat{a}_{0,V}) \\
&\quad + i(\hat{b}_{0,H}^\dagger \hat{b}_{0,V} - \hat{b}_{0,V}^\dagger \hat{b}_{0,H})] \\
&\quad + 2iGg(\hat{a}_{0,V}^\dagger \hat{b}_{0,H}^\dagger + \hat{a}_{0,V} \hat{b}_{0,H} - \hat{a}_{0,H}^\dagger \hat{b}_{0,V}^\dagger - \hat{a}_{0,H} \hat{b}_{0,V}), \\
\hat{S}_{z,+} &= \hat{S}_z^a + \hat{S}_z^b \\
&= i(\hat{a}_{0,V}^\dagger \hat{a}_{0,H} - \hat{a}_{0,H}^\dagger \hat{a}_{0,V}) - i(\hat{b}_{0,H}^\dagger \hat{b}_{0,V} - \hat{b}_{0,V}^\dagger \hat{b}_{0,H}). \tag{30}
\end{aligned}$$

The mean-value of the combined Stokes operators can

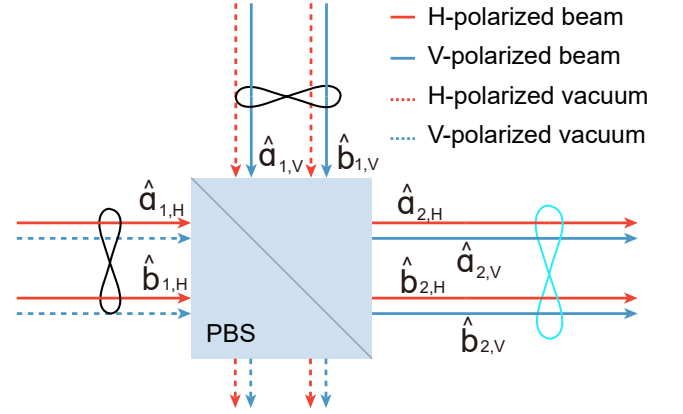


FIG. 7. Schematic of polarization-entangled beam generation based on PBSs. Blue and red lines represent horizontally (H)- and vertically (V)- polarized beams, respectively. Dashed lines represent vacuum states. The beams generated from the PAs are quadrature-squeezed beams in a specific polarization mode, but are vacuum beams in the other polarization mode. Two sets of perpendicularly-polarized twin squeezed beams are mixed by PBSs, and the twin beams in one output arm are entangled in both polarization modes.

be calculated as:

$$\begin{aligned}
 \langle \hat{S}_{x,-} \rangle &= \langle \hat{S}_{x,+} \rangle = 0, \\
 \langle \hat{S}_{y,-} \rangle &= 2|\alpha|^2, \\
 \langle \hat{S}_{y,+} \rangle &= 2|\alpha|^2(G^2 + g^2), \\
 \langle \hat{S}_{z,-} \rangle &= \langle \hat{S}_{z,+} \rangle = 0.
 \end{aligned} \tag{31}$$

We then analyze the noise of the six combined Stokes operators, yielding:

$$\begin{aligned}
 \Delta^2 \hat{S}_{x,-} &= \langle \hat{S}_{x,-}^2 \rangle - \langle \hat{S}_{x,-} \rangle^2 = 2|\alpha|^2, \\
 \Delta^2 \hat{S}_{x,+} &= \langle \hat{S}_{x,+}^2 \rangle - \langle \hat{S}_{x,+} \rangle^2 = 2|\alpha|^2(G^4 + g^4 + 6G^2g^2), \\
 \Delta^2 \hat{S}_{y,-} &= \langle \hat{S}_{y,-}^2 \rangle - \langle \hat{S}_{y,-} \rangle^2 = 2|\alpha|^2, \\
 \Delta^2 \hat{S}_{y,+} &= \langle \hat{S}_{y,+}^2 \rangle - \langle \hat{S}_{y,+} \rangle^2 = 2|\alpha|^2(G^4 + g^4 + 6G^2g^2), \\
 \Delta^2 \hat{S}_{z,-} &= \langle \hat{S}_{z,-}^2 \rangle - \langle \hat{S}_{z,-} \rangle^2 = 2|\alpha|^2(G^4 + g^4 + 6G^2g^2), \\
 \Delta^2 \hat{S}_{z,+} &= \langle \hat{S}_{z,+}^2 \rangle - \langle \hat{S}_{z,+} \rangle^2 = 2|\alpha|^2.
 \end{aligned} \tag{32}$$

The methods of Resolve Magnetic Field Direction Degeneracy

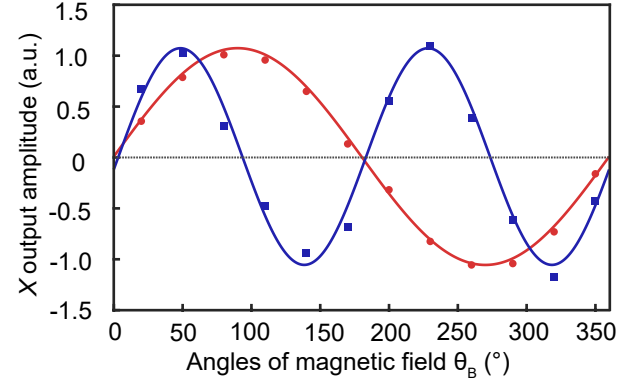


FIG. 8. **Breaking magnetic field direction degeneracy via dual spin projections.** The X output of $\langle \hat{F}_x \rangle$ (red) and $\langle \hat{F}_z \rangle$ (blue) vary with θ_B at $\Omega_B = \Omega_m$. The data points are experimentally measured, and the curves are fitted according to $\sin(\theta)$ and $\sin(2\theta)$.

In the main text, the magnetic field direction is extracted from the X output of the lock-in amplifier, whose amplitude is proportional to the spin projection $\langle \hat{F}_x \rangle$. As shown in Eq. (1), scales as $\sin(\theta_B)$. Since $\sin(\theta_B) = \sin(\pi - \theta_B)$, a single measurement of cannot distinguish between field angles θ_B and $\pi - \theta_B$. To break this degeneracy, we introduce a second spin projection measurement. By measuring $\langle \hat{F}_z \rangle$ simultaneously with $\langle \hat{F}_x \rangle$, we obtain an independent signal whose amplitude follows a different angular dependence, specifically $\sin(2\theta_B)$, as derived from the spin precession equations. Because $\sin(\theta_B)$ and $\sin(2\theta_B)$ have distinct symmetries in $[0, 2\pi]$, the pair of measured amplitudes provides a one-to-one mapping to the absolute field angle over the full range.